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A theory of global economic growth in the very long-run: is a grand innovation slowdown inevitable?

Abstract: The paper shows how the original semi endogenous and balanced growth model of Phelps (1966), and my extended version of it (Gomulka, 1990), could be useful in explaining the key 'stylized facts' of global long-term growth so far, and in predicting its dynamics in the future. During the last two centuries the sector of R&D and education, producing qualitative changes, has been expanding in the world's most developed countries much faster than the sector producing conventional goods. The extended model is used to explore and evaluate. the consequences for the global long-term growth of the end of this unbalanced growth, of the completion of the catching up by most of the world's less developed countries, and of the expected eventual stabilization of the size of the world population. The theory yields a thesis, new in the literature, that the rate of global per capita GDP growth will eventually return to the historically standard very low level, thus implying that the world's technological revolution is going to be an innovation super-fluctuation.

Keywords: Endogenous growth theory, Phelps, Hat-shaped relationship, Long-term global growth slowdown, Key growth trends, Key growth stylized facts

JEL Codes: F01, O33, O41, O47

1 Introduction

Angus Maddison in his *Contours of the World Economy, I-2030 AD* (2007) shows that there was a high stability in the global per capita GDP trend growth rate in the Middle Ages 1000-1500, at a very low level of 0.05% per year, to be followed by only slightly higher rate of 0.07% a year in the protocapitalistic epoch 1500-1820. But during the capitalistic epoch 1820-2000 the trend rate exploded to a level about 17 times higher than in the preceding period. Under the two assumptions economists now usually make, constant returns to scale and labour-augmenting technological (and other qualitative) changes, the growth rate of GDP per unit of labour is, in the long term, determined fully by qualitative changes. These changes are produced by the R&D inventive activity, the educational activity, and by institutional reforms.

The key stylized fact of the long-term growth of the world economy is, therefore, that about two centuries ago a remarkably large acceleration in the percentage rate of technological and other qualitative changes started, and the new exceptionally high rate of these changes has since continued.

Following the empirical studies by economic historians and development economists, in my earlier publications Gomulka (1970, 1971, 1990), I accepted that the mechanisms of technological progress in the most developed countries, forming the *technology frontier area* (TFA), are quite different than in non-TFA countries, known also as *emerging economies*.

For the purposes in hand it will be useful to regard the TFA as a single economy. I shall also assume that the inventive activity of that imagined economy is the only factor capable of moving the world technological frontier outwards. The inventive activity of the firms operating behind the frontier will thus be ignored. As an approximation we can think of the United Kingdom and parts of continental Europe as the TFA for most of the nineteenth century, and of the United States, parts of Western Europe and Japan as the TFA for most of the twentieth century.

A family of macroeconomic models of innovation and growth, the so-called endogenous growth theory, was created in the second half of last century. Their primary purpose was to explain changes in the joint factor productivity, and to find the rates of GDP growth which would obtain when resources of labour and capital are distributed in an optimal manner between sector I, of conventional production, and sector II, mainly R&D and education, producing qualitative changes. As interpreted by Grossman and Helpman (1991) and noted by Jones (1995), in many initial models of that theory, and the model discussed in this paper does not belong to that category, permanent changes in certain policy variables have permanent effects on the rate of economic growth. According to Jones, such permanent changes have taken place, but „growth rates of GDP per capita show little or no persistent increase"(p. 495). During the last 20-25 years a second generation of endogenous growth models were developed, beginning with, among others, Peretto (IER 1996, JEG Dec 1998), Dinopoulos and Thompson (JEG 1998), Young (JPE 1998) and Howitt (JPE 1999). These models appear to perform better in empirical tests. Still, it remains to explain the causes and the consequences of the central feature of the world economy in general, and of its TFA part in particular, namely that during the last two centuries there has been an unbalanced growth: an expansion of sector II. much faster than that of sector I.

Two centuries is a relatively short period of time by historical standards. The unusually high innovation rate, supported by an unusually high growth rate of the world population during that period, is therefore still an exception in the history of humanity. Since the growth of the world population and much faster growth of sector II than sector I must eventually both come to an end, probably in the course of this century, there is a possibility that the exceptionally high rate of technological innovations, and of other qualitative changes, of the last two centuries will be followed by a declining rate. Such an innovation slowdown would have rendered the technological revolution to be at once a transitory phenomenon and one which would forever be seen as a huge innovation outburst or an innovation super-fluctuation.

The primary purpose of this paper is to investigate this prospect in order to identify the key assumptions and parameters which are to affect its likelihood and the time-scale. The analysis represents a development of the ideas by Edmund Phelps (1966). This in three directions. One is to strengthen the theoretical and empirical foundations of the model. The second is to consider the dynamics of growth in the TFA over the last two centuries, when sector II has been and still is expanding (much) faster than sector I. And the third is to note the future implications for the global growth rate of the *per capita* GDP of the expected stabilization of the size of the world population and of the substantial disappearance of the presently still strong duality of the world economy, as the per capita GDP and wealth in the TFA countries are yet much higher than in the non-TFA countries.

2 The key stylized facts of economic growth

The statistical data on global long-term economic growth have certain fundamental characteristics, termed 'stylized facts', which the growth theory must explain first and foremost. Possibly the best known such 'facts' were originally those of Kaldor (1961). Now we have also five 'facts' of Easterly and Levine (2001) and six of Jones and Romer (2009). In Gomulka (2017) I present and discuss these much different two lists, adding to them one of my own Gomulka (2009). This particular list consists of seven 'facts'. These are as follows:

With respect to all countries:

1. The great acceleration in the growth rate of world GDP per capita, and still more per working hour, took place some two centuries ago, and a historically exceptionally high growth rate has since continued;
2. Over the past two centuries there has been a large variation in the rate of per capita growth between countries, leading to the very high degree of duality of the world economy by the end of the 20th century.

With respect to the TFA countries:

3. During the past two to three centuries, there has been a far more rapid growth of inputs of labour and capital in the sector producing qualitative changes than the growth of inputs in the sector producing conventional goods;
4. The growth rates of inputs in both sectors have been stable over time. Likewise, the growth rate of the ratio Y/L, output per manhour, has been stable, although very much higher (an order of magnitude greater) than during the many centuries that preceded it;
5. The rate of growth of the ratio Y/L has been and is relatively stable over time, differs to a small extent between countries, and depends weakly on the ratio of investment to the gross domestic product (GDP).

With respect to non-TFA countries:

6. The rate of growth of Y/L varies strongly over time and between countries;
7. The growth rate of Y/L is strongly dependent on the level of investment as a fraction of the GDP.

At the level of firms in the TFA countries we observe a huge variation in the ratio of R&D expenditures to sales and a huge variation in the rate of return on such expenditures. This makes it difficult to provide microeconomic foundations to a macro growth theory. However,, as noted in facts 3 to 5, during the last two centuries there has been little variation over time and across countries in respect to some key macro variables. This suggests that a macroeconomic approach to a theory of long- term growth for the TFA should be successfully attempted. In non-TFA countries we have a completely different set of data: a large variation over time and across countries in respect to key macro variables and a marginal contribution of their own inventive activity to the world inventive output. This suggests a fundamental role there of factors determining international technology transfer from the TFA, hence the key role in those countries of institutions and economic policy, to explain economic growth.

Since Phelps's model is a point of departure for this analysis, I shall begin by presenting it in some detail. However, it should be stressed from the outset that that model and its optimal growth path apply only to the TFA, and only to a future target situation of balanced growth. The model does not say anything about the growth path for the global economy during the transitional period of some probably three-four centuries, during which a gradual adjustment takes place of the distribution of global resources of capital and labour between sectors I and II, from their very low, highly suboptimal levels in sector II about two centuries ago to much higher, optimal ones, in a century or two.

The pace of this redistribution of resources has been so far quite stable (stylized facts 3 and 4). The purpose of my extended model is to study the growth dynamics during the adjustment period, assuming that the pace of redistribution will continue unchanged. As no attempt is made to explain this pace of redistribution, the model is not fully endogenous. Phelps's model of the end-point economy is neoclassical, but my growth model of the transition to that endpoint state is, using the Nelson-Winter terminology, evolutionary. Still, I assume in my extended model that the technology function which Phelps proposed applies during the entire adjustment process.

3 The Phelps model of innovation and balanced growth

The key equations of the model are as follows:

$$Y = F\left(K, TN\right) \quad (1)$$

$$\dot{T} = H\left(E, T\right) \quad (2)$$

$$E = M^{\beta} R^{\mu} L^{\gamma} \quad (3)$$

$$L = N + R = L_0 exp\left(nt\right) \quad (4)$$

According to (1), the net output Y of the conventional sector is dependent on the capital stock K and the 'effective' labour TN, where T represents an index of the quality of capital and labour, and N represents labour input in terms of man-hours. Qualitative changes are thus assumed to be purely labour-saving (or labour-augmenting). Equations (2) and (3) represent an embedded two-level production function for the output of sector II, where in (2) the dot over T denotes the time derivative. In the original model, sector II is limited to the production of new technology, E is the amount of research produced when researchers R are equipped with capital M and selected from a total labour force L. This research, in turn, brings new technology ΔT, and in (2) this addition is assumed to be influenced positively by T itself. The reason is that innovation also builds on accumulated past research.

In this paper I shall continue to use the term „technology" for the index of all qualitative changes, which apart from technological innovations include also improvements in human capital and in institutions. Therefore, persons R shall include also teachers.

In (1) and (2) constant returns to scale are assumed, but the elasticities of substitution between K and N and between E and T need be neither unitary nor constant. In (3) the partial elasticities with respect to inputs R and M are constant and, to be consistent with the optimal macro conditions (8) and (9) implied by the model, the sum of these elasticities need be less than 1 (See also equation (15) and the comment on that equation). This particular feature of the theory means that the productivities of these two inputs decline as their size increases. The empirical evidence reported and discussed recently in a paper by Bloom *et al.* (2017), fully supports this important feature.

The elasticity of substitution between any two of the three 'factors' in (3) was assumed by Phelps to be unitary. This assumption is highly restrictive and it will be dispensed with later *in the paper.*

Let the partial elasticity of the function F with respect to K be denoted by a, and the partial elasticity of the function H with respect to E by b. The assumption of constant returns to scale implies that:

$$a = a\,(K/TN),\ b = b\,(E/T),\ \text{and}\ 0 < a, b < 1$$

The Phelps technology production function has two important and intuitively appealing properties. One is that the same research effort will be more productive if it is spread evenly over a longer period of time rather than being concentrated in a short period. To see this, consider a variable period Δt, a steady research flow E, and a constant total research effort $E\Delta t$. Denoting the latter by c, we have that $E = c/\Delta t$ and $\Delta T = H(c/\Delta t, T)\Delta t$. Hence $\partial\Delta T/\partial\Delta t = (1 - b)\ H > 0$, which confirms that ΔT increases with Δt, given c. An implication of this property is that research effort allocated evenly over a period of time is assumed to be more productive than an equivalent total research effort which proceeds in fits and starts.

Another important property of the Phelps technology function is that it attempts to capture the inherent heterogeneity of people with respect to their inventive ability. If we assume that in the inventive activity most inventive persons are employed first, the research capability of a given number of such persons can be expected to increase as the total pool from which they are selected increases. This is the reason why L is an argument in the E function. Specification (3) is rather *ad hoc,* but we shall provide its theoretical justification.

3.1 The optimum research intensity and the equilibrium innovation rate

'Empirical evidence tells us that in the past two centuries or so the technology-producing sector has usually been expanding much faster than the conventional sector. It is instructive, however, to consider first the case of balanced growth. Accordingly, suppose that Y, K, and M all change at a common constant growth rate, to be denoted by g, and that L, N, and R change at another constant rate n. From (1) we have:

$$g = \alpha + n \tag{5}$$

where, $\alpha = T'$ the growth rate of all qualitative changes. in short the innovation rate. Thus indeed in the long run the *per capita* GDP growth rate is determined fully by this innovation rate.

Important notation: Here and throughout the paper the upper-case comma denotes the time derivative of the indicated variable divided by the variable itself, or the growth rate of that variable.

Given the assumption of constant returns to scale, it follows from (2) that $\alpha = H\left(E/T,1\right)$. Thus the rate α is constant, a requirement of balanced growth, only if T is proportional to E, say $T = \eta E$.

We also note that under balanced growth, gross investment in fixed capital equals $\dot{K} + \delta K + \dot{M} + \delta M, or \left(g+\delta\right)\left(K+M\right)$, where δ is the depreciation rate. Therefore the level of consumption is as follows:

$$C = F\left\{K, \eta E\left(M, L-N, L\right)N\right\} - \left(g+\delta\right)\left(K+M\right) \tag{6}$$

This level is at a maximum if the inputs *K, M,* and *N* are chosen to meet these first-order optimality conditions:

$$F_K = g + \delta \tag{7}$$

$$\frac{M}{K} = \frac{1-a}{a}\beta \tag{8}$$

$$\frac{R}{N} = \mu \tag{9}$$

Condition (7) gives the optimal capital intensity in the conventional sector, while (8) and (9) give the optimal sectoral distribution of the two resources, capital and labour. Using these conditions we can find gross capital investment in each sector. We can also find the optimal (balanced-growth) research intensity, i.e. the expenditure on wages and investment in the technology sector as a proportion of the conventional output. Gross investment is, in the conventional sector:

$$\left(g+\delta\right)K = F_K K = \left(F_K \frac{K}{Y}\right)Y = aY$$

and, in the technology sector:

$$\left(g+\delta\right)M = \left(g+\delta\right)\frac{M}{K}K = \left(1-a\right)\beta Y$$

Subtracting total investment from output gives consumption. Now, suppose that consumption is the same as the total wage income and that wage rates are the same in both sectors. Condition (9) enables us to find the wage income in each sector. Thus we have obtained both the investment and the wage element of the R&D expenditure. The research intensity i - the share of total conventional output devoted to the technology sector - is in this case:

$$i = \left(1-a\right)\left\{\beta + \frac{\mu}{1+\mu}\left(1-\beta\right)\right\} \tag{10}$$

Since, in this model, a is the share of gross investment in the conventional sector, it can be expected to be less than 0.5. If presently observed values of *M/K* and *R/N* in the TFA are any indication of their optimal values, then by (8) and (9) both μ and β are small and, consequently, both 1+μ and 1−β are close to unity. The optimal research intensity can therefore be approximated as follows:

$$i \approx \left(1-a\right)\left(\beta+\mu\right)$$

Of central interest, however, is the magnitude of the equilibrium innovation rate α. We obtain it by recalling that $T = \eta E$, from which it follows that $\alpha = T' = E'$. From (3) we have in turn that $E' = \beta M' + \left(\mu+\gamma\right)n$. However, according to (5), $M' = \alpha + n$. Therefore $\alpha = \beta\left(\alpha+n\right) + \left(\mu+\gamma\right)n$ which gives:

$$\alpha = \frac{\beta+\mu+\gamma}{1-\beta}n = \alpha^* \tag{11}$$

The asterisk indicates that this is the equilibrium rate.

Two important implications of the result (11) can be noted immediately. One is that if β and μ are significantly less than unity, and there are indeed good grounds to place them between zero and 0.1, then the heterogeneity of the labour force with respect to inventive ability, represented by γ, may be a key factor determining the innovation rate. The other implication is that if the population of the TFA ceases to grow, so that $n = 0$, the innovation rate would be still positive at any finite time, but it would be falling continuously and be zero in plus infinity. So, in this case a long-term equilibrium does not exist.

4 Human inventive and innovative heterogeneity and technological progress in the technology sector itself: two generalizations

Inventive ability is known to differ substantially between individuals. Figure 1 shows a possible distribution of the working population with respect to this ability v. What matters for us is not the innate or natural inventive ability but the actual inventive ability, given possible

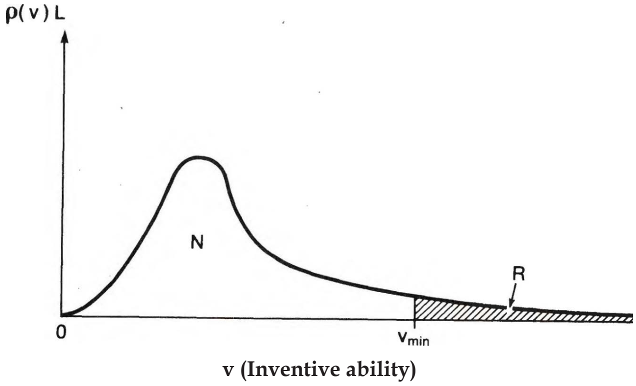

Fig. 1: The Distribution of the total working population *N* and *R* with respect to inventive ability *v*.

environmental influences such as quality of schooling, family circumstances, and attitudes to learning, as well as the system of incentives and values encouraging potential inventors to make use of their potential. If the 'screening methods' of the 'appointments committees' are appropriate, the research workers would be represented by the shaded area in Figure 1 below the upper tail of the distribution $L\rho(v)$.

Our first substantial modification of the Phelps model is to assume that this tail is a Pareto-type function, an assumption often adopted in economics (for example, to describe the upper tail of the distribution of income or wealth). In this case, $R/L = \text{prob}(v>v_{min}) = C_1(v_{min})^{-\lambda}$ where $\lambda > 1$ and C_1 is a positive constant. It follows that, for $v > v_{min}$, the underlying density distribution is $\rho(v) = \lambda C_1 v^{-\lambda-1}$. The total inventive ability of our *R* researchers can now be calculated:

$$V = L\int_{v_{min}}^{\infty} v\rho(v)\,dv = C_2 R^{1-1/\lambda} L^{1/\lambda} \quad (12)$$

where $C_2 = \{\lambda/(\lambda-1)\}C_1^{1/\lambda}$. This result justifies specification (3). Moreover, writing (3) as $E = C^{\mu}M^{\beta}V^{\nu}$, we have that $\mu = \nu(1-1/\lambda)$ and $\gamma = \nu/\lambda$. Consequently, in this case $\mu+\gamma$ in expression (11) for α^* would be equal to ν; the innovation rate α^* would thus be independent of the ability variation parameter λ.

Our second modification of the Phelps model is to regard T as an index of quality of the standard inputs, labour and capital, and to extend the R&D sector by the educational activities, to form a Q sector. When the labour input in the quality producing sector II is expressed in units of ability-hours, as in (12), rather than in man-hours, as in the original Phelps model, then the *E* function need not be of the very restrictive Cobb-Douglas form to permit balanced growth. The least restrictive specification that would still be satisfactory is:

$$E = E(M, TV) \quad (13)$$

where *V* is multiplied by *T* to account for technological change also enhancing the research and education capability of the researchers and teachers themselves. Equation (13) is our third modification of the model. Specifications (1) and (13) are now symmetric. Consequently, the elasticity of substitution between *M* and *V*, as between *K* and *L* in (1), need be neither unitary nor constant. Parameter b continues to be the elasticity of *E* with respect to *M*, and ν now stands for the elasticity of *E* with respect to *TV*.

It is interesting to note the implication of replacing (3) by (13) for the magnitude of α^*, the key variable of the model. Let ε be the scale elasticity, assumed constant, of the *E* function. Hence:

$$E = T^{\varepsilon}V^{\varepsilon}E(M/TV, 1) = T^{\varepsilon}V^{\varepsilon}e(m) \quad (14)$$

Where $m = M/TV$. On a balanced growth path *m* is a constant and therefore $E' = \varepsilon(T' + V')$. However, $E' = T'$ and $V' = n$. Hence the balanced growth innovation rate would be:

$$\alpha^* = \frac{\varepsilon}{1-\varepsilon}n = \frac{\beta+\nu}{1-\beta-\nu}n \quad (15)$$

This result is similar to (11) where, incidentally, $\beta+\mu+\gamma$ is the same as ε in (15). In particular, despite allowing for (labour-saving) technological change in the Q sector, the equilibrium innovation rate still remains proportional to the population growth rate. Conditions (7)-(9) for the optimal distribution of capital and labour between the two sectors, conventional production and quality enhancing, also remain unchanged. However, for (15) to make economic sense, research activity must be subject to diminishing returns to scale ($\varepsilon < 1$).

5 Price's two laws and the 'technological revolution': the case of unbalanced growth

The term 'technological revolution' is one of those which are used often without being defined precisely. The term seems intuitively clear enough; it means a period of 'unusually' rapid innovation in a particular sector, country, or the world as a whole. Some authors distinguish major bursts of world innovative activity, such as that based on the steam engine and consequent mechanization, or electrical power and its applications, or inventions in electronics and telecommunication, or, recently, microelectronics and the use of robots. They refer to these bursts of innovations as technological revolutions - first, second, and so forth - in their own right. Such distinctions are sometimes useful to a social scientist by their virtue as indicators of the changing content of the innovation flow, with its implications for changes in the skills required, social stratification, social mobility, and the rate of spread of information and ideas. However, for the economist it is the rate of innovation flow as such, rather than the flow's specific content, which is of central interest.

What is meant, then, by an 'unusually' high rate of innovation? It must be a rate which cannot be sustained 'forever', i.e. an innovation rate which is greater than the balanced-growth innovation rate, or α^* in (15). In terms of our two-sector economy, technological revolution can therefore be defined as a prolonged period of economic growth in which the technology sector is expanding faster than the conventional sector. Such unbalanced growth cannot be sustained for ever, but as long as it lasts it does give rise to $\alpha > \alpha^*$.

A convenient measure of the expansion of any sector of economic activity is a weighted sum of the growth rates of the inputs employed in that sector - labour and capital in the case of our present model. The data on the growth rates of these two inputs in the technology sector vary in quality among countries and between different periods. However, such records as we have indicate that, over the last two to three centuries, the world Q sector II has been expanding (i) nearly exponentially and (ii) much faster than the conventional sector I. The empirical propositions (i) and (ii) are among the key stylized facts concerning technological change and long-term growth that have been (relatively) well established. According to the science historian, Derek de Solla Price:

> "*many numerical indicators of the various fields and aspects of science... show with impressive consistency and regularity that if any sufficiently large segment of science is measured in any reasonable way, the normal mode of growth is exponential*." (Price 1963: 4-5)

Price suggests that this steady exponential growth of the size of world science has been maintained for the past two to three centuries. Because of this long period of validity, he calls it the '*fundamental law of any analysis of science*' (Price 1963: 5). We shall refer to it as Price's first empirical law. His second empirical law is as follows:

> "*depending on what one measures and how, the crude size of science in manpower or in publications tends to double within a period of 10 to 15 years. The 10-year period emerges from the catchall measures that do not distinguish low-grade work from high but adopts a basic, minimal definition of science; the 15- year period results when one is more selective, counting only on some more stringent definition of published scientific work and those who produce it.*" (Price 1963: 6)

The steady doubling every 10 to 15 years gives the growth rate of the world scientific membership as between 4.7 and 7.2 per cent per annum. Judging from the detailed country data for the past 50 years or so, the total labour input in both research and development has been increasing about as rapidly as the number of scientists alone. These data also indicate that the non-personnel (real) expenditure on R&D has been expanding somewhat faster than the R&D personnel. These two growth rates in the period 1750-1975 have apparently been much higher than the growth rates of labour and capital in the conventional sector, which were roughly 0.7 per cent (the growth rate of the world population) and 1.7 per cent (the growth rate of the world GDP) respectively. It is this wide disparity in the sectoral growth rates, in favour of the technology sector, which above all underlines the phenomenon of 'technological revolution'. Such a disparity cannot be maintained for ever; in fact there has already been a significant slow-down in world R&D growth since about 1970. The balanced-growth solution of the previous section is relevant only with reference to an equilibrium configuration that was present in the distant past and will emerge in the (possibly less distant) future. However, the past two or three centuries represent a period of highly unbalanced growth that needs separate consideration. We shall do this in this section.

We retain the model specifications (1)-(4) of the previous section, except that (3) is replaced by (13). We also retain the assumption that the capital-to-output ratio in the conventional sector is constant. However, neither the savings ratio nor the growth rates of total output and its components need be constant. Let N with subscript 1 be the number of workers engaged in producing capital goods for the quality enhancing sector. The assumption that the capital- to-output ratio is constant implies that, as in the previous section, the productivity of these workers is proportional to the aggregate level of technology *T*. Hence the growth rate of capital employed in the quality enhancing sector is

$$M' = \alpha + n_1 \tag{16}$$

The growth rate of the labour input in the quality enhancing sector is

$$R' = n_2 \tag{17}$$

Price's two empirical laws are

(i) that n_1 and n_2 have both been significantly greater than *n*, and

(ii) that they have been approximately constant.

Given n_1 and n_2, we can now obtain the innovation rate from equations (2) and (13).

Assume $H(E,T) = E^b T^{1-b}$ and $E(M,TV) = M^\beta (TV)^\nu$ where *V* is given by (12) and the elasticities β, b, and ν are all constant. With these specifications the system determining α is as follows:

$$\alpha' + b\alpha = bE' \tag{18}$$

$$E' = \beta M' + \nu(\alpha + V') \tag{19}$$

$$V' = \frac{\lambda - 1}{\lambda} R' + \frac{1}{\lambda} n \tag{20}$$

The growth rate of research effort is, in (18), the propelling force that determines the dynamics of the innovation rate. This growth rate is in turn determined by the growth rates of Q inputs, the measure of the labour input taking due account of the variation in inventive ability, and the effect of innovation on efficiency in the Q sector itself.

Superimposing the stylized facts (16) and (17) on the system (18)-(20) yields the following differential equation for $\alpha(t)$:

$$(1-\beta-\nu)\alpha + \frac{1}{b}\alpha' = \beta n_1 + \left\{\frac{1}{\lambda} n + \left(1 - \frac{1}{\lambda}\right) n_2\right\}\nu \tag{21}$$

Hence

$$\alpha' = b(1-\beta-\nu)\left(\alpha^*_{TR} - \alpha\right) \tag{22}$$

where

$$\alpha^*_{TR} = \frac{\beta n_1 + \nu\left\{(1/\lambda)n + (1-1/\lambda)n_2\right\}}{1-\beta-\nu} \tag{23}$$

From (22) it follows that $\alpha(t)$ approaches α^*_{TR} with time. This α^*_{TR} stands for the 'equilibrium' component of the innovation rate in the course of the technological revolution. Equation (22) can be solved numerically to give α as a function of time:

$$\alpha(t) \text{ increaseing if } \alpha < \alpha^*_{TR} \text{ and declining if } \alpha > \alpha^*_{TR} \tag{24}$$

A growth slow-down in Q activity has taken place in the TFA during the last half a century. If the slow-down means that the optimal ratios of *R/N* and *M/K* are about to be or have already been achieved in the TFA, we can use the optimality conditions (8) and (9) for estimating the values of the parameters that appear in (23). These conditions are that $\nu(1-1/\lambda) = R/N$ and $\beta(1/a - 1) = M/K$. Guided by empirical evidence we also assume that *R/N* = *M/K* and $n_1 = n_2$. Consequently, the following relationship can be obtained:

$$\alpha^*_{TR} = \frac{(\lambda-1)n_1 + (1-a)n}{(\lambda-1)(1-a)(N/R) - (\lambda-1)a - \lambda(1-a)} \tag{25}$$

Tab. 1: The values of λ as implied by (25), given α^*_{TR}, n_1, and *R/N*, and assuming that *a* =0. 2 and *n* = 0.7 per cent

α^*_{TR} (%)	n_1 (%)	λ				
		R/N = 0.01	R/N = 0.02	R/N = 0.03	R/N = 0.04	R/N = 0.05
1	4	1.02	1.04	1.06	1.09	1.13
	6	1.02	1.04	1.07	1.11	1.15
	8	1.02	1.04	1.07	1.12	1.19
2	4	1.01	1.03	1.05	1.06	1.08
	6	1.01	1.03	1.05	1.07	1.08
	8	1.01	1.03	1.05	1.07	1.10
3	4	1.01	1.02	1.04	1.05	1.07
	6	1.01	1.02	1.04	1.06	1.08
	8	1.01	1.02	1.04	1.06	1.09

Note: The values of λ are rounded to two decimal places.

It should be noted that the higher is the value of λ the lower would be the proportion of highly innovative individuals; the case of $\lambda = \infty$ is the limiting situation where no very innovative talent is present.

According to Price, the rate n_1 has been somewhere in the range 4-8 per cent. Given this range, it is instructive to find the values of λ for which the rate α^*_{TR}, as given by (25), would be approximately equal to the innovation rate actually observed. These values are presented in Table 1. Since our knowledge of the optimal ratio *R/N* is uncertain, the table provides the values of λ for a range of *R/N* from 1 to 5 per cent.

6 A consistency test of the model and Lotka's law

The instructive point of this numerical example is the result that, for the probable values of a_{TR} and *R/N*, the model we discuss predicts l to be only somewhat greater than unity. If there was independent evidence indicating that the λ actually observed is in fact not far from unity, the model would pass an important empirical test.

The trend growth rate of GDP per man-hour, which can be taken as a measure of α, was 2.3 per cent per annum in the United States in the period 1870-1970 (Maddison, 1979). Since the growth rate was fairly stable during that period, it can also be taken as a measure of α^*_{TR}.

An indication of the value of λ is provided by studies of the frequency distribution of scientific productivity. A pioneer investigation of this type was made by Lotka (1926). The result of his investigation, later repeated and confirmed by several others, is the finding that the number of scientists producing *m* papers within their lifetime is approximately proportional to $1/m^2$. The number of publications or the number of inventions is, of course, only one of several possible measures of inventive power. The measure may be a poor guide for judging the weight of the contribution to science or technology of any particular individual, but a good guide for the 'representative' scientist (inventor).

If we take *m* as a measure of the inventive ability, denoted by v in equation (12), Lotka's law asserts that our frequency distribution r(n), the distribution specified in (12) as $C_1\ v^{-\lambda-1}$, is proportional to v^{-2}, implying that λ = 1. Several other investigators have since repeated such publication counts. According to Price, they all confirm Lotka's result, '*which does not seem to depend upon the type of science or the date of the index volume*' (Price 1963: 43). Moreover, Lotka's law is known to overestimate somewhat the proportion of researchers with a high *m* (Price 1963: 46-9). This in turn implies that the 'true empirical' l is in fact somewhat greater than unity. It is interesting, indeed remarkable, that such values of λ also happen to be the requirement of the theoretical model discussed in the previous section. Lotka's law thus seems to provide empirical support for this particular theory of technological change and economic growth.

7 The Hat-Shape Relationship and the hypothesis of innovation limits to growth

The story of technological change and growth that is told by the theory of this paper is one in which the technological revolution is a phenomenon of the TFA when the key resource ratios *R/N* and *M/K* are rising fairly fast to reach their optimum levels, and when the key growth rates n_1, n_2 and a are temporarily significantly higher than their balanced growth magnitudes *n* and α^*. The last two centuries are not the only ones when inventive activity, in terms of the inputs used, has been expanding faster than conventional activity. The history of science and technology provides ample evidence of significant bursts of inventive and innovative work in Europe in the Middle Ages, as well as in the ancient civilizations of the Middle East, China, and the Mediterranean. However, what makes the present technological revolution qualitatively quite unique is the circumstance that the growth rates *n*, n_1, and n_2 have apparently all been much higher than ever before over a prolonged period, giving rise to a correspondingly much higher innovation rate with profound implications for the pace of economic and social change in much of the world.

In the past century or two the relative size of sector II has been rising rapidly, but this change of size apparently did not influence the innovation rate very much, which remained fairly stable in the TFA. This stylized empirical fact agrees well with our equations (22) and (23), since the growth rates of inputs in that sector, rather than their levels, influence the innovation rate. In equation (23), these levels could influence α only through the parameters β and ν. Therefore, we can deduce that these parameters have been almost independent of the ratios *M/K* and *R/N* respectively and that the actual α was near to α^*_{TR}.

Given the apparent stability of β and ν so far, it is fair to assume that the two parameters will remain in future about the same as they were in the past. However, the future will bring about two important new phenomena: (i) an inevitable fall in the employment growth rates n_1 and n_2 to about *n,* as the technology sector ceases to claim an increasing share of resources, and (ii) an equally inevitable fall of the rate *n* itself to about zero as the size of the world population must eventually stabilize. It is interesting, in the light of our theory, to find what the impact of these two phenomena on the innovation rate in the TFA will be.

7.1 Case (i): end of the faster growth of the technology sector

From (23) it follows that for $n_1 = n_2 = n$ the target innovation rate would be

$$\alpha^* = \frac{\beta+\nu}{1-\beta-\nu}n \qquad (26)$$

The actual rate α(t) would be falling from α^*_{TR} to α^*. Let us express α^* in terms of α^*_{TR}:

$$\alpha^* = \frac{(\beta+\nu)n}{n_1+\nu\{(1/\lambda)n+(1-1/\lambda)n_2\}}\alpha^*_{TR} \qquad (27)$$

To illustrate the possible size of the fall, suppose that on a balanced growth path *M/K* = *R/N*= constant. Suppose also that in the past $n_1 = n_2$, n = 0.7 per cent, and a = 0.2. On using the optimality conditions, namely that $\beta = \{a/(1-a)\}M/K$ and $\nu = \{\lambda/(\lambda-1)\}R/N$, expression (27) implies that:

$$\alpha^* = \frac{\lambda+\{(\lambda-1)/(1-a)\}an}{1+\{(\lambda-1)/(1-a)\}n_1/n}\alpha^*_{TR} \qquad (28)$$

Lotka's law suggests that the value of λ is not much greater than unity; suppose that it is at most 1.2. The ratio n_1/n is unlikely to have been greater than 10. Substituting these two numbers into (28) gives the lower limit for α^*, which turns out to be about 0.35 α^*_{TR}. Thus, the upper limit for the innovation slow-down is from α^*_{TR} down to 0.35 α^*_{TR}. The size of the slow-down is sensitive to the value of λ. Taking λ = 1.1 and retaining the values of the other parameters gives $\alpha^* = 0.49\alpha^*_{TR}$.

7.2 Case (ii): end of the world population growth

Should the population of the TFA cease to grow, the model predicts that while the innovation rate would be positive at any finite time, it would be falling continuously from α^*_{TR} to near zero after a sufficiently long period. This is clearly an interesting prediction, especially since an end to population growth in the TFA, and eventually in the whole world, may not be far off. It is therefore important to discuss the model's implications for the innovation rate and economic growth in this case.

If the population is constant and a constant proportion of it is engaged in Q-type activities, our variable *V*, denoting the total number of researchers and teachers corrected for their inventive and educational ability, is also constant. The index of quality is nevertheless increasing. This is because technological progress still takes place, increasing not only the output of the conventional sector, and therefore investment *M*, but also the productivity of inputs *M* and *V* in the Q sector. Both *M* and *TV* are in fact rising at a common rate equal to α. However, the scale elasticity β + ν of the *E* function must be less than unity if equation (23) is to make economic sense. However, if $\beta + \nu < 1$, the research output would grow at a rate less than α. The ratio *E/T* would therefore be declining. Hence $\alpha = H(E/T)$ would also be declining.

It should be noted that absolute annual additions to the technology level, if following the rule that $\Delta T = H(E,T)$, would continue to be increasing with time. However, these additions would be increasing at a falling percentage rate, to become eventually nearly constant. Total conventional output, capital stock, and consumption would all also continue to increase. However, instead of increasing at a nearly geometric rate, as they did when the technology sector II itself had been expanding nearly exponentially, they would be increasing at a nearly arithmetic rate. The innovation limits to growth should therefore be understood to mean a very long-term growth slow-down, both technological and economic, but not necessarily an end to economic growth. The model does not imply that there is any finite upper limit to the level of technology or conventional output; such a limit is known to arise if some essential inputs were both non-reproducible and difficult to substitute for by reproducible inputs, with the relevant elasticities of substitution being less than unity. In our model the labour input is the only natural resource, but it is one which is reproducing itself; it is not non-reproducible.

It seems obvious, or at least possible, that if the world is finite, everything is finite, including the scientific and technological knowledge that is still to be discovered. In the model the innovation limits of this ultimate kind are implicitly assumed to be so distant as to have no impact on the inventive productivity of our researchers. However, this factor may be expected to reinforce the innovation slow-down in due course.

7.3 The Hat-Shape Relationship

In (Gomulka, 1990) I discussed the variation of innovation rates among countries at different levels of development in any relatively short period of time, such as a decade. I noted that such a cross-country variation tends to form a hat-shaped pattern, with the medium-developed countries tending to experience faster innovation than both the least and the most developed countries.

Our discussion in this paper is limited to the TFA of the world. The central question is how the area's innovation rate changes over time in the course of centuries. This is thus 'one-country' dynamic analysis. This analysis indicates that the pattern of change of the innovation rate over time may also be eventually hat-shaped. The First Hat-Shape Relationship is an empirical law that is given a theoretical interpretation. The Second Hat-Shape Relationship seen in Figure 2 is in part a prediction based on a particular model of innovation and growth. Its acceleration and steady growth segments correspond well to the past reality. However, its slowdown part is yet to be tested.

The five periods distinguished in Figure 2 have the following characteristics:

(i) The growth rates n_1, n_2, and n are all very low, close to zero. Consequently the innovation rate is also low.

(ii) $(n_1, n_2) \gg n$, and the population growth rate n is high. The actual innovation rate increases from the very low level of the previous period, approaching the target innovation rate α^*_{TR} at the end of period (ii).

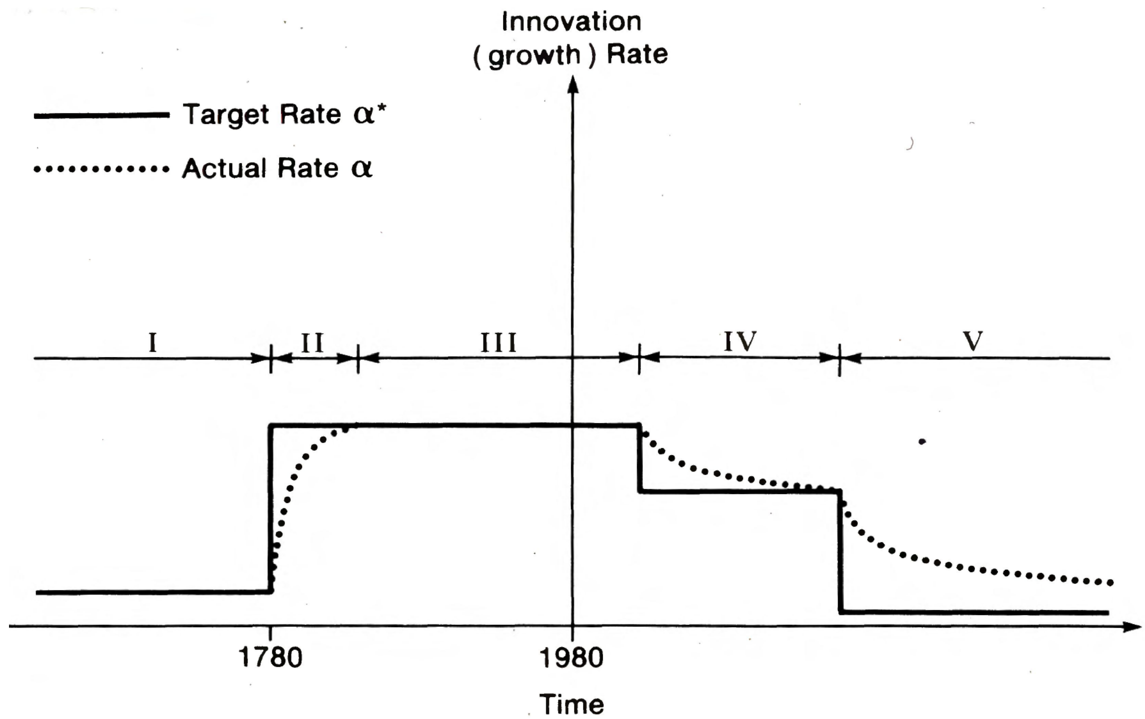

Fig. 2: The target and actual innovation rates over time in the TFA. The dates and magnitudes are chosen for illustrative purposes.

(iii) $(n_1, n_2) \gg n$ and $\alpha = \alpha^*_{TR}$: This is a period of balanced growth in the sense that the trend growth rate of per capita GDP is fairly stable, but it is unbalanced in the sense that the sector upgrading the quality of the labour and producible material inputs is expanding much faster than the conventional sector.

(iv) $n_1 = n_2 = n$, but n is still high. In those circumstances the target innovation rate may continue to be high, though lower than in periods (ii) and (iii). The actual rate is higher than the target rate, but approaches gradually the latter, triggering off a global innovational slowdown.

(v) $n_1 = n_2 = n = 0$: The target rate drops to zero, strengthening the slowdown.

In periods (ii) and (iii) the sector producing qualitative changes is expanding much faster than does the conventional sector. This is the time of the technological revolution. In period (iv) the two sectors expand at a common growth rate, which is itself falling. Period (v) is the same as period (iv), except that the labour force ceases to grow.

8 The plausibility of the innovation slow-down hypothesis

Econometric estimates of production functions for national economies and major conventional sectors suggest that slower growth of inputs virtually always causes slower growth of outputs. A slowdown in the trend growth of research inputs, especially labour, is clearly inevitable. Since the late 1960s there has been a marked fall in growth rates of the industrial R&D expenditure and the employment of labour in OECD countries. But is the link between the growth rates of inputs and outputs in the technology sector similar to that apparently observed in the conventional sector? Moreover, even if some innovation slowdown already occurs or will occur, how plausible is it that it will be of the kind suggested by the theory discussed in this paper?

The answer to the second question cannot be definitive. Our growth data and econometric estimates are not precise enough to be otherwise.. But it is interesting that the proposed model is capable of generating the type of innovation and growth pattern that has been observed in the past. In particular, the model appears consistent with Price's two laws of growth of science, Lotka's law concerning the distribution of inventive ability, and the stylized facts of economic growth in the TFA. Such a broad agreement with key facts relating to innovation and growth over a period of centuries suggests that the theory has passed a test important enough to be taken seriously both as a plausible interpretation of economic growth in the past and as a useful vehicle for making predictions about the growth in the future. The recent evidence discussed by Bloom *et al.* (2017), provides additional support for this theory.

9 Remarks concerning 21st c. growth

Recently, ten economists attempted to answer the question related to the global economy in the 21st c., will the trends of the 20th c. continue? Their answers were published in a book edited by Ignacio Huerta (2013) Among the five trends of the 20th c. which I discuss elsewhere (Gomulka, 2017), there are three those of Daron Acemoglu (2013), one of the ten economists. To these I added two. One concerns the dynamics of the sector II producing qualitative changes. By the end of the 20th c. the most developed economies have already (nearly) fully employed their potential innovation pool, so their strongly unbalanced growth during the 19^{th} and 20^{th} centuries- sector II expanding much faster than the conventional sector I, came essentially to an end.

A situation typical to the early 20th c. in the TFA can be observed now in the emerging economies, which are still far from full use of their innovative potential. According to the theory presented in this paper, increasing engagement of that resource has the potential to underpin the global GDP *per capita* growth rate close to its current level for the better part of 21^{st} c. The growing divergence in terms of the per capita GDP (PPP) between the TFA countries and the non-TFA countries, observed during the 20 c., has started to be replaced by a growing convergence. This dramatic new trend gives support to this prediction. The key players in that process are China and India.

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