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Poisson area-biased Ailamujia Distribution and its applications in environmental and medical sciences

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ABSTRACT

In this paper, a new Poisson area-biased Ailamujia distribution has been formulated to analyse count data. It was created by combining two distributions: the Poisson and area-biased Ailamujia distributions, using the compounding technique. Several distributional properties of the formulated distribution were studied. Its ageing characteristics were determined and expressed explicitly. A variety of diagrams were used to demonstrate the characteristics of the probability mass function (pmf) and the cumulative distribution function (cdf). The parameter of the developed model was estimated by employing the maximum likelihood estimation approach. Finally, two data sets were used to demonstrate the effectiveness of the investigated distribution.

Key words: compound technique, Poisson distribution, area-biased Ailamujia distribution, reliability analysis, order statistics, maximum likelihood estimator.

Mathematics subject classification: 60E05, 62E15.

1. Introduction

In probability distributions, discrete distributions are very essential. Researches are focused extensively in past years to build new discrete models for assessing count data. There are a variety of procedures for developing new distributions in the statistics literature. Extensions to classical distributions can be made by adding additional parameters to them. Transmutation, discretization of continuous distributions, Marshall-Olkin method, compounding, and other approaches were examples. Classical distributions frequently fail to offer an acceptable fit to observable data. This became imperative for researchers to investigate new probability models in order to overcome

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the drawbacks of classical distributions. The compounding of distributions has attracted the attention of researchers over the last decade. The compounding approach is most commonly used when the parameter of one distribution is a random variable that follows another distribution, as in the case of count data. The compounding of distributions occurs when two separate distributions are combined. It makes no odds whether they are discrete or continuous in character. Based upon parent distribution, the resultant distribution from compounding may be continuous or discrete.

The concept of weighted models can be traced back from Fisher (1935). Later on weighted models were briefly discussed by C.R. Rao (1964), when sample observations have an unequal probability of choosing. Thus, in such situation we add weights to the distribution to model bias.

Suppose Y denotes random variable with pmf $p(y)$, then pmf of weighted variable Y_w is defined by

$$P(y;\theta)=\frac{w(y)f(y)}{E[w(y)]};y>0$$

where $w(y)=y^k$ is a non–negative weight function. For $k=2$ we get area-biased distributions.

In this study, we have used compounding approach to create a new distribution by combining Poisson and area-biased Ailamujia distribution. The newly established distribution is called "Poisson area-biased Ailamujia distribution". Compounding distributions have extensive applications in several sectors of research such as biomedicine, insurance, engineering, and communications, among others. Researchers in this field have worked extensively, and they have made significant contributions to compounding research that has been tracked back to 1920. The inception of compounding models has been traced from Greenwood and Yule (1920). Sankaran (1970), Gerstenkorn (1993,1996), Mahmodi et al. (2010), Zamani and Ismail (2010), Gupta and Ong (2004), Shanker (2017), Shi(2012), Subhradev sen (2018), Giovani Carrara Rodrigues et al. (2018), Shanker et al. (2019), This study proposes a novel probability model known as the Poisson area-biased Ailamujia distribution, which is derived via the compounding process, and discusses its many mathematical aspects.

2. Definition of Poisson Area-Biased Ailamujia Distribution

Consider a random variable Y follows Poisson distribution i:e $Y\sim P(\lambda)$ and assume that the parameter of $P(\lambda)$ follows area-biased Ailamujia distribution with parameter θ. The distribution obtained by compounding Poisson with area-biased Ailamujia distribution follows a discrete distribution whose probability mass function is denoted as PABAD (Y,θ). The probability function of the obtained model PABAD (θ) is given by the following theorem.

Theorem 2.1. The probability mass function of a discrete Poisson area-biased Ailamujia distribution PABAD(Y,θ) is given as

$$P(Y=y)=\frac{1}{6}\left(\frac{2\theta}{2\theta+1}\right)^4\frac{(y+1)(y+2)(y+3)}{(2\theta+y)^y} \quad ;y=0,1,2..\theta>0$$

Proof: The probability mass function of the discrete Poisson area-biased Ailamujia distribution PABAD(Y,θ) may be obtained as

If $Y\sim P(\lambda)$, the probability mass function (pmf) of the Poisson distribution is given by

$$f(Y|\lambda)=\frac{e^{-\lambda}\lambda^y}{y!};y=0,1,2,...;\lambda>0$$

As the parameter λ follows area-biased Ailamujia distribution with probability density function (pdf)

$$g(\lambda;\theta)=\frac{(2\theta)^4}{6}\lambda^3e^{-2\theta\lambda};\lambda>0,\theta>0$$

We have

$$P(Y=y)=\int_0^\infty f(Y|\lambda).g(\lambda;\theta)d\lambda$$

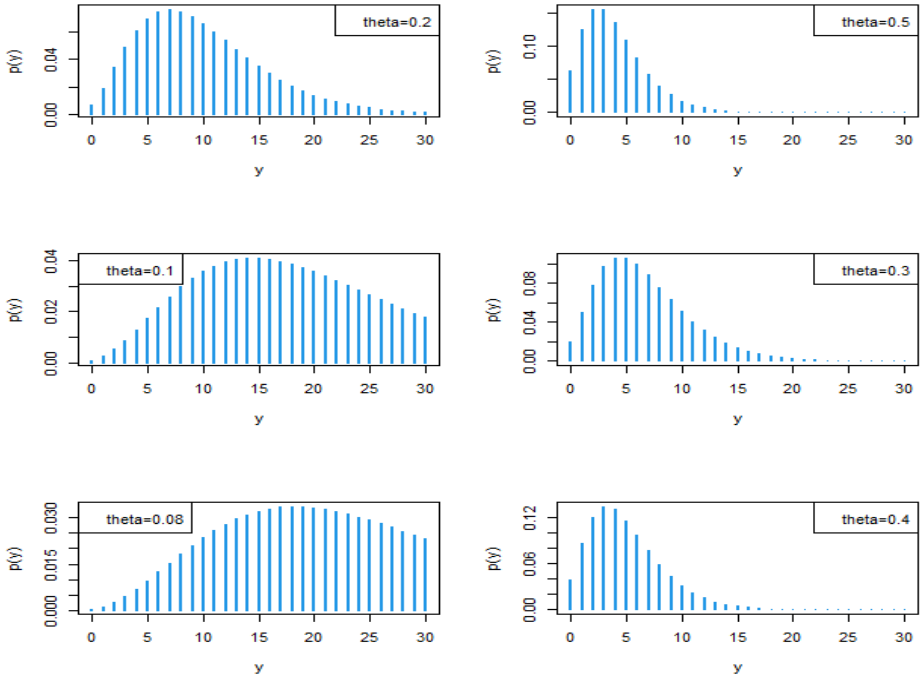
$$=\int_0^\infty\frac{e^{-\lambda}\lambda^y}{y!}\frac{(2\theta)^4}{6}\lambda^3e^{-2\theta\lambda}d\lambda$$

$$=\frac{(2\theta)^4}{6y!}\int_0^\infty\lambda^{y+3}e^{-(2\theta+1)\lambda}d\lambda$$

$$=\frac{(2\theta)^4}{6y!}\frac{(y+3)!}{(2\theta+1)^{y+4}}$$

$$=\frac{1}{6}\left(\frac{2\theta}{2\theta+1}\right)^4\frac{(y+1)(y+2)(y+3)}{(2\theta+1)^y} \quad ;y=0,1,2,...;\theta>0 \tag{2.1}$$

The following six graphs illustrate the behaviour of pmf of the Poisson area-biased Ailamujia distribution for different values of parameter



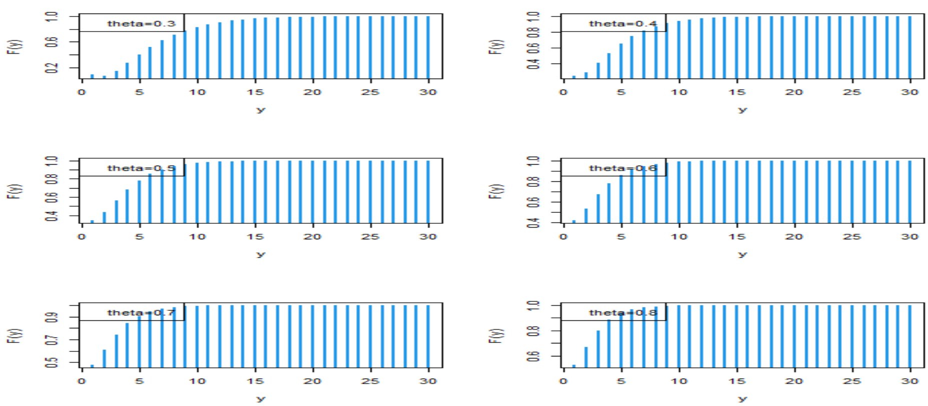
The corresponding cumulative distribution function (cdf) of the discrete Poisson area-biased Ailamujia distribution is given as

$$F(Y=y)=p_r(Y\le y)=1-p_r(Y>y)$$

$$=1-\sum_{w=y+1}^{\infty}P(w)$$

$$=1-\frac{\left\{\begin{array}{l}8\theta^3y^3+4\theta^2(7\theta+3)y^2+\left(208\theta^3+60\theta^2+24\theta+12\right)y\\+\left(52\theta^2+16\theta+6\right)(2\theta+1)\end{array}\right\}}{6(2\theta+1)^{y+4}}\;;y=0,1,2,..;\theta>0 \qquad (2.2)$$

The following six graphs illustrate the behaviour of cdf of the Poisson area-biased Ailamujia distribution for different values of parameter



3. Statistical Measures of Poisson Area-Biased Ailamujia Distribution

In this section several statistical measures of the Poisson area-biased Ailamujia distribution has been studied. They include are moments, moment generating function (mgf) and probability generation function (pgf).

3.1. Moments of Poisson Area-Biased Ailamujia Distribution.

The r^{th} factorial moment of the Poisson area-biased Ailamujia distribution is denoted as $\mu_{(r)}'$ and can be obtained by

$$\mu_{(r)}' = E\left[E\left(Y^{(r)} \middle| \lambda\right)\right], \text{ where } Y^{(r)} = Y(Y-1)(Y-2)\ldots(Y-r+1)$$

$$= \frac{(2\theta)^4}{6} \int_0^\infty \left[\sum_{y=0}^{\infty} y^{(r)} \frac{e^{-\lambda}\lambda^y}{y!}\right] \lambda^3 e^{-2\theta\lambda} d\lambda$$

$$= \frac{(2\theta)^4}{6} \int_0^\infty \left[\lambda^r \sum_{y=r}^{\infty} \frac{e^{-\lambda}\lambda^{y-r}}{(y-r)!}\right] \lambda^3 e^{-2\theta\lambda} d\lambda$$

Taking $y+r$ in place of y within the bracket, we get

$$\mu_{(r)}' = \frac{(2\theta)^4}{6} \int_0^\infty \left[\lambda^r \left(\sum_{y=0}^{\infty} \frac{e^{-\lambda}\lambda^y}{y!}\right)\right] \lambda^3 e^{-2\theta\lambda} d\lambda$$

$$\mu_{(r)}' = \frac{(2\theta)^4}{6}\int_0^\infty \lambda^{3+r} e^{-2\theta\lambda} d\lambda$$

$$= \frac{(2\theta)^4}{6}\frac{\Gamma(r+4)}{(2\theta)^{r+4}} = \frac{(r+3)!}{6(2\theta)^r} \tag{3.1}$$

Substituting $r = 1,2,3,4$ in (3.1), the first four factorial moments can be obtained, and using the relationship between factorial moments and moments about origin, the first four moments about origin of the PABAD (2.1) are obtained as

$$\mu_1' = \frac{2}{\theta}, \qquad \mu_2' = \frac{5+2\theta}{\theta^2},$$

$$\mu_3' = \frac{2\theta^2+15\theta+15}{\theta^3}, \qquad \mu_4' = \frac{4\theta^3+70\theta^2+180\theta+105}{2\theta^4}.$$

The moments about mean of the Poisson area-biased Ailamujia distribution are obtained by using the relationship between moments about mean and moments about origin

$$\mu_2 = \frac{2\theta+1}{\theta^2}$$

$$\mu_3 = \frac{2\theta^3+3\theta+1}{\theta^3}$$

$$\mu_4 = \frac{-\left(28\theta^3-70\theta^2-36\theta-9\right)}{2\theta^4}$$

The coefficient of variation (C.V), coefficient of skewness $\left(\sqrt{\beta_1}\right)$, coefficient of kurtosis (β_2), index of dispersion (γ) of the Poisson area-biased Ailamujia distribution are determined as

$$C.V = \frac{\sigma}{\mu_1'} = \frac{\sqrt{1+2\theta}}{2}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{(\mu_2)^{\frac{3}{2}}} = \frac{\left(2\theta^3+3\theta+1\right)}{(5+2\theta)^3}$$

$$\beta_2 = \frac{\mu_4}{(\mu_2)} = \frac{-\left(28\theta^3-70\theta^2-36\theta-9\right)}{2(5+2\theta)^2}$$

$$\gamma = \frac{\sigma^2}{\mu_1'} = \frac{2\theta+1}{2\theta}$$

Table1. The numerical values of the mean, variance, skewness, kurtosis, coefficient of variation and index of dispersion for some values of parameter θ

θ	μ	σ^2	$\sqrt{\beta_1}$	β_2	C.V	γ
0.5	4.00	8.000	0.012	0.569	0.707	2.000
0.6	3.333	6.111	0.013	0.647	0.741	1.833
0.7	2.857	4.897	0.014	0.718	0.774	1.714
0.8	2.500	4.062	0.015	0.783	0.806	1.625
0.9	2.222	3.456	0.016	0.840	0.689	1.555
1	2.00	3.000	0.017	0.887	0.836	1.500
2	1.00	1.250	0.031	0.845	0.866	1.250
3	0.666	0.777	0.048	-0.037	1.118	1.166
4	0.500	0.562	0.064	-1.535	1.322	1.125
5	0.400	0.440	0.078	-3.468	1.658	1.100

3.2. Generating Functions (pgf, mgf, ch.f) of Poisson Area-Biased Ailamujia Distribution

In this section we study pgf, mgf and characteristics function (ch.f) of the Poisson area-biased Ailamujia distribution.

Theorem.3.2.1. If $Y \sim PABAD(\theta)$ then the probability generating function $P_Y(t)$ is

$$p_Y(t)=\frac{1}{6}\left(\frac{2\theta}{2\theta+1}\right)^4\left\{\frac{6t^4}{(2\theta+1-t)^4}+\frac{12t^3}{(2\theta+1-t)^3}+\frac{11t^2}{(2\theta+1-t)^2}+\frac{18t}{(2\theta+1-t)}+6\right\}$$

Proof: The probability generating function (pgf) of the Poisson area-biased Ailamujia distribution is defined as

$$P_Y(t)=E(t)=\sum_{y=0}^{\infty}t^yP(y)$$

$$=\sum_{y=0}^{\infty}\frac{1}{6}\left(\frac{2\theta}{2\theta+1}\right)^4\frac{\left(y^3+6y^2+11y+6\right)}{(2\theta+1)^y}t^y$$

$$=\frac{1}{6}\left(\frac{2\theta}{2\theta+1}\right)^4\sum_{y=0}^{\infty}\left(y^3+6y^2+11y+6\right)\left(\frac{t}{2\theta+1}\right)^y$$

$$=\frac{1}{6}\left(\frac{2\theta}{2\theta+1}\right)^4\left\{\sum_{y=0}^{\infty}y^3\left(\frac{t}{2\theta+1}\right)^y+6\sum_{y=0}^{\infty}y^2\left(\frac{t}{2\theta+1}\right)^y+11\sum_{y=0}^{\infty}y\left(\frac{t}{2\theta+1}\right)^y+6\sum_{y=0}^{\infty}\left(\frac{t}{2\theta+1}\right)^y\right\}$$

$$=\frac{1}{6}\left(\frac{2\theta}{2\theta+1}\right)^4\left\{\frac{(2\theta+1)\left(t^3+4t^2+t\right)}{(2\theta+1-t)^4}+\frac{t^2(2\theta+1)+t(2\theta+1)^2}{(2\theta+1-t)^3}+\frac{(2\theta+1)t}{(2\theta+1-t)^2}+\frac{(2\theta+1)}{(2\theta+1-t)}\right\}$$

$$=\frac{1}{6}\left(\frac{2\theta}{2\theta+1}\right)^4\left\{\frac{6t^4}{(2\theta+1-t)^4}+\frac{12t^3}{(2\theta+1-t)^3}+\frac{11t^2}{(2\theta+1-t)^2}+\frac{18t}{(2\theta+1-t)}+6\right\}$$

Theorem 3.2.2. If $Y \sim PABAD(\theta)$ then the moment generating function $M_Y(t)$ is

$$M_Y(t)=\frac{1}{6}\left(\frac{2\theta}{2\theta+1}\right)^4\left\{\frac{6e^{4t}}{\left(2\theta+1-e^t\right)^4}+\frac{12e^{3t}}{\left(2\theta+1-e^t\right)^3}+\frac{11e^{2t}}{\left(2\theta+1-e^t\right)^2}+\frac{18e^t}{\left(2\theta+1-e^t\right)}+6\right\}$$

Proof: Since the moment generating function is a generalization of the probability generating function with the relationship given as

$$M_Y(t)=P_Y\left(e^t\right)$$

So that

$$M_Y(t)=\frac{1}{6}\left(\frac{2\theta}{2\theta+1}\right)^4\left\{\frac{6e^{4t}}{\left(2\theta+1-e^t\right)^4}+\frac{12e^{3t}}{\left(2\theta+1-e^t\right)^3}+\frac{11e^{2t}}{\left(2\theta+1-e^t\right)^2}+\frac{18e^t}{\left(2\theta+1-e^t\right)}+6\right\}$$

Similarly, the relationship between mgf and ch.f is defined as

$$M_Y(t)=\phi_Y(it)$$

$$\phi_Y(it)=\frac{1}{6}\left(\frac{2\theta}{2\theta+1}\right)^4\left\{\frac{6e^{4it}}{\left(2\theta+1-e^{it}\right)^4}+\frac{12e^{3it}}{\left(2\theta+1-e^{it}\right)^3}+\frac{11e^{2it}}{\left(2\theta+1-e^{it}\right)^2}+\frac{18e^{it}}{\left(2\theta+1-e^{it}\right)}+6\right\}$$

4. Reliability Measures of Poisson Area-Biased Ailamujia Distribution

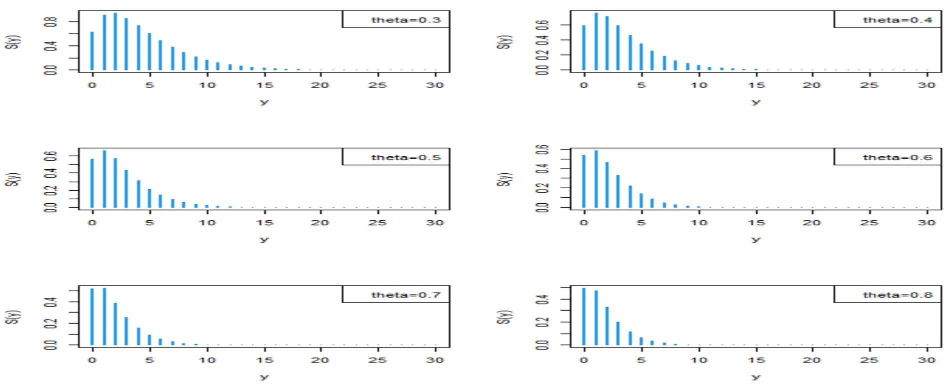
The reliability of the majority of the system decreases with time. So, the chance that a device that is operating until period "t" would fail after that period is referred to as the device's reliability. Suppose Y is a continuous random variable with cdf $F(y);\ y>0$. Then its reliability function, which is also called survival function, is defined as

$$S(y)=p_r(Y>y)=\int_y^{\infty} f(y)dy=1-F(y)$$

The survival function of the discrete Poisson area-biased Ailamujia distribution is given as

$$R(y,\theta)=S(y,\theta)=\frac{\begin{Bmatrix}8\theta^3y^3+4\theta^2(7\theta+3)y^2+\left(208\theta^3+60\theta^2+24\theta+12\right)y\\+\left(52\theta^2+16\theta+6\right)(2\theta+1)\end{Bmatrix}}{6(2\theta+1)^{y+4}} \tag{4.1}$$

The following six graphs show the behaviour of the survival function of the Poisson area-biased Ailamujia distribution for different values of parameter.



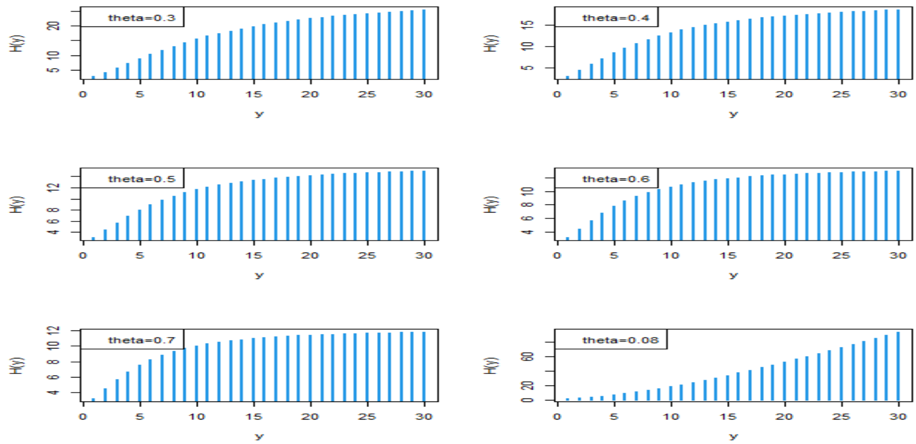
The hazard rate function is described as an indicator of the system's proclivity to collapse within a certain time interval. The hazard rate function of a random variable y is given as

$$H(y,\theta)=\frac{f(y,\theta)}{S(y,\theta)} \tag{4.2}$$

Substituting (2.2) and (4.1) into (4.2), we get

$$H(y,\theta)=\frac{(2\theta)^4\left(y^3+6y^2+11y+6\right)}{\begin{Bmatrix}8\theta^3y^3+4\theta^2(7\theta+3)y^2+\left(208\theta^3+60\theta^2+24\theta+12\right)y\\+\left(52\theta^2+16\theta+6\right)(2\theta+1)\end{Bmatrix}}$$

The following six graphs show the behaviour of the hazard function of the Poisson area-biased Ailamujia distribution for different values of parameter.

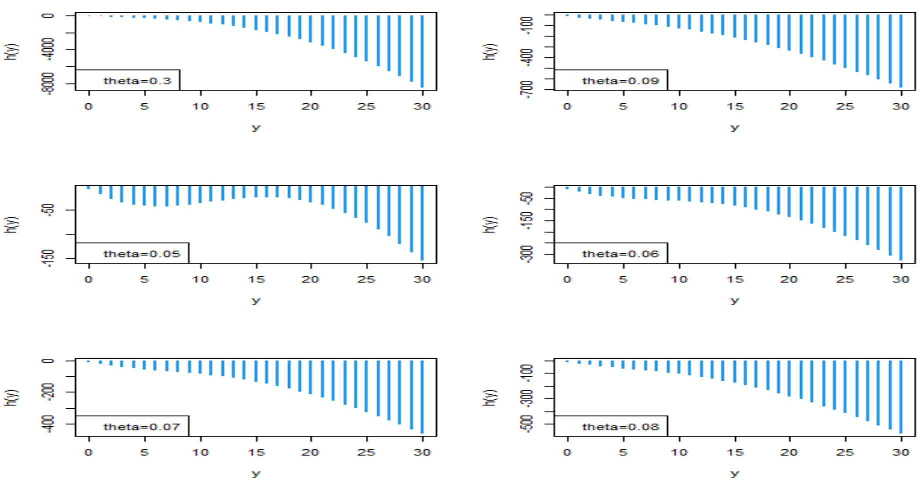


The reverse hazard rate function denoted as h_r is given by

$$h_r(y,\theta)=\frac{f(y,\theta)}{F(y,\theta)}$$

$$h_r(y,\theta)=\frac{(2\theta)^4\left(y^3+6y^2+11y+6\right)}{6(2\theta+1)^{y+4}-\left\{\begin{array}{l}8\theta^3y^3+4\theta^2(7\theta+3)y^2+\left(208\theta^3+60\theta^2+24\theta+12\right)y\\+\left(52\theta^2+16\theta+6\right)(2\theta+1)\end{array}\right\}}$$

The following six graphs shows the behaviour of reverse hazard function of the Poisson area-biased Ailamujia distribution for different values of parameter



5. Recurrence Relation of Poisson Area-Biased Ailamujia Distribution.

If $Y \sim PABAD(\theta)$ then probability mass function of Y is

$$P(Y=y)=\frac{1}{6}\left(\frac{2\theta}{2\theta+1}\right)^4\frac{(y+1)(y+2)(y+3)}{(2\theta+1)^y}\quad ;y=0,1,2,...,;\theta>0$$

The recurrence relation of the Poisson area-biased Ailamujia distribution is given by

$$\frac{P(y+1)}{P(y)}=\frac{(y+2)(y+3)(y+4)}{(y+1)(y+2)(y+3)}\left\{\frac{(2\theta+1)^y}{(2\theta+1)^{y+1}}\right\}$$

$$P(y+1)=\frac{(y+4)}{(y+1)}\frac{1}{(2\theta+1)}P(y)$$

This represents the recurrence relation.

6. Method of Estimation

6.1. Method of Moments (MOM)

Suppose $y_1, y_2,..., y_n$ denotes a random sample of size n from the Poisson area-biased Ailamujia distribution. Now, to obtain sample moments, we replace population moments with sample moments.

$$\mu_1' = \frac{1}{n}\sum_{i=1}^{n} y_i$$

$$\bar{y}=\frac{2}{\theta}\Rightarrow\hat{\theta}=\frac{2}{\bar{y}}$$

Theorem 6.1.1. The MOM estimator $\hat{\theta}$ of θ is positively biased.

Proof: Let us suppose $\hat{\theta}=\varphi(\bar{y})$, where $\varphi(u)=\frac{2}{u}, u>0$ so that $\varphi''(u)=\frac{4}{u^3}>0$

Then, $\varphi(u)$ is strictly convex. Hence by Jensen's inequality, we have

$$E(\varphi(\bar{u}))>\varphi(E(\bar{u}))$$

Thus,

$$\varphi(E(\bar{u})) = \varphi(\mu) = \varphi\left(\frac{2}{\theta}\right) = \theta$$

We obtain

$$E(\hat{\theta}) > \theta$$

Theorem 6.1.2. The MOM estimator $\hat{\theta}$ of θ is consistent and asymptotically normal

$$\sqrt{n}(\hat{\theta} - \theta) \xrightarrow{d} N(0, \upsilon^2(n))$$

where $\upsilon^2(n) = \frac{\theta^2(2\theta+1)}{4}$

Proof: Consistency: since $\mu < \infty$, then $\bar{Y} \xrightarrow{p} \mu$. Also, since $\varphi(\mu)$ is a continuous function at $u = \mu$, then $\varphi(\bar{Y}) \to \varphi(\mu)$, $\hat{\theta} \xrightarrow{p} \theta$.

Asymptotically normality:

As $\sigma^2 < \infty$, then by applying central limit theorem we have

$$\sqrt{n}(\bar{Y} - \mu) \xrightarrow{d} N(0, \sigma^2)$$

since $\varphi(\mu)$ is differentiable function and $\varphi'(\mu) \neq 0$. Then, by applying the delta method we have

$$\sqrt{n}(\varphi(\bar{Y}) - \varphi(\mu)) \xrightarrow{d} N(0, [\varphi'(\mu)]^2 \sigma^2)$$

Finally, we have

$$\varphi(\bar{Y}) = \hat{\theta}, \quad \varphi(\mu) = \theta$$

and

$$\varphi'(\mu) = \frac{-2}{\mu^2} = -\frac{\theta^2}{2}$$

The theorem follows, as a result of this asymptotic $100(1-\alpha)\%$ the confidence interval for θ is given as

$$\hat{\theta} \pm Z_{\frac{\alpha}{2}} \frac{v(\hat{\theta})}{\sqrt{n}}$$

where $Z_{\frac{\alpha}{2}}$ denotes the $\left(1 - \frac{\alpha}{2}\right)$ percentile of the standard normal distribution.

6.2. Maximum Likelihood Estimation of Poisson Area-Biased Ailamujia Distribution.

Let $y_1, y_2, \ldots, y_n$ denote a random sample of size n from the Poisson area-biased Ailamujia distribution. Then, its likelihood function is given by

$$l = \prod_{i=1}^{n} p(y_i, \theta)$$

$$= \prod_{i=1}^{n} \frac{1}{6}\left(\frac{2\theta}{2\theta+1}\right)^4 \frac{\left(y_i^3 + 6y_i^2 + 11y_i + 6\right)}{(2\theta+1)^{y_i}}$$

$$= \left(\frac{1}{6}\right)^n \left(\frac{2\theta}{2\theta+1}\right)^n \prod_{i=1}^{n} \frac{\left(y_i^3 + 6y_i^2 + 11y_i + 6\right)}{(2\theta+1)^{y_i}}$$

The log likelihood function is obtained as

$$\log l = -n\log(6) + n\log\left(\frac{2\theta}{2\theta+1}\right) + \sum_{i=1}^{n} \log\left(y_i^3 + 6y_i^2 + 11y_i + 6\right) - \sum_{i=1}^{n} \log(2\theta+1) y_i$$

Differentiate w.r.t to θ, we get

$$\frac{\partial \log l}{\partial \theta} = \frac{4n}{\theta} - \frac{8n}{(2\theta+1)} - 2\sum_{i=1}^{n} \frac{y_i}{(2\theta+1)}$$

Substituting $\frac{\partial \log l}{\partial \theta} = 0$, we get the required $\hat{\theta}_{mle}$

$$\hat{\theta} = \frac{2}{\bar{y}}$$

7. Application to real data sets

In this section the goodness of fit of area-biased Poisson Ailamujia distribution (PABAD) has been proposed for two real count data sets. And we show that the established distribution perform better than size-biased Poisson Ailamujia distribution (PSBAD), Poisson Ailamujia distribution (PAD) and Poisson distribution (PD), Poisson Lindley distribution (PLD) and Poisson Shunkar distribution (PSD).

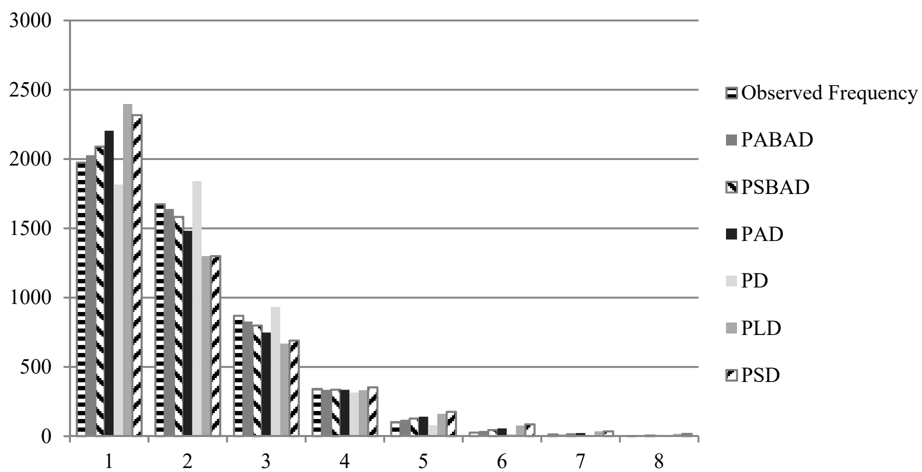
Data set 1: The first data set represents the number of micronuclei after exposure at dose 4 Gy of γ radiation, counted using the cytochalasim B method and available in reference (10).

In order to compare the above distribution models, we consider the criteria like AIC (Akaike Information criterion), AICC (corrected Akaike information criterion), BIC (Bayesian information criterion). Among the above distributions, the better distribution is considered to have lesser values of AIC, AICC.

Table 7.1. Number of micronuclei

Number of micronuclei	Observed frequency	Expected frequency					
		PABAD	**PSBAD**	**PAD**	**PD**	**PLD**	**PSD**
0	1974	2027.28	2089.44	2203.66	1816.07	2396.79	2316.91
1	1674	1638.94	1582.51	1481.90	1839.91	1300.33	1299.64
2	869	828.12	799.09	747.49	932.13	668.82	689.95
3	342	334.74	336.23	335.13	314.81	332.15	353.33
4	102	118.39	127.33	140.85	79.74	160.91	176.42
5	26	38.29	45.01	56.84	16.16	76.53	86.44
6	13	11.61	15.15	22.30	2.73	35.88	35.88
7	2	3.35	4.92	8.57	0.40	16.63	16.63
Total	5002	5000.72	4999.76	4999.84	5001.12	4988.04	4975.2
ML estimates (Standard Error)		1.9739 (0.031)	1.4804 (0.024)	0.9869 (0.0170)	1.0131 (0.014)	1.3873 (0.022)	1.3197 (0.018)
$-\log L$		6740.37	6752.8	6794.42	6767.91	6918.36	6931.20
AIC		13482.3	13507.2	13590.2	13537.2	13836.72	13864.1
AICC		13482.6	13507.8	13590.8	13537.82	13838.73	13864.41
BIC		13489.5	13514.3	13597.4	13544.4	13845.25	13870.2
χ^2		11.25	32.98	105.07	92.73	281.35	273.89
df		5	5	7	4	7	7
p-value		0.1280	$2.6*10^{-5}$	$9.6*10^{-20}$	$3.3*10^{-17}$	$3.4*10^{-66}$	$2.4*10^{-62}$

The following histogram represents the number of micronuclei for the proposed model when compared with other models.

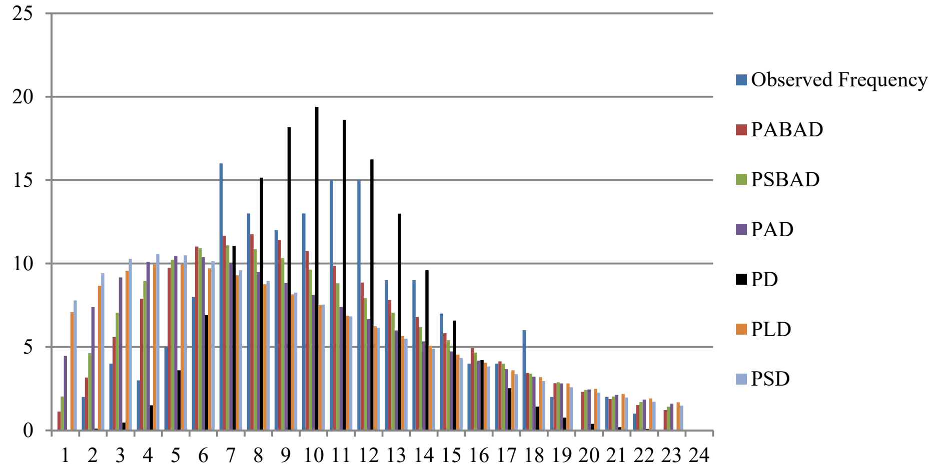


Data set 2: Data on the macroscopic fresh-water fauna in dredge samples from the bottom of water ber Lake is due to Juday (1942) and Thomas (1949).

Table 7.2. Microcalanus Nauplii

Microcalnus Nauplii	Observed frequency	Expected frequency					
		PABAD	PSBAD	PAD	PD	PLD	PSD
0	0	1.13	2.03	4.46	0.02	7.09	7.79
1	2	3.17	4.63	7.38	0.10	8.67	9.4
2	4	5.60	7.06	9.16	0.47	9.56	10.30
3	3	7.81	8.96	10.11	1.50	9.94	10.58
4	5	9.76	10.24	10.46	3.60	9.97	10.49
5	8	11.07	10.92	10.39	6.10	9.71	10.13
6	16	11.67	11.09	10.03	11.05	9.29	9.60
7	13	11.76	10.86	9.49	15.15	8.75	8.96
8	12	11.42	10.35	8.83	18.18	8.15	8.26
9	13	10.75	9.64	8.12	19.39	7.57	7.54
10	15	9.86	8.81	7.40	18.62	6.88	6.83
11	15	8.86	7.92	6.68	16.24	6.25	6.14
12	9	7.82	7.05	5.99	12.99	5.65	5.50
13	9	6.71	6.20	5.34	9.60	5.07	4.90
14	7	5.82	5.40	4.79	6.59	4.54	4.34
15	4	4.93	4.66	4.18	4.21	4.05	3.83
16	4	4.13	3.99	3.68	2.53	3.60	3.37
17	6	3.43	3.40	3.22	1.43	3.19	2.96
18	2	2.83	2.88	2.81	0.77	2.82	2.59
19	0	2.31	2.42	2.45	0.39	2.48	2.26
20	2	1.88	2.03	2.13	0.19	2.18	1.97
21	1	1.51	1.70	1.85	0.09	1.9	1.70
22	0	1.21	1.40	1.60	0.04	1.68	1.48
Total	150	145.52	143.63	140.41	149.98	138.93	140.89
ML estimates (Standard Error)		0.2083 (0.0101)	0.1562 (0.008)	0.1041 (0.006)	9.6000 (0.2529)	0.1907 (0.0120)	0.2024 (0.0125)
$-\log L$		435.85	443.80	459.20	441.62	467.25	461.16
AIC		873.71	889.60	920.41	885.24	936.51	924.33
AICC		873.74	889.63	920.44	885.27	936.53	924.36
BIC		876.72	892.61	923.42	888.25	939.52	927.34
χ^2		23.38	32.45	57.54	109.12	75.22	71.37
df		11	12	11	9	13	12
p-value		0.31875	0.03275	$3.0*10^{-5}$	$1.3*10^{-15}$	$9.5*10^{-8}$	$4.0*10^{-7}$

The following histogram represents the number of micronuclei for the proposed model when compared with other models.



From Table 1 and 2, it has been observed that the discrete Poisson area-biased Ailamujia distribution have the lesser AIC, AICC, $-2\log l$, BIC and χ^2 values along with higher p-values as compared to size-biased Poisson Ailamujia distribution (PSBAD), Poisson Ailamujia distribution (PAD), Poisson distribution (PD), Poisson Lindley distribution (PLD) and Poisson Shanker distribution (PSD). It is evident from the above arguments that the proposed distribution provides better fit than the compared ones.

8. Concluding Remarks

The aim of this study is to use compounding to develop a new distribution for count data termed the “Poisson area-biased Ailamujia distribution”. Different distributional features of the newly formed distribution have been obtained and analysed. The parameter of the proposed distribution has been estimated by the known method of maximum likelihood estimation. Eventually, the model's efficiency was assessed using two count data sets, and it was revealed that the Poisson area-biased Ailamujia distribution provides an appropriate fit for the two count data sets.

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