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A Renaissance mathematician's art

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ABSTRACT

Piero della Francesca is best known as a painter but he was also a mathematician. His treatise *De prospectiva pingendi* is a superb example of a union between the fine arts and mathematical sciences of arithmetic and geometry. In this paper, I explain some reasons why his painting is considered as a part of perspective and, therefore, can be identified with a branch of geometry.

KEYWORDS

Piero della Francesca; Euclid; geometry

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Judith V. Field in her *Piero Della Francesca. A mathematician's art* shows that Piero della Francesca (c. 1412–1492) was not only a brilliant painter but also a brilliant mathematician. Moritz Cantor in the nineteenth century was so impressed by the achievements of Piero della Francesca in mathematics and geometry that in his *Vorlesungen über Geschichte der Mathematik* (1894) he devoted far more attention to Piero della Francesca than any other algebraicist of his time.

Euclid is considered the first theoretician of perspective in the history of mankind. However, it was only the theory of the center projection, whose theoretical foundations were formulated in the period of the Italian Renaissance, that contributed to the unusual development of painting techniques. Linear perspective is thought to have been devised about 1415 by Italian Renaissance architect Filippo Brunelleschi and later documented by architect and writer Leon Battista Alberti in 1435 in his work *Della pittura*. It is a system of creating an illusion of depth on a flat surface. The three components essential to the linear perspective system are orthogonals (parallel lines), the horizon line, and a vanishing point. So as to appear farther from the viewer, objects in the compositions are rendered increasingly smaller as they near the vanishing point. Filippo Brunelleschi created two perspective panels in an elaborate experiment showing the miraculous perceived depth that could be achieved by drawing a piece of architecture a certain way — in his most famous example, the Church of St Giovanni, Florence's Baptistery. Linear perspective was first applied by Masaccio in his *Trinity altarpiece* in the church of Santa Maria Novella in Florence.

Field emphasizes that in Piero della Francesca's time, space did not exist and it was defined as extension, and measured by body (Field, 2005: 95). This definition comes from the work of Aristotle. One of the consequences of this philosophy was its avoidance of the notion of infinity. Universe, which is spherical, must be finite. In this way, as in the Euclid system, the concept of measurement by some fixed measure is thus always a finite procedure. The concept of line and plane is also finite. In today's terms the line from the point *A* to *B* should be called the line segment *AB*.

The above assumptions are applicable in the interpretation of theorems from the treatise of Piero della Francesca *De prospectiva pingendi* written between 1474 until 1482. Proposition 1.13 of Piero della Francesca is known as the first new European theorem in geometry after Fibonacci. The proof refers to the similarity of the triangles. In *Elements* discussion of these issues is included in the Book VI, Proposition 4 to 8.[1] It concerns perspective construction of a horizontal square of definite side seen from the elevation of the definite point. At the same time this proposition can be used to the interpretation

[1] See: http://aleph0.clarku.edu/~djoyce/java/elements/toc.html (30.06.2019).

of Renaissance painting Piero della Francesca's, applying the strict rules of geometry and perspective. Using the line segments one can show how to draw in perspective a surface of undefined shape, which is located in profile as a straight line. It means that a horizontal line *BC* can be foreshortened into a vertical line *EB*. The line *AD* represents a hypothetical observer and the point *A* is the position of the eye in relation to the line *EB*. The vertical line *BF* represents the picture plane.

According to Proposition 1.13 we add a square *BCGF* that represents the object to be drawn in reality in a horizontal plane (Field, 2005: Appendix 8). Then, we draw from the point *A* visual rays to the corners of the square.

Then we construct the parallel half-line from the point *A* to the line segment *BC* and divide the line segment *BC* into two equal parts in *I*. From this point we construct the line perpendicular to the point *A'*, and then draw the line from the point *E* to the point *K*, again parallel to *BC*. The final construction looks as follows (Fig. 1):

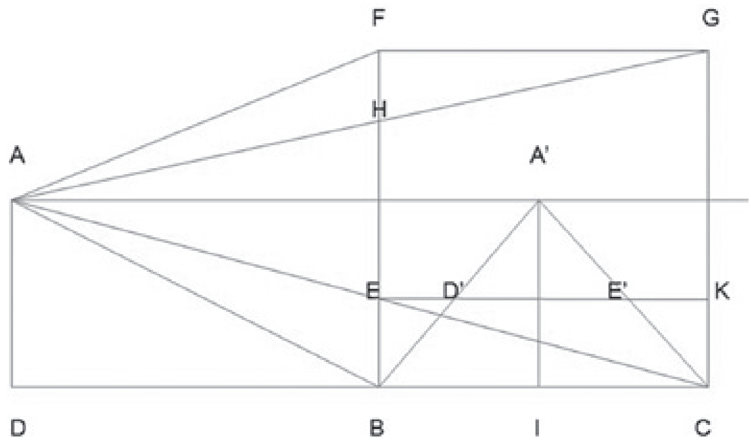

Fig. 1

Finally, we draw the line *A'B* and *A'C* (Mirek, 2014). The goal of design is to show, that from the point of view of *A*, the perspective image of the side *BC* of the square *BCGF* is *EB*, of the side *FG* is *FH*, while the farthest side of the *CG* is *EH*. What may amaze is Piero's assertion that the line segment *D'E'* is also the perspective image of the side of *CG*, so *EH* = *D'E'*.

What is important about the proposition is that it may be used in the interpretation of the painting of Piero della Francesca. The *Flagellation of Christ*, located in Urbino Galleria Nazionale delle Marche, it is perceived as an example of an ideal application of geometry in the arts. The height of the panel is about one braccio (58.4 cm). While, the width of this one was determined with the help of a compass and it is equal to the length of the diagonal of a square of one braccio. On the basis of the Pythagorean Theorem is known that the diagonal of a square of side 1 is an irrational number, so the image width is $\sqrt{2}$, which is 1.4142... braccio.

The composition is split between two scenes, separated by the colonnade supporting on the right side of the Praetorium where we see Jesus. Christ is captured as a small figure in the background, while the three much larger men are standing in the foreground. The man at the extreme right standing in profile corresponds to the line *AD* in accordance with the previous diagram.

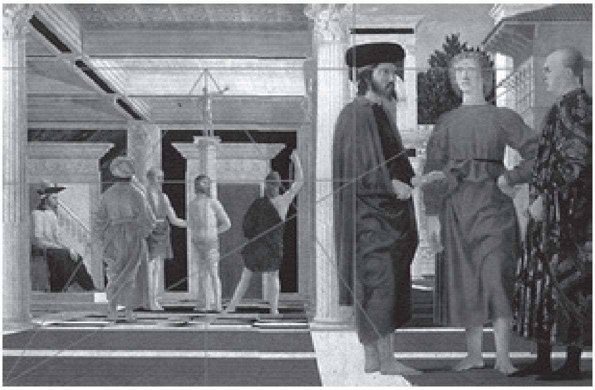

Fig. 2

It is worth noting that the lower-right corner of the square perfectly closes a space of the Praetorium loggia, outlined by a white marble. The diagonal of the square cross above the head of Christ, manacled to a column, setting the center of the scene, depicted on the left panel's side. However, what we want is the line segment *EK*. In the case of the square *BCGF* the line passes exactly through the heads of the flagellators and the head of Christ himself. Thus the line segment *D'E'* would be another clue of where to find the perspective center of the picture. In addition, the line *EK* passes through the headgear of Pilate.

The proposition was used in other paintings of Francesca. The setting of the *Baptism of Christ* provides a similar guidance. As in the *Flagellation of Christ* the composition is split between two scenes, this time separated by a tree and a clay bank. Carter has shown that one can use the rectangle to determine the height of Christ (Carter, 1981). The method refers to Euclidean Proposition 16, Book 4, which involves constructing a fifteen-sided figure, equilateral and equiangular. However, what interests us is the first part of the construction. According to Carter's explanation:

> the figure of Christ coincides with a vertical line bisecting the panel. Using the top side of the rectangle we construct an equilateral triangle, and we find that its apex falls at the point where the central vertical axis passes through the tip of Christ's right foot (Carter, 1981: 151).

Then we locate the center of the triangle and find it to be precisely at the fingertips of Christ's hands in prayer. Now we can return to the Proposition 1.13 and its application. As has been noted the *Baptism* follows in some ways a pattern of the *Flagellation*. This time, however, the three figures in the form of angels appear on the left side of the panel. One may now start a construction from the angel at the extreme left standing in profile. The lower-left corner of the square at the point *B* has been determined by an end of the clay bank. What is most striking is the fact that the line *EK* fits in perfectly with the center of the triangle and a vertical line bisecting the panel (Fig. 3). Thus the painting's composition is based on two geometric visions which are complementary to each other.

Fig. 3

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