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When Competence Hurts: Revelation of Complex Information

Abstract: Even when information is complex and the information processing capacity of economic agents uncertain, noisy messages do not necessarily indicate bad news. I exploit this intuition to examine a simple sender – receiver persuasion game in which effective communication about the state of the world depends not only on the sender's efforts but also on the complexity of that state and the receiver's competence. In this environment, the sender-optimal equilibria maximise the amount of noise. The receiver faces a "competence curse" whereby the smart types might end up with less information and a lower payoff than those who are somewhat less competent.

Keywords: communication, sender-receiver model, information revelation

JEL Classification Code: D83

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Kompetencje a rozumienie złożonych informacji

Streszczenie: Kiedy informacje przekazywane między agentami ekonomicznymi są złożone, a stopień rozumienia ich – niepewny, „szum" w informacji niekoniecznie oznacza złe wieści. W artykule zbadano prostą grę w perswazję między nadawcą a odbiorcą, w której efektywna komunikacja zależy nie tylko od wysiłków nadawcy, ale również od złożoności świata i kompetencji odbiorcy. W tym modelu równowaga optymalna z punktu widzenia

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nadawcy maksymalizuje ilość szumu w komunikacji. Odbiorca staje się ofiarą „przekleństwa kompetencji" – typ odbiorcy lepiej radzącego sobie ze złożonym przekazem może uzyskać mniej informacji i niższą wypłatę niż odbiorca o niższych kompetencjach komunikacyjnych.

Słowa kluczowe: komunikacja, model nadawcy-odbiorcy, ujawnanie informacji

Kod klasyfikacji JEL: D83

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Introduction

It is hardly possible to imagine communication between two people that would allow for a perfect exchange of any given information. Misunderstanding, misinterpretation or just imprecision might arise due to exogenous frictions, such as the sender's ability to formulate the message, the receiver's competence to absorb and correctly interpret the information content of the message, or just the complexity of the matter discussed. It is interesting to consider how such ”language barriers” (a term coined by Blume and Board [2013]) influence the players' incentives to exchange information. In particular, it is interesting to examine whether a higher level of competence in understanding complex messages leads to more informative communication and better decisions.

The issue of competence is particularly important in complex real-life choices. Between 2006 and 2010, more than a million households in Poland, Croatia, Romania, and other Eastern European countries took out mortgage loans denominated in the Swiss franc, to escape high borrowing costs in their home countries. As the franc appreciated until 2011 and then soared even further in 2015 (when the Swiss National Bank unpegged it from the euro), Swiss – franc borrowers were left not only with monthly instalments doubled but also with mortgages worth more than the underlying properties. The dissatisfied borrowers complained about being misinformed, claiming in the European Court of Justice that the bank's presentation "was made in a biased manner, emphasizing the advantages (…), while failing to point out the potential risks or the likelihood of those risks materializing."[1] The Court emphasised that "a term under which the loan must be repaid (…) must be understood by the consumer both at the formal and grammatical level, and also in terms of its actual effects."[2]

The issue of a customer's financial (il) literacy became central to the discussion on the unfortunate mortgage holders. Polish data indicates Swiss – franc borrowers were on *average* relatively wealthy,[3] suggesting their finan-

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1 European Court of Justice [2017, para. 11].

2 *Ibid*., para. 51.

3 The National Bank of Poland estimates that franc borrowers have 30% higher annual income and a more than twice as high a level of liquid financial assets as mortgage holders with loans denominated in the Polish zloty. See National Bank of Poland [2015].

cial literacy might have been relatively high. However, since the banks varied in their loan policies, there is a substantial concern that well-informed risk lovers were pooled with some risk-averse victims of misinformation.

The mortgage example clearly shows that even when information transmitted between the parties must be truthful, a lack of congruence between the receiver and the sender might attenuate communication, if only the latter can manipulate the information content of her message. Anticipating this, the receiver would not only take the strategic incentives into consideration when interpreting the message but might also find it worthwhile to hide his competence in order to enhance the informativeness of the sender's message.

In this paper, I examine a persuasion game in which the state of the world might be difficult to transmit. The sender perfectly observes the state of the world and its complexity, i.e. how difficult it is to understand the state of the world. The sender then chooses a simple or a complex message, with the latter bearing a small cost. Overly complex and overly simplistic messages are a source of noise in this setup. The sender's goal is to persuade the receiver to take some action A that yields the latter an uncertain payoff. As an alternative, the receiver can take an outside option $\varnothing$ with a payoff of 0.

I show that when there is uncertainty about the complexity of information, noise is no longer perceived purely as "bad news". This is because the noise might come from two different sources: the exogenous complexity of information required for successful communication or the endogenous sender's incentive to obfuscate an unfavourable state of affairs.

I concentrate on sender-optimal equilibria and show that there are three types of equilibrium profiles. If the prior probability of obtaining a positive payoff is sufficiently high, the receiver is willing to choose action A upon hearing noise and the sender can sustain her mostly preferred noisy equilibrium in which little information is transmitted. If the prior is less favourable for the receiver, the receiver is warier and the sender's best option is to send as much information as possible and refrain from issuing extra noise. This leads to either an informative or a semi-informative equilibrium. The surprising result, however, is that for a given belief, more competent receiver types might end up in the worse, noisy equilibrium than slightly less competent types, who are guaranteed to end up in the informative outcome. Therefore, competence becomes a curse.

To understand the result, suppose the sender tries to "sell" noise as a good signal and issue intentionally complex messages. Since information revelation must be truthful, announcements are more likely to be correctly understood if the receiver is more competent. As a result, upon hearing noise the high type would be less likely to expect a state to be low than high. Thus, noise becomes a favourable message and the sender has no incentive to transmit any information. This equilibrium cannot be sustained for a less competent receiver, precisely because of his limited understanding of complex messages. The incompetent type is warier and unwilling to choose A upon hearing noise. Therefore, the sender has no choice, but to persuade him with an informative announcement.

In a comparative statics exercise, I show that the utility loss associated with an equilibrium change is discrete and negative – in other words, the smart receiver would have a strong incentive to ”play dumb”. While in the standard setup, this is not possible, I also examine a game in which the sender is uncertain about the receiver's competence, which might be either high or low. I show that for a relevant range of prior beliefs the competent receiver strictly benefits from extra uncertainty, as he now ends up in the unique semi-informative outcome. The low type's outcome is the same so he has no strict incentive to disturb the pooling equilibrium.

Set-up

The model is an augmented version of the Dewatripont and Tirole [2005] set-up, with an additional dimension of uncertainty.[4] There are two players, a sender (she) and a receiver (he). The receiver is going to choose between a known status-quo that yields payoff (normalized to) 0 to both players, and some risky action A. Action A yields a certain payoff 1 for the sender and an uncertain payoff that depends on the unknown state of the world ρ for the receiver. The payoff is either ρ_H in state H or ρ_L in state L, with $\rho_H > 0 > \rho_L$. The prior probability of a high state is $\alpha \in (0,1)$.

The sender has information about the state of the world $\omega \in \{H, L\}$ which she might communicate to the receiver. However, the state of the world could be either simple to transmit, which will be denoted by complexity parameter $n = 1$ and happens with probability q, or complex, which we would denote by $n = 2$ and prob. $1 - q$. We shall assume that complexity is independent of the state realization.

After observing the state realization and its complexity, the sender decides to send a simple $(m = 1)$ or complex $(m = 2)$ message to the receiver. The sender cannot lie, but she can choose the level of message complexity in order to influence the receiver's beliefs.

The crucial assumptions regard the interaction between the complexity of the state and the message. Any message could be understood as the truth $\{H, L\}$ or noise. In particular, simple messages in simple states perfectly reveal the state – that is, the message is understood as H or L – while simple messages in complex states are never sufficient and regarded as noise. Complex messages in both simple and complex states are understood with probability x, while with probability $1 - x$ the receiver regards the message as noise. I shall call x the receiver's competence and assume it is observable by both parties.[5]

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[4] The model is loosely related to the *original* Dewatripont and Tirole [2005] set-up, but closely related to the idea mentioned in footnote (32) of their article.

[5] I interpret competence as e.g. financial literacy, similarly to Bucher-Koenen and Koenen [2015]. Thus, more experienced financial traders simply have higher x. Another, very different idea was employed by Inderst and Ottaviani [2012], where financial literacy was associated with the customers' level of strategic "sophistication". In other words, financial novices were considered to be naïve, i.e., unaware of the existing conflict of interest.

The timing of the model is as follows:

1. Nature chooses:
   a) state $\omega \in \{H, L\} \sim (\alpha, 1-\alpha)$
   b) complexity of the state $n \in \{1,2\} \sim (q, 1-q)$ (known by the sender)
   c) competence of the receiver $x \in (0,1)$;
2. The sender observes (ω, n) and chooses message complexity $m \in \{1,2\}$.
3. The receiver absorbs the message as $\{H, L, noise\}$ and updates his beliefs about state ω.
4. The receiver takes action $a \in \{\emptyset, A\}$.

It is crucial to notice that the sender chooses her message after learning the state of the world (ω, n). In other words, she does not commit to her strategy ex-ante.

The rationale of the model is the following: the sender decides to transmit some potentially complex matter e.g. technical information that requires some expertise to be understood. The information communicated by the sender must be truthful, but it could be noisy. In particular, the sender can exploit the receiver's (lack of) competence by issuing a "too complex" or "overly simplistic" message. If the complexity of the message is smaller than the complexity needed to understand a given state (i.e. $m < n$), the message becomes noise. But also, if a simple state is obfuscated by a complicated announcement, the receiver would only understand it with probability x. Intuitively, a simple message about a simple state is always understood perfectly.

It is crucial that the sender's announcement does not convey any signal about either the complexity of the state or of the message itself. In particular, if the receiver hears a noisy message, he cannot tell whether that was because of mismatched complexities $(m < n)$ or his own small competence x.

The assumption about the receiver's inability to differentiate between "simple noisy" and "complicated noisy" messages is strong but could be treated as a useful benchmark. Consider the motivating example of a client trying to get a mortgage in a bank and the bank sees whether a particular product is suitable for the client or not. The bank is required to present truthful arguments but is not prohibited from presenting additional arguments that may be irrelevant for the client but obfuscate the message. In a simple state, the bank can only select relevant arguments that would persuade the client to make a correct decision – this would correspond to a simple message – or present some more arguments, including also irrelevant ones. In a complex state, there are many arguments required to understand the correct decision, and the effort required to understand the truth is non-zero. The bank can cite only relevant arguments or present irrelevant arguments or too few arguments. The client cannot tell whether the arguments are too simple or just irrelevant.

Without further assumptions, there is a plethora of equilibria in the game. To limit the equilibria, I shall assume that any message is in principle cheap, but complex messages are a bit more costly to send. In particular, sending a message of complexity $m = 1$ costs 0, while sending a message of complexity $m = 2$ costs $c > 0$. If a complex message is more costly than simple messages, the cost plays the role of Ockham's razor – this is a bit similar to the assumption in Verrecchia [1983].[6]

Throughout the paper, I concentrate only on the most interesting case when the cost of sending the more complex message is positive, but small, that is, $c < \min(x, 1-x)$. Otherwise, if the cost is greater than any potential gain from complexity, only simple messages shall be used.

Equilibrium condition and notation

In order to establish the properties of a perfect Bayesian equilibrium of game Γ_x with known competence x, one needs to specify:

1. The sender's message function $\sigma^S : \{H, L\}\{1,2\} \rightarrow \{1,2\}$, that for every pair (ω, n) (i.e. the state and competence needed to understand it) observed by S, defines her message complexity $m \in \{1,2\}$. For notational convenience, I shall describe the sender's strategy as a quadruple: $\big((H,1),(H,2),(L,1),(L,2)\big) \mapsto \big(m_{H1}, m_{H2}, m_{L1}, m_{L2}\big)$.
2. The receiver's beliefs μ in each of his information sets (which correspond to messages he understands) $\{H, L, noise\}$. The believes must follow the Bayes rule whenever possible.
3. the receiver's action function, i.e. $a : \{H, L, noise\} \rightarrow \{\varnothing, A\}$.

Lemma 1. (trivial) In any perfect Bayesian equilibrium, the receiver's beliefs upon hearing messages H and L are trivial, i.e. $\mu(\omega = H \mid H) = \mu(\omega = L \mid L) = 1$. Therefore, his optimal actions are $a(H) = A$, $a(L) = \varnothing$. The only nontrivial action is:

$$a(noise) = \begin{cases} A & \text{if } \mu(H \mid noise)\rho_H + \mu(L \mid noise)\rho_L \geq 0 \\ \varnothing & \text{otherwise.} \end{cases}$$

Proof. Since the revelation is truthful, upon hearing a non-noisy message the receiver is certain about the state, which trivially determines his optimal actions. The action upon hearing noise is a result of the Bayes rule. Since for any $x < 1$ and any sender's strategy, the probability of receiving a noisy message is positive, then for every strategy profile $\big(m_{H1}, m_{H2}, m_{L1}, m_{L2}\big)$ the receiver can calculate the posterior probability of the state being H when noise was heard. Then, in any perfect Bayesian equilibrium, the receiver would take action A upon hearing noise only if according to his Bayesian beliefs: $\mu(H \mid noise)\rho_H + \mu(L \mid noise)\rho_L \geq 0$.

Lemma 2. In any perfect Bayesian equilibrium with costly complex messages, the sender's strategy choice must have $m_{H1} = 1$ and $m_{L2} = 1$.

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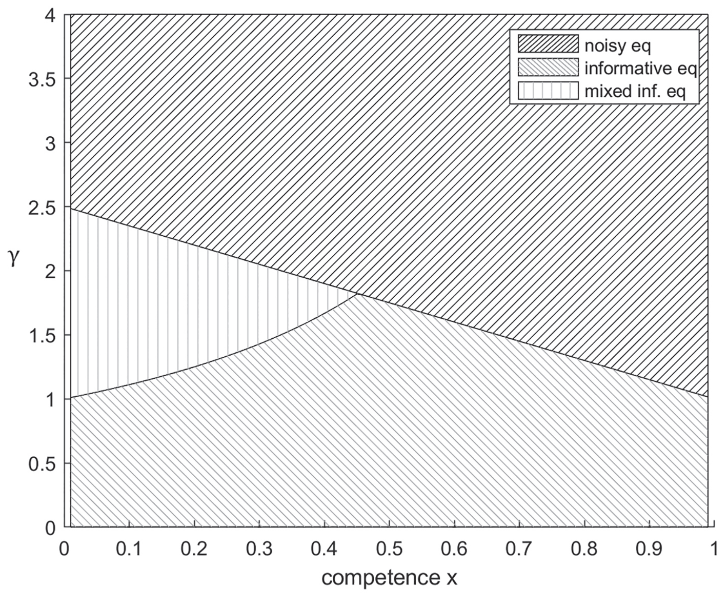
[6] It would be convenient to assume that complex messages are lexicographically less preferred than simple or noisy messages. This would allow us to remove the cost from the model. However, this would also prevent the existence of any equilibrium for some parameter values.

Proof. Recall that the sender chooses her actions already knowing (ω, n). In state $(H,1)$, the choice of sending a simple message, one that would surely be understood by the receiver and induce action A, strictly dominates the choice of a complex message, which is not only more costly but also leaves the possibility of misunderstanding. Similarly, in state $(L,2)$ the complex message is more costly, and in no way can it induce a better action than the simple (here meaning: noisy) message.

Define $\gamma_\alpha = \frac{\alpha}{1-\alpha}$ as the ratio of prior probabilities of states H and L. Note that γ_α is strictly increasing in α and define $\gamma_\rho = \frac{-\rho_L}{\rho_H}$, as the benchmark prior γ_α, which makes the receiver indifferent between action A and the null action. Note that γ_α and γ_ρ summarise the uncertainty about the state along two different dimensions, while γ_α is directly related to the probability distribution, γ_ρ depends exclusively on the payoffs. Denote the ratio of the two parameters by $\gamma = \frac{\gamma_\alpha}{\gamma_\rho} = -\frac{\rho_H \alpha}{\rho_L (1-\alpha)}$. Large values of γ mean that the prior is strong (or the gain in the high state is high), while small values of γ indicate that the prior is weak (or the punishment in the low state is substantial). The parameter γ summarises the gains or losses from uncertainty in the model. Observe that a priori the receiver would take action A only if $\gamma > 1$.

Figure 1. Sender-best equilibria in the game for given x and γ



Source: own calculation.

Even for a positive cost of a complex message, there might exist one or three Bayesian equilibria in the game (see: Proposition A in the Appendix, where full characterisation is provided). To explicitly analyse the competence effect, I shall impose an equilibrium selection rule in order to enable comparative statics between unique outcomes. Following the approach of Bayesian persuasion models, we shall concentrate on sender-optimal equilibria.[7] It is not difficult to verify that, in fact, the sender-optimal equilibrium is the one maximising the amount of noise.

Proposition 1. Let $c < \min(x, 1-x)$. Depending on (x,γ), the sender-optimal equilibrium in the communication game is one of three possible forms:

1) A noisy equilibrium in pure strategies, for $\gamma \geq \frac{1-qx}{1-q}$:
   a) The sender's strategy is $\big((H,0),(H,1),(L,0),(L,1)\big) \mapsto (1,1,2,1)$,
   b) The receiver's beliefs are $\mu(H \mid noise) = \frac{\alpha(1-q)}{\alpha(1-q)+(1-\alpha)(1-qx)}$, $\mu(L \mid noise) = \frac{(1-\alpha)(1-qx)}{\alpha(1-q)+(1-\alpha)(1-qx)}$, $\mu(H \mid H) = \mu(L \mid L) = 1$,
   c) The receiver's actions are $a(H) = A$, $a(L) = \varnothing$ and $a(noise) = A$;
2) An informative equilibrium in pure strategies, for $\gamma < \min\left(\frac{1-qx}{1-q}, \frac{1}{1-x}\right)$:
   a) The sender's strategy is $\big((H,0),(H,1),(L,0),(L,1)\big) \mapsto (1,2,1,1)$,
   b) The receiver's beliefs are $\mu(H \mid noise) = \frac{\alpha(1-x)}{1-\alpha x}$, $\mu(L \mid noise) = \frac{1-\alpha}{1-\alpha x}$, $\mu(H \mid H) = \mu(L \mid L) = 1$,
   c) The receiver's actions are $a(H) = A$, $a(L) = \varnothing$ and $a(noise) = \varnothing$;
3) A semi-informative equilibrium in mixed strategies, for $\frac{1}{1-x} \leq \gamma < \frac{1-qx}{1-q}$:
   a) The sender's strategy is $\big(1, 2, (1-r, r), 1\big)$ with $r = \frac{(1-q)\big(\gamma(1-x)-1\big)}{q(1-x)}$,
   b) The receiver's actions are $a(H) = A$, $a(L) = \varnothing$ and $a(noise) = (A, \varnothing)$ with probabilities $\left(\frac{c}{1-x}, 1-\frac{c}{1-x}\right)$.

Proof. See the Appendix.

The noisy equilibrium $\{(1,1,2,1), A\}$, is preferred by the sender whenever it can be supported by the receiver's beliefs. If the noisy equilibrium fails to exist, the existing informative or mixed semi-informative equilibrium is unique, therefore it is sender-optimal. The sender-optimal equilibria are pictured in the right panel of Figure 1.

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[7] Glazer and Rubinstein [2012] propose a different approach where the receiver commits to a "persuasion codex". With such an assumption, the selected equilibria would be optimal for the receiver.

Receiver's expected payoff in the game

We may now analyse the receiver's gain from the game and how it depends on his language competence x. Assume $\gamma \in \left(\frac{1-qx}{1-q}, \frac{1}{1-q}\right)$ is fixed. Depending on x, up to three different (sender-optimal) equilibria are possible. Notice that for a given γ, a higher competence may be a burden for the receiver, as it might result in a noisy equilibrium $\{(1,1,2,1), A\}$, while for somewhat lower values of x the unique equilibrium is either informative $\{(1,2,1,1), \varnothing\}$ or semi-informative $\left\{(1,2,(1-r,r),1), \left(\frac{c}{1-x}, 1-\frac{c}{1-x}\right)\right\}$. We shall see this intuition formalised in the theorem below.

Theorem 1. Assume the prior $\gamma \in \left(\frac{1-qx}{1-q}, \frac{1}{1-q}\right)$ is given. A marginal change in the competence x around $x' = \frac{1}{q}\left(1-\gamma(1-q)\right)$ brings a discontinuous decrease in the receiver's expected utility. As a result, a marginally more competent receiver gets a strictly lower payoff than a less competent type.

Proof. Let us analyse the receiver's expected payoff in the three equilibria:

$$E^R_{\text{info eq}}\rho(x) = \left(\alpha q + \alpha(1-q)x\right)\rho_H$$
$$E^R_{\text{mixed}}\rho(x) = \left(\alpha q + \alpha(1-q)x\right)\rho_H$$
$$E^R_{\text{noisy eq}}\rho(x) = \alpha\rho_H + \rho_L(1-\alpha)(1-qx)$$

The fact that in the mixed equilibrium, the expected payoff has the same functional form as in the informative equilibrium shall not be surprising. By definition of a mixed equilibrium, the receiver is indifferent between his two choices, conditional on noise. In particular, his utility is the same as if he had chosen $A(noise) = \varnothing$, which results in a payoff as above.

Take $x' = \frac{1}{q}\left(1-\gamma(1-q)\right)$, that is the benchmark receiver, for whom the noisy equilibrium could be sustained. The expected gain/loss for an equilibrium switch in the neighbourhood of x' is:

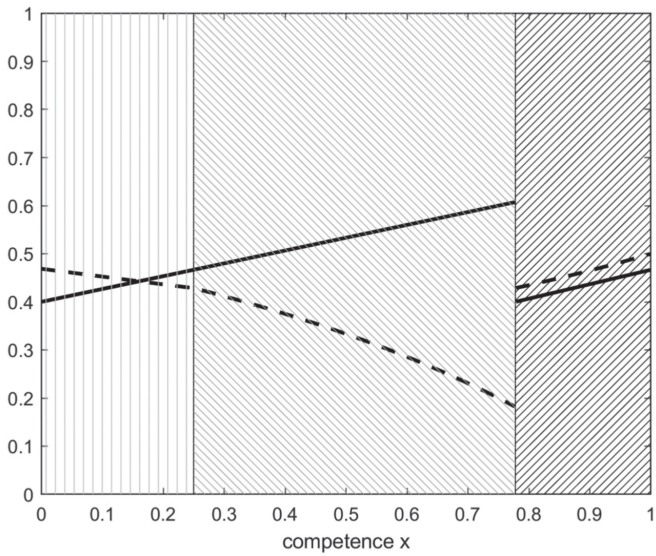
$$\begin{aligned} E\rho\left(x \searrow x'\right) - E\rho\left(x \nearrow x'\right) &= \alpha(1-q)(1-x)\rho_H + \rho_L(1-\alpha)(1-qx) = \\ &= \alpha\rho_H\left[(1-q)(1-x) - \frac{1}{\gamma}\left(1-qx\right)\right] \\ &= -\alpha\rho_H(1-q)x' < 0. \end{aligned}$$

The first transformation exploits the definition of $\gamma = \frac{-\rho_H\alpha}{\rho_L(1-\alpha)}$, while the second uses the fact, that $x' = \frac{1}{q}\left(1-\gamma(1-q)\right)$ is equivalent to $\frac{1}{\gamma} = \frac{1-q}{1-qx'}$. Thus, a marginal change in x around the threshold results in a discrete decrease in utility.

Theorem 1 shows that for a small ϵ, two receiver types $x'+\epsilon$ and $x'-\epsilon$ not only end up in different equilibria but also the more competent receiver is strictly worse off. If it was possible, he would rather decrease his competence to $x'-\epsilon$ to induce an informative equilibrium than remain more competent, but less informed. This is due to a change in the equilibrium profile. Within a given class of equilibrium, the receiver always benefits from greater competence, as all three functions $E^R_{\text{info eq}}\rho(x)$, $E^R_{\text{mixed}}\rho(x)$ and $E^R_{\text{noisy eq}}\rho(x)$ are increasing in x.

The discontinuous jump in utility is accompanied by a change in beliefs. As competence decreases around the threshold x', the posterior belief $\mu(H \mid noise)$ – which is an increasing function of x in the noisy equilibrium – also plummets, up to a point where it is no longer profitable for the receiver to choose A upon hearing noise and he would rather take the $\varnothing$ action instead. The sender is then forced to switch to an informative strategy (1,2,1,1) and a new equilibrium arises. Source: own calculation. presents the changes in both beliefs and the expected utility as a function of x for an exemplary choice of parameters.

Figure 2. Receiver's expected utility (solid line) and belief $\mu(H \mid noise)$ as a function of competence x (plotted for $\rho_H = \frac{4}{3}$, $\rho_L = -1$, $\alpha = 0.5$)



Source: own calculation.

It must be noted that the other possibility of a change in the equilibrium profile – that is, when the equilibrium is switched from an informative to a mixed semi-informative – does not result in a discrete change of utility, which is continuous and increasing in x for $x < x'$. In fact, since the receiver's utility in both equilibria shares the same functional form, the semi-informative equilibrium is a natural extension of the informative equilibrium whenever the latter cannot be sustained. Therefore, the receiver of type $x \in (0, x')$ has no incentive to reduce his competence, even if that was possible.

Change in prior signal

A similar reasoning applies to changes in γ. Just like with competence, having a higher initial prior does not necessarily benefit the receiver. In particular, if the receiver faces an increase in γ, he might end up in a worse equilibrium. This is quite intuitive as a more favourable prior makes the receiver more likely to choose A, thus decreasing the sender's incentives to transmit information.

The prior information γ and the communication competence x are substitutes. It would be interesting – but beyond the scope of this paper – to examine a model in which the two types of communication skills are substantially different; while one dimension represents the stock of knowledge, the other describes the ability to absorb new knowledge. In reality, those two dimensions of information literacy are distinct skills.

Private information about competence

I have shown that the receiver might face a "competence curse." In particular if his competence is so high that it induces an uninformative equilibrium, the receiver might be worse off than with somewhat lower x. However, reducing x is hardly possible.

Assume that competence becomes the receiver's private information. To simplify, let us consider a case in which competence may be either x_L with probability π or x_H with probability $1-\pi$ and denote the equilibrium probability of type i choosing A upon hearing noise as b_i. To make things interesting, assume $x_L < \bar{x} < x_H$, where $\bar{x}$ satisfies $\frac{1}{1-\bar{x}} = \frac{1-q\bar{x}}{1-\bar{q}}$, and probability π is separated from 0 and 1 – so that the two – types case does not trivially collapse to the one – type setup.[8]

It can be shown (see: Appendix) that even though there are multiple equilibria in the setting, as long as the noisy profile $\{(1,1,2,1), b_L = 1, b_H = 1\}$ can be sustained – that is for $\gamma > \frac{1-qx_L}{1-q}$ —it remains the sender-best equilibrium. If $\gamma < \frac{1}{1-x_L}$ the unique equilibrium is the informative one in which neither receiver-type chooses A unless he is certain about the state. The only interesting

[8] $\bar{x}$ is a crossing point of the indifference curves that define equilibrium conditions.

case is therefore $\frac{1}{1-x_L} < \gamma < \frac{1-qx_L}{1-q}$. Indeed, in this range, the high type might benefit from the uncertainty. The only (therefore, sender-optimal) equilibrium is $\left\{ \left(1,2,(1-r,r),1\right), b_L = \frac{c}{\pi(1-x_H)}, b_H = 0 \right\}$.[9] Notice that the high type is not only better off than without uncertainty about x but also his outcome is higher than the low type's payoff. If the sender attempts to persuade the low type, she must send at least a semi-informative message. The more competent type "freerides" and can now enjoy the more favourable outcome. The low type, on the other hand, enjoys the same outcome as if he played single-handedly.

This outcome would persist if the receiver could send a cheap-talk message. Notice that the high type would always want to send the same message as the low type, as it is in his best interest to be pooled. If, instead, the types were able to credibly certify their types at no cost, the low type would be able to separate from the high type but have no strong incentives to do so. In fact, he is indifferent between being certified or not.

Conclusion and discussion

I presented a simple model of persuasion with a binary state of the world. I show that when information about the state of the world is simple or difficult to transmit, the sender can exploit uncertainty about its complexity to hide "bad news" with a noisy message. There are three types of sender-best equilibria in this setup. In a noisy equilibrium, the sender minimises the informativeness of his messages, but the receiver is willing to take the sender-preferred action A even upon hearing a noisy message. In an informative equilibrium, the receiver takes action only if he is certain about the state being H, therefore the sender has no incentive to issue noise. In a semi-informative equilibrium, the players choose fully mixed strategies, but the receiver remains wary and is "persuaded" by noise only with a small probability.

I show that in this setup, the receiver faces a "competence curse" under which more competent types may end up in a worse, noisy equilibrium, while if the competence falls below a certain threshold, the unique equilibrium is informative or semi-informative. The more competent type has therefore a strong incentive to hide his competence.

To see why higher types face a "competence curse," observe that for more competent receivers, noise might be a favourable message. Assume that the sender chooses a strategy $(1,1,2,1)$ and the receiver anticipates it. In state $(L,1)$, the receiver hears a signal that he correctly understands as L with probability x. High values of x are a sign of competence, therefore the receiver is relatively effective in identifying a low state correctly with certainty. As a result, he believes noise is less likely to arise in the low state. As $\mu(L \mid noise)$ is low,

9 More specifically, $r = \frac{q - x_L - qx_L(1-x_L)}{q(1-x_L)}$.

$\mu(H \mid noise)$ must be relatively high. Therefore the receiver becomes less sceptical upon seeing noise, and competence becomes a curse. Due to the receiver's good understanding of low states, the sender is able to "sell" the noisy message as a favourable signal and maintain the equilibrium in which little information is transmitted.[10]

This article contributes to the growing literature on communication with limited information processing abilities. The main reference is a model by Dewatripont and Tirole [2005] that inspired the current model. As in their setup, I examine a sender – receiver game in which the former tries to persuade the latter to take some action. I modify their framework by adding another dimension of uncertainty, which is the complexity of the state and the sender's message. Furthermore, I examine how the equilibrium changes with the receiver's competence in understanding complex messages and what his incentives are to signal his abilities.

The idea that the receiver has some intellectual, time or attention constraints appeared in a famous model of rational inattention developed by Sims [2003]. Glazer and Rubinstein [2004] derived optimal mechanisms of persuading a receiver that can understand only a single argument, which has a similar flavour to my assumption that simple messages are always understood. Guembel and Rossetto [2009]; Bucher-Koenen and Koenen [2015] define "competence" as the probability of a correct message, similarly to my approach. However, they examine cheap-talk (Crawford and Sobel [1982]) communication, while I concentrate on truthful information revelation. A close reference is, therefore, Persson [2018], who builds on the Dewatripont and Tirole [2005] framework to examine the issue of investment in communication with several senders competing for the attention of one receiver – or, in a similar manner – a monopoly sender communicating about several aspects of the good. In her setup, information overload arises endogenously as a result of the receiver's limited attention. If the prior is favourable, i.e. if without communication the receiver took the sender's preferred action, experts send irrelevant cues to prevent the receiver from discovering potentially unfavourable news. I describe a similar equilibrium (however, in a much simpler setup), but competence is exogenous in my setup. While my model shares the idea of "strategic obfuscation," contrary to Persson [2018], I discover that "smarter" types can end up with a worse outcome.

The result that competence can be harmful appeared in multiple articles, mostly within the cheap-talk literature. Moreno de Barreda [2010] shows that, in a Crawford-Sobel framework, the receiver's private information might

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10 As explained in the Appendix, whenever $\gamma < \frac{1}{1-x}$, the sender-optimal noisy equilibrium co-exists with a sender-suboptimal informative equilibrium. One rationale for sustaining a noisy equilibrium – if my equilibrium selection criterion was to be relaxed – is the receiver's overconfidence. In particular, if the receiver believes in his high ability to recognise the low state from an overly complex message in $(L, 1)$, the sender has no incentive to reduce the amount of noise in equilibrium.

reduce the informativeness of the partition used by the sender, while Rantakari [2016] shows that a similar logic applies to endogenous (and ex-post) information acquisition. Ishida and Shimizu [2016] show that the receiver's prior knowledge impedes communication in a model with a discrete action choice, like mine. Li and Madarász [2008] – also within the cheap-talk framework – show that extra information about the conflict of interest can decrease communication. However, the mechanism of cheap-talk games is quite different than in my model of information disclosure, where the messages are constrained to be truthful.

Outside of the cheap-talk literature, the models by Kessler [1998] (focusing on contracts) and Roesler & Szentes [2017] (bilateral trade) share a similar message: that being "too informed" may not be optimal for the receiver. Also, noiseless communication may not be optimal for welfare – see Fishman and Hagerty [1990]; Goldstein and Leitner [2018]; and Blume et al. [2007].

Contrary to those articles, this paper falls into the broad literature of truthful, albeit not necessarily complete, information transmission. The classic unravelling mechanism, as in Milgrom [1981] and Grossman [1981], is disturbed in my model by the presence of uncertainty about the complexity of the state. The setup resembles that in Shin [1994], who introduces uncertainty about the expert's information space, or to Morgan and Stocken [2003], who have uncertainty about incentives. While their research questions are different from mine, we share the intuition that another dimension of uncertainty undermines the classic "no news is bad news" result [Milgrom, 1981]. In my model, uncertainty about the information complexity required to understand the state of the world, combined with (a lack of) competence in communication, makes uninformative messages look favourable, thus enabling the sender to maintain a noisy equilibrium. On the other hand, my informative equilibrium has a flavour similar to that of Dziuda [2011]. However, I concentrate explicitly on the issue of the receiver's competence.

The model also corresponds to the general class of Bayesian persuasion games, as described in the seminal paper by Kamenica and Gentzkow [2009]. As in Kamenica and Gentzkow [2009]; Rayo and Segal [2010]; and Alonso and Câmara [2014], the sender can benefit from non-full disclosure. The main difference between my approach and Bayesian persuasion models is that I have no commitment in the sender's strategy. Notice that if the sender was able to commit, he could commit to a non-expensive strategy of always sending a simple message, regardless of the state. Such a strategy would have no strategic bite – the posterior belief would be the same as the prior, resulting in the receiver taking action A whenever $\gamma > 1$. While the class of Bayesian persuasion games is extensive, one might imagine multiple setups in which the sender is unable to commit *ex ante*.

The main limitation of this article is the specific nature of binary complexity that drives the discrete jump in utility. It would be interesting, but beyond the scope of this paper, to examine how this issue could be tackled in a more realistic scenario of continuous complexity of the state and messages. This would necessarily include some more sophisticated assumptions about the receiver's understanding of messages of varying complexity.

One can also argue that my assumption about the non-observability of the complexity of the message is too strong. While, indeed, in some real-life applications the receiver might to some extent observe message complexity, my model serves as a useful benchmark for describing the mechanism of "strategic obfuscation." As long as the receiver has some uncertainty about a possible source of noise, the sender has incentives to hide the bad news.

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Appendix

General characterisation of Bayesian equilibria in the game

Proposition A. For $c < \min(x, 1-x)$ there are four types of perfect Bayesian equilibria of the game with known competence x:

1. The informative equilibrium in pure strategies $\{(1,2,1,1), a(noise) = \varnothing\}$, that exists whenever $\gamma < \frac{1}{1-x}$;
2. The noisy equilibrium in pure strategies $\{(1,1,2,1), a(noise) = A\}$, that exists whenever $\gamma > \frac{1-qx}{1-q}$;
3. The mixed semi-informative equilibrium $\left\{(1,2,(1-r,r),1),\ P\big(a(noise) = A\big) = \frac{c}{1-x},\ P\big(a(noise) = \varnothing\big) = 1 - \frac{c}{1-x}\right\}$ with $r = \frac{(1-q)\big(\gamma(1-x)-1\big)}{q(1-x)}$, that exists for $\gamma \in \left(\frac{1}{1-x}, \frac{1-qx}{(1-q)(1-x)}\right)$;

4. The mixed semi-noisy equilibrium $\left\{(1,(p,1-p),2,1),\ P\big(a(noise)=A\big)=1-\frac{c}{x},\right.$ $\left. P\big(a(noise)=\varnothing\big)=\frac{c}{x}\right\}$ with $p=\frac{\big(1-\gamma(1-q)(1-x)-qx\big)}{\gamma x(1-q)}$, that exists for $\gamma \in \left(\frac{1-qx}{1-q},\frac{1-qx}{(1-q)(1-x)}\right)$.

Proof. I shall analyse which strategy profiles might arise in an equilibrium. By Lemma 2, there are only four feasible strategies of the sender: (1,1,1,1), (1,2,1,1), (1,1,2,1), (1,2,2,1). Notice also that in any perfect Bayesian equilibrium the receiver's beliefs must be consistent with the sender's strategy. Let us assume $c<\min(x,1-x)$, which means there is at least some incentive to invest in a costly message. Examine four cases:

1. The sender uses strategy (1,1,1,1). Such strategy is consistent with the receiver's beliefs $\mu(H\,|\,noise)=\alpha$ and $\mu(L\,|\,noise)=1-\alpha$. Assume that the receiver chooses $a(noise)=A$. The sender can deviate to (1,1,2,1), generating noise with positive probability and increasing his expected payoff by $(1-\alpha)q(1-x)$, or, putting things simply, by $1-x$, conditional on the state being $(L,1)$. Assume the receiver takes action $\varnothing$ when hearing noise. The sender can benefit from deviating to (1,2,1,1), i.e. more informative message, that brings a payoff increase of $\alpha(1-q)x$. Thus strategy (1,1,1,1) is never optimal.
2. The sender uses strategy (1,2,1,1). This strategy is consistent with the receiver's beliefs $\mu(H\,|\,noise)=\frac{\alpha(1-x)}{1-\alpha x}$ and $\mu(L\,|\,noise)=\frac{1-\alpha}{1-\alpha x}$. Assume the receiver takes A when hearing noise. Then the sender has an incentive to deviate to a more noisy message (1,2,2,1) (with an extra benefit of $1-x$ in a state $(L,1)$). In the other case, when the receiver takes $\varnothing$ upon hearing noise, there is no incentive to deviate. The appropriate beliefs imply $\gamma<\frac{1}{(1-x)}$ and in such a case the strategies $\{(1,2,1,1),\ a(noise)=\varnothing\}$ constitute an equilibrium.
3. The sender uses strategy (1,1,2,1). Upon hearing noise the receiver would take action A if $\gamma\geq\frac{1-qx}{1-q}$ and $\varnothing$ otherwise. In the latter case, i.e. $a(noise)=\varnothing$, the sender has an incentive to deviate from costly (1,1,2,1) to less costly (1,1,1,1), saving c in $(L,1)$. If $a(noise)=A$, there is no incentive to deviate and the profile $\{(1,1,2,1), a(noise)=A\}$ constitutes an equilibrium when $\gamma\geq\frac{1-qx}{1-q}$.
4. The sender uses strategy (1,2,2,1). Upon hearing noise the receiver would take action A if $\gamma\geq\frac{1-qx}{(1-q)(1-x)}$ and $\varnothing$ otherwise. If $a(noise)=A$ the sender

has an incentive to deviate from the more costly (1,2,2,1) to the less costly (1,1,2,1). In the second case, when $a(noise) = \varnothing$, the sender has an incentive to deviate from the costly (1,2,2,1) to the less costly (1,2,1,1). Both deviations bring benefit c in either $(H, 2)$ or $(L, 1)$. Thus, (1,2,2,1) is not used in any equilibrium.

Notice that the analysis above could be also performed taking a purely interim point of view, i.e. analysing just the actual choice in critical states $(H, 2)$ and $(L, 1)$. This approach would be used to examine mixed strategies. Since the choice is made after the state is realised, the choices of m_{H2} and m_{L1} are interdependent only through the beliefs they induce in the equilibrium. The mixed strategy could be arbitrary $\big(1,(p,1-p),(1-r,r),1\big)$. In any mixed equilibrium in which at least one of p,r is interior, the receiver must be indifferent between choosing A and $\varnothing$, therefore p,r must satisfy:

$$\gamma = \frac{qr(1-x)+(1-q)}{(1-q)(1-x+px)}. \tag{1}$$

The receiver's response is $(b, 1-b)$ where $b = P\big(a(noise) = A\big)$.

Assume the receiver plays according to strategy $(b, 1-b)$ with $b \in (0, 1)$. Consider the state $(H, 2)$ and the sender's choice of $(p, 1-p)$ that costs $c(1-p)$. Notice that the sender's payoff is linear in p.

$$E\big(\text{payoff in } (H, 2)\big) = \big(-x(1-b)+c\big)p + \alpha(1-q)\big(x+(1-x)b-c\big) \tag{2}$$

If $b = 1-\frac{c}{x}$ then the sender's choice of p could be arbitrary, as the payoff is constant in p. If, $b < 1-\frac{c}{x}$ then the sender finds it optimal to choose $p = 0$ and if $b > 1-\frac{c}{x}$ then the optimal choice is $p = 1$.

Similarly:

$$E\big(\text{payoff in } (L, 1)\big) = \big((1-x)b-c\big)r \tag{3}$$

Generically, for a given pair (c,x), it cannot simultaneously hold that $b = 1-\frac{c}{x}$ and $b = \frac{c}{1-x}$ as long as $c \neq x(1-x)$. Therefore, at most one of the formulas (2) and (3) can be independent of p or r and allow for an interior choice of the parameter. Therefore mixing would be performed only in one of the critical states $(H, 2)$ and $(L, 1)$. In the other state, the incentives would drive the receiver to choose a corner solution from a set $\{0,1\}$. This is quite clear if we observe that the sender's decision is indeed a linear programming problem.

The first type of mixed equilibrium is of the form $\Big\{\big(1,(p,1-p),2,1\big),(b_1,1-b_1)\Big\}$ with $b_1 = 1-\frac{c}{x}$ and exists whenever $\frac{1-qx}{1-q} \le \gamma \le \frac{1-qx}{(1-q)(1-x)}$. Notice that for a small c, the probability of the receiver taking action upon hearing noise is close to 1, therefore this equilibrium is relatively noisy. We shall call it a semi-noisy mixed equilibrium.

The second type of mixed equilibrium is of the form $\{(1,2,(1-r,r),1),(b_2,1-b_2)\}$ with $b_2=\frac{c}{1-x}$ and exists whenever $\frac{1}{(1-x)}\le\gamma\le\frac{1-qx}{(1-q)(1-x)}$. Whenever c is small, b_2 is close to zero. Therefore this equilibrium would be labelled as a semi-informative mixed equilibrium.

In any mixed equilibrium, condition (1) must be satisfied, thus the mixed equilibria can only be sustained within some subset of the (x,γ) space.

In the unlikely case of $c=x(1-x)$, the equilibrium is $\{(1,(p,1-p),(1-r,r),1),(x,1-x)\}$. In this equilibrium, the probability of the receiver accepting action A is exactly equal to his competence.

Proof of Proposition 1

All Bayesian equilibria are characterised in Proposition A above. I will therefore focus only on the selection of the sender-best equilibria across the three possibilities whenever the equilibria are multiple.

Notice that the sender's payoff from an arbitrary pure or mixed strategy of the general form $\{(1,(p,1-p),(1-r,r),1),(b,1-b)\}$, is:

$$Eu^S(\text{eq. profile})=\alpha q+\alpha(1-q)\left[(1-p)\left(x+(1-x)b\right)+pb\right]+$$
$$+(1-\alpha)qr(1-x)b+(1-\alpha)(1-q)b-c\left(\alpha(1-q)(1-p)+r(1-\alpha)q\right).$$

The mixed strategy payoff is quite easy to derive. For the mixed equilibria, recall that by the definition of equilibrium b, the payoff must be independent of p in a semi-noisy equilibrium and of r in the semi-informative equilibrium.

$$Eu^S(\text{info eq.})=\alpha q+\alpha(1-q)(x-c),$$
$$Eu^S(\text{noisy eq.})=\alpha+(1-\alpha)\left((1-qx)-cq\right),$$
$$Eu^S(\text{semi-info eq.})=\alpha q+\alpha(1-q)\left[x+(1-x)b-c\right]+(1-\alpha)(1-q)b,$$
$$Eu^S(\text{semi-noisy eq.})=\alpha q+\alpha(1-q)\left[x+(1-x)b-c\right]+$$
$$+(1-\alpha)\left[q\left((1-x)b-c\right)+(1-q)b\right].$$

It is clear that $Eu^S(\text{noisy eq.})>Eu^S(\text{info eq.})$as $x-c<x+c<1$ for $c<\min(x, 1-x)$. Notice also that the mixed profiles are increasing in b, therefore:

$$Eu^S(\text{semi-noisy eq.})<\alpha q+\alpha(1-q)(1-c)+(1-\alpha)(1-qx-qc)<Eu^S(\text{noisy eq})$$
$$Eu^S(\text{semi-info eq.})<\alpha q+\alpha(1-q)(1-c)+(1-\alpha)(1-q)<Eu^S(\text{noisy eq})$$

Therefore $Eu^S(\text{noisy eq.})$ dominates all other payoffs.