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Aging, Migration and Monetary Policy in Poland*

Wpływ procesu starzenia i migracji na polską politykę pieniężną

Abstract

Poland faces a particularly sharp demographic transition. The old-age dependency ratio is expected to increase from just above 20% in 2000 to over 60% in 2050. At the same time, the country has recently witnessed a huge wave of immigration, mostly from Ukraine. In this paper, we investigate how aging and migration will affect the Polish economy and what consequences these adjustments have for monetary policy. Using a general equilibrium model with life-cycle considerations, we show that the decline in the natural rate of interest (NRI) due to demographic processes is substantial, amounting to around 1.5 percentage points, albeit spread over a period of 40 years. The impact of migration flows is relatively small and cannot significantly alleviate the downward pressure on the NRI induced by populating aging. If the central bank is slow in learning about the declining NRI, an extended period of inflation running below the target is likely. In this case, the probability of hitting the zero lower bound (ZLB) becomes a major constraint on monetary policy, while it could remain under control if the central bank uses demographic trends to update the NRI estimates in real time.

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Streszczenie

Polska stoi w obliczu szczególnie ostrych przemian demograficznych. Oczekuje się, że wskaźnik obciążenia demograficznego wzrośnie z nieco powyżej 20% w 2000 r. do ponad 60% w 2050 r. Polska doświadczyła ostatnio także ogromnej fali imigracji, głównie z Ukrainy. W niniejszym artykule zbadano, jak starzenie się i migracje wpłyną na polską gospodarkę i jakie konsekwencje mają te dostosowania dla polityki pieniężnej. Stosując model równowagi ogólnej z uwzględnieniem cyklu życia, pokazano, że spadek naturalnej stopy procentowej (NSP) spowodowany procesami demograficznymi jest znaczny i wynosi ponad 1,5 punktu procentowego, choć rozłożony jest na 40 lat. Wpływ przepływów migracyjnych jest stosunkowo niewielki i nie może znacząco złagodzić presji na spadek wskaźnika NSP wywołany starzeniem się populacji. Jeśli bank centralny dowiaduje się o malejącym wskaźniku NSP z opóźnieniem, prawdopodobny jest dłuższy okres utrzymywania się

Słowa kluczowe:
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inflacji poniżej celu. W tym przypadku prawdopodobieństwo osiągnięcia zerowej dolnej granicy dla stóp procentowych staje się głównym ograniczeniem polityki pieniężnej, podczas gdy może pozostać pod kontrolą, jeśli bank centralny wykorzysta trendy demograficzne do aktualizacji bieżących szacunków NSP.

Introduction

Poland faces a sharp demographic transition. As evidenced in Figure 1, the country is currently undergoing a rapid drop in the number of people entering the working-age phase of life and future fertility rates are projected to remain persistently low. Moreover, mortality rates are consistently falling and the probability of reaching the retirement age is expected to increase from 75% in 2000 to almost 95% around 2100 (Figure 2).[1] These two forces reinforce each other in leading to a rapid increase in the old-age dependency ratio (Figure 3), which is projected to reach 80% by 2060. To make the situation even more interesting, huge migration flows have affected the country recently. Between 2014 and 2020 an estimated 1 million migrants, mainly from Ukraine, joined the labour force.

Several consequences of demographics have been discussed in the literature so far, including its impact on potential growth, pension system sustainability, structure and the volume of fiscal expenditures and housing markets. Recent findings have also pointed to potential consequences for monetary policy, which are at the centre of our interest in this paper. Given the particularly sharp demographic transition and large migration flows, we intend to check how strong the consequences can be in Poland.

Economic theory predicts that a decrease in the population growth rate should lead to a drop in the natural rate of interest (NRI) through higher capital per worker. Moreover, an increase in longevity lengthens the planning horizon of households, inducing them to save more, and thus exerting further downward pressure on the NRI. Longer living households, dependent largely on accumulated wealth and asset income during their retirement, may prefer lower and more stable inflation rates, influencing politicians' and central bankers' preferences towards the inflation-output trade-off – see **Bullard et al.** [**2012**] and **Vlandas** [**2016**], in contrast with **Juselius and Takats** [**2015**]. Last but not least, for given inflation targets, lower NRIs translate into lower nominal interest rates, leaving less space for conventional monetary policy during slowdowns in economic activity and thus increasing the risk of hitting the zero lower bound (ZLB) constraint on the nominal interest rate.

We ask the following questions: What is the quantitative impact of aging and migrations on the Polish economy, and in particular on monetary policy? Should the monetary authorities be concerned about the scale and speed of the decline in the NRI? In particular, what happens if Narodowy Bank Polski observes it with a lag? How do the demographic processes and migrations affect the probability of hitting the ZLB? What is the role of economic openness, and in particular the concurrent demographic transition in the euro area?

The impact of demographics on monetary policy has been tackled in a limited number of papers to date. **Kara and von Thadden** [**2016**] calibrate a Blanchard-Yaari overlapping generations model to the euro area and project a decrease in the natural interest rate for the euro area from 3.9% in 2008 to 3.0% in 2030. They conclude that such adjustments are not important within the horizon that is relevant for monetary policy and do not call for an adjustment in its conduct, consistent with the outlook of some central bankers, e.g. **Bean** [**2004**]. **Carvalho et al.** [**2016**] calibrate a similar model to the average of several developed countries and simulate a more significant decline of the equilibrium interest rate (1.5 percentage points between 1990 and 2014). In contrast to **Kara and von Thadden** [**2016**], they conclude that low and declining real interest rates carry important challenges for monetary authorities.

1 These estimates do not yet take into account the effects of the COVID-19 pandemic, which can significantly affect the described projections if the virus is not successfully contained with vaccines.

Bielecki et al. [2022] construct a closed-economy model of the euro area and show that demographic forces can have a long-lasting impact on the NRI and generate a small but persistent deflationary bias. Regarding Poland, Bielecki et al. [2016] and Acedański and Włodarczyk [2018] use calibrated life-cycle models and show that aging will lead to a decline in the NRI. However, these two papers are focused on the implications of aging for the pension system and tackle neither monetary policy nor migration.

We offer the following answers to the questions enumerated above. First of all, we find that the impact of aging on the Polish economy will be substantial from the monetary policy perspective. In particular, given the currently available demographic projections, the equilibrium interest rate in Poland is projected to decline by around 1.5 percentage points between 2010 and 2050. International financial flows can reduce the impact of aging on the NRI only to a limited extent, even if demographic developments abroad are less dramatic. Increased immigration can somewhat alleviate the downward pressure on the NRI, but the size of this compensating impact is not large for a reasonable magnitude of population inflows. Observing the declining NRI by the central bank in real time makes the process relatively harmless for price stability. However, if the monetary authority learns about the impact of demographic processes on the NRI only slowly, a prolonged period of below-target inflation follows. This deflationary bias may be sizable and amount to almost 1 percentage point during the transition process. According to our simulations, demographic developments are expected to substantially increase the ZLB probability in Poland. This is particularly pronounced under the NRI learning scenario.

The paper is organised as follows. The next section presents the model used in our analysis. Section 3 documents the construction of demographic input data and our calibration strategy. In Section 4 we discuss our simulation results, in particular pertaining to the natural interest rate, the zero lower bound, the role of central bank learning, and the extent of open-economy spillovers. This section also contains a discussion of alternative simulations accounting for the role of migration and changes in fertility rates. Section 5 concludes.

Model

We construct a small open economy model with overlapping generations as well as real and nominal frictions to analyse the implications of demographic transition for monetary policy. The model economy is populated by households facing age- and time-dependent mortality risk, five types of firms, and a monetary policy authority. The model also has a closed economy variant, where we shut down international trade and cross-border financial flows.

Households

Optimisation problem

Each household consists of a single agent who appears in our model at age 20 and is assigned age index $j = 1$. Agents can live up to 99 years ($j = J = 80$), at each year subject to age- and time-dependent mortality risk $\omega_{j,t}$. Hence, at each time period the model economy is populated by 80 cohorts of overlapping generations, with the size of cohort j denoted by $N_{j,t}$.

A j-aged household maximises its expected remaining lifetime utility that depends on consumption $c_{j,t}$ and hours worked $h_{j,t}$ according to

$$U_{j,t} = \mathbb{E}_t \sum_{i=0}^{J-j} \beta^i \frac{N_{j+i,t+i}}{N_{j,t}} \exp\left\{\varepsilon_{u,t}\right\} \left[\ln c_{j+i,t+i} - \phi_{j+i} \frac{h_{j+i,t+i}^{1+\varphi}}{1+\varphi} \right] \tag{1}$$

where $\varepsilon_{u,t}$ is a preference shock, β denotes the discount factor, the ratio $N_{j+i,t+i} / N_{j,t} \equiv (1-\omega_{j,t}) \ldots (1-\omega_{j+i-1,t+i-1})$ represents the probability of surviving for at least i more years, ϕ_j is the age-dependent labour disutility parameter, and φ is the inverse of the Frisch elasticity of labour supply.

All households face the following budget constraint

$$P_t c_{j,t} + A_{j,t} = W_t z_j h_{j,t} + R_t^a A_{j-1,t-1} + Beq_t \tag{2}$$

where P_t denotes the aggregate price level, $A_{j,t}$ stands for the nominal stock of assets that are managed by investment funds and yield the gross nominal rate of return R_t^a, W_t is the nominal wage per effective hour, while z_j represents age-specific labour productivity. Our model features exogenous retirement upon reaching age 64 ($j = JR = 45$) and hence we set $z_j = 0$ for $j \geq JR$. Finally, since agents can die before reaching their maximum age, they leave unintentional bequests, which are redistributed equally across all living agents in the form of lump-sum transfers Beq_t.

Demography and aggregation

Demographic processes are governed by changes in the size of initial young cohorts $N_{1,t}$ and mortality risk $\omega_{j,t}$, both of which are assumed to be exogenous. The total number of agents alive N_t and the population growth rate n_t are given by

$$N_t = \sum_{j=1}^{J} N_{j,t} \quad \text{and} \quad n_t = \frac{N_t}{N_{t-1}} - 1 \tag{3}$$

where the number of agents evolves according to

$$N_{j,t} = (1 - \omega_{j-1,t-1}) N_{j-1,t-1} \tag{4}$$

To better capture the impact of expected demographic changes, we allow population growth in the steady state to differ from zero. As then the number of agents within each cohort becomes nonstationary, it is useful to define the size of cohorts relative to that of the youngest one

$$N_{j,t}^{rel} = \frac{N_{j,t}}{N_{1,t}} \tag{5}$$

and the growth rate of initial young $n_{1,t}$

$$n_{1,t} = \frac{N_{1,t}}{N_{1,t-1}} - 1 \tag{6}$$

This allows us to rewrite equations (4) and (3) in relative terms

$$N_t^{rel} = \sum_{j=1}^{J} N_{j,t}^{rel} \quad \text{and} \quad n_t = \frac{N_t^{rel}}{N_{t-1}^{rel}} \left(1 + n_{1,t}\right) - 1 \tag{7}$$

$$N_{j,t}^{rel} = \frac{\left(1 - \omega_{j-1,t-1}\right) N_{j-1,t-1}^{rel}}{1 + n_{1,t}} \tag{8}$$

Then the allocations of all households can be aggregated to the following per capita variables

$$c_t = \sum_{j=1}^{J} \frac{N_{j,t} c_{j,t}}{N_t} = \sum_{j=1}^{J} \frac{N_{j,t}^{rel} c_{j,t}}{N_t^{rel}} \tag{9}$$

$$h_t = \sum_{j=1}^{J} \frac{N_{j,t} z_j h_{j,t}}{N_t} = \sum_{j=1}^{J} \frac{N_{j,t}^{rel} z_j h_{j,t}}{N_t^{rel}} \tag{10}$$

$$A_t = \sum_{j=1}^{J} \frac{N_{j,t} A_{j,t}}{N_{t+1}} = \sum_{j=1}^{J} \frac{N_{j,t}^{rel} A_{j,t}}{N_{t+1}^{rel}\left(1+n_{1,t+1}\right)} \tag{11}$$

$$Beq_t = \sum_{j=1}^{J} \frac{\left(N_{j-1,t-1} - N_{j,t}\right) R_t^a A_{j-1,t-1}}{N_t} = \sum_{j=1}^{J} \frac{\left[N_{j-1,t-1}^{rel} - \left(1+n_{1,t}\right) N_{j,t}^{rel}\right] R_t^a A_{j-1,t-1}}{\left(1+n_{1,t}\right) N_t^{rel}} \tag{12}$$

Firms

There are five types of firms in the economy: investment funds, final goods producers, capital producers, intermediate goods producers, and importers. The first three groups are perfectly competitive, while intermediate goods producers and importers operate in a monopolistically competitive environment. All firms are risk-neutral, i.e. they maximise the expected present value of future profits, discounting them using nominal interest rates.

Investment funds

Investment funds use household savings to buy and manage a portfolio of assets, transferring every period the earned gross return back to households. The portfolio consists of domestic and foreign bonds, domestic physical capital and shares of intermediate goods producers. Investment funds maximise the expected present value of gross return

$$\mathbb{E}_t \frac{1}{R_t} \left[\begin{array}{c} \left[R_{t+1}^k + (1-\delta) Q_{t+1}\right] k_t + R_t B_t + S_{t+1} \mathbf{\Gamma}_t R_t^* B_t^* \\ + \int_0^1 \left[\left(1+n_{t+1}\right) P_{t+1}^d(i) + F_{t+1}(i)\right] d_t(i) di \end{array} \right] \tag{13}$$

where R_{t+1}^k is the nominal rental rate on capital, while Q_t is the nominal price of a unit of capital k_t, which depreciates at rate δ. R_t and R_t^* denote the gross nominal interest rates on domestic bonds B_t and foreign bonds B_t^* respectively. The nominal exchange rate is denoted by S_t, and $\mathbf{\Gamma}_t$ is an international risk premium term that depends on the foreign debt-to-GDP ratio and is defined as follows:

$$\mathbf{\Gamma}_t = \gamma \left(\exp\left\{ -\frac{S_t B_t^*}{P_{H,t} gdp_t} \right\} - 1 \right) + \exp\left\{ \varepsilon_{\Gamma,t} \right\} \tag{14}$$

where gdp_t denotes the real aggregate output to be defined later, $P_{H,t}$ is the producer price index for domestic production, $\varepsilon_{\Gamma,t}$ is a risk premium (exchange rate) shock, and γ is a risk premium parameter. Finally, $d_t(i)$ stands for the number of shares issued by intermediate goods producing firm i that are traded at price $P_t^d(i)$ and yield dividends $F_t(i)$.[2]

The balance sheet of investment funds can be written as

$$A_t = Q_t k_t + B_t + S_t B_t^* + \int_0^1 P_t^d(i) d_t(i) di \tag{15}$$

Since we assume that all revenue from asset management is transferred back to households, the ex-post rate of return on assets is given by

$$R_t^a A_{t-1} = \left[R_t^k + (1-\delta) Q_t\right] k_{t-1} + R_{t-1} B_{t-1} + S_t \mathbf{\Gamma}_{t-1} R_{t-1}^* B_{t-1}^* \tag{16}$$
$$+ \int_0^1 \left[\left(1+n_t\right) P_t^d(i) + F_t(i)\right] d_{t-1}(i) di$$

2 We multiply the proceeds from selling shares by the population growth rate to ensure that the mass of firms is linked to the mass of households. This captures the fact that the number of firms in the economy depends on the size of the population.

Final goods producers

Final goods producers serving the domestic market purchase domestic and foreign composites of goods, and produce a homogeneous final good according to the following CES aggregator:

$$y_t = \left[\eta^{\frac{1}{\phi}} y_{H,t}^{\frac{\phi-1}{\phi}} + (1-\eta)^{\frac{1}{\phi}} y_{F,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{17}$$

where η reflects the home bias and ϕ is the elasticity of substitution between home-made and imported composites, which are in turn made of intermediate inputs according to

$$y_{H,t} = \left[\int_0^1 y_{H,t}(i)^{\frac{1}{\mu}} di \right]^{\mu} \tag{18}$$

$$y_{F,t} = \left[\int_0^1 y_{F,t}(i)^{\frac{1}{\mu}} di \right]^{\mu} \tag{19}$$

where μ is the gross markup that depends on the elasticity of substitution between intermediate varieties. The associated price indices are

$$P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{\frac{1}{1-\mu}} di \right]^{1-\mu} \tag{20}$$

$$P_{F,t} = \left[\int_0^1 P_{F,t}(i)^{\frac{1}{1-\mu}} di \right]^{1-\mu} \tag{21}$$

Similarly, producers of final export goods aggregate domestically produced intermediate inputs according to

$$y_{H,t}^{*} = \left[\int_0^1 y_{H,t}^{*}(i)^{\frac{1}{\mu}} di \right]^{\mu} \tag{22}$$

and sell them abroad at a foreign currency denominated price $P_{H,t}^{*}$ defined as

$$P_{H,t}^{*} = \left[\int_0^1 P_{H,t}^{*}(i)^{\frac{1}{1-\mu}} di \right]^{1-\mu} \tag{23}$$

facing the following demand function:

$$y_{H,t}^{*} = \left(\frac{P_{H,t}^{*}}{P_t^{*}} \right)^{-\phi^{*}} y_t^{*} \tag{24}$$

where y_t^{*} is foreign output, P_t^{*} denotes the foreign aggregate price level, and ϕ^{*} is the price elasticity of export demand.

Capital producers

Capital producers buy undepreciated capital and combine it with investment goods inv_t subject to investment adjustment costs. The new capital created in this way is resold during the same period. The aggregate law of motion for capital is then

$$\left(1+n_{t+1}\right)k_t = (1-\delta)k_{t-1} + \left[1 - S_k\left(\frac{inv_t}{inv_{t-1}}\right)\right]inv_t \tag{25}$$

where the investment adjustment cost function is of the following form:

$$S_k\left(\frac{inv_t}{inv_{t-1}}\right) = \frac{S_1}{2}\left[\frac{inv_t}{inv_{t-1}} - \left(1+n_t\right)\right]^2 \tag{26}$$

for $S_1 \geq 0$, so that the cost is zero in the steady state.

Importers

Importers of foreign intermediate varieties purchase each of them at price P_t^* expressed in foreign currency and then resell them to domestic final goods producers at prices denominated in local currency. They face demand for their products consistent with aggregation (19) and are subject to the Calvo friction, so that each importer in every period faces constant probability θ_F of not being able to reoptimise, in which case the price is fully indexed to domestic steady-state inflation. We assume that importers are owned by foreign agents so their profits are transferred abroad.

Intermediate goods producers

Intermediate goods producers supply domestic and foreign markets with output produced according to the Cobb-Douglas production function:

$$y_{H,t}(i) + y_{H,t}^*(i) = \varepsilon_{z,t} k_t(i)^{\alpha} h_t(i)^{1-\alpha} \tag{27}$$

where $\varepsilon_{z,t}$ is a productivity shock. They face schedules implied by aggregation (18) and (22), and set their prices separately for the domestic and foreign markets in the currency of the destination country. Their pricing decisions are subject to the Calvo friction, with θ_H and θ_H^* representing the probability of not receiving the reoptimisation signal, in which case prices are fully indexed to local steady-state inflation.

Monetary authority

The monetary authority sets the nominal interest rate according to the standard Taylor-like rule that takes into account the zero lower bound constraint:

$$R_t = \begin{cases} R_t^{cb}, & R_t^{cb} > 1 \\ 1, & R_t^{cb} \leq 1 \end{cases} \tag{28}$$

$$R_t^{cb} = R_{t-1}^{\gamma_R}\left[\tilde{R}_t^e\left(\frac{\pi_t}{\pi_{ss}}\right)^{\gamma_\pi}\left(gdp_t / gdp_{t-1}\right)^{\gamma_y}\right]^{1-\gamma_R} \exp\left\{\varepsilon_{R,t}\right\} \tag{29}$$

where gdp_t is aggregate output that we define below, $\pi_t \equiv P_t / P_{t-1}$ is the gross rate of inflation, π_{ss} is the inflation target, and $\varepsilon_{R,t}$ is a monetary policy shock. The coefficients γ_R, γ_π and γ_y control the degree of interest rate smoothing, the response to deviations of inflation from the target, and the response to deviation of output growth from its potential, respectively.

The variable $\tilde{R}_t^e$ describes the central bank perception of the natural nominal interest rate $\tilde{R}_t \equiv \pi \tilde{r}_t$, where $\tilde{r}_t$ denotes the natural real interest rate (NRI). This natural level is defined as a hypothetical value of the real interest rate that would be observed under fully flexible prices (i.e. $\theta_H = \theta_H^* = \theta_F = 0$) and absent stochastic shocks, but with demographic changes taken into account. Unless indicated otherwise, the perception of the monetary authority is assumed to be consistent with current economic developments, i.e. $\tilde{R}_t^e = \tilde{R}_t$. Alternatively, we assume that these perceptions are linked to the actual value with a constant gain learning process as in Evans and Honkapohja (2001):

$$\tilde{R}_t^e = \tilde{R}_{t-1}^e + \lambda\left(\pi \tilde{r}_{t-1} - \tilde{R}_{t-1}^e\right) \tag{30}$$

so that the central bank observes the true NRI only with a lag, and updates its current guess with a fraction λ of the previous forecast error.[3] This way of formulating the feedback rule ensures the long-run consistency of the equilibrium with central bank targets, but also allows us to model imperfect knowledge of the monetary authority.

Market clearing conditions

The model is closed with a standard set of market clearing conditions. The equilibrium on the final goods market implies

$$y_t = c_t + inv_t \tag{31}$$

The market clearing conditions for capital can be written as

$$\int_0^1 k_t(i)di = k_{t-1} \tag{32}$$

while that for labour is

$$\int_0^1 h_t\left(i\right)di = h_t \tag{33}$$

This allows us to write the aggregate production function as

$$gdp_t \equiv y_{H,t}\Delta_{H,t} + y_{H,t}^*\Delta_{H,t}^* = \varepsilon_{z,t} k_{t-1}^{\alpha} h_t^{1-\alpha} \tag{34}$$

where $\Delta_{H,t} \equiv \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{\frac{\mu}{1-\mu}} di$ and $\Delta_{H,t}^* \equiv \int_0^1 \left(\frac{P_{H,t}^*(i)}{P_{H,t}^*}\right)^{\frac{\mu}{1-\mu}} di$ measure the price dispersion in domestic and export markets.

Since domestic bonds are traded only between (identical) investment funds, we have

$$B_t = 0 \tag{35}$$

Without losing generality, the number of shares issued by each intermediate goods producing firms can be normalised to unity, which gives

$$d_t(i) = 1 \tag{36}$$

3 The resulting estimate of the unobserved quantity is equivalent to an exponentially weighted average of all its all past values.

Finally, foreign bond holdings, i.e. the economy's net foreign assets position, evolves according to

$$\left(1+n_{t+1}\right)S_t B_t^* = S_t \Gamma_{t-1} R_{t-1}^* B_{t-1}^* + S_t P_{H,t}^* y_{H,t}^* - P_{F,t} y_{F,t} \tag{37}$$

Exogenous shocks

The model economy is driven by the following exogenous variables. Demographic processes are characterised by the the growth rate of initial young $n_{1,t}$ and age-specific mortality risk $\omega_{j,t}$, all of which are treated as deterministic, i.e. known to all optimising agents. Additionally, the economy is hit by stochastic shocks to productivity $\varepsilon_{z,t}$, household preferences $\varepsilon_{u,t}$, risk in the international financial market $\varepsilon_{\Gamma,t}$, monetary policy $\varepsilon_{m,t}$, as well as foreign output y_t^*, inflation π_t^*, and interest rate R_t^*. The three foreign variables may also have a deterministic component that reflects the consequences of demographic processes abroad, which we obtain by simulating the closed economy variant of the model described above. These processes are exogenous to the domestic economy due to our small open economy assumption. Stochastic fluctuations in the foreign variables are described by a VAR (1) process. Monetary policy shocks are assumed to be white noise. All other stochastic shocks are modeled as independent AR (1) processes.

Calibration, data sources and solution method

We calibrate the model such that the home country resembles the Polish economy and the rest of the world is represented by the euro area. The chosen values of structural parameters are reported in Table 1. They are based on the previous literature, complemented with econometric estimates performed outside the model and a moment matching exercise. Some of the parameters are calibrated such that the means of key variables implied by our demographic transition scenario during the 2003–2012 period coincide with actual Polish data covering this period. This targeted 10-year data sample can be considered as one during which the real interest rate in Poland was close to its equilibrium level.[4] Throughout, the time frequency is annual.

We set the discount factor to 0.977, which allows us to match the average real interest rate of 2.1% observed in Poland from 2003 to 2012. Our calibration of the Frisch elasticity of labour supply of 0.25 is consistent with estimates from the microeconomic literature (see e.g. **Peterman** [**2016**]. The age profiles of productivity z_j are taken from **Kolasa** [**2017**]. Since these profiles are expressed in per household and not per hour terms, we adjust the age specific labour disutility parameters ϕ_j so that working hours in the initial steady state (i.e. before the demographic transition) are constant over the life cycle. The resulting age profiles are presented in Figures 4 and 5.

Physical capital is assumed to depreciate at a standard rate of 10% annually. The capital elasticity of output is calibrated at 0.25, which is consistent with labour income share estimates for Poland used by **Gradzewicz et al.** [**2018**], corrected for the effect of markups estimated by **Gradzewicz and Mućk** [**2019**]. It also ensures that the investment rate is close to the average values observed in the Polish economy. The investment adjustment cost function curvature is set to 4 and taken from an estimated DSGE model for Poland documented in **Brzoza-Brzezina et al.** [**2016**]. Since our model does not feature nominal wage rigidities, we calibrate the quarterly Calvo probability faced by all intermediate goods producing firms at a somewhat high value of 0.9.

The home bias parameter in the final goods basket is set to a value that ensures a share of imports in domestic demand of around 25%, which corresponds to the mean observed in the data, corrected for the import content of exports estimated for Poland by the OECD at 30%. The elasticity of substitution between home and foreign goods is calibrated at a standard value of 1.5. The elasticity of risk premium on international financial

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4 Over this period inflation fluctuated around the central bank target of 2.5%, suggesting an approximately neutral stance of monetary policy. Before 2003 Poland underwent a long-lasting disinflation, and after 2012 it experienced a long period of very low (even negative) inflation.

markets is set to 0.013, which helps match the average international investment position of –55% observed in the Polish economy over the 2003–2012 period.

The parametrisation of the monetary policy feedback rule is based on econometric estimates using an extended sample of detrended quarterly data covering the 1999–2016 period (i.e. since the introduction of inflation targeting in Poland), converted to annual frequency. Whenever we allow the central bank to learn about changes in the natural interest rate only gradually, we calibrate the gain parameter at 0.08. This value follows the empirical literature documenting the observed speed of learning [**Branch, Evans, 2006**; **Malmendier, Nagel, 2016**; **Milani, 2011**] and implies an annual learning rate of 8%.

In our simulations, we treat Poland as a small open economy that is affected by demographic processes and usual business cycle fluctuations in the euro area, manifesting themselves from the Polish perspective by exogenous movements in foreign output, inflation and the nominal interest rate. The responses of these three variables to a demographic transition are generated using the closed economy version of the model described in the previous section, with some of the parameters recalibrated to fit the euro area data – see the bottom panel of Table 1. This concerns the parametrisation of the monetary policy rule and the Calvo probability, which are taken from **Christoffel et al.** [**2008**], as well as the discount factor, that we reset to 0.992 to match the average real interest rate of 1.2%, observed in the euro area over the 1999–2008 period, i.e. since its creation until the financial crisis.[5] For lack of appropriate estimates of age-specific productivity for the euro area, we use calculations for the United States from **Gourinchas and Parker** [**2002**] as existing evidence points at relatively small differences in the life cycle income profiles between developed economies [**OECD, 1998**].

Apart from the demographic transition scenario, which is purely deterministic, we use our model in a stochastic context. For this purpose, fluctuations in foreign output, inflation and the interest rate are modelled as a first-order VAR, whose parameters we estimate using detrended estimates of euro-area aggregates from the ECB Area Wide Model database that cover the 1970–2012 period. Table 2 reports the properties of shocks driving the model small open economy. Four of them, namely the monetary shock as well as the three shocks describing foreign variables, are estimated outside the model as described above. The remaining three are determined in a moment matching exercise, in which we use detrended data on Polish real GDP, HICP inflation, the short-term nominal interest rate, and the CPI-based real effective exchange rate over the 1999–2016 period. More specifically, we minimise the distance between the model-based standard deviations, first-order autocorrelation and correlation with output and their respective data counterparts. All model-based moments are calculated using the first-order Taylor series approximation of the model's equilibrium conditions around the point defined as the mean of the state variables in our demographic transition scenario over the 2003–2012 period. We use the same weights for all matched moments, except the volatility and inertia of the nominal interest rate, for which we assign much higher weights so that the match is exact. It is important to achieve a perfect match for this variable as one of our goals is to evaluate the impact of demographic transition on the probability of hitting the ZLB. As it can be seen from Table 3, the achieved fit is very good for most other moments.

Our demographic scenarios use past data and projections on mortality rates and the sizes of the cohorts of 20-year olds. We rely on the United Nations World Population Prospects 2019, covering the periods 1950–2020 (actual data) and 2020–2100 (projections). Since the data are available for individual countries in five-year intervals and for broad five-year age groups, we first interpolate them to annual frequency using cubic splines, and then use the population-weighted averages to construct the aggregated demographic data for the euro area.[6] The resulting mortality rates and rates of growth of the population of 20-year-olds are smoothed

5 As in the case of Poland, this is the longest period when the interest rate can be considered close to its equilibrium value. Inflation was relatively stable in this period. After 2008 the euro area faced a prolonged crisis that pushed interest rates down for cyclical rather than structural reasons.

6 To ensure the validity of this approach, we cross-check the constructed data with comparable data from Eurostat that are available at the annual frequency but cover a shorter time span. We find no significant differences between the actual and constructed population structures.

using the Hodrick-Prescott filter (with a smoothing parameter of 1600) to avoid spurious jumps in the demographic input data produced by data revisions or by splicing data from not fully compatible sources. At the end of our projection horizon, we assume that mortality rates stabilise while the rate of change of the 20-year-old cohort size stays at the level projected for 2080. To ensure that the model predictions for the years that we focus on (i.e. 2000 and beyond) are not contaminated by frontloading effects and initial conditions, we start our deterministic simulations in 1900. To that end, we construct artificial population data for the 1900–1970 period and backcast the sizes of historical 20-year-old cohorts while holding the historical mortality rates from 1970 to closely approximate the actual population structure. Since, due to the presence of migration flows, we cannot exactly match the population structure in each year, we have chosen 2010 as our base year,[7] while the modelled population structures for other years are constructed using data on mortality rates and projected changes in the number of 20-year-olds over time.

We use the following methods to solve the model. The demographic transition scenarios presented in Sections 4.1, 4.2, 4.3 and 4.5 are obtained under the assumption of perfect foresight (deterministic simulations).[8] The stochastic simulations used in Section 4.4 are performed using first-order Taylor series approximations to the model's equilibrium conditions, taken independently at each point of a given perfect foresight scenario, and using appropriately defined news shocks to take into account the ZLB constraint.[9]

Effects of demographic change

In this section we seek to answer several important questions about the consequences of the demographic transition for monetary policy. As it is well known from the literature, population aging can have a sizable impact on savings and the equilibrium interest rate. We start by documenting this effect for Poland. Then we move to analysing various consequences for monetary policy: we compare the impact of the demographic change on inflation under an alternative assumption where the central bank notices the decline of the NRI only with a lag. Third, we assess how much the declining NRI raises the probability of hitting the zero lower bound on the nominal interest rates. This is done both for the case when the central bank observes the declining NRI in real time and when it gradually learns about its change. Fourth, we show how our results depend on the assumptions about the demographic transition in the euro area. Finally, we check how our main findings change if migration flows are taken into account.

Impact of demographic transition

We begin with describing the impact that the demographic transition exerts on the main macroeconomic variables. To this end we run a deterministic simulation, assuming that the demographic processes in the euro area and Poland, as described in Section 3, are known to all agents. Since our focus is on Poland, we do not discuss EA developments in detail. We note that the real interest rate in the EA declines by just over 1 percentage point between 2000 and 2030 and kindly refer readers interested in a more detailed discussion to our companion paper **Bielecki et al. [2020]**.

Figure 6 presents the demographic transition in Poland. The upper-left panel shows the old-age dependency ratio. The economy faces a sharp increase in this ratio, resulting from lower birth rates and higher life expectancy. Workers facing a longer expected time in retirement increase savings. This results in higher asset holdings and lowers the real interest rate. The decline is substantial though spread over time. Between 2000

7 This choice is dictated by the fact that significant migration flows from new EU member states including Poland to EU-15 countries occurred after the 2004–2007 rounds of enlargement. The stock of migrants stabilised shortly after the global financial crisis of 2008–2009.

8 For numerical reasons, we work with two model variants: closed economy for the euro area and small open economy for Poland. To generate each scenario, we first simulate the euro area model. Then, we use the obtained series of GDP, inflation and interest rate as exogenous foreign variables when simulating the model for Poland.

9 The simulations are run using the Dynare OBC package [**Holden, 2021**].

and 2050 the interest rate declines by approximately 1.5 percentage points. Other developments include a decline in the labour supply, an increasing real wage and consequently a higher capital-labour ratio chosen by firms. With less labour the economy produces fewer goods so ultimately GDP per capita and capital per capita are lower after the transition.

Given qualitatively similar demographic developments, it should not come as a surprise that the NRI declines in both the EA and Poland. However, two important differences should be mentioned. First, the demography in Poland is even worse than in the EA. As a consequence, the adjustment process is deeper. Second, Poland is a small open economy. This means that, from its perspective, savings can be imported or exported abroad. This is exactly what happens. A substantial part of the savings generated by the demographic transition is invested abroad. As a result, the country's foreign debt-to-GDP ratio declines significantly – by over 20 percentage points.

Consequences for inflation

In the past, equilibrium interest rates were frequently assumed to be constant (or at least stationary) over time. For instance, until the mid-2000 s many economists and central bankers placed NRIs in the United States, the UK and the euro area in the range of 2%–3% [Laubach, Williams, 2003; Holston et al., 2017]. With relatively rare exceptions, Taylor rules that were calibrated or estimated for these (and several other developed) countries assumed a constant intercept, and hence implied a constant NRI, see e.g. Taylor [1993]; Smets and Wouters [2003] for standard rules and Orphanides and Williams [2002]; Trehan and Wu [2007] for exceptions with time-varying NRI. Such an assumption seemed warranted by long periods of real interest rates that seemed stationary. Things changed somewhat after the global financial crisis erupted and NRIs apparently declined in most countries. However, even now it is far from evident whether the declining interest rates were a symptom of long-term trends, or of cyclical developments.

Given this experience, coupled with the intrinsically unobservable nature of the equilibrium interest rate, it seems plausible to assume that the long-term decline of the NRI that we expect to be happening due to demographic reasons, could be noticed by central banks only with a lag – see Orphanides and Williams [2002] for a discussion. This could clearly lead to an overly restrictive monetary policy with all its consequences. In this section we analyse such a scenario.

As explained in Section 2, our monetary policy rule was designed to account for a declining NRI. In the baseline calibration, we assume that the central bank observes the NRI in real time. Now we compare this result to the case when the monetary authority follows a learning process, as described by equation (30), so that every period the central bank's guesses of this unobservable variable are updated for fraction λ of the last period forecast error. The resulting expected NRI is equivalent to the exponentially weighted average of all its past values.

Figure 7 documents our findings by comparing the inflation rate under the baseline and learning assumptions. In the baseline scenario, inflation hovers around the central bank's target. Any small deviations result from the bank's inability to simultaneously neutralise the stickiness of domestic and imported goods prices. However, if the central bank fails to timely account for the demographic trends, monetary policy becomes much more restrictive and results in a deflationary bias. Even though the NRI drops at an average rate of only 0.04 percentage point per year, the permanent bias in monetary policy has a substantial impact on inflation. In particular, inflation remains permanently below the target and the gap in the analysed period is almost 1 percentage point at maximum. As the natural rate stabilises after 2050, the deviations in its perception stop being permanently biased and inflation returns to a level close to its target.[10]

[10] These results would be qualitatively unchanged if we employed a more sophisticated learning framework, such as Holston et al. [2017] – see the Online Appendix for Bielecki et al. [2022].

Consequences for hitting the zero lower bound

During the last decade interest rates in many countries have hit the zero lower bound (ZLB). Something that looked like a textbook curiosity has become a part of central bank reality. While the affected banks managed to develop alternative tools to at least partially overcome the consequences of the constraint (e.g. quantitative easing), it seems they still prefer to use the short-term interest rate as the main policy instrument. Keeping this in mind, we now check whether the lower NRI substantially raises the probability of hitting the ZLB. If this is the case, central banks might have to consider increasing their inflation targets to compensate for the declining NRI.

We proceed as follows. We run stochastic simulations with productivity, time preference, risk premium and foreign shocks calibrated as described in Section 3 for 100,000 periods. This exercise is repeated in the vicinity of every point on our deterministic path. The exercise is done three times. First, simulations are based on our baseline scenario and assume that the central bank knows the NRI. Second, we check what happens if the bank learns about this unobservable variable as described in Section 2.3. Since learning can result in a long period when the economy faces persistently low inflation, we expect the probability of hitting the ZLB to be higher at some points. Each time we approximate the probability of hitting the ZLB by calculating the frequency of periods during which the nominal interest rate is constrained by this limit.

Our findings are summarised in Figure 8. According to the baseline scenario, the probability of hitting the ZLB in the early 2000 s was relatively low (1% – 2%). This result is consistent with the findings of **Brzoza-Brzezina et al. [2016]** and is a consequence of the relatively high NRI and a high inflation target. However, as the equilibrium real rate declines, the probability increases to over 7% in 2050 and finally stabilises slightly below 5% in the new steady state. These values can hardly be seen as alarming, in particular when taking into account the probability that by 2050 the ZLB problem may be non-existent due to technological progress (e.g. if cash is not used).

Even more interesting and possibly disturbing are the results from the learning scenario. Not only does the probability increase to much higher levels (almost 15% in the 2040 s), but the numbers for the contemporaneous times are quite high. By 2020 the probability of hitting the ZLB exceeds 6%. These results show that a slowly, but permanently declining equilibrium interest rate, if not properly accounted for, can result in a serious deterioration of the quality of monetary policy.[11]

This finding stipulates our main conclusion for monetary policy. If Narodowy Bank Polski is aware of the consequences presented in this paper, the demographic transition is relatively harmless from its point of view. However, if the bank fails to timely account for the declining NRI, monetary policy may become suboptimal to a substantial degree – too contractionary and possibly leading to liquidity traps.[12]

The impact of foreign demography

Another interesting question is how demographic developments abroad modify the adjustment in Poland. In order to take a deeper look into the matter we run two additional experiments and compare them to our baseline results. First, we switch off the demographic transition in the euro area. Second, we assume that the demographic prospects in the euro area are the same as in Poland. These scenarios can be seen as two extremes, with our baseline placed somewhere in between.

The findings are presented in Figure 9. A small open economy can adapt to variations in the savings-investment balance not only via the interest rate, but also by exporting/ importing capital. This effect could already be observed when analysing the path of foreign debt in Section 4.1, where foreign indebtedness declines by over 20% of GDP in the studied period. This means that additional savings are partially exported (invested)

11 We do not discuss in detail the adverse consequences of hitting the ZLB. This has been done in many studies – see, e.g., **Gust et al. [2012]** and **Ireland [2011]** for the United States, and **Brzoza-Brzezina et al. [2016]** for Poland.

12 A formal, quantitative assessment of policy suboptimality is left for further research.

abroad. Clearly, the second adjustment channel dominates the adjustment in our case. Without demographic changes in the euro area, savings from Poland are exported to an even larger extent as they flow to where the return is not undermined by demographic developments. As a result, foreign debt declines much more than in the baseline scenario, with the economy ultimately achieving an NFA surplus. If the demographics in the EA behaves in the same way as in Poland, the return on assets declines similarly in both economies and, as a consequence, net international capital flows are limited and Poland's foreign debt is almost constant. As expected, our baseline scenario is placed between the two extremes.

The impact of foreign developments on the interest rate is much less pronounced. As can be expected, the demographic transition in the EA adds to the NRI decline in Poland, but the difference is not large.

The impact of migration and fertility

In this section, we quantitatively examine the impact of a recent wave of immigration to Poland, stemming mostly from Ukraine, other former Soviet economies and Southeast Asian countries, and the role of a potentially higher fertility rate.

In a relatively short period of time, starting around 2014, Poland has seen an influx of approximately 1 million migrants, mostly young individuals. Our baseline scenario incorporates past migration flows (prior to 2010) but does not incorporate this recent wave of immigration to Poland, which we now capture in an alternative scenario. We do it by decomposing the age distribution of migrants into two waves, approximating the distribution of the new wave of young migrants via a normal distribution over age – see Figure 10.[13] We assume that immigrants from the previous waves have already been accounted for in the domestic demographic data. To ensure consistency with our modelling framework, we assume that an immigrant who was e.g. 34 years old in 2018 arrived in Poland 14 years earlier at age 20. While this is not an entirely innocuous assumption, the fact that young workers are characterised by lower productivity and that migrants typically arrive with a relatively low level of assets regardless of age makes the error introduced by this way of constructing the immigration inflow less severe. We also assume that all of thus included migration is permanent, which additionally justifies our assumption on restricting our attention to young individuals as older cohorts are more likely to treat work abroad as an additional and transitory source of income that is mostly spent in the home country.

The implications of this alternative scenario are depicted in Figure 11. As expected, immigration increases the real interest rate during roughly the first three decades of our simulation. Quantitatively, however, this effect is not very large, never exceeding 0.1 percentage point. Since migrants are relatively young and start accumulating significant savings only after reaching middle age, the foreign debt of the Polish economy is initially higher than in the baseline scenario. It starts decreasing rapidly once the wave of migration becomes older. Overall, we can conclude that while immigration can somewhat alleviate the downward pressure on the natural interest rate, its magnitude would have to be very large to make a sizable impact on the evolution of this variable.

A permanently higher fertility rate can generate more profound effects, albeit significantly delayed in time. In the second alternative scenario, we make use of the high fertility variant of the United Nations World Population Prospects (UN WPP), which mechanically assumes that throughout the 2020–2100 period the total fertility rate is higher by 0.5 percentage points than in the baseline variant. As such a large increase in fertility rates seems to be unattainable even after applying a wide variety of pro-natalist policies, we can treat this scenario as a reasonable upper bound on the process. The results of this scenario are depicted in Figure 12. As the evolution of the demographic variables is assumed to be known well in advance by economic agents, the path of the real interest rate is already slightly higher even before 2040, although the effect is quite small, of

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[13] The data on the age structure of immigrants into Poland come from the Polish Social Insurance Fund (SIF) and encompass only immigrants who have been legally employed. The data has been adjusted upwards to reflect the SIF's estimate that as of the end of 2019 the total number of immigrants was slightly above 2 million.

the order of 0.1 percentage point. As additional newborns from 2020 onward will enter the workforce with a lag of two decades, appreciable changes can only be observed from 2040 onward, with the real interest rate up to 0.4 percentage points higher than in the baseline. Since the evolution of the domestic interest rate is now almost identical to that of its foreign counterpart, the foreign debt-to-GDP ratio exhibits only minor fluctuations in the range between 55% and 60%.

While we view the high fertility scenario as overly optimistic, our model offers an alternative interpretation of the results. Since foreign-born workers are indistinguishable from the domestic workforce, this scenario can also be viewed as a simulation where immigration does not come in a single wave but serves as a regular source of inflow of young workers. To put the numbers in a perspective, the high fertility scenario implies about 100,000 more births per year, which roughly corresponds to an annual increase in insured foreign-born workers in the 2015–2020 period. Therefore, we conclude that regular immigration can to an extent alleviate downward pressures on the natural real rate of interest, but the effect would still be modest and delayed in time.

As demonstrated in similar modelling setups by **Carvalho et al.** [**2016**] and **Bielecki et al.** [**2020**], downward pressure on the NRI associated with population aging could also be alleviated by increasing the retirement age. Such a reform was indeed introduced in Poland in 2013, only to be completely reversed in 2017, and seems unlikely to be undertaken again in the foreseeable future. However, to the extent that increasing longevity will result in a voluntary postponement of retirement decisions, our simulations may overestimate the magnitude of the fall in the real interest rate over longer horizons.

Conclusions

How does aging affect monetary policy in Poland? What role will be played by migration flows? Given the findings from the literature, showing that demographic processes affect the natural rate of interest and monetary policy, we turn to the case of a country with a particularly sharp aging process and particularly large migration flows. To this end, we use an overlapping generations framework calibrated to projected birth and mortality rates as well as migration flows in Poland.

Our findings are as follows. First, due to aging, the equilibrium interest rate declines by more than 1.5 percentage points between 2000 and 2050. This could potentially not affect monetary policy if the central bank adjusts interest rates to follow the declining natural rate. However, the natural rate is not directly observable, and it seems possible that monetary policy responds to the decline only with a lag. Second, with a lower natural rate the probability of hitting the zero lower bound increases.

Both problems matter. Learning about the declining natural rate can result in a long period of low inflation, ranging up to 1 percentage point below the central bank target. As expected, the annual probability of hitting the ZLB also increases, from 1%–2% in the early 2000 s to almost 5% after the transition. When learning is taken into account, the probability of hitting the ZLB exceeds 6% by 2020 and rises to almost 15% in the 2040s. These developments are more serious than those found in earlier studies for the euro area. This is because aging is faster in Poland than in the EA.

Next, we check how important foreign demography is for Poland. To this end we simulate our model under various demographic scenarios for the euro area. The main conclusion is that the role played by foreign demography translates primarily into Poland's net foreign asset position (NFA), and much less into the equilibrium interest rate. In an open economy, capital flows to where the return is higher. In our baseline scenario, Poland's demography worsens more than in the euro area, hence Poland's NFA position improves by over 20%. If the euro area's demography worsened exactly as in Poland, the impact on the NFA would vanish as capital would find no reason to flow abroad. As a consequence, the domestic equilibrium interest rate would decline even more. However, this last effect would be quantitatively small.

Last but not least, we take a deeper look into the role that migration can play. Poland recently accepted some 1 million immigrants, mainly from Ukraine. We simulate our model adding migration flows after 2015 to the baseline scenario and comparing the results. As can be expected, the inflow of (mainly young) migrants improves the situation somewhat. However, quantitatively the impact is rather small and much delayed.

Our main conclusion is that the demographic transition will have a significant impact on monetary policy in Poland. In particular, if the central bank fails to timely account for the declining NRI, monetary policy may become too restrictive and possibly lead to ZLB traps. On the other hand, if the central bank is aware of the consequences presented in this paper, the demographic transition should not lead to deflationary biases, but can still raise the ZLB probability. Given the sharp demographic developments, migration does not seem to offer a quantitatively significant solution to the problems described in this paper.

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Tables and figures

Table 1. Calibrated structural parameters

Parameter	Value	Description
β	0.977	Discount factor
φ^{-1}	0.25	Frisch elasticity of labour supply
δ	0.1	Capital depreciation rate
α	0.25	Capital share in output
S_1	4	Investment adjustment cost curvature
μ	1.02	Product markup
θ_H, θ_H^*, θ_F	0.9[4]	Calvo probabilities
η	0.785	Home bias in final goods
ϕ	1.5	Elasticity of substitution between domestic goods and imports
ϕ^*	1.5	Elasticity of export demand
γ	0.013	Risk premium parameter
π_{ss}	1.025	Inflation target
γ_R	0.82[4]	Interest rate smoothing
γ_π	1.67	Reaction to inflation
γ_y	0.33	Reaction to GDP growth
λ	0.08	Learning parameter if NRI imperfectly perceived
β^*	0.992	Discount factor (EA)
θ^*	0.75[4]	Calvo probability (EA)
γ_R^*	0.87[4]	Interest rate smoothing (EA)
γ_π^*	1.9	Reaction to inflation (EA)
γ_y^*	0.15	Reaction to GDP growth (EA)
π_{ss}^*	1.02	Inflation target (EA)

Source: Author's own elaboration.

Table 2. Calibrated stochastic shocks

Parameter	Value	Description
ρ_z	0.88	Inertia of productivity shocks
ρ_u	0.42	Inertia of preference shocks
ρ_Γ	0.29	Inertia of risk premium shocks
σ_R	0.006	Standard dev. of monetary shocks
σ_{y^*}	0.011	Standard dev. of innovations to foreign output
σ_{π^*}	0.006	Standard dev. of innovations to foreign inflation
σ_{R^*}	0.012	Standard dev. of innovations to foreign interest rate
σ_z	0.007	Standard dev. of innovations to productivity shocks
σ_u	0.051	Standard dev. of innovations to preference shocks
σ_Γ	0.029	Standard dev. of innovations to risk premium shocks
$corr\left(y^*,\pi^*\right)$	0.38	Correlation of innovations to foreign output and inflation
$corr\left(y^*,R^*\right)$	0.59	Correlation of innovations to foreign output and interest rate
$corr\left(\pi^*,R^*\right)$	0.55	Correlation of innovations to foreign inflation and interest rate

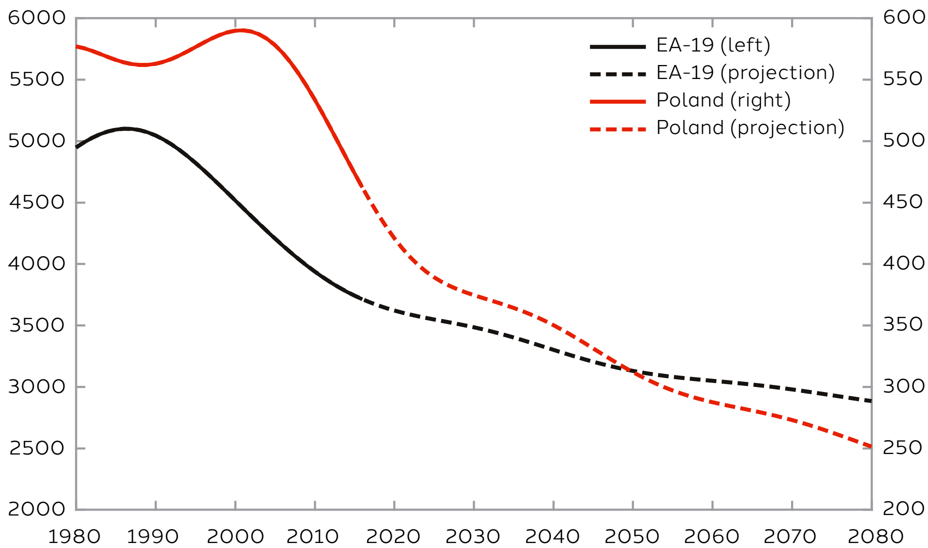
Source: Author's own elaboration.

Table 3. Matched data moments

Variable	Standard dev. Autocorrelation Corr. with GDP					
	Model	Data	Model	Data	Model	Data
GDP	1.75	1.84	0.79	0.68	1.00	1.00
Inflation	1.50	1.77	0.25	0.37	0.39	0.72
Interest rate	1.97	1.97	0.34	0.34	0.38	0.57
Real exchange rate	5.55	5.55	0.46	0.22	0.01	0.31

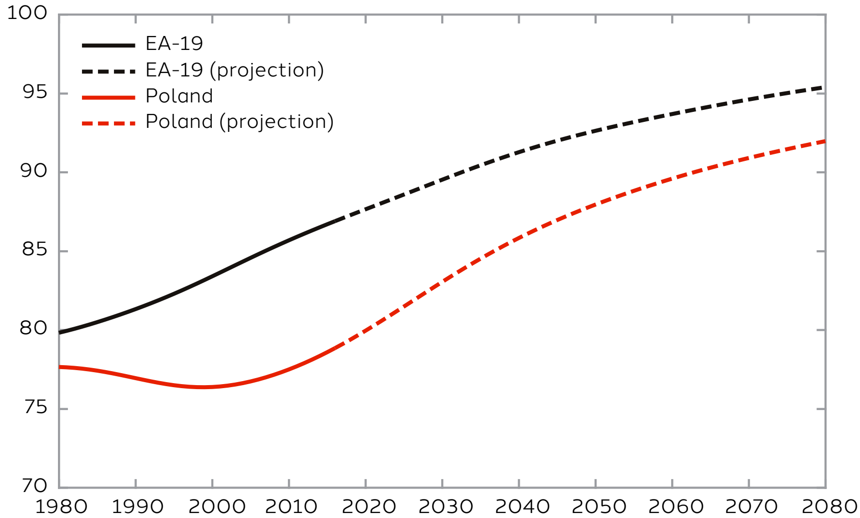
Source: Author's own elaboration.

Figure 1. Population size of 20-year-olds, EA-19 and Poland (in thousands)



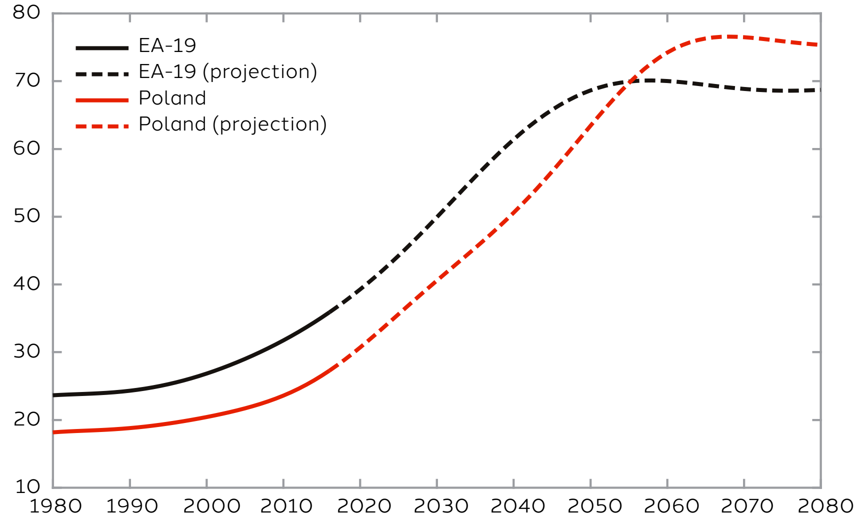
Source: Author's own elaboration.

Figure 2. Probability of survival from age 20 to 65, EA-19 and Poland (in %)



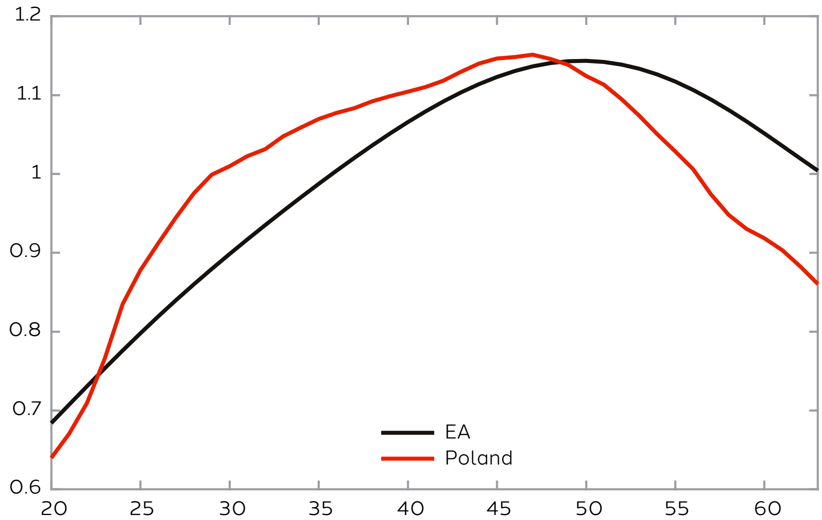
Source: Author's own elaboration.

Figure 3. Old-age dependency ratio, EA-19 and Poland (in %)



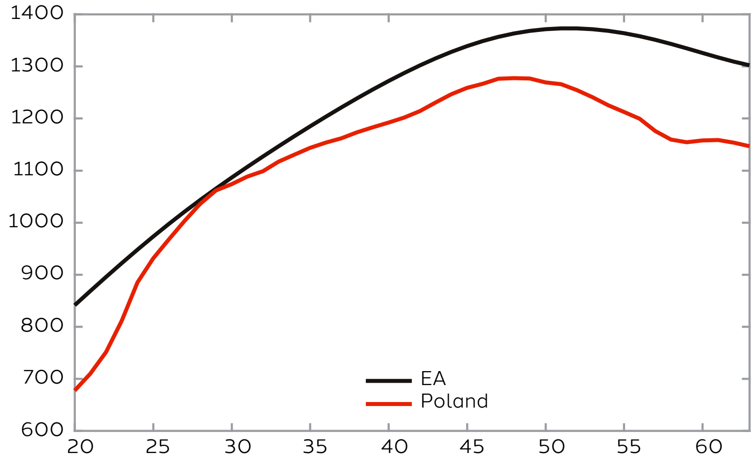
Source: Author's own elaboration.

Figure 4. Age-dependent productivity profiles



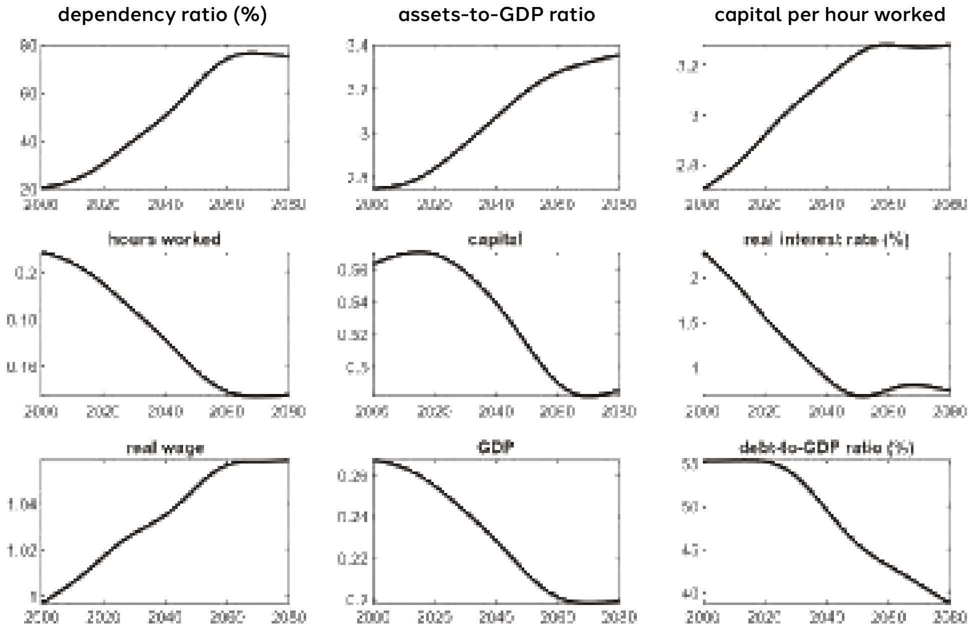
Source: Author's own elaboration.

Figure 5. Age-dependent labour disutility



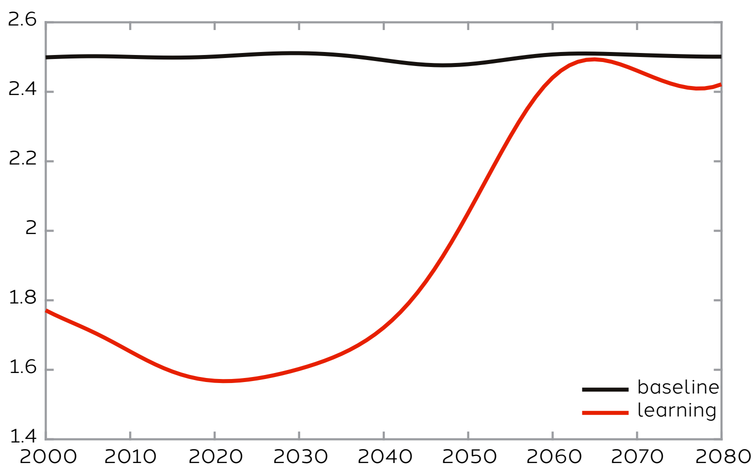
Source: Author's own elaboration.

Figure 6. Impact of demographic transition on Poland



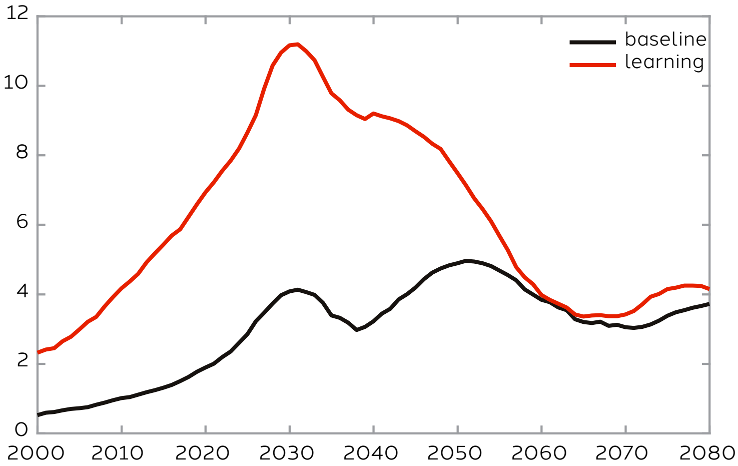
Source: Author's own elaboration.

Figure 7. Inflation under learning about NRI (in %)



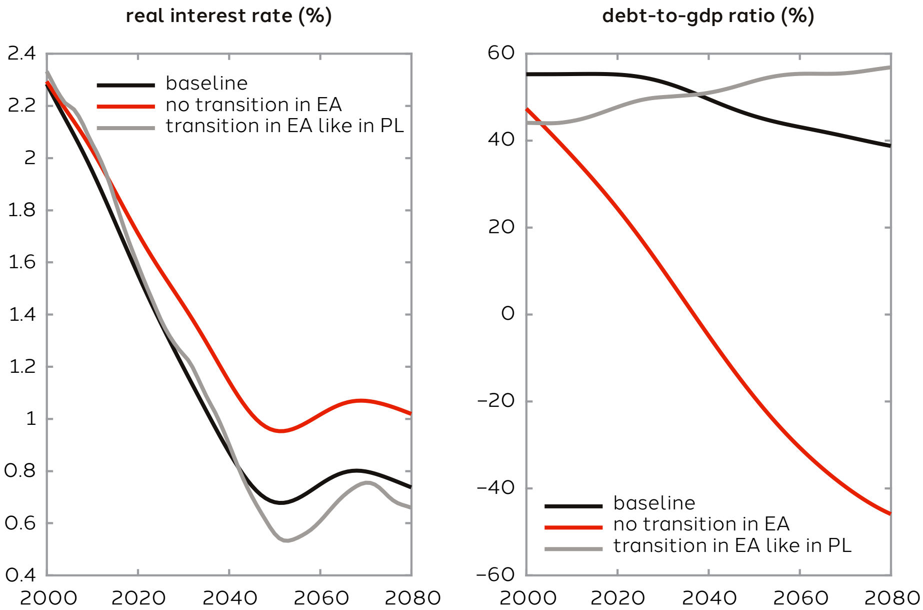
Note: black line – baseline model; red line – model with NRI learning. Inflation in percent (vertical axis).
Source: Author's own elaboration.

Figure 8. Probability of hitting the ZLB (in %)



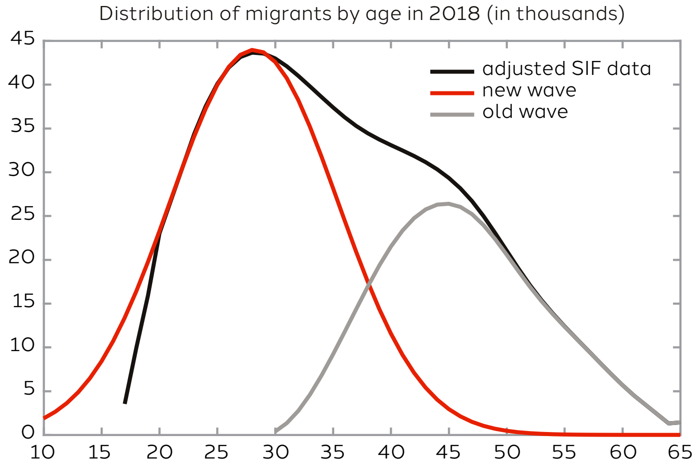
Note: black line – baseline scenario; red line – scenario with NRI learning. Probability in percent (vertical axis).
Source: Author's own elaboration.

Figure 9. Effects of foreign demography



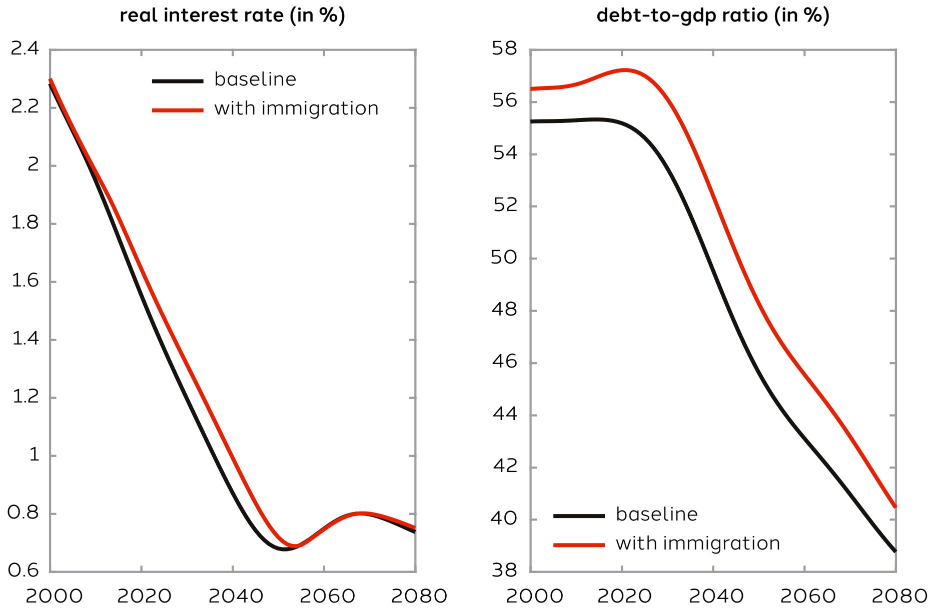
Note: black line – baseline model; red line – model with constant demography in the EA; grey line – model with demography in the EA same as in Poland.
Source: Author's own elaboration.

Figure 10. Distribution of immigrants by age and decomposition into two waves



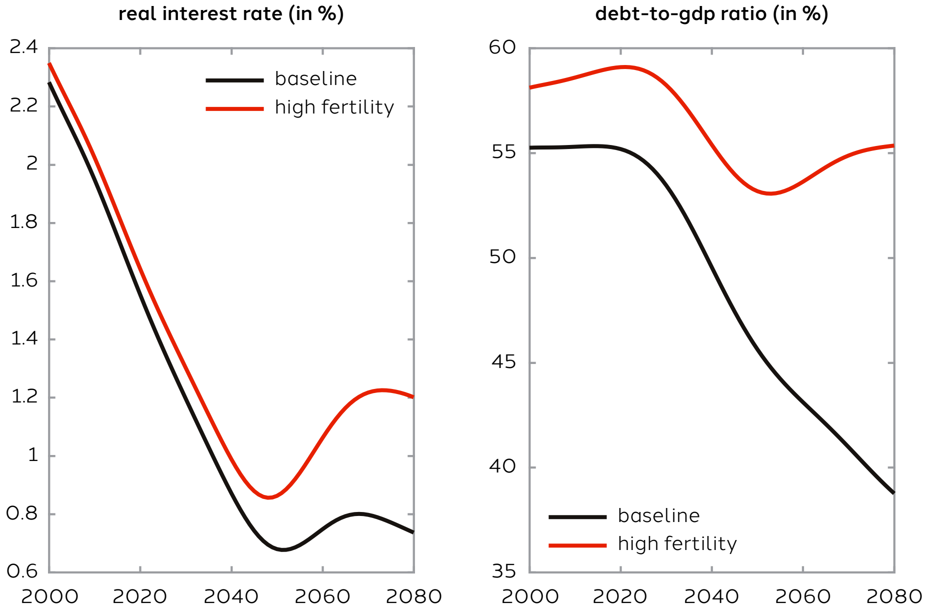
Note: black line – SIF data interpolated and adjusted upward by estimated total number of immigrants; red line – normal distribution approximating age distribution of new migrants; grey line – residual capturing age distribution of migrants from previous waves.
Source: Author's own elaboration.

Figure 11. Effects of immigration



Note: black line – baseline scenario; red line – increased immigration scenario.
Source: Author's own elaboration.

Figure 12. Effects of higher fertility



Note: black line – baseline scenario; red line – high fertility scenario.
Source: Author's own elaboration.

Appendix. Complete list of model equations

Households

$$c_{j,t} + a_{j,t} = w_t z_j h_{j,t} + \frac{R_t^a}{\pi_t} a_{j-1,t-1} + beq_t \tag{A.1}$$

$$a_{0,t} = 0 \tag{A.2}$$

$$a_{J,t} = 0 \tag{A.3}$$

$$1 = \beta\left(1 - \omega_{j,t}\right)\mathbb{E}_t\left[\frac{c_{j,t}}{c_{j+1,t+1}} \frac{R_{t+1}^a}{\pi_{t+1}}\right] \tag{A.4}$$

$$h_{j,t} = \left(\frac{w_t z_j}{\phi_j c_{j,t}}\right)^{1/\varphi} \qquad z_{j \geq JR} = 0 \tag{A.5}$$

Demography

$$N_{1,t}^{rel} = 1 \tag{A.6}$$

$$N_{j,t}^{rel} = \frac{\left(1 - \omega_{j-1,t-1}\right) N_{j-1,t-1}^{rel}}{1 + n_{1,t}} \tag{A.7}$$

$$N_t^{rel} = \sum_{j=1}^{J} N_{j,t}^{rel} \tag{A.8}$$

$$n_t = \frac{N_t^{rel}}{N_{t-1}^{rel}}\left(1 + n_{1,t}\right) - 1 \tag{A.9}$$

Aggregation

$$c_t = \frac{\sum_{j=1}^{J} N_{j,t}^{rel} c_{j,t}}{N_t^{rel}} \tag{A.10}$$

$$h_t = \frac{\sum_{j=1}^{J} N_{j,t}^{rel} z_j h_{j,t}}{N_t^{rel}} \tag{A.11}$$

$$a_t = \frac{\sum_{j=1}^{J} N_{j,t}^{rel} a_{j,t}}{N_{t+1}^{rel}\left(1+n_{1,t+1}\right)} \tag{A.12}$$

$$beq_t = \frac{\sum_{j=1}^{J}\left(N_{j-1,t-1}^{rel}/\left(1+n_{1,t}\right)-N_{j,t}^{rel}\right)\left(R_t^a/\pi_t\right)a_{j-1,t-1}}{N_t^{rel}} \tag{A.13}$$

Financial intermediary

$$a_t = q_t k_t + s_t b_t^* + p_t^d \tag{A.14}$$

$$R_t q_t = \mathbb{E}_t\left[\left(r_{t+1}^k + (1-\delta)q_{t+1}\right)\pi_{t+1}\right] \tag{A.15}$$

$$R_t s_t = \mathbb{E}_t\left[s_{t+1}\Gamma_t R_t^* \pi_{t+1}/\pi_{t+1}^*\right] \tag{A.16}$$

$$R_t p_t^d = \mathbb{E}_t\left[\left(p_{t+1}^d\left(1+n_{t+1}\right)+f_{t+1}\right)\pi_{t+1}\right] \tag{A.17}$$

$$\frac{R_t^a}{\pi_t}a_{t-1} = \left(r_t^k + (1-\delta)q_t\right)k_{t-1} + s_t\Gamma_{t-1}\frac{R_{t-1}^*}{\pi_t^*}b_{t-1}^* + \left(p_t^d\left(1+n_t\right)+f_t\right) \tag{A.18}$$

Final goods producers

$$y_t = \left[\eta^{\frac{\mu-1}{\mu}}\left(y_{H,t}\right)^{\frac{1}{\mu}} + \left(1-\eta\right)^{\frac{\mu-1}{\mu}}\left(y_{F,t}\right)^{\frac{1}{\mu}}\right]^{\mu} \tag{A.19}$$

$$y_{H,t} = \eta\left(p_{H,t}\right)^{\frac{\mu}{1-\mu}} y_t \tag{A.20}$$

$$y_{F,t} = \left(1-\eta\right)\left(p_{F,t}\right)^{\frac{\mu}{1-\mu}} y_t \tag{A.21}$$

Capital goods producers

$$\left(1+n_{t+1}\right)k_t = (1-\delta)k_{t-1} + \varepsilon_{i,t}\left[1-\frac{S_1}{2}\left(1+n_t\right)^2\left(\frac{inv_t}{inv_{t-1}}-1\right)^2\right]inv_t \tag{A.22}$$

$$1 = \varepsilon_{i,t}q_t\left[1-\frac{S_1}{2}(1+n_t)^2\left(\frac{inv_t}{inv_{t-1}}-1\right)^2 - S_1(1+n_t)^2\left(\frac{inv_t}{inv_{t-1}}-1\right)\frac{inv_t}{inv_{t-1}}\right] \tag{A.23}$$

$$+\mathbb{E}_t\left[\frac{\pi_{t+1}}{R_t}\varepsilon_{i,t+1}q_{t+1}S_1\left(1+n_{t+1}\right)^3\left(\frac{inv_{t+1}}{inv_t}-1\right)\frac{inv_{t+1}^2}{inv_t^2}\right]$$

Importers

$$\tilde{p}_{F,t} = \mu_F \frac{\Omega_{F,t}}{\mathbf{Y}_{F,t}} \tag{A.24}$$

$$\Omega_{F,t} = \left(p_{F,t}\right)^{\frac{\mu_F}{\mu_F - 1}} y_{F,t} + \theta_F \mathbb{E}_t \left[\frac{\pi^*_{t+1}}{R^*_t} \left(\frac{\pi_{ss}}{\pi_{t+1}} \right)^{\frac{\mu_F}{1-\mu_F}} \Omega_{F,t+1} \right] \tag{A.25}$$

$$\mathbf{Y}_{F,t} = \varepsilon^{\tau}_{F,t} \tau_F \left(p_{F,t}\right)^{\frac{\mu_F}{\mu_F - 1}} y_{F,t} + \theta_F \mathbb{E}_t \left[\frac{\pi^*_{t+1}}{R^*_t} \left(\frac{\pi_{ss}}{\pi_{t+1}} \right)^{\frac{1}{1-\mu_F}} \mathbf{Y}_{F,t+1} \right] \tag{A.26}$$

Intermediate goods producers

$$\frac{r^k_t}{w_t} = \frac{\alpha}{1-\alpha} \frac{h_t}{k_{t-1}} \tag{A.27}$$

$$mc_t = \frac{1}{\varepsilon_{z,t}} \left(\frac{r^k_t}{\alpha} \right)^{\alpha} \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha} \tag{A.28}$$

$$y^*_{H,t} = \left(p^*_{H,t}\right)^{-\phi_y} y^*_t \tag{A.29}$$

$$\tilde{p}_{H,t} = \mu_H \frac{\Omega_{H,t}}{\mathbf{Y}_{H,t}} \tag{A.30}$$

$$\tilde{p}^*_{H,t} = \mu^*_H \frac{\Omega^*_{H,t}}{\mathbf{Y}^*_{H,t}} \tag{A.31}$$

$$f_t = p_{H,t} y_{H,t} + s_t p^*_{H,t} y^*_{H,t} - w_t h_t - r_{k,t} k_{t-1} \tag{A.32}$$

$$\Omega_{H,t} = mc_t \left(p_{H,t}\right)^{\frac{\mu_H}{\mu_H - 1}} y_{H,t} + \theta_H \mathbb{E}_t \left[\frac{\pi_{t+1}}{R_t} \left(\frac{\pi_{ss}}{\pi_{t+1}} \right)^{\frac{\mu_H}{1-\mu_H}} \Omega_{H,t+1} \right] \tag{A.33}$$

$$\mathbf{Y}_{H,t} = \varepsilon^{\tau}_{H,t} \tau_H \left(p_{H,t}\right)^{\frac{\mu_H}{\mu_H - 1}} y_{H,t} + \theta_H \mathbb{E}_t \left[\frac{\pi_{t+1}}{R_t} \left(\frac{\pi_{ss}}{\pi_{t+1}} \right)^{\frac{1}{1-\mu_H}} \mathbf{Y}_{H,t+1} \right] \tag{A.34}$$

$$\Omega^*_{H,t} = mc_t \left(p^*_{H,t}\right)^{\frac{\mu^*_H}{\mu^*_H - 1}} y^*_{H,t} + \theta^*_H \mathbb{E}_t \left[\frac{\pi_{t+1}}{R_t} \left(\frac{\pi^*_{ss}}{\pi^*_{t+1}} \right)^{\frac{\mu^*_H}{1-\mu^*_H}} \Omega^*_{H,t+1} \right] \tag{A.35}$$

$$\mathbf{Y}^*_{H,t} = \varepsilon^{\tau*}_{H,t} \tau^*_H \left(p^*_{H,t}\right)^{\frac{\mu^*_H}{\mu^*_H - 1}} y^*_{H,t} + \theta^*_H \mathbb{E}_t \left[\frac{\pi_{t+1}}{R_t} \left(\frac{\pi^*_{ss}}{\pi^*_{t+1}} \right)^{\frac{1}{1-\mu^*_H}} \mathbf{Y}^*_{H,t+1} \right] \tag{A.36}$$

Inflation and price dispersion dynamics

$$\left(p_{H,t}\right)^{\frac{1}{1-\mu_H}} = \theta_H \left(\frac{p_{H,t-1}\pi_{ss}}{\pi_t}\right)^{\frac{1}{1-\mu_H}} + \left(1-\theta_H\right)\left(\tilde{p}_{H,t}\right)^{\frac{1}{1-\mu_H}} \tag{A.37}$$

$$\left(p_{F,t}\right)^{\frac{1}{1-\mu_F}} = \theta_F \left(\frac{p_{F,t-1}\pi_{ss}}{\pi_t}\right)^{\frac{1}{1-\mu_F}} + \left(1-\theta_F\right)\left(\tilde{p}_{F,t}\right)^{\frac{1}{1-\mu_F}} \tag{A.38}$$

$$\left(p^*_{H,t}\right)^{\frac{1}{1-\mu^*_H}} = \theta^*_H \left(\frac{p^*_{H,t-1}\pi^*_{ss}}{\pi^*_t}\right)^{\frac{1}{1-\mu^*_H}} + \left(1-\theta^*_H\right)\left(\tilde{p}^*_{H,t}\right)^{\frac{1}{1-\mu^*_H}} \tag{A.39}$$

$$\Delta_{H,t} = \left(1-\theta_H\right)\left(\frac{\tilde{p}_{H,t}}{p_{H,t}}\right)^{\frac{\mu_H}{1-\mu_H}} + \theta_H \Delta_{H,t-1}\left(\frac{p_{H,t}}{p_{H,t-1}}\right)^{\frac{\mu_H}{\mu_{H-1}}}\left(\frac{\pi_{ss}}{\pi_t}\right)^{\frac{\mu_H}{1-\mu_H}} \tag{A.40}$$

$$\Delta^*_{H,t} = \left(1-\theta^*_H\right)\left(\frac{\tilde{p}^*_{H,t}}{p^*_{H,t}}\right)^{\frac{\mu^*_H}{1-\mu^*_H}} + \theta^*_H \Delta^*_{H,t-1}\left(\frac{p^*_{H,t}}{p^*_{H,t-1}}\right)^{\frac{\mu^*_H}{\mu^*_H-1}}\left(\frac{\pi^*_{ss}}{\pi^*_t}\right)^{\frac{\mu^*_H}{1-\mu^*_H}} \tag{A.41}$$

Monetary policy

$$R^{cb}_t = R^{\gamma_R}_{t-1}\left[\tilde{R}^e_t\left(\frac{\pi_t}{\pi_{ss}}\right)^{\gamma_\pi}\left(gdp_t / gdp_{t-1}\right)^{\gamma_y}\right]^{1-\gamma_R} \exp\left\{\varepsilon_{R,t}\right\} \tag{A.42}$$

$$R_t = \begin{cases} R^{cb}_t, & R^{cb}_t > 1 \\ 1, & R^{cb}_t \leq 1 \end{cases} \tag{A.43}$$

Alternative assumptions about $\tilde{R}^e_t$

$$\tilde{R}^e_t = \pi_{ss}\tilde{r}_t = \mathbb{E}_t\left[\frac{R^a_{t+1}}{\pi_{t+1}}\right]\pi_{ss} \tag{A.44}$$

$$\tilde{R}^e_t = \tilde{R}^e_{t-1} + \lambda\left(\pi_{ss}\tilde{r}_{t-1} - \tilde{R}^e_{t-1}\right)$$

Market clearing

$$y_{H,t}\Delta_{H,t} + y^*_{H,t}\Delta^*_{H,t} = \varepsilon_{z,t}k^\alpha_{t-1}h^{1-\alpha}_t \tag{A.45}$$

$$y_t = c_t + inv_t \tag{A.46}$$

$$gdp_t = y_{H,t}\Delta_{H,t} + y^*_{H,t}\Delta^*_{H,t} \tag{A.47}$$

$$dy_t = -\frac{s_t b^*_t}{p_{H,t}gdp_t} \tag{A.48}$$

$$\Gamma_t = 1 + \gamma\left(\exp(dy_t) - 1\right) + \exp\left\{\varepsilon_{\Gamma,t}\right\} \tag{A.49}$$