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The Evolution of Fiscal Policy and Public Debt Dynamics: The Case of Sweden

Ewolucja polityki fiskalnej i dynamika długu publicznego: przypadek Szwecji

Abstract

The aim of the paper is to evaluate the long-run sustainability of fiscal policy in Sweden using a time-varying fiscal reaction function. Sufficient sustainability conditions are discussed, and it is demonstrated that fiscal sustainability is consistent with various types of transient fiscal policy and temporary debt expansions. The fiscal response to the debt-to-income ratio is estimated recursively using constant gain least squares. The evolution of fiscal policy in Sweden is evaluated over the period from 1850 to 2014. It is demonstrated that a sustainable fiscal policy can be an outcome of a long-run process of learning. In this process, fiscal authorities are forced to adapt their policies to changes in the economic and political environment. It is concluded that fiscal sustainability should be judged by the eventual ability of fiscal authorities to stabilise debt rather than by transient debt dynamics.

Streszczenie

Celem analizy jest ocena długookresowej stabilności polityki fiskalnej w Szwecji na podstawie fiskalnej funkcji reakcji z parametrami zmiennymi w czasie. W artykule omówiono wystarczające warunki stabilności i wykazano, że stabilność fiskalna może obejmować różne typy przejściowej polityki fiskalnej i okresy tymczasowej ekspansji długu. Reakcję polityki fiskalnej na zmiany stosunku długu do dochodów oszacowano rekursywnie za pomocą metody najmniejszych kwadratów ze stałymi wagami. Następnie zbadano ewolucję polityki fiskalnej Szwecji w okresie 1850–2014. Pokazano, że stabilna polityka fiskalna może być wynikiem długookresowego procesu uczenia się. W procesie tym władze fiskalne są zmuszone do dostosowywania swojej polityki do zmian w otoczeniu ekonomicznym i politycznym. Sformułowano wniosek, że stabilność fiskalną należy oceniać na podstawie ostatecznej zdolności władz do stabilizacji długu, a nie przejściowych zmian dynamiki długu.

Introduction

The fiscal history of many economies includes episodes of public debt expansion when the debt-to-income ratio followed an explosive path. Although such fiscal expansions led to defaults in many countries, there are also examples of countries such as Sweden and Britain where fiscal authorities stabilised public debt by implementing timely fiscal adjustments and policy framework modifications.

In the literature, fiscal policy is usually modelled by a constant or time-varying policy rule. Such a rule is often used to test long-run fiscal sustainability defined as the historical dynamics of public debt consistent with the intertemporal budget constraint of the public sector regardless of implemented policy regimes [**Bohn, 2008**]. In this framework, recurrent debt expansions can be explained by the adaptive learning of fiscal authorities about the changing economic environment.

In models of adaptive learning, economic agents use recursive econometric procedures, such as recursive or constant gain least squares, in order to evaluate changes in the economic environment and update policy rules [**Evans, Honkapohja, 2001**]. Such an approach implies that policy rules evolve with changing economic conditions but the adaptation of policy to new economic conditions is lagged.

Under the adaptive learning of fiscal authorities, an adverse economic shock can cause an episode of unsustainable debt dynamics that ends when the information of the shock is absorbed and the policy rule is updated. In this framework, long-run sustainability can be interpreted as an outcome of adaptive learning in the evolving economic environment. Episodes of debt expansion are consistent with long-run sustainability as long as the eventual debt dynamics is consistent with the intertemporal budget constraint of the public sector.

In this paper, econometric methods for the recursive evaluation of fiscal policy are applied in order to explain the evolution of fiscal policy and debt dynamics in Sweden, which provides an interesting example of an economy with historical economic data of high quality. Although Sweden has not directly participated in any war since 1815 and its fiscal authorities have not defaulted on their debt since 1818, the subsequent fiscal history of Sweden includes several known episodes of debt expansion that can be explained by economic developments and the evolution of fiscal policy.

The objective of this paper is to approximate the time-varying response of the primary surplus to changes in the debt-to-income ratio using a long historical time series. The analysis of the fiscal response to increases in the debt-to-income ratio contributes to the explanation of known debt expansions in Sweden and to the understanding of how these episodes of transient explosive dynamics can be reconciled with long-run fiscal sustainability.

In the theoretical part of the paper, a set of conditions sufficient for long-run fiscal sustainability is specified. These conditions are alternative to the sustainability conditions proposed by **Bohn** [**1998**] and **Canzoneri et al.** [**2001**] as they imply long-run sustainability even if there are episodes when fiscal authorities respond to debt expansions by running primary deficits. Under these conditions, expectations of future primary surpluses are crucial for long-run fiscal sustainability.

In addition, the paper extends the existing methods of analysing fiscal policy and debt dynamics by implementing the recursive identification of a time-varying fiscal reaction function. Although some approaches to testing fiscal sustainability include regime-switching policy functions (see, e.g., **Aldama, Creel** [**2017**], [**2019**]), these methods require a complete specification of the model describing a policy rule. In the framework of recursive identification, the evolution of fiscal policy can be tracked without assuming that the complete model specification is known.

The empirical results, based on recursive identification of the fiscal reaction function, are consistent with the long-run sustainability conditions specified in the theoretical part of the paper. Although the full-sample estimates of the constant parameter policy function confirm the long-run fiscal sustainability, the recursive estimates show that the fiscal response to debt expansions in Sweden was small over a large part of the historical time series and in some episodes debt expansions were followed by primary deficits. The evidence of the long-run fiscal sustainability found in the full-sample estimates can be explained by a sustainable pol-

icy regime based on explicit policy rules that was eventually adopted in Sweden as a result of fiscal reforms in the 1980s and 1990s.

The paper is organised as follows. Section 2 provides a review of selected studies of fiscal sustainability and the fiscal policy of Sweden. The theoretical underpinnings of the analysis presented in this paper are described in Section 3. The empirical model and the data used for estimation are discussed in Section 4, while the results are given in Section 5. Section 6 concludes.

Literature review

The fiscal reaction function, describing the relationship between the primary surplus and the debt-to-income ratio, was introduced in **Bohn** [**1998**]. The paper specified sufficient conditions for long-run fiscal sustainability using a constant-parameter reaction function and tested sustainability for the United States.

An alternative set-up was considered by **Canzoneri et al.** [**2001**], who formulated sufficient conditions for long-run fiscal sustainability using a time-varying fiscal reaction function. The crucial assumption in **Canzoneri et al.** [**2001**] is that the ratio of the primary surplus to income is a non-decreasing function of the debt-to-income ratio and that an increase in the debt-to-income ratio causes an increase in the surplus-to-income ratio infinitely often. This assumption is very general, allowing for various types of transient debt dynamics when fiscal policy responds to economic and political shocks. Nevertheless, empirical studies of fiscal sustainability show that in some policy regimes the surplus-to-income ratio can have a negative response to the debt-to-income ratio (see, e.g., **Claeys** [**2008**]).

The sustainability conditions specified in **Bohn** [**1998**] and **Canzoneri et al.** [**2001**] are sufficient but not necessary for long-run sustainability to hold. In this paper, a set of alternative sustainability conditions is specified. These conditions do not include the restriction of the primary surplus being a non-decreasing function of debt. Fiscal sustainability can be maintained even if there are episodes of expanding debt followed by decreasing surpluses. But in this case sustainability depends on the expectation of the private sector that the primary surplus will eventually be an increasing function of debt.

Some further studies of sustainability based on the assumption of time-varying policy regimes are as follows. **Davig** [**2005**] confirms the long-run sustainability of US debt using a Markov-switching autoregressive model of debt dynamics allowing for periodic debt expansions. **Davig and Leeper** [**2006**] consider a model with fiscal policy switching between two regimes: active (unsustainable) and passive (sustainable). **Aldama and Creel** [**2017**; **2019**] use the Markov-switching framework in order to derive sufficient conditions for long-run (or global) sustainability and test for sustainability in France and the United States.

There are numerous analyses of sustainability based on Swedish data. This paper is related to analyses carried out in **Claeys** [**2008**] and **Bystrov and Mackiewicz** [**2018**]. **Claeys** [**2008**] tests for fiscal sustainability in Sweden over the period from 1970 to 2006, using a time-invariant fiscal policy rule, and explains debt expansions and contractions using regime-switching models. **Bystrov and Mackiewicz** [**2020**] implement tests of recurrent explosive behaviour in order to identify episodes of explosive debt dynamics in the United States, UK, and Sweden from 1792 to 2012, but conclude that transient explosive debt dynamics can be consistent with long-run sustainability.

The modern fiscal policy regime emerged in Sweden in the aftermath of the Napoleonic wars and evolved over two centuries adapting to transformations in the structure of the economy (see **Schön** [**1989**; **2010**]). Swedish fiscal policy changed as a result of policy learning in the aftermath of economic crises [**Jonung, 2015**]. The policy learning mechanism is described in **Jonung** [**2015**] as a process of adaptation by policy makers to new circumstances based on earlier experience and new information. The adaptation mechanism can be econometrically evaluated in a time-varying policy reaction function: the parameters of the policy reaction function should be drifting.

Theoretical background

Sustainability and Fiscal Reaction Function

The period-by-period government budget identity (written in real terms) is

$$D_{t+1} = (1+r_{t+1})D_t - S_{t+1}, \tag{1}$$

where D_t is the real end-of-period debt, S_{t+1} is the real primary surplus (taxes minus non-interest spending, $S_{t+1} = T_{t+1} - G_{t+1}$) accumulated over period $t+1$ and r_{t+1} is the real rate of return on government securities.

Bohn [1995] specifies the general intertemporal budget constraint in a stochastic economy as

$$D_t = \sum_{i=1}^{\infty} E_t\left[u_{t,i} S_{t+i}\right], \tag{2}$$

which is satisfied if and only if

$$\lim_{T \to \infty} E_t\left[u_{t,T} D_{t+T}\right] = 0, \tag{3}$$

where $u_{t,T}$ is the marginal rate of substitution between periods t and $t+T$, which is the product of one-period marginal rates of substitution: $u_{t,T} = \prod_{j=0}^{T-1} u_{t+j,1}$. The rate of return r_{t+1} must satisfy the Euler equation $E_t\left[u_{t,1}(1+r_{t+1})\right] = 1$ for every t. For more details, see Bohn [1995; 1998]. A fiscal policy that satisfies the intertemporal budget constraint (2) is said to be sustainable.

The budget equation (1) in a growing economy can be written in the ratio form

$$d_{t+1} = \frac{1+r_{t+1}}{1+y_{t+1}}d_t - s_{t+1}, \tag{4}$$

where $d_t = D_t / Y_t$ is the (end-of-period) ratio of debt to aggregate income, $s_{t+1} = S_{t+1} / Y_{t+1}$ is the ratio of the primary surplus to income, and y_{t+1} is the real income growth rate, $1+y_{t+1} = Y_{t+1} / Y_t$.

The transversality condition (3) can also be rewritten in the ratio form:

$$\lim_{T \to \infty} E_t\left[u_{t,T} \prod_{j=1}^{T}(1+y_{t+j})d_{t+T}\right] = 0. \tag{5}$$

Following Bohn [1998, 2008], the fiscal reaction function or fiscal policy rule can be written as

$$s_{t+1} = \theta d_{t+1}^* + \mu_{t+1}, \tag{6}$$

where $d_{t+1}^* = \frac{1+r_{t+1}}{1+y_{t+1}}d_t$ denotes the debt-to-income ratio at the start of period $t+1$, μ_{t+1} is a component representing other (temporary) determinants of the primary surplus (e.g., temporary government outlays, output gap). Bohn [1998; 2008] proves that if $\theta > 0$, μ_{t+1} is a bounded stochastic process and the stream of aggregate income Y_t has a finite present value, then the fiscal policy described by rule (6) satisfies the intertemporal budget constraint (2). Canzoneri et al. [2001] describe the sufficient conditions for the time-varying fiscal policy rule

$$s_{t+1} = \theta_{t+1} d_{t+1}^* + \mu_{t+1}, \tag{6'}$$

to ensure fiscal sustainability. In particular, Canzoneri et al. [2001] assume that θ_t is either a deterministic time-varying or a bounded random variable such that $0 \le \theta_t < 1$ and $\limsup_{t \to \infty} \theta_t > 0$ almost surely (with probability one). The condition on the limit superior (lim sup) guarantees that the time-varying parameter θ_t is positive infinitely often. This implies that a sustainable policy is implemented infinitely often.

Canzoneri et al. [2001] assume that the response parameter θ_t is non-negative, i.e., an increase in debt does not cause a decrease in the primary surplus. However, this assumption is difficult to reconcile with extended episodes of primary deficits despite increasing debt-to-income ratios in many economies. As the sustainability conditions in Bohn [1998; 2008] and Canzoneri et al. [2001] are sufficient, but not necessary, it is possible to specify alternative sustainability conditions, allowing for negative values of the response parameter θ_t. The possibility that the response parameter takes negative values allows for explaining some episodes of explosive debt dynamics. The proposition below gives an example of such conditions.

Proposition 1: If the primary surplus to income ratio is defined by equation (6′) where

1) $\{\theta_t\}$ is a bounded deterministic sequence, such that $\theta_t < 1$ and $\liminf_{t \to \infty} \theta_t > \underline{\theta} > 0$

2) $\{u_{t,j}\}$, $\{r_t^*\}$, and $\{\mu_t\}$ are bounded stochastic processes such that $u_{t,j} > 0$, $(1 + r_{t+j}) > 0$ and $(1 + y_{t+j}) > 0$.

3) the one-period rate of return on government securities satisfies the Euler equation: $E_t\left[u_{t+i,1}(1 + r_{t+i+1})\right] = 1$.
   There is no arbitrage. (Hence, $E_t\left[\prod_{i=0}^{T-1} u_{t+i,1}(1 + r_{t+i+1})\right] = 1$, see, e.g., Hansen and Renault [2010])

4) $\sup_T \prod_{i=0}^{T-1} u_{t+i,1}(1 + r_{t+i+1}) = \omega < \infty$

5) the stream of future incomes has a finite present value,

$$V_t = Y_t \sum_{i=1}^{\infty} E_t\left[u_{t,i} \prod_{j=t+1}^{t+i}(1 + y_j)\right] < \infty,$$

then the government policy satisfies the intertemporal budget constraint (2).

The proof of the proposition is available in Appendix A. Assumption 1) guarantees that the sequence $\{\theta_t\}$ will be bounded away from zero eventually resulting in a sustainable policy. This assumption is more restrictive than the assumption on the limit superior in Proposition 1 in Canzoneri et al. [2001]. But it makes it possible to relax the assumption of non-negative θ_t and account for episodes of explosive debt dynamics.

If $\liminf_{t \to \infty} \theta_t = \limsup_{t \to \infty} \theta_t = \overline{\theta}$ and $\overline{\theta} \in (0,1)$, then fiscal policy is sustainable and $\overline{\theta}$ can be interpreted as the long-run fiscal response parameter. But the existence of the positive limit $\overline{\theta} = \lim_{t \to \infty} \theta_t$ is not necessary for long-run sustainability.

Recursive identification of fiscal reaction function

Under the assumption of evolving fiscal policy, the fiscal response parameter should be changing over time. Moreover, the underlying law of motion for the response parameter can be changing as well. Recursive identification (described, e.g., in Ljung, Soderstrom [1983]; Kushner, Yin [2003]) makes it possible to track the fiscal response parameter without assuming that the underlying law of motion is known.

Let us rewrite the fiscal reaction function as

$$s_t = x'_t \beta_t,$$

where $x'_t = (d_t^*, z'_t)$ and $\beta_t = (\theta_t, \delta'_t)'$; z_t is a vector of variables, other than the debt-to-income ratio, that determine the primary surplus and δ_t is the corresponding vector of time-varying parameters.

A recursive algorithm can be applied in order to track the time-varying vector of parameters β_t using the following equations (see Ljung, Soderstrom [1983] or Kushner, Yin [2003]):

$$\hat{\beta}_t = \Pi_B\left[\hat{\beta}_{t-1} + \lambda_t R_t^{-1} x_t\left(s_t - x'_t \beta_{t-1}\right)\right],$$

$$R_t = \Pi_R\left[R_{t-1} + \lambda_t\left(x_t x'_t - R_{t-1}\right)\right],$$

where matrix R_t is a recursive estimator of $E_t\left[x_t x'_t\right]$ and $\lambda_t \in (0,1)$ is a "gain" parameter that determines the discount or forgetting factor $(1 - \lambda_j)$ for past estimates of β_t $(j = 1, \dots t)$. If the gain parameter λ_t is equal to $1/t$, then the algorithm is the recursive least squares. If the gain parameter is constant $(\lambda_t = \lambda)$, then the algorithm is called the constant gain least squares.

Π_B and Π_R are projections onto compact sets B and R that are determined by restrictions imposed onto vector β_t and matrix R_t. The restrictions are usually imposed in order to guarantee the numerical stability of the algorithm. Alternatively, the restrictions can be implied by economic theory.

Ljung and Soderstrom [**1983**] and **Kushner and Yin** [**2003**] provide alternative sets of assumptions under which sequences $\{\hat{\beta}_t\}$ and $\{R_t\}$ computed by the recursive least squares method $(\lambda_t = 1/t)$ converge almost surely (with probability one) to a stable stationary point of the projected differential equation

$$\frac{d}{d\tau}\beta(\tau) = \Pi_B\left[R(\tau)^{-1}\overline{R}\left(\overline{\beta} - \beta(\tau)\right)\right],$$

$$\frac{d}{d\tau}R(\tau) = \Pi_R\left[\overline{R} - R(\tau)\right],$$

where matrix $\overline{R} = \underset{m,n\to\infty}{plim}\frac{1}{m}\sum_{t=n}^{n+m-1}E_t\left[x_t x'_t\right]$ and parameter vector $\overline{\beta} = \overline{R}^{-1}\overline{Q}$ ($\overline{Q} = \underset{m,n\to\infty}{plim}\frac{1}{m}\sum_{t=n}^{n+m-1}E_t\left[x_t s_t\right]$) represent the corresponding convergence point. The existence of the convergence point implies that there is a long-run fiscal response parameter $\overline{\theta}$.

If some of the explanatory variables in vector x_t are endogenous (their values are determined simultaneously with the explained variable s_t) and there is a vector of predetermined instrumental variables, w_t, then a modification of the recursive algorithm, using instrumental variables, can be implemented:

$$\hat{\beta}_t = \Pi_B\left[\hat{\beta}_{t-1} + \lambda_t R_t^{-1} w_t\left(s_t - x'_t\hat{\beta}_{t-1}\right)\right], \tag{7}$$

$$R_t = \Pi_R\left[R_{t-1} + \lambda_t\left(w_t x'_t - R_{t-1}\right)\right], \tag{8}$$

where matrix R_t is a recursive estimator of $E_t\left[w_t x'_t\right]$. The associated projected differential equation can be written as

$$\frac{d}{d\tau}\beta(\tau) = \Pi_B\left[R(\tau)^{-1}\overline{R}\left(\overline{\beta} - \beta(\tau)\right)\right],$$

$$\frac{d}{d\tau}R(\tau) = \Pi_R\left[\overline{R} - R(\tau)\right],$$

where matrix $\overline{R} = \underset{m,n\to\infty}{plim}\frac{1}{m}\sum_{t=n}^{n+m-1}E_t\left[w_t x'_t\right]$ and parameter vector $\overline{\beta} = \overline{R}^{-1}\overline{Q}$ ($\overline{Q} = \underset{m,n\to\infty}{plim}\frac{1}{m}\sum_{t=n}^{n+m-1}E_t\left[x_t s_t\right]$).

For the constant gain least squares $(\lambda_t = \lambda)$ **Kushner and Yin** [**2003**] specify assumptions under which sequences $\{\hat{\beta}_t\}$ and $\{R_t\}$ converge weakly (in distribution) to the stable stationary solution of the projected differential equation, i.e., $\{\hat{\beta}_t\}$ and $\{R_t\}$ spend nearly all of their time (asymptotically) in any arbitrarily small neighbourhood of the stationary solution $(\overline{\beta}, \overline{R})$. The assumptions specified in **Kushner and Yin** [**2003**] are very general: the data-generating process $\{s_t, x_t\}$ does not have to be stationary – weak convergence can be proven under any condition that guarantees that the averages of second-order statistics over long time intervals converge. These assumptions do not exclude transient explosive dynamics of the data-generating process.

Unlike the popular Kalman filter, the constant gain algorithm does not rely on complete specification of the data-generating process including the law of motion for time-varying parameters. The constant gain algorithm adapts to abrupt changes as well as to the smooth dynamics of parameters by measuring the responses of a target variable to signal variables. The robustness of the constant gain algorithm does not come without a cost: the parameter uncertainty is greater than in the case of the Kalman filter. Nevertheless, for the mod-

elling of the fiscal reaction function over the period from 1820 to 2014, the robustness to structural changes is of greater importance.

The constant gain least squares approach can also explain large deviations from the stationary solution $\left(\overline{\beta}, \overline{R}\right)$ of the corresponding differential equation. The application of the constant gain algorithm in order to explain the long-run evolution of the fiscal policy in Sweden is described in the empirical part of the paper.

Methodology and data

Empirical model

In this paper, the empirical specification of the fiscal reaction function proposed in **Claeys** [**2008**] is used. This specification is a modification of the fiscal reaction function used by **Bohn** [**1998**; **2008**]. It augments the fiscal reaction function by the output gap and allows for the first-order autocorrelation of the residual term:

$$s_t = \omega + \theta d_t^* + \gamma \tilde{y}_t + u_t,$$

$$u_t = \rho u_{t-1} + \varepsilon_t,$$

where ω is an intercept, $\tilde{y}_t$ is the output gap, and u_t is a residual term that follows a first-order autoregressive process. The autoregressive residual term u_t accounts for implementation lags of fiscal policy resulting from lengthy legislative processes. Combining these two equations gives

$$s_t = \left(1-\rho\right)\left(\omega + \theta d_t^* + \gamma \tilde{y}_t\right) + \rho s_{t-1} + \varepsilon_t. \tag{9}$$

The structural parameters ρ, ω, θ, and γ can be computed from the reduced form

$$s_t = \beta_0 + \beta_1 d_t^* + \beta_2 \tilde{y}_t + \beta_3 s_{t-1} + \varepsilon_t, \tag{10}$$

using the relations: $\omega = \beta_0 / \left(1-\beta_3\right)$, $\theta = \beta_1 / \left(1-\beta_3\right)$, $\gamma = \beta_2 / \left(1-\beta_3\right)$, and $\rho = \beta_3$.

The constant gain least squares algorithm is applied to the time-varying version of equation (9):

$$s_t = \beta_{0t} + \beta_{1t} d_t^* + \beta_{2t} \tilde{y}_t + \beta_{3t} s_{t-1} + \varepsilon_t. \tag{11}$$

Since debt d_t^* and output gap $\tilde{y}_t$ can be endogenous in equations (9) and (10), the instrumental variables method is used both for model estimation under the assumption of constant parameters and for model estimation under the assumption of time-varying parameters. The instrumental variables include lags of the debt d_t^* and the output gap $\tilde{y}_t$.

Data description

The data are collected from the database of the historical monetary statistics (https://www.riksbank.se/en-gb/statistics/historical-monetary-statistics-of-sweden/) of the central bank of Sweden (Riksbank).

The database includes historical annual time series of prices, monetary aggregates, national accounts and fiscal variables. In this paper, annual time series from 1820 to 2014 are used. By 1820 old war debts were written off and since then Sweden has never directly participated in any war. The historical time series cannot be extended beyond 2014 because of changes in definitions and the methodology of data collection.

The definitions of the variables and their transformations are described in Table 1. The output gap is computed using the one-sided Hodrick-Prescott filter that generates recursive estimates of the output gap depending on the current and past observations of the output [**Stock, Watson, 1999**]. For a robustness check, the output gap is also computed using the measure proposed by **Hamilton** [**2018**].

Table 1. Data description

Y_t	real GDP, expenditure approach, mln SEK, base year 2000
P_t	GDP deflator, base year 2000
$P_t Y_t$	nominal GDP, expenditure approach, mln SEK
$P_t S_t$	end-of-year nominal primary surplus, mln SEK
$P_t D_t$	end-of-year nominal debt, mln SEK
$R_t D_{t-1}$	nominal interest payments on government debt, mln SEK
$s_t = \dfrac{P_t S_t}{P_t Y_t} \times 100$	end-of-year surplus-to-GDP ratio, percent
$d_t^w = \left(1 + R_t\right) \dfrac{P_{t-1} Y_{t-1}}{P_t Y_t} \dfrac{D_{t-1}}{Y_{t-1}} \times 100$	start-of-year debt-to-GDP ratio, percent
$\tilde{y}_t = \left(\log\left(Y_t\right) - \log\left(\bar{Y}_t\right)\right) \times 100$	output gap, where $\bar{Y}_t$ is the potential output estimated using the one-sided Hodrick-Prescott or Hamilton filter, percent

Source: Authors' own elaboration.

Figure 1. Primary surplus-to-income and public debt-to-income ratios
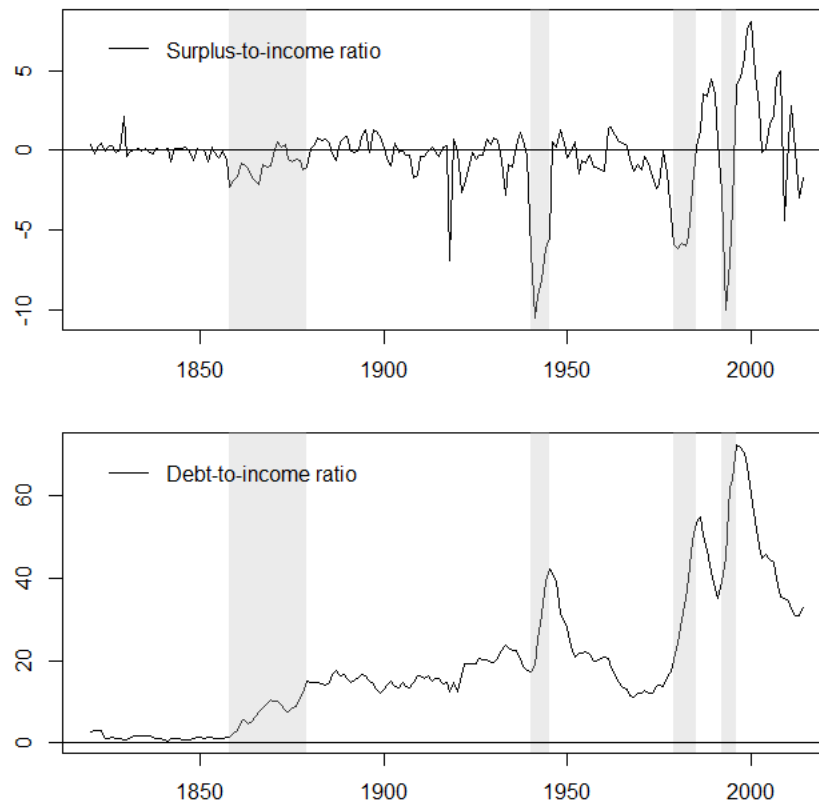
Source: Authors' own elaboration.

Figure 1 shows the dynamics of the ratios of primary surplus and public debt to GDP in Sweden in the 1820–2014 period. The four known episodes of debt expansion are marked with grey areas. The first period occurred in the mid-19th century (1858–1878) and is associated with large infrastructure investments financed by public debt issues (see **Schön** [**1989**]). The Swedish famine of 1867–1869 also contributed to debt expansion during that period, as relief help was partially financed by government borrowings. The second episode of debt expansion lasted from 1940 to 1945 when public debt was used to finance defence expenditure during World War II. The 1979–1985 period followed a second oil price shock when an expansionary fiscal policy response resulted in rising public debt (see **Jonung** [**2015**]). The last episode of debt expansion (1992–1996) was associated with the Swedish Banking Rescue when the government financed the rescue of the banking system by a debt issue (see **Englund** [**1999**]).

The debt expansions followed adverse economic shocks and can be explained by increased government expenditures aimed to mitigate the consequences of adverse shocks. Nevertheless, each debt expansion was followed by fiscal adjustments that reduced the debt-to-income ratio.

Results and discussion

Full-sample estimates

In the first stage, the fiscal reaction function was estimated using the full sample from 1820 to 2014. The estimation was carried out using the two-stage least squares (2SLS) method, where the first lags of the debt-to-GDP ratio and the output gap were used as instruments for the corresponding variables (as in **Claeys** [**2008**]). The full sample estimates for the structural parameters are provided in Table 2:

Table 2. Full-sample (1820–2014) estimates of structural parameters

Parameter/Statistic	Estimate (Standard Error)	
	Hodrick-Prescott Filter	Hamilton Filter
ω	−1.972023* (1.310585)	−2.128492*** (0.064760)
θ	0.076637* (0.042139)	0.081153*** (0.027080)
γ	3.660276** (1.740958)	0.645405** (0.299082)
ρ	0.810457*** (0.051046)	0.708512*** (0.045578)
Breusch-Godfrey test (2 lags)	0.035334	1.908600
Ljung-Box test (5 lags)	4.786800	2.276600
Wu-Hauman test	4.456000**	2.730000*
Andrews supF test	38.433000***	45.244000***

*, **, *** indicate significance for significance levels of 10%, 5% and 1% respectively.

Source: Authors' own elaboration.

For both versions of the fiscal reaction function, including the Hodrick-Prescott and Hamilton measures of the output gap, the null of no residual autocorrelation cannot be rejected in the Breusch-Godfrey and the Ljung Box tests for the 5-percent level of significance, using 2 and 5 lags of residuals. The Wu-Hausman test indicates endogeneity of debt and output gap (for the 5-percent significance level in the case of the Hodrick-Prescott filter and 10 percent in the case of the Hamilton filter).

For the parameter θ, the null hypothesis of zero response ($H_0 : \theta = 0$) is rejected in favour of the alternative of sustainability ($H_0 : \theta > 0$) given the 10-percent significance level for the model using the Hodrick-Prescott measure of output gap and the 1-percent significance level for the model using the Hamilton measure of output gap. Nevertheless, the estimated equation does not pass tests for parameter stability. The null hypothesis of stability in the supF test of **Andrews** [**1993**] is rejected for both versions of the model (see Table 2).

Although the full-sample estimates of the fiscal response function, based on a long time series, indicate long-run fiscal sustainability, these estimates are not stable over time as they average over various fiscal policy regimes.
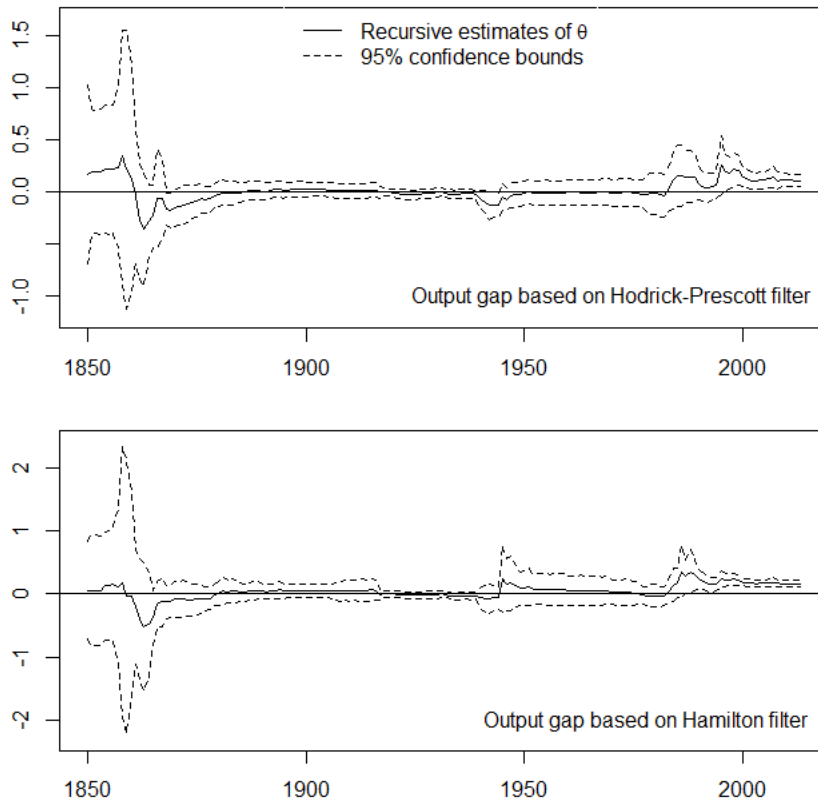
Recursive estimates

The recursive estimates were obtained using the constant-gain instrumental variables estimator based on the equations (7)–(8). The initial estimates, $\hat{\beta}_0$ and R_0 were provided by the instrumental variable estimator applied to equation (9) using the sub-sample from 1820 to 1850. The optimal value of the gain parameter λ was obtained by searching over all $\lambda \in (0,1)$ and was selected so that it minimised the mean square forecast error over the sub-sample from 1851 to 2014 (as proposed in **Branch and Evans** [**2006**]). The optimal value of

λ is equal to 0.015 – it is the constant weight (gain) of a new observation. Past estimates of β_t are discounted with a factor of 0.985.

The projection method implemented in the recursive algorithm (7)–(8) is described in **Ljung and Soderstrom** [**1983**]. Following the theoretical considerations from Section 2, the fiscal response parameter was bounded to a closed interval imposing the restriction $\theta_t \leq 0.99$. The autoregressive parameter was also bounded away from 1: $\rho_t \leq 0.99$. But none of these restrictions was binding, i.e., for every estimation period t parameters θ_t and ρ_t were below the upper bound: $\theta_t < 0.99$ and $\rho_t < 0.99$.
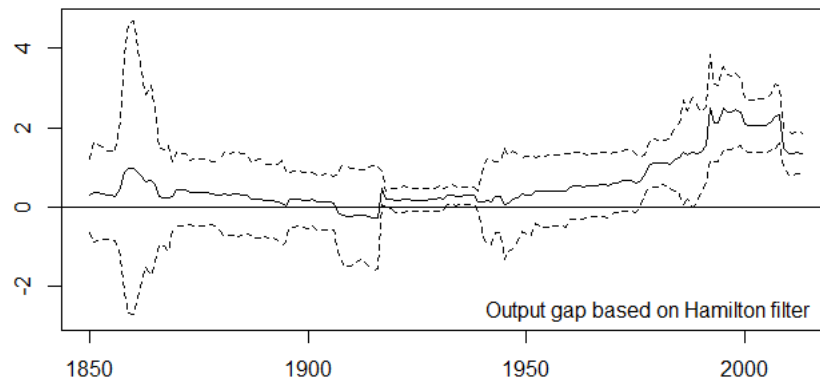
Figure 2. Recursive estimates of fiscal response parameter
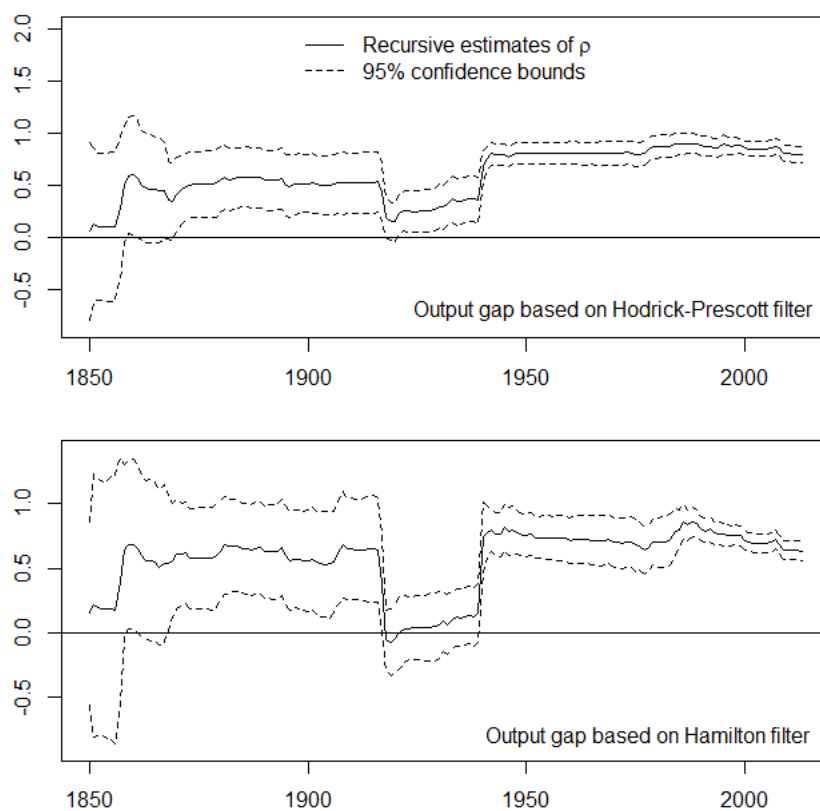

Source: Authors' own elaboration.

Figure 3. Recursive estimates of response to output gap
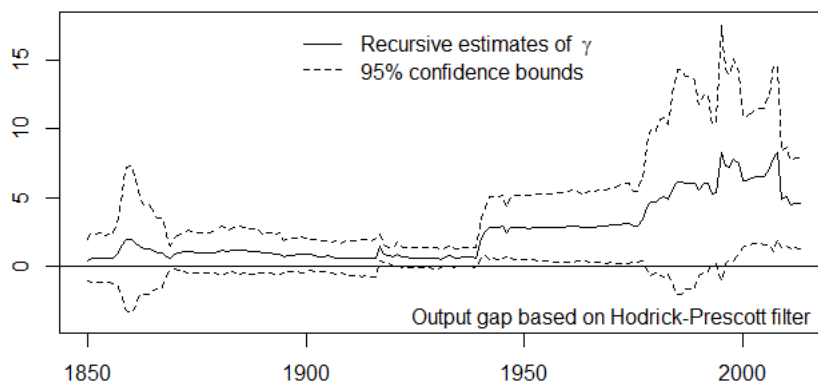
Source: Authors' own elaboration.

Figure 4. Recursive estimates of persistence parameter

Source: Authors' own elaboration.

Figure 2 shows recursive estimates of the fiscal response parameter (θ_t) and recursive 95-percent confidence bounds for one-sided hypothesis testing. The recursive estimate θ_t and its conditional variance are estimates of the fiscal response based on past observations (up to t). The weight of past observations is discounted with a constant factor.

For the majority of observations over the period from 1850 to 2014, the recursive estimates of the fiscal response parameter were not significantly different from zero. The insignificant fiscal response in a given period implies that the current stance of fiscal policy does not meet the sufficient conditions of fiscal sustainability stated in **Bohn** [**1998**].

For the model using the Hodrick-Prescott output gap there are two episodes when the estimated fiscal response parameter θ_t was significantly below the zero line, which means that increasing the debt-to-income ratio was associated with increasing the primary deficit after controlling for cyclical fluctuations of output.

The first episode occurred in 1868–1869 and can be explained by poor harvests and the famine of 1867–1869. The natural disaster caused recession and required additional government expenditures in order to help the victims. The second episode of negative fiscal response occurred in 1940–1944. Although Sweden officially maintained neutrality during World War II, its overseas trade was squeezed, causing a major recession, while military expenditure increased in preparation for possible involvement in the war. For the model using the Hamilton output gap the estimated fiscal response parameter was negative in these sub-periods, but it was not significantly different from zero.

The expansionist fiscal policy continued after World War II, with primary deficits recorded 17 times over the 1946–1973 period. The role of countercyclical fiscal policy increased considerably, as demonstrated by a structural shift in the recursive estimates of the response to the output gap (see Figure 3), while recursive estimates of the response to the debt-to-income ratio were negligible over this period (Figure 2). Nevertheless, the debt-to-income ratio shrank from 42.30 percent in 1945 to 11.91 percent in 1973 because of an accelerated nominal income growth during that period. The fast growth of nominal GDP led to a stabilisation of the debt-to-income ratio even though the fiscal response was weak. The nominal income growth was associated with expansionist fiscal as well as monetary policies (see **Schön** [**1989**]). The debt-to-income ratio decreased as a result of fast real growth as well as accelerated inflation.

The recursive estimates of the persistence parameter ρ_t have shifted upwards since World War II, fluctuating between 0.6 and 0.9 (Figure 4). The increased persistence of primary surpluses (or deficits) can be explained by an increased role of the welfare system and electoral processes in determining the fiscal policy stance. Fiscal policy is an outcome of lengthy legislative procedures in an elected parliament that may not be able (or willing) to dismiss decisions made by previous legislatures.

The largest debt expansions occurred in 1979–1985 in the aftermath of an oil price shock in 1979 and in 1992–1996 after the Swedish Banking Rescue (see **Englund** [**1999**]). These expansions are associated with an increasing role of the output gap in explaining the dynamics of the primary surplus (Figure 3). For the reaction function including the Hodrick-Prescott output gap, the dynamics of the recursive fiscal response (Figure 2) shows that both crises were followed by stabilising policies but the significance of recursive estimates θ_t is only detectable from 1996 when fiscal consolidation was achieved and a new fiscal framework, based on explicit fiscal rules, was put in place. The results differ for the reaction function using the Hamilton output gap, as the recursive estimates of the fiscal response parameter are significantly different from zero from 1987 onwards.

The analysis described above was extended by considering total government liabilities including public debt as well as the stock of base money. This included the full-sample estimation and recursive identification of the policy reaction function for total liabilities. Although the recursive estimates of the policy response parameter for total liabilities behave similarly to the estimates of the corresponding parameter for public debt, the dynamics differs in some sub-periods, indicating that the fiscal and monetary authorities coordinated policy to stabilise total liabilities rather than public debt. The detailed results of this analysis are presented in Appendix B.

Conclusions

In this paper, the long-term evolution of fiscal policy in Sweden is examined using a recursive identification framework. A time-varying fiscal response function is estimated with the constant gain least squares method, using instrumental variables, which makes it possible to track changes in the policy response to the level of public debt.

Inferences are based on a proposition presented in the paper. This proposition specifies a set of sufficient conditions for fiscal sustainability, which is an alternative to sufficient conditions proposed by **Bohn** [**1998**] and **Canzoneri et al.** [**2001**]. The conditions discussed in this paper allow for a transient negative response of the primary surplus to the debt-to-income ratio, which is often observed in empirical studies.

For the majority of observations over the examined period from 1850 to 2014, the response of the primary surplus to the debt-to-income ratio was not significantly greater than zero. The stabilisation of the debt-to-income ratio resulted from a mix of discretionary fiscal and monetary policies, fast economic growth and inflation shocks.

In the baseline model, the fiscal response parameter was positive and significantly different from zero since 1996 when explicit fiscal policy rules were put in place in the aftermath of the major fiscal crisis caused by the Swedish Banking Rescue. The consistently sustainable fiscal policy emerged as an outcome of long-run learning processes where adverse economic and political shocks caused debt expansions and forced fiscal authorities to adapt fiscal policy to new economic conditions.

The main theoretical assumption of long-run fiscal sustainability is that the private sector expects a stabilising fiscal response to debt expansions in the future. The analysis of historical time series for Sweden demonstrates that defaults can be avoided even if the fiscal response is small or negative for prolonged periods of time. This can be explained by the ability to maintain the trust of the private sector that debt will sooner or later be stabilised by appropriate policy adjustments and shows the importance of reputation and expectations in debt management.

Future research could be aimed at developing a complete theoretical framework combining adaptive learning of fiscal authorities and recursive identification of the policy function. Another interesting direction for research is the analysis of interactions between fiscal and monetary policies in stabilising government liabilities.

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Appendix A: Proof of Proposition 1

In order to prove that the policy defined by (6′) satisfies the sustainability condition (5), we have to prove that $z_T = E_t \left[u_{t,T} \prod_{j=t+1}^{t+T} (1+y_j) d_{t+T} \right]$ converges to zero as $T \to \infty$. That is, for any $\varepsilon > 0$, there must be a value N^* such that $|z_T| < \varepsilon$ for all $T \geq N^*$. Define $\delta_{t,T} = \prod_{j=t+1}^{t+T} (1+r_j)$, $\gamma_{t,T} = \prod_{j=t+1}^{t+T} (1+y_j)$, $\phi_{t,T} = \prod_{j=t+1}^{t+T} (1-\theta_j)$. By iterating on (4) and (6′), we obtain

$$d_{t+T} = \delta_{t,T} \frac{\phi_{t,T}}{\gamma_{t,T}} d_t - \sum_{i=1}^{T} \delta_{t+i,T} \frac{\phi_{t+i,T-i}}{\gamma_{t+i,T-i}} \mu_{t+i} \qquad (A1)$$

Using the relations $z_T = E_t \left[u_{t,T} \gamma_{t,T} d_{t+T} \right]$ and (A1) we get

$$z_T = E_t [u_{t,T} \delta_{t,T} \phi_{t,T} d_t] - \sum_{i=1}^{T} E_t \left[u_{t,T} \delta_{t+i,T} \frac{\phi_{t+i,T} \gamma_{t,T}}{\gamma_{t+i,T}} \mu_{t+i} \right].$$

The modulus inequality and the triangle inequality imply that

$$|z_T| \leq E_t \left| u_{t,T} \delta_{t,T} \phi_{t,T} d_t \right| + \sum_{i=1}^{T} E_t \left| u_{t,T} \delta_{t+i,T} \frac{\phi_{t+i,T} \gamma_{t,T}}{\gamma_{t+i,T}} \mu_{t+i} \right|. \qquad (A2)$$

Consider the first summand in the right-hand side of inequality (A2). Assumptions 1) and 2) in Proposition 1 imply that this term can be written as

$$E_t \left| u_{t,T} \delta_{t,T} \phi_{t,T} d_t \right| = E \left[u_{t,T} \delta_{t,T} \right] \phi_{t,T} d_t$$

for any non-negative initial debt-to-income ratio d_t. Assumption 3) implies that $E_t \left[u_{t,T} \delta_{t,T} \right] = 1$. Consider $\phi_{t,T} = \prod_{j=1}^{T} (1-\theta_{t+j})$. Assumption 1) implies that there is N such that $\theta_{t+k} > \underline{\theta} > 0$ for all $k \geq N$. In the decomposition $\phi_{t,T} = \prod_{j=1}^{N-1} (1-\theta_{t+j}) \prod_{k=N}^{T} (1-\theta_{t+k})$ the first term is bounded, $\prod_{j=1}^{N-1} (1-\theta_{t+j}) < \infty$, and the second term converges to zero, $\lim_{T\to\infty} \prod_{k=N}^{T} (1-\theta_{t+k}) = 0$ because $0 < (1-\theta_{t+k}) < 1$ for all $k \geq N$. This implies that $\lim_{T\to\infty} \phi_{t,T} = 0$ – the first summand in the right-hand side of inequality (A2) converges to zero.

Consider the second summand in the right-hand side of inequality (A2). Observing that $u_{t,T} = u_{t,i} u_{t+i,T}$ and $\gamma_{t,T} / \gamma_{t+i,T} = \gamma_{t,i}$ for any $i = 1,\ldots,T$, and using Assumptions 1), 2) and 4) we get

$$\sum_{i=1}^{T} E_t \left| u_{t,T} \delta_{t+i,T} \frac{\phi_{t+i,T} \gamma_{t,T}}{\gamma_{t+i,T}} \mu_{t+i} \right| \leq \omega \sum_{i=1}^{T} \phi_{t+i,T} E_t \left[u_{t,i} \gamma_{t,i} \mu_{t+i} \right],$$

where $\omega < \infty$. If Assumptions 2) and 5) hold, then $\lim_{i\to\infty} E_t \left[u_{t,i} \gamma_{t,i} \mu_{t+i} \right] = 0$, i.e., for any $\epsilon > 0$ there is M such that $E_t \left| u_{t,i} \gamma_{t,i} \mu_{t+i} \right| < \epsilon$ for all $i \geq M$. Assumption 1) implies that there is N such that $\theta_{t+i} > \underline{\theta} > 0$ for all $i \geq N$. This

means that as *T-i* tends to infinity $\phi_{t+i,T} = \prod_{j=t+i+1}^{t+T}\left(1-\theta_j\right)$ is dominated by $\left(1-\underline{\theta}\right)^{T-i}$: $0 < \phi_{t+i,T} < \left(1-\underline{\theta}\right)^{T-i}$ for $i \geq N$. As $\lim_{T\to\infty}\sum_{i=N}^{T}\left(1-\underline{\theta}\right)^{T-i} = 1/\underline{\theta}$ is finite, the sum of the dominated series is also finite: $\lim_{T\to\infty}\sum_{i=N}^{T}\phi_{t+i,T} =$
$= \varphi < \infty$. Let $N^* = max\{M,N\}$. Consider a decomposition

$$\sum_{i=1}^{T}\phi_{t+i,T}E_t\left[u_{t,i}\gamma_{t,i}\mu_{t+i}\right] = \phi_{t+N^*,T}\sum_{i=1}^{N^*-1}\phi_{t+i,N^*}E_t\left[u_{t,i}\gamma_{t,i}\mu_{t+i}\right] + \sum_{i=N^*}^{T}\phi_{t+i,T}E_t\left[u_{t,i}\gamma_{t,i}\mu_{t+i}\right] \quad .$$

The first component in the decomposition above converges to zero as $\lim_{T\to\infty}\phi_{t,T} = 0$ and $\sum_{i=1}^{N^*-1}\phi_{t+i,N^*}$ $E_t\left[u_{t,i}\gamma_{t,i}\mu_{t+i}\right] < \infty$. For the second component, we have $\sum_{i=N^*}^{T}\phi_{t+i,T}E_t\left[u_{t,i}\gamma_{t,i}\mu_{t+i}\right] \leq \varphi\epsilon$. This implies that for large enough *T* the absolute value of z_T will be less than ε provided that one picks $\epsilon < \varepsilon/(\omega\varphi)$. QED.

Appendix B: Analysis of total liabilities

Consider a period-by-period government budget identity that includes changes in the stock of base money:

$$P_{t+1}D_{t+1} = \left(1+R_{t+1}\right)P_tD_t - P_{t+1}S_{t+1} - \left(P_{t+1}M_{t+1} - P_tM_t\right), \tag{B1}$$

where P_tD_t is the end-of-period nominal debt, $P_{t+1}S_{t+1}$ is the nominal primary surplus (taxes minus non-interest spending, $P_{t+1}S_{t+1} = \left(P_{t+1}T_{t+1} - P_{t+1}G_{t+1}\right)$ accumulated over period *t*, R_{t+1} is the nominal rate of return on government securities and $\left(P_{t+1}M_{t+1} - P_tM_t\right)$ is the change in the stock of nominal base money. Let us rescale the variables in equation (B1) and solve this equation with respect to the ratio of total liabilities to GDP:

$$\frac{D_{t+1} + M_{t+1}}{Y_{t+1}} = \left(1+R_{t+1}\right)\frac{P_tY_t}{P_{t+1}Y_{t+1}}\frac{D_t}{Y_t} - \frac{S_{t+1}}{Y_{t+1}} + \frac{P_tY_t}{P_{t+1}Y_{t+1}}\frac{M_t}{Y_t}.$$

Figure B1 shows the dynamics of base money-to-output, debt-to-output and total liabilities-to-output from 1850 to 2014.

Figure B1. Dynamics of government liabilities
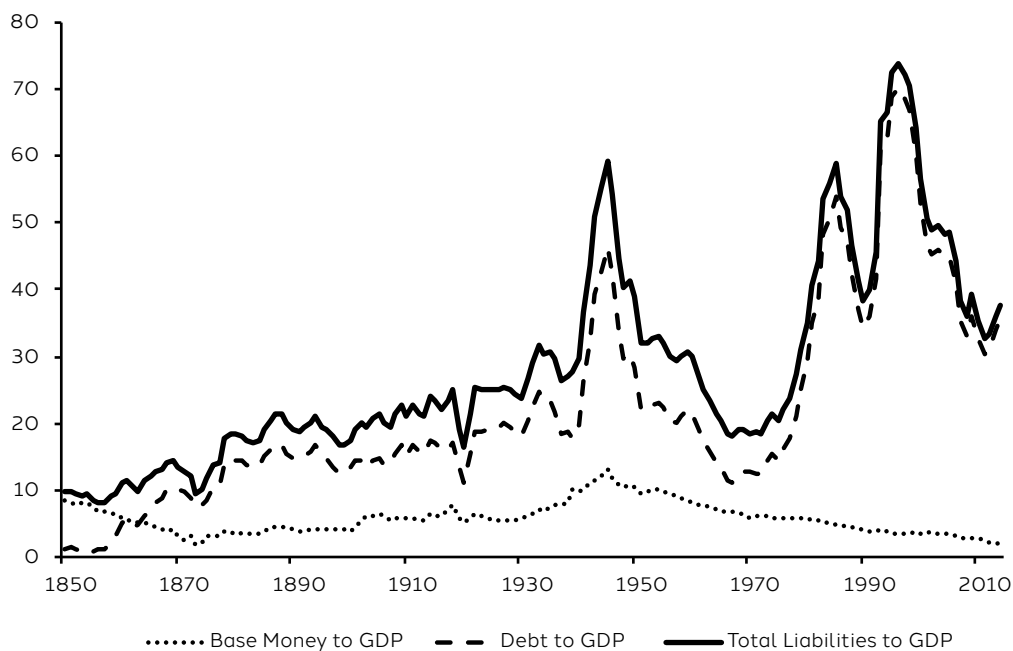
Source: Authors' own elaboration.

The dynamics of the ratio of base money to nominal income contributed to stabilising total liabilities in the middle of the 19th century and in the aftermath of WWII, but it also contributed to the fast growth of total liabilities in the inter-war period and during WWII.

Define the real rate of interest, r_{t+1}, by identity $(1+r_{t+1})=(1+R_{t+1})\dfrac{P_t}{P_{t+1}}$, the real growth rate of GDP, y_t, by identity $(1+y_{t+1})=\dfrac{Y_{t+1}}{Y_t}$ and the ratio of base money to GDP by $m_t=\dfrac{M_t}{Y_t}$. The ratio of total liabilities to GDP, $w_{t+1}=\dfrac{D_{t+1}+M_{t+1}}{Y_{t+1}}$, can be expressed as

$$w_{t+1}=\frac{1+r_{t+1}}{1+y_{t+1}}w_t-s_{t+1}^{*}, \tag{B2}$$

where $s_{t+1}^{*}=s_{t+1}-\dfrac{r_{t+1}}{1+y_{t+1}}m_t$ is the primary surplus ratio corrected for central bank transfers to fiscal authorities. The equation (B2) for total liabilities is analogous to the equation (4) for debt. The policy reaction function for total liabilities, analogous to the reaction function (6), can be written as

$$s_{t+1}^{*}=\theta w_{t+1}^{*}+\mu_{t+1}, \tag{B3}$$

where $w_{t+1}^{*}=\dfrac{1+r_{t+1}}{1+y_{t+1}}w_t$ denotes the debt-to-income ratio at the start of period $t+1$, μ_{t+1} is a component representing other (temporary) determinants of s_{t+1}^{*}. The empirical specification of equation (B3), analogous to equation (9), can be written as

$$s_t^{*}=(1-\rho)\left(\omega+\theta w_t^{*}+\gamma\tilde{y}_t\right)+\rho s_{t-1}^{*}+\varepsilon_t.$$

The full-sample or recursive estimates of structural parameters ρ, ω, θ, and γ can be obtained using the methods described in the main part of the paper. The full-sample estimates are given in Table B. As for the debt, the reaction function is estimated using output gap measures based on the one-sided Hodrick-Prescott filter and the Hamilton filter. For both versions of the constant-parameter reaction function the hypothesis of long-run sustainability cannot be rejected at the 5-percent level of significance. Nevertheless, both versions of the reaction function do not pass the the Andrews stability test assuming the 5-percent level of significance.

Table B. Full-sample (1820–2014) estimates of structural parameters

Parameter/Statistic	Estimate (Standard Error)	
	Hodrick-Prescott Filter	Hamilton Filter
ω	−2.368942* (1.310585)	−2.628319*** (0.853316)
θ	0.083151* (0.045677)	0.090830*** (0.029657)
γ	3.621146** (1.742039)	0.628516** (0.297472)
ρ	0.811017*** (0.051210)	0.710215*** (0.045578)
Breusch-Godfrey test (2 lags)	0.055598	2.051200
Ljung-Box test (5 lags)	5.185000	2.020000
Wu-Hauman test	4.364000**	2.960000*
Andrews supF test	37.132000***	39.422000***

*, **, *** indicate significance for the significance level of 10%, 5% and 1% respectively.
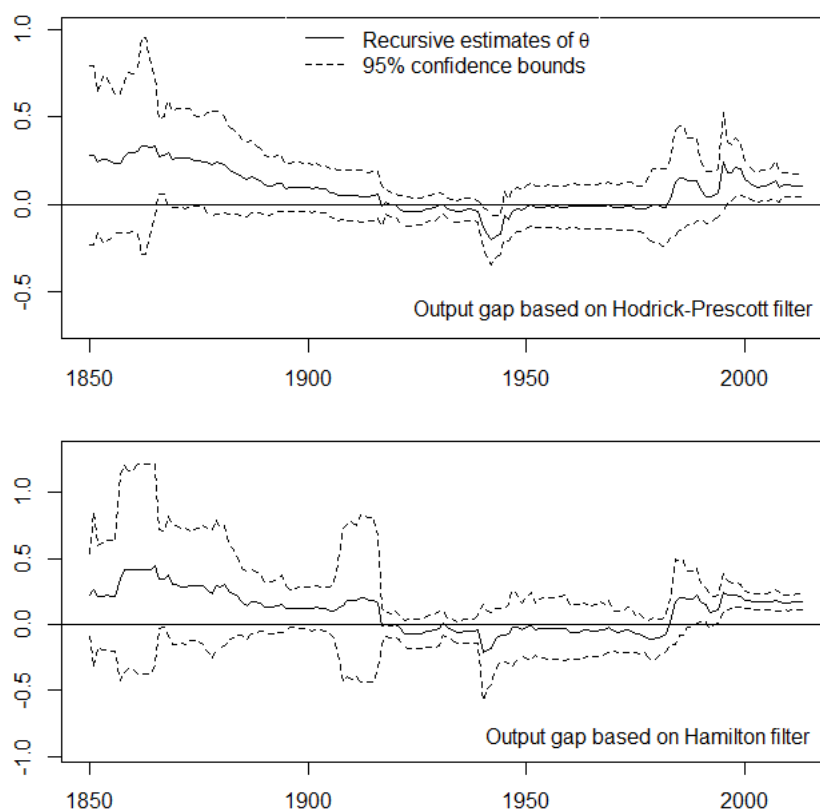
Source: Authors' own elaboration.

Figure B2 shows the recursive estimates of the fiscal response parameter (θ) and recursive 95-percent confidence bounds for the reaction function including the Hodrick-Prescott and the Hamilton estimates of the

output gap. Both figures show qualitatively similar results with the recursive estimates of the response parameter being positive from 1850 to 1917. However, because of large estimation errors at the beginning of the sample, these estimates are not significantly different from zero for most observations from 1850 to 1917. The recursive estimates of the response parameter are below zero for most observations from the end of WWI to the beginning of the 1980s. After the fiscal and monetary reforms of the mid-1980 s and early 1990s, the response parameter stabilised at positive values significantly above zero.

The qualitative difference between the results in this appendix and the baseline results in the paper concerns the dynamics of the response parameter in the 19th century: as the ratio of base money to output was decreasing from the 1850s to the 1870s, the response parameter for total liabilities was positive, while the response parameter for the debt-to-output ratio fell below zero during structural reforms and large government investments financed by debt issues in the 1860s and 1870s.

Figure B2. Recursive estimates of response parameter

Source: Authors' own elaboration.