Andrzej Buszko
Warmia and Mazury University in Olsztyn
ORCID ID: 0000-0003-0600-4646

Does the shadow economy violate
the Walras equilibrium?

ABSTRACT

The paper is devoted to examining the role of the shadow economy in the context of market equilibrium. The Walras theorem was used in order to highlight the role of the shadow economy in market equilibrium violation. Habitually, a majority of theories as well as models focus just on observed variables and latent factors are omitted. The shadow economy is an unprejudiced typical case in this respect. It exists in any economy and it very often influences market mechanisms significantly. A mathematical model was employed to exhibit dependence between the shadow economy and market equilibrium. Whenever the shadow economy is ignored in modeling market equilibrium, the market is proved to be balanced. The situation changes when the shadow economy is taken into account. Market equilibrium is difficult to be achieved.

Keywords: shadow economy, market, equilibrium, model
JEL Classification: C10, C15, C20

Introduction

The shadow economy exists in any country and just varies in its level and category [Godfrey, 2011]. Even though the main roots and effects of the shadow economy are generally recognised, still little research is devoted to its impact on market equilibrium as well as economic theory in particular. It should be emphasised that the shadow economy violates theories of economy in many ways. This is mainly the case since that important part of real economy is ignored [Schneider, Klinglmair, 2004]. In many countries, especially less developed ones or under transformation, the shadow economy constitutes more than 25% of GDP [Schneider, 2015].

Just a few researchers assume illegal activity as a key actor fostering market mechanisms and the concepts of any social sciences. Theory is to be a crucial approach to explain real behaviour of both individuals and companies [Kawulich, 2009]. Theory helps us to clarify market mechanisms and identify the role of economic categories in any science as well [McClelland, Fine, 2008]. Theory usually comes first and then it is verified by data and proposed mathematical models. Mathematical models come after mathematical modelling. They can describe both the real and conceptual world. Thanks to the quantitative approach, any idea may be explained more thoroughly [Blum et al., 2007]. Moreover, modelling can be used to strengthen the decision-making process at the macro and micro level. If this happens, theory is grounded and occupies a sound place in any science.

The roots of the shadow economy are numerous and they are connected mainly with a high level of taxes, contradicted law system, invalid institutional order, high level of unemployment, business cycles, organised crime, history and national culture [Gutmann, 1977; Hatipoglu, Ozbek, 2007; Ihrig, Moe, 2004; Rosser Jr, Rosser, Ahmed, 2000]. Even though there are some academics voting for positive effects of the shadow economy, overall, this economic category has proved to be devastating for social activity. The shadow economy hampers development, innovation, creativity, activates criminality, corruption, budget deficit and in general worsens the business environment [Giuliano, Ruiz-Arranz, 2009; Lackó, 1996; Quarishi, 2016; Tanzi, 1983].

Commonly, key assumptions for development and analyses of economic processes neglect shadow economy influences. They just focus on observed and legal variables [Hicks, 1959; Karakoc, 2017; Lechner, 2019; Lucas, 1997; Lumineau, Quelin, 2012; Menger, 2004; North, 1990; Seade, 1977; Stiglitz, 2003; Veblen, 2009; Williamson, 2007; Winkler, 2009]. This attitude is typical in nearly all recognised research. Therefore, it is not surprising when theory does not meet reality, especially in those countries where the shadow economy occupies a sound part of the economy. This is because investigators consider a minor part of the economy. Persuasive mechanisms are ignored.

Even though models and theory are used interchangeably, they have not only two different meanings but play dissimilar roles in scientific research. Theory is an objective set of beliefs which aim to explain reality [Leijonhufvud, 1997]. On the contrary, models are constructed to prove theory mechanisms. Whenever they cannot do it, they are supposed to create an idea

for a new paradigm. Commonly, models refer to crucial aspects of any theory assumptions. It is probably arguable, but I vote for testing models and theories verification, as nothing exists forever.

The Walras theorem and the shadow economy performance

Equilibrium is a fundamental economic concept. It attracts attention of many scholars, academics, and researchers. This category is applied in other sciences like biology, physics, medicine, sociology, psychology, and even in philosophy. Equilibrium plays an essential role not only in the mainstream of economic sciences but relates to any discussion on improving market failure and conducting reforms. That way this category affirms transformation processes in many countries, especially in those where reforms have proved to be successfully conducted. Faltering evolution of economics from a principally diversified set of disciplines to a unified branch of modern science is based on a meticulously accepted scheme. This change was supported by protagonists of several Nobel prizes, starting already in the 1950s. They employed both the mathematical and psychological approach mainly, albeit overall market equilibrium has occupied a vital grabbing part in the history of modern economics and goes much beyond its formal treatment. One of the first initiators of economic equilibration as well as market equilibrium was Walras. He was the first who formalised the general market equilibrium model. His approach was based on mathematical treatments in a purely competitive market-oriented economy. Walras focused on irrevocable disequilibrium behaviour and capital accumulation. Such a market equilibrium is thus track dependent; this is due to the fact that this model refers to exchange economies as well as the fact that mixed extensions of finite games have an odd number of equilibria, provided that certain regularity conditions are fulfilled [Zierhut, 2020]. In other words, Walras's theory may be characterised by the legal part of the economy, free competition, uniform prices, the public sector (government intervention), international trade but it does not address the role the shadow economy. Walras developed exchange economy assumptions with its enlargement through production associated with capital formulation [Davar, 2005]. Walras believed in market equilibrium achieved by separate commodity markets when effective demand was equal to effective supply [Schumpeter,1954]. By the way, this theorem may be not valid whenever the problem of the shadow economy is raised. The demand and supply from the shadow economy can disturb the legal market mechanisms and the observed equilibrium is hardly achieved. Additionally, the shadow economy may interrupt the circulation of money and credit motion. Money from illegal activity can flow to legal markets unexpectedly and quite often at a large scale. In such a case the determination of prices and raw materials as well as final goods circulations will be under invisible pressure, so the general law of capital equilibrium formulation may start to be questioned [Walras, 1954]. Furthermore, the shadow economy can be used as a tool for arranging speculative bubbles. The bigger the shadow economy level, the more speculative

bubbles can occur and interfere with market equilibrium. This is done especially whenever real and financial markets are discussed. Money from illegal activity jumps into financial markets skillfully and it is very difficult to be detected [Button, 2014]. So, taking those theoretical assumptions into consideration, Walras's theory may stay irrelevant to reality. I do believe that some opponents to the Walras theorem like Pareto [1968], Hicks [1967], even Lange [1942, focused on more persuasive factors shaping market equilibrium than Walras did. But all of them ignored the impact of the shadow economy.

It should be stressed that companies select their asset payoffs strategically. Managers focus on present income; profit maximisation is their natural objective. Quasilinear utility guarantees a single-valued mapping from choices to market prices. In this framework attempt, closer attention was not put to incomplete markets, even though some Walrasian followers postulated that the more incomplete market becomes, the larger set of equilibria rises. Incomplete markets occupy more attention since they are realistic [Akerlof, 1970]. I would argue that complete markets like the concept of Arrow and Debreu do not exist, and they are just handy for some general theoretical divagation [Arrow, Debreu, 1954]. Nowadays the shadow economy is supposed to be considered as a very important feature creating incomplete markets.

An interesting approach, similar to Walras's way of thinking was provided by Bertrand. He considered a duopoly market, where companies were price-oriented. Firms competed on prices, producing standardised goods at undistinguishable and persistent marginal costs. That is why consumers preferred to buy products at the lowest price offered by the company. Just to follow this idea, companies minimise the cost until the level of marginal costs. In this way market equilibrium can be achieved, but on the other hand, companies will not be rewarded by profits earning [Dastidar, 1995]. In the longer run firms face the problem with sound financial standing and their insolvency starts to be a main concern. This situation is coined 'Bertrand's paradox.' Developing this idea, it might be stated that market equilibrium (based upon Bertrand's vision) is just an attempt to describe a not characteristically stable situation of the market. It is hard to believe that even two competitors are symmetric in their action and the Bertrand game admits a unique coalition-proof Bertrand equilibrium. The fundamental assumption should be referred to discrepancy of the company's action. So, the key question is devoted to the speed of price adjustment. For some companies, even though they produce the same products (like their competitors), have similar technology and staff, their market price modification can be slower than for the others. Quicker adopters earn more than just followers. Additionally, some of them may consider the shadow economy performance. They cannot compete officially, and illegal activity may be the only choice. So, the general idea of Bertrand's model is supposed to be modified in the following way.

There is a set of companies numbered by N {1,…N} with N ≥ 1. Price makers deliver their products or services in a certain period of time, ordered by rising cost of production. This time varies from 0 to T, T > 0. The market demand is D as well as the cost of production Cp is equal, it means:

$$D = Cp$$

Each company's (price-maker) cost of production function is unremitting, positive into-nation, strictly convex and meets the formula Cp(0) = 0.

The market demand is not split (as Routledge suggests) but devoted to the lowest offered market price [Routledge, 2013].

There is a set of buyers numbered by 1, …,A with A ≥ 1 delivered by price-markers numbered by 1,…, P with P ≥ 1.

Let S be a supply level S ≥ 0

Assumption 1

Therefore, each company offers different prices but a tendency to market equilibrium is observed, hence:

$$\acute{A} = \lim_{p \to Cp} \frac{Ni}{n}, \; ni \; nj$$

So, each market entity exploits its chances to make a profit in a certain time and performs based upon the following formula:

$$\tilde{N} = \lim_{t \to tk} min \frac{\tilde{N}xi}{N}, \; Cp \, (0) = 0$$

If we assume that the vector of costs is given, hence the performance of the company might be foreseen, and the following formula related to the Bertrand market equilibrium can be formulated:

$$S(N) = \begin{cases} S(ni) \text{ for } ni < nj \text{ with } \lim_{cj \to ci} Cn \\[2mm] \dfrac{1}{n}S \text{ for } ni = nj \\[2mm] \leq 0 \text{ for } ni > nj \text{ with } \lim_{ci \to cj} Cn \end{cases}$$

Focusing on a single company operating in a legal industry, it gains the profits until its costs of production are lower than the other ones. Such a statement might be in a way discussable, since the firm's performance depends on the market size. If the market is big and even some producers cannot meet demand, there is space for those with higher costs. But this situation is not an everlasting one, just for the time being. It should be presumed that companies with higher costs of production in a homogenous market try to reduce their costs of production, otherwise if they want to survive, they switch to the shadow economy activity. Those companies are likely to hide some costs or escape from levies, taxes supposed to be paid. The border line of such a step is described by this equation:

$$S\,(i)[ni, nj] = \frac{1}{N}S \; \text{ for } \; ni = nj$$

Therefore, in fully competitive markets the shadow economy performance is supposed to be taken into account. Moreover, excluding this economic category, all real market mechanisms and company behaviours cannot be explained correctly.

In addition, companies (those willing to stay in the official part of the economy) try to avoid homogenous production and compete just on prices only. They seek for diversification and other tools to gain market advantage. Those tools can be classified as legal and illegal ones, tightly connected with the shadow economy performance, and creating incomplete markets. When it comes to the shadow economy and incomplete markets creation, market equilibrium may be devastated by corruption, rent-seeking behaviour, and misleading transaction costs.

Assumption 2

Given a set of companies A and B operating in the legal economy A {1,…A} with A ≥ 1, B {1, … B} with B≥1, and a set of companies A˚ and B˚ active in the shadow economy sphere A˚ {1, … Å} with Å ≥ 1, B˚ {1, … B˚} with B˚ ≥ 1, respectively, a single company has its own strategy Š {1, … Š} with Š ≥1 and gains economic benefits Q {1,…Q} with Q ≥ 1. Benefits are understood not only as pure profit, but it can be considered as income for company owners, value increase of the company, share of the market, payrolls for employees, etc. All of them run their business at a certain level of risk R, calculated by themselves, hence the risk level constitutes set R {0,…R}, associated with the size of shadow economy SE.

Each company, no matter if from the legal or the shadow economy sphere, tries to maximise its economic benefits based upon the employed strategy and minimise the risk.

That is why some companies will stay in the legal economy, since they are hesitating to increase the risk of activity. This situation remains unchangeable, even if their lawful activity is accompanied by a relative high level of the shadow economy performance. The same issue applies to illegal motion. Some companies are not going to be traced and they will stay in the shadow economy area. In this way they attempt to capture a low level of risk. The effects of the company performance can be evaluated within a certain period of time and illustrated by an adjustable function. It this case I propose:

$f(x) = \sin x$ with adjusted parameters, taking into consideration amplitude, phases, and frequency.

The effects of the company's business activity vary, especially in a very unpredictable environment and due to the impact of the shadow economy performance. That is why from time to time business activity goes up and down. This is a characteristic trend, applying to any company, functioning in the shadow or legal economy. There is not a single strategy that can guarantee enduring and increasing profits all the time. Let us refer to the l'Hospital postulation:

$$\lim_{x \to a} f(x) = 0$$

$$\lim_{x \to a} g(x) = 0$$

$$\text{and } \lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Therefore, market equilibrium is supposed to be considered in a different way, employing other factors as well. For each company there is a possibility of operating legally or in the shadow economy, so the $sin(x)$ function may be foreseen with adjusted parameters, including maximum or minimum economic benefits, maximum or minimum risks and the adopted strategy.

If the situation with the shadow economy level (*SE*) changes, the business opportunities are accordingly modified. The $sin(x)$ function with adjusted parameters is already taken into consideration due to the amplitude, phases, and frequency. Amplitude, phases, and frequency are shaped by the level of the shadow economy, risk, and benefits. Whenever illegal economic benefits are going up, in other words they are improved, the risk of such an operation is going down but meanwhile legal economic benefits are worsened, then the shadow economy is going up. More entities are interested in operating in the framework of the shadow economy.

When the shadow economy level increases, there are better opportunities for market entities from the group coined A˚ and B˚. The less market space is provided for legal operating firms, their economic benefits are reduced. A contrary case is applied for the shadow economy. Entities that constitute this group can maximise benefits associated with risk lowering.

Contrary to that is the situation when the shadow economy ia going down. Companies belonging to the legal group are increasing the activity and those from the shadow economy are minimsing their motion, since their risk of identification is increasing significantly and economic benefits are going down. Legal entities are not afraid of lawful risks.

As soon as the shadow economy is going down, more market space for legal operations is foreseen. Companies from the shadow economy reduce their performance, the risk for them is higher. Authorised firms earn more, the risk of their activity is going down. Moreover, companies from the shadow economy try to switch their activity to legal industries and they are more willing to cooperate with the sanctioned, legally functioning entities.

Taking all the assumptions into account, the following model of market equilibrium can be presented.

Model

Demand is characterised by the following formula:

$$D(x) = -ax + b$$

Supply is categorised by:

$$S(x) = cx + d$$

Where *S* is supply, *D* – demand, *x* – price, *a, b, c, d* – formula parameters

If we assume that

$$f'(xo) = \lim_{x \to xo} \frac{f(x) - f(xo)}{x - xo}$$

we can present market equilibrium referring to the Walras theorem as:

$$f'(xo) = E(D - S), \text{ where E is price elasticity.}$$

It means that the price increase is correlated with the difference between demand and supply. Taking all the assumptions together, the Walras model can be defined as follows:

$$\begin{cases} D(x) = -ax + b \\ S(x) = cx + d \\ f'(xo) = E(D - S) \end{cases}$$

therefore,

$$f'(xo) = E(-ax + b - cx - d)$$

if we take into consideration that:

$$D = E(a + c) \text{ and } S = E(b + d),$$

it should be notified that both D and S are > 0
that the subsequent assumption is supposed to be created:

$$f'(xo) = -Dx + Q$$

if prices change during a certain period of time, the subsequent formula is supposed to be generated:

$$X(t) = C e^{-Dt} + \frac{S}{D}$$

C – is the parameter of equation
If we assume:

$$Xo = C + \frac{S}{D} \text{ hence } C = Xo + \frac{S}{D}$$

and finally, such an equation can be made:

$$X(t) = (Xo - \frac{S}{D})e^{-Dt} + \frac{S}{D}$$

In the market-oriented economy, market equilibrium is settled down when the level of supply meets the level of demand. This presumption can be written as:

$$S = D \text{ and } X(eq) = \frac{S}{D}$$

Where $X(eq)$ is the price equilibrium.

Developing this way of thinking, such a formula can be summarised in the following way:

$$X_{(t)} = [X_{(0)} - X(eq)]e^{-et} + X(eq)$$

Demand is always > 0, and subsequently:

$$\lim_{t \to \infty} e^{-dt} = 0$$

It means that the Walras theorem in the market-oriented economy is still valid, so the price of a good or service tends to equilibrate exponentially. Such a situation is vital without considering the shadow economy existence. If we add the shadow economy category, the situation changes. The demand in the official market can be sometimes 0 or even less than 0. This is because supply can come from the shadow economy, and even all demand might be satisfied from this source. That is why the Walras theorem is violated by the shadow economy and the market equilibrium calculated from the legal perspective could be hard to reach. Finally, market equilibrium is just a tendency shaped by demand and supply factors, not a permanent status. It could be even stated that the market is often imbalanced and seldom market equilibrium appears.

Summary

Market equilibrium is a fascinating economic category but still needs to be examined from many perspectives, including the role of the shadow economy. Even though the shadow economy is a recognised economic category, unfortunately it is studied mainly separately from other factors fostering the real economy. Its impact on market mechanisms is evident but it should be exhibited and evaluated. Such an approach is especially needed whenever the shadow economy occupies a sound part of business activity. If the shadow economy is ignored, all market reforms as well as social targets will be not accomplished. This means the loss of assets spent on reforms. I hope that this initial research opens a new area of discussions related to the role of the shadow economy violating economic theories. It would be necessary to indicate the effects of the shadow economy performance whenever market equilibrium is not met. Additionally, categories and the level of the shadow economies affecting market equilibrium are supposed to be detected as well. Probably a small level of the shadow economy does not disturb stability, it might even support development. Contrary to the situation when the economy faces the problem with a high level of the shadow economy, a given country remains in a permanent chaos. Market equilibrium will be occasionally indicated, then.

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