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Teaching assistant selection in Thailand by using an extended VIKOR based on piecewise linear approximation of fuzzy numbers

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Abstract

Because of the COVID-19 situation, selection for a teaching assistant position to get a TA scholarship in a university in Thailand needs to be performed online by the formed committee. Due to the online process and the limited number of scholarships offered by the university, beyond the face-to-face interview, multiple-criteria decision analysis can help to select a proper student. In this study, we use the extended VIKOR method with fuzzy numbers to help committees to select the students from the applicants. The criteria and the weights of the criteria are provided with the help of committees. Both trapezoidal and triangular linguistic variables are used to find the solution and to observe the range of the possible result. The different weights supporting the strategy of maximum group utility are varied to detect the potential alternatives. The ranking results are also compared with the one obtained from the TODIM approach to illustrate the appropriate alternative.

Keywords: *multi-criteria decision-making, extended VIKOR, teaching assistant, Thailand*

1. Introduction

In Thailand, students in universities, especially graduate students, intend to get a scholarship to study in master's or doctoral degree programs. Therefore, several students apply for a limited number of teaching assistant scholarships in order to pay less tuition fees or waive the tuition fee. Some scholarships in a university provide both tuition fees and personal expenses each month for a teaching assistant until the student graduates. Therefore, universities form a group of committees to select the applicants to get the scholarship with several criteria. Selecting a student to earn a scholarship from candidates is sometimes complicated. In this case, a multi-criteria decision-making (MCDM) method is adopted for the selection process. The multi-criteria decision scheme consists of several methods that help to choose and rank decent alternatives. For example, the analytic hierarchy process (AHP), one of the

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MCDM methods, focuses on the hierarchy of goals, criteria and alternatives, and the pair-wise comparison of decision-making components [2, 5, 19, 27, 28]. Another decision-making method is the technique for ordering performance by Ssimilarity to ideal solution (TOPSIS) which focuses on defining the positive and negative gains to get as close as possible to the positive earn and the farthest away from the negative option. This makes the VIKOR method more practical in analyzing data. For other MCDM approaches, for instance, fuzzy multiple-attribute decision-making (FMADM) [8] uses linguistic variables remodelled into fuzzy numbers to illustrate the uncertainty of the data. ELECTRE method [7, 23] is a decision-making method based on the sets of the concordance and discordance indices and matrices. The decision-making trial and evaluation laboratory (DEMATEL) method [4, 9] is the scheme that needs the perspective of decision-makers for compound problems. Evaluation based on the distance from average solution (EDAS) method [1] employs the average values of appraisal scores to rank alternatives. TODIM method [18, 22] is designed for capturing the attitude of the decision-makers.

The methods were applied by several authors. Irvanizam et al. [6] used both AHP and VIKOR schemes to rank alternatives. They employed AHP for having a pairwise criteria comparison matrix and then VIKOR to rank the alternatives. Irvanizam et al. [8] applied the fuzzy multi-attribute decision-making (FMADM) and simple additive weighting (SAW) methods to select a decent alternative to get a house from the Aceh government through a housing program. Irvanizam et al. [7] used ELECTRE with fuzzy sets to make a decision to distribute proper houses to decent alternatives in Aceh. Irvanizam et al. [9] combined DEMATEL and EDAS approaches to rank alternatives in order to receive help from the government to alleviate poverty. With a group conflict, Chinram et al. [3] presented intuitionistic fuzzy rough–EDAS (IFR- EDAS) scheme including a numerical example to a model. Qin [22] combined the TODIM method with the triangular fuzzy environment to select renewable energy alternatives. Irvanizam et al. [11] used an extended fuzzy TODIM method for multiple-attribute decision-making problems using dual-connection numbers. In this study, we apply the concept of the extension of the VIKOR method with fuzzy numbers to rank the students applying for the teaching assistant position because the VIKOR scheme can weigh and measure the disadvantages and advantages of each criterion in the direction of the current situation for the benefits of ranking alternatives. Moreover, in general, at least two committees are selected to interview the applicants. Then, the extended VIKOR method is needed for this situation. In addition, we study the difference of using the VIKOR scheme with the trapezoidal and triangular linguistic variables to seek for the proper alternative. This is because the trapezoidal fuzzy linguistic variables are satisfied for the problems that are more ambiguous for making a decision than using the triangular linguistic variables. Sometimes we may be unsure of which type to use for each problem. To the author's knowledge, there do not explicitly exist papers clearly comparing the results of these two different linguistic variables with an extended VIKOR scheme. Here we show the solution of both linguistic variables. The results from these two variables are compared in order to determine the appropriate result. Furthermore, we compare our ranking solutions with the TODIM method to grant a decent alternative.

In Section 2, we briefly introduce the VIKOR method and the extended VIKOR scheme with fuzzy number is presented in Section 3. The finding of the proper alternative using both trapezoidal and triangular linguistic variables is presented in Section 4. The crisp values of the weights of the criteria and decision matrices in Sections 4.1 and 4.2 are applied to the TODIM method in Section 4.3. The ranking result received from the TODIM approach is compared with our solutions. The conclusion is presented in the last section.

2. VIKOR method

The VIKOR method can help a decision-maker to select and rank alternatives for a problem with conflicting criteria. The n alternatives are represented by $A_1, A_2, \ldots, A_n$. To select a proper alternative, criteria are set depending on the problem. In this study, the m criteria of the problem are denoted by $C_1, C_2, \ldots, C_m$. Let f_{ij} be the value based on the jth criterion and the ith alternative. The VIKOR scheme is applied from the L_p-metric [25]:

$$L_{p,i} = \left(\sum_{j=1}^{m} \left(\frac{f_j^* - f_{ij}}{f_j^* - f_j^-} \right)^p \right)^{1/p}, \quad 1 \le p \le \infty,\ i = 1, 2, \ldots, n$$

where $f_j^* = \max_i f_{ij}$ and $f_j^- = \min_i f_{ij}$ if the jth criterion is a benefit criterion and $f_j^* = \min_i f_{ij}$ and $f_j^- = \max_i f_{ij}$ if the jth criterion is a cost, negative, criterion [26]. The alternatives are ranked from the values of $L_{1,i}$ (S_i), $L_{\infty,i}$ (R_i) and Q_i from the equations [25]

$$S_i = \sum_{j=1}^{m} w_j \left(\frac{f_j^* - f_{ij}}{f_j^* - f_j^-} \right), \quad R_i = \max_j w_j \left(\frac{f_j^* - f_{ij}}{f_j^* - f_j^-} \right) \tag{1}$$

$$Q_i = \nu \left(\frac{S_i - S^*}{S^- - S^*} \right) + (1 - \nu) \left(\frac{R_i - R^*}{R^- - R^*} \right), \quad i = 1, 2, \ldots, n \tag{2}$$

where w_j are the weights of criteria, $S^- = \max_i S_i$, $S^* = \min_i S_i$, $R^- = \max_i R_i$, $R^* = \min_i R_i$ and ν is a weight for the strategy of maximum group utility compromised when $\nu = 0.5$. The mininum of Q value provides the best alternative $A^{(1)}$ if $A^{(1)}$ satisfies the following two conditions:

1) $Q(A^{(2)}) - Q(A^{(1)}) \ge DQ$,

2) $S(A^{(1)})$ or $R(A^{(1)})$ is minimum,

where $DQ = 1/(n-1)$. If $A^{(1)}$ does not satisfy one of the two conditions, then

1. The set $\{A^{(1)}, A^{(2)}\}$ is the compromise solutions, if $A^{(1)}$ does not satisfy the second condition.
2. The set $\{A^{(1)}, A^{(2)}, \ldots, A^{(N)}\}$ is the compromise solution if the first condition is not satisfied, where N is the maximum number such that $Q(A^{(N)}) - Q(A^{(1)}) < DQ$ (these alternatives are close to each other).

Next, we propose the extended VIKOR scheme with both trapezoidal and triangular fuzzy linguistic variables.

3. Extended VIKOR scheme with fuzzy number

Sometimes, decision-making problems are uncertain. In this case, fuzzy numbers can help to solve the problem. In this study, we use fuzzy membership functions and linguistic variables to classify the significance of the criteria and evaluate alternatives with respect to criteria. Figures 1 and 2 show linguistic variables used to determine the important weight of criteria and to rate alternatives with respect to each criterion.

Figure 1 shows trapezoidal fuzzy linguistic variables, which are characterized by the trapezoidal membership function [24]

$$f(x) = \begin{cases} 0, & x \le a \text{ or } x \ge d \\ \dfrac{x-a}{b-a}, & a < x \le b \\ 1, & b < x \le c \\ \dfrac{d-x}{d-c}, & c < x \le d \\ 0, & d < x \end{cases}$$

A triangular membership function

$$f(x) = \begin{cases} 0, & x \le a \\ \dfrac{x-a}{b-a}, & a < x \le b \\ \dfrac{c-x}{c-b}, & b < x \le c \\ 0, & c < x \end{cases}$$

is shown in Figure 2 as the triangular fuzzy linguistic variables. For more details about triangular and trapezoid membership functions, please see [17]. Figures 1a and 2a show the linguistic variables for the important weight of each criterion beginning with very low and ending with very high, while Figures 1b and 2b illustrate the linguistic variables for rating alternatives with respect to each criterion starting with very poor and end with very good. The meaning of the linguistic variables and numerical scales are shown in Tables 1 and 2. Table 1 shows the numerical scale of Figure 1 and Table 2 demonstrates the numerical scale of Figure 2.

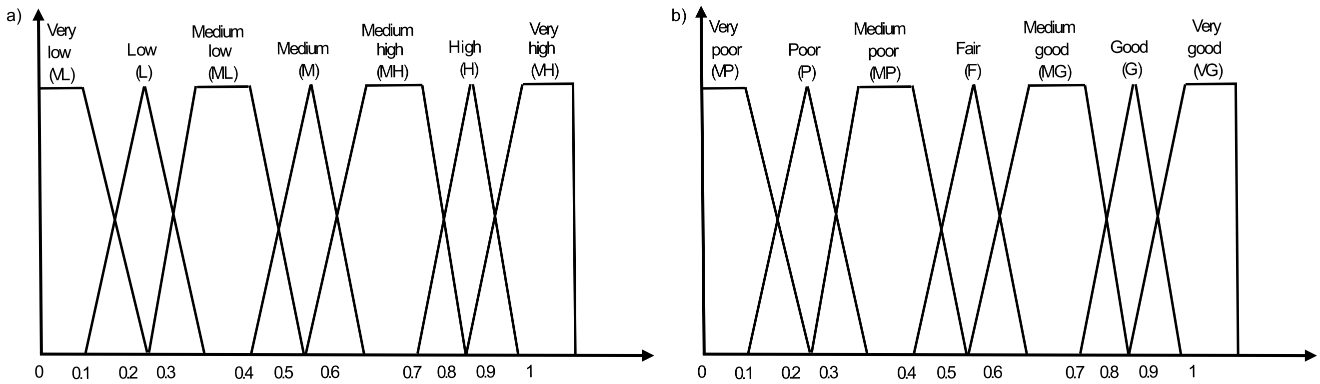

Figure 1. Trapezoidal fuzzy linguistic variables [24]: a) linguistic variables for the important weight of each criterion, b) linguistic variables for rating alternatives with respect to each criterion

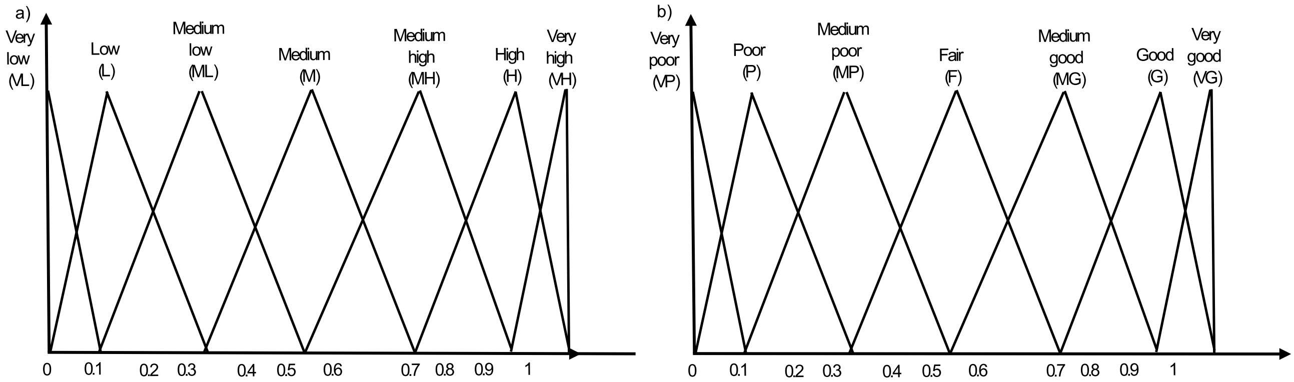

Figure 2. Triangular fuzzy linguistic variables [15]: a) linguistic variables for the important weight of each criterion, b) linguistic variables for rating alternatives with respect to each criterion

Table 1. Meaning of linguistic variables and numerical scales of trapezoidal fuzzy linguistic variables

Linguistic variables for selected criteria	Linguistic variables for the rating of alternatives	Trapezoidal fuzzy number
Very low (VL)	very poor (VP)	(0.0, 0.0, 0.1, 0.2)
Low (L)	poor (P)	(0.1, 0.2, 0.2, 0.3)
Medium low (ML)	medium poor (MP)	(0.2, 0.3, 0.4, 0.5)
Medium (M)	fair (F)	(0.4, 0.5, 0.5, 0.6)
Medium high (MH)	medium good (MG)	(0.5, 0.6, 0.7, 0.8)
High (H)	good (G)	(0.7, 0.8, 0.8, 0.9)
Very high (VH)	Vvery good (VG)	(0.8, 0.9, 1.0, 1.0)

Table 2. Meaning of linguistic variables and numerical scales of triangular fuzzy linguistic variables

Linguistic variables for selected criteria	Linguistic variables for the rating of alternatives	Triangular fuzzy number
Very low (VL)	very poor (VP)	(0.0, 0.0, 0.1)
Low (L)	poor (P)	(0.0, 0.1, 0.3)
Medium low (ML)	medium poor (MP)	(0.1, 0.3, 0.5)
Medium (M)	fair (F)	(0.3, 0.5, 0.7)
Medium high (MH)	medium good (MG)	(0.5, 0.7, 0.9)
High (H)	good (G)	(0.7, 0.9, 1.0)
Very high (VH)	very good (VG)	(0.9, 1.0, 1.0)

The numerical scales are assigned to criteria and each alternative with respect to each criterion by decision-makers. For trapezoidal fuzzy linguistic variables, let $x_{ijk} = (a_{ijk1}, a_{ijk2}, a_{ijk3}, a_{ijk4})$ and $y_{jk} = (b_{jk1}, b_{jk2}, b_{jk3}, b_{jk4})$, $i = 1, 2, \ldots, n, j = 1, 2, \ldots, m$ be the fuzzy rating and importance weight of the kth decision-maker, respectively. The elements of aggregated fuzzy rating $x_{ij} = (a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4})$ of alternatives with respect to each criterion and the elements of aggregated fuzzy weights $y_j = (b_{j1}, b_{j2}, b_{j3}, b_{j4})$ of each criterion can be calculated from the equations

$$a_{ij1} = \min_k\{a_{ijk1}\}, \quad a_{ij2} = \frac{1}{K}\sum_{k=1}^{K} a_{ijk2}, \quad a_{ij3} = \frac{1}{K}\sum_{k=1}^{K} a_{ijk3}, \quad a_{ij4} = \max_k\{a_{ijk4}\}, \tag{3}$$

$$b_{j1} = \min_k\{b_{jk1}\}, \quad b_{j2} = \frac{1}{K}\sum_{k=1}^{K} b_{jk2}, \quad b_{j3} = \frac{1}{K}\sum_{k=1}^{K} b_{jk3}, \quad b_{j4} = \max_k\{b_{jk4}\}, \tag{4}$$

where K is the total number of decision-makers.

Let $D = [x_{ij}]$ be the fuzzy decision matrix of elements x_{ij} and $W = [y_j]$ be the vector of the weights of criteria, y_j. Then, the elements x_{ij} of the matrix D and y_j of the vector W are defuzzified into non-fuzzy crisp values f_{ij} and w_j, respectively, by using a defuzzification technique [16, 21]. In this work, we use the centre-of-area defuzzification method. The obtained values f_{ij} are then used in Section 2 to rank the alternatives.

For the triangular case, we apply the same process as the trapezoidal linguistic variables except for the number of the tuples changed from 4 tuples to be 3 tuples. a_{ij3} and b_{ij3} in (3) and (4) are modified to be $a_{ij3} = \max_k\{a_{ijk3}\}$ and $b_{ij3} = \max_k\{b_{jk3}\}$ as follows

$$a_{ij1} = \min_k\{a_{ijk1}\}, \qquad a_{ij2} = \frac{1}{K}\sum_{k=1}^{K} a_{ijk2}, \qquad a_{ij3} = \max_k\{a_{ijk3}\} \tag{5}$$

$$b_{j1} = \min_k\{b_{jk1}\}, \qquad b_{j2} = \frac{1}{K}\sum_{k=1}^{K} b_{jk2}, \qquad b_{j3} = \max_k\{b_{jk3}\} \tag{6}$$

The processes in Sections 2 and 3 are used to find the result in the next section.

4. Numerical and comparative analyses

To select a suitable applicant for teaching assistance, three decision-making committees (M_1, M_2 and M_3) are formed to evaluate and rank the alternatives. In this study, five possible alternatives A_i, $i = 1, 2, \ldots, 5$ are considered and nine criteria $C_j, j = 1, 2, \ldots, 9$ are chosen for the evaluation: monthly income of student's family (C_1), monthly income of a student (C_2), student's learning ability (C_3), English proficiency (C_4), human relations (C_5), punctuality (C_6), moral cognition (C_7), obedience (C_8) and grade point average (C_9). To have a proper alternative, in this study, we employ two different linguistic variables characterized by trapezoidal and triangular membership functions. In Section 4.1, we show the results using the trapezoidal linguistic variable while the triangular variable is presented in Section 4.2.

4.1. Trapezoidal linguistic variables

The weights of the nine criteria and the rating alternatives with respect to each criterion are characterized by using the trapezoidal linguistic variables provided in Figures 1, where the numerical scales of the trapezoidal variables are presented in Table 1. Each decision-making committee assigns a vague level of importance for all nine criteria in Table 3 and the ratings of the five teaching assistants alternatives by the decision-makers under the nine criteria are illustrated in Table 6. The linguistic evaluations are transformed into trapezoidal fuzzy numbers as shown in Tables 4 and 5 for the important weight of criteria and Tables 7 and 8 for rating alternatives with respect to the criteria.

Table 3. Weight of criteria C_1–C_9 from three committees

Committee	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
M_1	MH	H	H	M	M	H	M	H	MH
M_2	H	MH	VH	ML	MH	VH	M	H	H
M_3	VH	VH	H	L	H	H	MH	MH	H

Table 4. Numerical scales of weight of criteria $C_1 - C_5$ from three committees

Committee	C_1	C_2	C_3	C_4	C_5
M_1	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.4, 0.5, 0.5, 0.6)	(0.4, 0.5, 0.5, 0.6)
M_2	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.8, 0.9, 1.0, 1.0)	(0.2, 0.3, 0.4, 0.5)	(0.5, 0.6, 0.7, 0.8)
M_3	(0.8, 0.9, 1.0, 1.0)	(0.8, 0.9, 1.0, 1.0)	(0.7, 0.8, 0.8, 0.9)	(0.1, 0.2, 0.2, 0.3)	(0.7, 0.8, 0.8, 0.9)

Table 5. Numerical scales of weight of criteria $C_6 - C_9$ from three committees

Committee	C_6	C_7	C_8	C_9
M_1	(0.7, 0.8, 0.8, 0.9)	(0.4, 0.5, 0.5, 0.6)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)
M_2	(0.8, 0.9, 1.0, 1.0)	(0.4, 0.5, 0.5, 0.6)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)
M_3	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)

Table 6. Rating alternatives with respect to criteria $C_1 - C_5$

Alternative	Committee	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
	M_1	VP	VP	G	F	VG	MG	VG	VG	G
A_1	M_2	P	VP	VG	MG	VG	F	G	VG	VG
	M_3	VP	P	G	F	G	F	MG	G	G
	M_1	MP	P	G	F	MG	MG	G	G	G
A_2	M_2	P	P	G	MG	G	MG	G	G	MG
	M_3	MP	P	MG	F	G	MG	MG	MG	F
	M_1	F	MP	MP	MG	MG	G	F	F	VG
A_3	M_2	MP	MP	G	MG	MG	MG	F	MG	G
	M_3	F	MP	G	G	G	MG	F	F	G
	M_1	MP	MG	VG	G	MG	G	F	G	G
A_4	M_2	F	F	G	MG	MG	G	F	MG	MG
	M_3	MG	F	G	G	F	VG	MP	F	G
	M_1	VG	VG	G	G	MG	VG	MP	MP	G
A_5	M_2	G	G	G	VG	MG	VG	F	F	MG
	M_3	VG	VG	VG	G	F	G	MP	MP	F

Table 7. Numerical scale of rating alternatives with respect to criteria $C_1 - C_5$

Alternative	Committee	C_1	C_2	C_3	C_4	C_5
	M_1	(0.0, 0.0, 0.1, 0.2)	(0.0, 0.0, 0.1, 0.2)	(0.7, 0.8, 0.8, 0.9)	(0.4, 0.5, 0.5, 0.6)	(0.8, 0.9, 1.0, 1.0)
A_1	M_2	(0.1, 0.2, 0.2, 0.3)	(0.0, 0.0, 0.1, 0.2)	(0.8, 0.9, 1.0, 1.0)	(0.5, 0.6, 0.7, 0.8)	(0.8, 0.9, 1.0, 1.0)
	M_3	(0.0, 0.0, 0.1, 0.2)	(0.1, 0.2, 0.2, 0.3)	(0.7, 0.8, 0.8, 0.9)	(0.4, 0.5, 0.5, 0.6)	(0.7, 0.8, 0.8, 0.9)
	M_1	(0.2, 0.3, 0.4, 0.5)	(0.1, 0.2, 0.2, 0.3)	(0.7, 0.8, 0.8, 0.9)	(0.4, 0.5, 0.5, 0.6)	(0.5, 0.6, 0.7, 0.8)
A_2	M_2	(0.1, 0.2, 0.2, 0.3)	(0.1, 0.2, 0.2, 0.3)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)
	M_3	(0.2, 0.3, 0.4, 0.5)	(0.1, 0.2, 0.2, 0.3)	(0.5, 0.6, 0.7, 0.8)	(0.4, 0.5, 0.5, 0.6)	(0.7, 0.8, 0.8, 0.9)
	M_1	(0.4, 0.5, 0.5, 0.6)	(0.2, 0.3, 0.4, 0.5)	(0.2, 0.3, 0.4, 0.5)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)
A_3	M_2	(0.2, 0.3, 0.4, 0.5)	(0.2, 0.3, 0.4, 0.5)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)
	M_3	(0.4, 0.5, 0.5, 0.6)	(0.2, 0.3, 0.4, 0.5)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)
	M_1	(0.2, 0.3, 0.4, 0.5)	(0.5, 0.6, 0.7, 0.8)	(0.8, 0.9, 1.0, 1.0)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)
A_4	M_2	(0.4, 0.5, 0.5, 0.6)	(0.4, 0.5, 0.5, 0.6)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)
	M_3	(0.5, 0.6, 0.7, 0.8)	(0.4, 0.5, 0.5, 0.6)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.4, 0.5, 0.5, 0.6)
	M_1	(0.8, 0.9, 1.0, 1.0)	(0.8, 0.9, 1.0, 1.0)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)
A_5	M_2	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.8, 0.9, 1.0, 1.0)	(0.5, 0.6, 0.7, 0.8)
	M_3	(0.8, 0.9, 1.0, 1.0)	(0.8, 0.9, 1.0, 1.0)	(0.8, 0.9, 1.0, 1.0)	(0.7, 0.8, 0.8, 0.9)	(0.4, 0.5, 0.5, 0.6)

We apply equations (3) and (4) to the elements in Tables 4 and 5 to calculate aggregated fuzzy weight and to Tables 7 and 8 to determine the aggregated fuzzy rating of alternatives, which are shown in Tables 9, 10. The crisp values of the weight of each criterion and decision matrix D are presented in Table 11 by using the centre of area defuzzification method [16, 21]. Next, we caculate the best f_j^* and worst f_j^- values provided in Section 2. Because Thailand is a developing country, to select a student to be a TA, if two students have similar properties, accept the financial position, the one who has financial problems will be selected so that both of them can have a chance to get their degrees. Therefore, if students have a chance to study without having a scholarship, the first and second criteria (C_1 and C_2, respectively) are considered to be opposite to the benefit criterion for them. The best and worst values of all criterion ratings are shown in Table 12.

Table 8. Numerical scale of rating alternatives with respect to the criteria C_6–C_9

Alternative	Committee	C_6	C_7	C_8	C_9
A_1	M_1	(0.5, 0.6, 0.7, 0.8)	(0.8, 0.9, 1.0, 1.0)	(0.8, 0.9, 1.0, 1.0)	(0.7, 0.8, 0.8, 0.9)
	M_2	(0.4, 0.5, 0.5, 0.6)	(0.7, 0.8, 0.8, 0.9)	(0.8, 0.9, 1.0, 1.0)	(0.8, 0.9, 1.0, 1.0)
	M_3	(0.4, 0.5, 0.5, 0.6)	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)
A_2	M_1	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)
	M_2	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)
	M_3	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.4, 0.5, 0.5, 0.6)
A_3	M_1	(0.7, 0.8, 0.8, 0.9)	(0.4, 0.5, 0.5, 0.6)	(0.4, 0.5, 0.5, 0.6)	(0.8, 0.9, 1.0, 1.0)
	M_2	(0.5, 0.6, 0.7, 0.8)	(0.4, 0.5, 0.5, 0.6)	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)
	M_3	(0.5, 0.6, 0.7, 0.8)	(0.4, 0.5, 0.5, 0.6)	(0.4, 0.5, 0.5, 0.6)	(0.7, 0.8, 0.8, 0.9)
A_4	M_1	(0.7, 0.8, 0.8, 0.9)	(0.4, 0.5, 0.5, 0.6)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)
	M_2	(0.7, 0.8, 0.8, 0.9)	(0.4, 0.5, 0.5, 0.6)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)
	M_3	(0.8, 0.9, 1.0, 1.0)	(0.2, 0.3, 0.4, 0.5)	(0.4, 0.5, 0.5, 0.6)	(0.7, 0.8, 0.8, 0.9)
A_5	M_1	(0.8, 0.9, 1.0, 1.0)	(0.2, 0.3, 0.4, 0.5)	(0.2, 0.3, 0.4, 0.5)	(0.7, 0.8, 0.8, 0.9)
	M_2	(0.8, 0.9, 1.0, 1.0)	(0.4, 0.5, 0.5, 0.6)	(0.4, 0.5, 0.5, 0.6)	(0.5, 0.6, 0.7, 0.8)
	M_3	(0.7, 0.8, 0.8, 0.9)	(0.2, 0.3, 0.4, 0.5)	(0.2, 0.3, 0.4, 0.5)	(0.4, 0.5, 0.5, 0.6)

Table 9. Aggregated fuzzy weight W of criteria C_1–C_5 and aggregated fuzzy rating of alternatives A_i

	C_1	C_2	C_3	C_4	C_5
W	(0.50, 0.77, 0.83, 1.00)	(0.50, 0.77, 0.83, 1.00)	(0.70, 0.83, 0.87, 1.00)	(0.10, 0.33, 0.37, 0.60)	(0.40, 0.63, 0.67, 0.90)
A_1	(0.00, 0.07, 0.13, 0.30)	(0.00, 0.07, 0.13, 0.30)	(0.70, 0.83, 0.87, 1.00)	(0.40, 0.53, 0.57, 0.80)	(0.70, 0.87, 0.93, 1.00)
A_2	(0.10, 0.27, 0.33, 0.50)	(0.10, 0.20, 0.20, 0.30)	(0.50, 0.73, 0.77, 0.90)	(0.40, 0.53, 0.57, 0.80)	(0.50, 0.73, 0.77, 0.90)
A_3	(0.20, 0.43, 0.47, 0.60)	(0.20, 0.30, 0.40, 0.50)	(0.20, 0.63, 0.67, 0.90)	(0.50, 0.67, 0.73, 0.90)	(0.50, 0.67, 0.73, 0.90)
A_4	(0.20, 0.47, 0.53, 0.80)	(0.40, 0.53, 0.57, 0.80)	(0.70, 0.83, 0.87, 1.00)	(0.50, 0.73, 0.77, 0.90)	(0.40, 0.56, 0.63, 0.80)
A_5	(0.70, 0.87, 0.93, 1.00)	(0.70, 0.87, 0.93, 1.00)	(0.70, 0.83, 0.87, 1.00)	(0.70, 0.83, 0.87, 1.00)	(0.40, 0.57, 0.63, 0.80)

Table 10. Aggregated fuzzy weights W of criteria C_6–C_9 and aggregated fuzzy rating of alternatives

	C_6	C_7	C_8	C_9
W	(0.70, 0.83, 0.87, 1.00)	(0.40, 0.53, 0.57, 0.80)	(0.50, 0.73, 0.77, 0.90)	(0.50, 0.73, 0.77, 0.90)
A_1	(0.40, 0.53, 0.57, 0.80)	(0.50, 0.77, 0.83, 1.00)	(0.70, 0.87, 0.93, 1.00)	(0.70, 0.83, 0.87, 1.00)
A_2	(0.50, 0.60, 0.70, 0.80)	(0.50, 0.73, 0.77, 0.90)	(0.50, 0.73, 0.77, 0.90)	(0.40, 0.63, 0.67, 0.90)
A_3	(0.50, 0.67, 0.73, 0.90)	(0.40, 0.50, 0.50, 0.60)	(0.40, 0.53, 0.57, 0.80)	(0.70, 0.83, 0.87, 1.00)
A_4	(0.70, 0.83, 0.87, 1.00)	(0.20, 0.43, 0.47, 0.60)	(0.40, 0.63, 0.67, 0.90)	(0.50, 0.73, 0.77, 0.90)
A_5	(0.70, 0.87, 0.93, 1.00)	(0.20, 0.37, 0.43, 0.60)	(0.20, 0.36, 0.43, 0.60)	(0.40, 0.63, 0.67, 0.90)

Table 11. Crisp values $f_{ij}, i = 1, 2, \dots 5, j = 1, 2, \dots, 9$, of weight of each criterion and decision matrix D

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
W	0.78	0.78	0.85	0.35	0.65	0.85	0.58	0.73	0.73
A_1	0.13	0.13	0.85	0.58	0.88	0.58	0.78	0.88	0.85
A_2	0.30	0.20	0.73	0.58	0.73	0.65	0.73	0.73	0.65
A_3	0.43	0.35	0.60	0.70	0.70	0.70	0.50	0.58	0.85
A_4	0.50	0.58	0.85	0.73	0.60	0.85	0.43	0.65	0.73
A_5	0.88	0.88	0.85	0.85	0.60	0.88	0.40	0.40	0.65

Table 12. The best and worst values of all criterion ratings

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
f_j^*	0.13	0.13	0.85	0.85	0.88	0.88	0.78	0.88	0.85
f_j^-	0.88	0.88	0.60	0.58	0.60	0.58	0.40	0.40	0.65

Next, the values of S_i, R_i and Q_i are computed from (1) as shown in Table 13 when $\nu = 0.5$. The ranking of the alternatives is provided in Table 14. The best alternative ranking by the S values has the smallest S number and the next selected alternative is the next higher S number, etc. The same process is used for alternative ranking by R and Q values. If two alternatives have the same values of S (or R or Q), they are placed in the same rank. From the Q values and the process of consideration provided in Section 2, therefore A_1, A_2 and A_4 are our best solutions (these three alternatives are "in closeness") when $\nu = 0.5$. Table 15 shows the Q values with different weights for the strategy of maximum group utility ν and then Table 16 illustrates the alternative ranking by Q for different values of ν. Because S and R do not depend on ν, they are not varied with the variable ν. If the weight for the strategy of maximum group utility is small and the weight of the individual regret $(1 - \nu)$ is large, then A_4 are the best. Oppositely, if ν is large and the weight of the individual regret is small, then A_1 becomes the best solution. Notice that the best solutions go around A_1 and A_4. Figure 3 shows the values of Q of the alternatives A_1–A_5 for different ν. From the graph, the values of Q are smallest at A_4 from $\nu = 0$ to $\nu = 0.6$ and then the smallest value of Q is at A_1 when ν is greater than or equal 0.7. The proper alternatives depend on the value of ν.

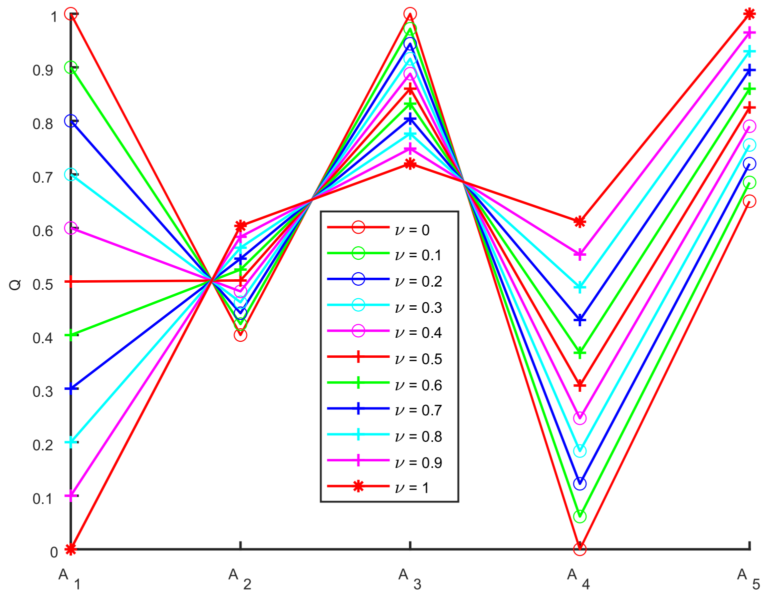

Figure 3. The values of Q depending on the alternatives A_1–A_5 for different ν

Table 13. The values of S, R and Q of all alternatives when $\nu = 0.5$

	A_1	A_2	A_3	A_4	A_5
S	1.20	3.04	3.40	3.07	4.25
R	0.85	0.73	0.85	0.65	0.78
Q	0.50	0.50	0.86	0.31	0.83

Table 14. The alternatives ranked by the values of S, R and Q when $\nu = 0.5$

By	Ranked alternatives				
S	A_1	A_2	A_4	A_3	A_5
R	A_4	A_2	A_5	A_1, A_3	A_1, A_3
Q	A_1, A_2, A_4	A_1, A_2, A_4	A_1, A_2, A_4	A_5	A_3

Table 15. The values of Q with different weights for the strategy of maximum group utility ν

	A_1	A_2	A_3	A_4	A_5
$Q\ (\nu = 0.1)$	0.90	0.42	0.97	0.06	0.69
$Q\ (\nu = 0.5)$	0.50	0.50	0.86	0.31	0.83
$Q\ (\nu = 0.9)$	0.10	0.58	0.75	0.55	0.97

Table 16. The alternatives ranked by the values of Q with different ν

By	Ranked alternatives				
$Q\ (\nu = 0.1)$	A_4	A_2	A_5	A_1	A_3
$Q\ (\nu = 0.5)$	A_1, A_2, A_4	A_1, A_2, A_4	A_1, A_2, A_4	A_5	A_3
$Q\ (\nu = 0.9)$	A_1	A_4	A_2	A_3	A_5

4.2. Triangular linguistic variables

In this section, we repeat the process described in Section 4.1 employing triangular fuzzy linguistic variables to find the best alternative. We compare the results given in both sections, considering Table 3 for the weight of criteria from three committees and Tables 6 and 7 for rating alternatives with respect to each measure in this section. Therefore, the numerical scales of the weight of criteria and of rating alternatives with respect to the criteria with the triangular fuzzy numbers are illustrated in Tables 17, 18 and 19, 20, respectively.

Table 17. Numerical scales of weight of criteria C_1–C_5 from three committees using triangular fuzzy numbers

Committee	C_1	C_2	C_3	C_4	C_5
M_1	(0.5, 0.7, 0.9)	(0.7, 0.9, 1.0)	(0.7, 0.9, 1.0)	(0.3, 0.5, 0.7)	(0.3, 0.5, 0.7)
M_2	(0.7, 0.9, 1.0)	(0.5, 0.7, 0.9)	(0.9, 1.0, 1.0)	(0.1, 0.3, 0.5)	(0.5, 0.7, 0.9)
M_3	(0.9, 1.0, 1.0)	(0.9, 1.0, 1.0)	(0.7, 0.9, 1.0)	(0.0, 0.1, 0.3)	(0.7, 0.9, 1.0)

Table 18. Numerical scales of weight of criteria C_6–C_9 from three committees using triangular fuzzy numbers

Committee	C_6	C_7	C_8	C_9
M_1	(0.7, 0.9, 1.0)	(0.3, 0.5, 0.7)	(0.7, 0.9, 1.0)	(0.5, 0.7, 0.9)
M_2	(0.9, 1.0, 1.0)	(0.3, 0.5, 0.7)	(0.7, 0.9, 1.0)	(0.7, 0.9, 1.0)
M_3	(0.7, 0.9, 1.0)	(0.5, 0.7, 0.9)	(0.5, 0.7, 0.9)	(0.7, 0.9, 1.0)

The aggregated fuzzy weight and fuzzy rating of alternatives are shown in Tables 21-22. By using the centre of area defuzzification method, the crisp values of the weight of each criterion and decision matrix are presented in Table 23. Then the best and worst values of all criterion ratings are illustrated in Table 24. Next, the values of S, R and Q are calculated and provided in Table 25 when $\nu = 0.5$. Since we write our own code to calculate the values in the tables, the numbers obtained by pressing a calculator may differ from those calculated on the computer only the digits after the second decimal point onwards. The alternative ranking is shown in Table 26. Notice that for the triangular linguistic variables, the alternative A_4 is the best solution for this problem when $\nu = 0.5$ because $Q(A_1) - Q(A_4) = 0.25 \geq DQ$ and $R(A_4)$ is minimum. Tables 27 and 28 show the values of Q of the alternatives and alternative ranking for different values of ν, respectively. Figure 4 illustrates the values of Q depending on the alternatives A_1–A_5 for different ν for the triangular case. Similar to the trapezoidal linguistic variable, the value of

Q is smallest at the alternative A_4 from $\nu = 0$ to $\nu = 0.6$ and then Q is smallest at A_1 from $\nu = 0.7$ until $\nu = 1$. Notice that from these two linguistic variables approach the best alternative goes around A_1 and A_4 depending on the variable ν.

Table 19. Numerical scale of rating alternatives with respect to the criteria C_1–C_5 using triangular fuzzy numbers

Alternative	Committee	C_1	C_2	C_3	C_4	C_5
	M_1	(0.0, 0.0, 0.1)	(0.0, 0.0, 0.1)	(0.7, 0.9, 1.0)	(0.3, 0.5, 0.7)	(0.9, 1.0, 1.0)
A_1	M_2	(0.0, 0.1, 0.3)	(0.0, 0.0, 0.1)	(0.9, 1.0, 1.0)	(0.5, 0.7, 0.9)	(0.9, 1.0, 1.0)
	M_3	(0.0, 0.0, 0.1)	(0.0, 0.1, 0.3)	(0.7, 0.9, 1.0)	(0.3, 0.5, 0.7)	(0.7, 0.9, 1.0)
	M_1	(0.1, 0.3, 0.5)	(0.0, 0.1, 0.3)	(0.7, 0.9, 1.0)	(0.3, 0.5, 0.7)	(0.5, 0.7, 0.9)
A_2	M_2	(0.0, 0.1, 0.3)	(0.0, 0.1, 0.3)	(0.7, 0.9, 1.0)	(0.5, 0.7, 0.9)	(0.7, 0.9, 1.0)
	M_3	(0.1, 0.3, 0.5)	(0.0, 0.1, 0.3)	(0.5, 0.7, 0.9)	(0.3, 0.5, 0.7)	(0.7, 0.9, 1.0)
	M_1	(0.3, 0.5, 0.7)	(0.1, 0.3, 0.5)	(0.1, 0.3, 0.5)	(0.5, 0.7, 0.9)	(0.5, 0.7, 0.9)
A_3	M_2	(0.1, 0.3, 0.5)	(0.1, 0.3, 0.5)	(0.7, 0.9, 1.0)	(0.5, 0.7, 0.9)	(0.5, 0.7, 0.9)
	M_3	(0.3, 0.5, 0.7)	(0.1, 0.3, 0.5)	(0.7, 0.9, 1.0)	(0.7, 0.9, 1.0)	(0.7, 0.9, 1.0)
	M_1	(0.1, 0.3, 0.5)	(0.5, 0.7, 0.9)	(0.9, 1.0, 1.0)	(0.7, 0.9, 1.0)	(0.5, 0.7, 0.9)
A_4	M_2	(0.3, 0.5, 0.7)	(0.3, 0.5, 0.7)	(0.7, 0.9, 1.0)	(0.5, 0.7, 0.9)	(0.5, 0.7, 0.9)
	M_3	(0.5, 0.7, 0.9)	(0.3, 0.5, 0.7)	(0.7, 0.9, 1.0)	(0.7, 0.9, 1.0)	(0.3, 0.5, 0.7)
	M_1	(0.9, 1.0, 1.0)	(0.9, 1.0, 1.0)	(0.7, 0.9, 1.0)	(0.7, 0.9, 1.0)	(0.5, 0.7, 0.9)
A_5	M_2	(0.7, 0.9, 1.0)	(0.7, 0.9, 1.0)	(0.7, 0.9, 1.0)	(0.9, 1.0, 1.0)	(0.5, 0.7, 0.9)
	M_3	(0.9, 1.0, 1.0)	(0.9, 1.0, 1.0)	(0.9, 1.0, 1.0)	(0.7, 0.9, 1.0)	(0.3, 0.5, 0.7)

Table 20. Numerical scale of rating alternatives with respect to the criteria C_6–C_9 using triangular fuzzy numbers

Alternative	Committee	C_6	C_7	C_8	C_9
	M_1	(0.5, 0.7, 0.9)	(0.9, 1.0, 1.0)	(0.9, 1.0, 1.0)	(0.7, 0.9, 1.0)
A_1	M_2	(0.3, 0.5, 0.7)	(0.7, 0.9, 1.0)	(0.9, 1.0, 1.0)	(0.9, 1.0, 1.0)
	M_3	(0.3, 0.5, 0.7)	(0.5, 0.7, 0.9)	(0.7, 0.9, 1.0)	(0.7, 0.9, 1.0)
	M_1	(0.5, 0.7, 0.9)	(0.7, 0.9, 1.0)	(0.7, 0.9, 1.0)	(0.7, 0.9, 1.0)
A_2	M_2	(0.5, 0.7, 0.9)	(0.7, 0.9, 1.0)	(0.7, 0.9, 1.0)	(0.5, 0.7, 0.9)
	M_3	(0.5, 0.7, 0.9)	(0.5, 0.7, 0.9)	(0.5, 0.7, 0.9)	(0.3, 0.5, 0.7)
	M_1	(0.7, 0.9, 1.0)	(0.3, 0.5, 0.7)	(0.3, 0.5, 0.7)	(0.9, 1.0, 1.0)
A_3	M_2	(0.5, 0.7, 0.9)	(0.3, 0.5, 0.7)	(0.5, 0.7, 0.9)	(0.7, 0.9, 1.0)
	M_3	(0.5, 0.7, 0.9)	(0.3, 0.5, 0.7)	(0.3, 0.5, 0.7)	(0.7, 0.9, 1.0)
	M_1	(0.7, 0.9, 1.0)	(0.3, 0.5, 0.7)	(0.7, 0.9, 1.0)	(0.7, 0.9, 1.0)
A_4	M_2	(0.7, 0.9, 1.0)	(0.3, 0.5, 0.7)	(0.5, 0.7, 0.9)	(0.5, 0.7, 0.9)
	M_3	(0.9, 1.0, 1.0)	(0.1, 0.3, 0.5)	(0.3, 0.5, 0.7)	(0.7, 0.9, 1.0)
	M_1	(0.9, 1.0, 1.0)	(0.1, 0.3, 0.5)	(0.1, 0.3, 0.5)	(0.7, 0.9, 1.0)
A_5	M_2	(0.9, 1.0, 1.0)	(0.3, 0.5, 0.7)	(0.3, 0.5, 0.7)	(0.5, 0.7, 0.9)
	M_3	(0.7, 0.9, 1.0)	(0.1, 0.3, 0.5)	(0.1, 0.3, 0.5)	(0.3, 0.5, 0.7)

Table 21. Aggregated fuzzy weight W of criteria C_1–C_5 and aggregated fuzzy rating of alternatives with triangular fuzzy numbers

	C_1	C_2	C_3	C_4	C_5
W	(0.50, 0.87, 1.00)	(0.50, 0.87, 1.00)	(0.70, 0.93, 1.00)	(0.00, 0.30, 0.70)	(0.30, 0.70, 1.00)
A_1	(0.00, 0.03, 0.30)	(0.00, 0.03, 0.30)	(0.70, 0.93, 1.00)	(0.30, 0.57, 0.90)	(0.70, 0.97, 1.00)
A_2	(0.00, 0.23, 0.50)	(0.00, 0.10, 0.30)	(0.50, 0.83, 1.00)	(0.30, 0.57, 0.90)	(0.50, 0.83, 1.00)
A_3	(0.10, 0.43, 0.70)	(0.10, 0.30, 0.50)	(0.10, 0.70, 1.00)	(0.50, 0.77, 1.00)	(0.50, 0.77, 1.00)
A_4	(0.10, 0.50, 0.90)	(0.30, 0.23, 0.90)	(0.70, 0.93, 1.00)	(0.50, 0.83, 1.00)	(0.30, 0.63, 0.90)
A_5	(0.70, 0.97, 1.00)	(0.70, 0.97, 1.00)	(0.70, 0.93, 1.00)	(0.70, 0.93, 1.00)	(0.30, 0.63, 0.90)

Table 22. Aggregated fuzzy weight W of criteria C_6–C_9 and aggregated fuzzy rating of alternatives with triangular fuzzy numbers

	C_6	C_7	C_8	C_9
W	(0.70, 0.93, 1.00)	(0.30, 0.57, 0.90)	(0.50, 0.83, 1.00)	(0.50, 0.83, 1.00)
A_1	(0.30, 0.57, 0.90)	(0.50, 0.87, 1.00)	(0.70, 0.97, 1.00)	(0.70, 0.93, 1.00)
A_2	(0.50, 0.70, 0.90)	(0.50, 0.83, 1.00)	(0.50, 0.83, 1.00)	(0.30, 0.70, 1.00)
A_3	(0.50, 0.77, 1.00)	(0.30, 0.50, 0.70)	(0.30, 0.57, 0.90)	(0.70, 0.93, 1.00)
A_4	(0.70, 0.93, 1.00)	(0.10, 0.43, 0.70)	(0.30, 0.70, 1.00)	(0.50, 0.83, 1.00)
A_5	(0.70, 0.97, 1.00)	(0.10, 0.37, 0.70)	(0.10, 0.37, 0.50)	(0.30, 0.70, 1.00)

Table 23. Crisp values $f_{ij}, i = 1, 2, \ldots, 5, j = 1, 2, \ldots, 9,$ of weight of each criterion and decision matrix D for the triangular case

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
W	0.79	0.79	0.88	0.33	0.67	0.88	0.59	0.78	0.78
A_1	0.11	0.11	0.88	0.59	0.89	0.59	0.79	0.89	0.88
A_2	0.24	0.13	0.78	0.59	0.78	0.70	0.78	0.78	0.67
A_3	0.41	0.30	0.60	0.75	0.76	0.76	0.50	0.59	0.88
A_4	0.50	0.48	0.88	0.78	0.61	0.88	0.41	0.67	0.78
A_5	0.89	0.89	0.88	0.88	0.61	0.89	0.39	0.32	0.67

Table 24. The best and worst values of all criterion ratings for the triangular case

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
f_j^*	0.11	0.11	0.88	0.88	0.89	0.89	0.79	0.89	0.88
f_j^-	0.89	0.89	0.60	0.59	0.61	0.59	0.39	0.32	0.67

Table 25. The values of S, R and Q of all alternative when $\nu = 0.5$ for the triangular case

	A_1	A_2	A_3	A_4	A_5
S	1.23	2.60	3.07	2.85	4.41
R	0.88	0.79	0.88	0.67	0.79
Q	0.50	0.51	0.79	0.25	0.79

Table 26. The alternatives ranked by the values of S, R and Q when $\nu = 0.5$ for the triangular case

By	Ranking alternatives				
S	A_1	A_2	A_4	A_3	A_5
R	A_4	A_2, A_5	A_2, A_5	A_1, A_3	A_1, A_3
Q	A_4	A_1	A_2	A_3, A_5	A_3, A_5

Table 27. The values of Q with different weights for the strategy of maximum group utility ν for the triangular case

	A_1	A_2	A_3	A_4	A_5
Q ($\nu = 0.1$)	0.90	0.57	0.96	0.05	0.63
Q ($\nu = 0.5$)	0.50	0.51	0.79	0.25	0.79
Q ($\nu = 0.9$)	0.10	0.45	0.62	0.46	0.96

Table 28. The alternatives ranked by the values of Q with different ν for the triangular case

By	Ranking alternatives				
Q ($\nu = 0.1$)	A_4	A_2	A_5	A_1	A_3
Q ($\nu = 0.5$)	A_4	A_1	A_2	A_3, A_5	A_3, A_5
Q ($\nu = 0.9$)	A_1	A_2	A_4	A_3	A_5

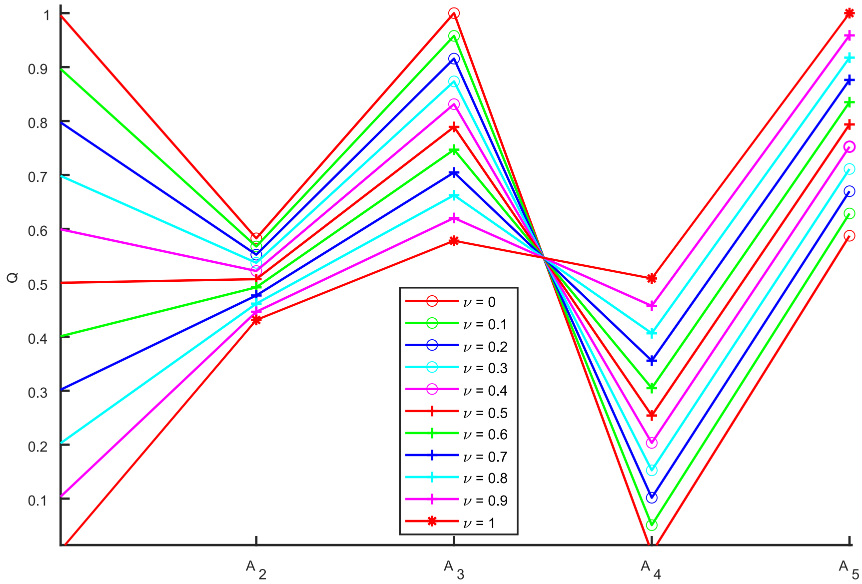

Figure 4. The values of Q depending on the alternatives A_1-A_5 for different ν for the triangular case

4.3. Comparison of the ranking results with TODIM scheme

In this section, we compare our results with the ranking obtained from the TODIM method. The crisp values of the weights of the criteria and the decision matrices D in Tables 11 and 23 are taken to the TODIM approach with the equation [18]:

$$\xi(A_i) = \frac{\Phi(A_i) - \min\limits_{1 \leqslant l \leqslant n} \Phi(A_l)}{\max\limits_{1 \leqslant l \leqslant n} \Phi(A_l) - \min\limits_{1 \leqslant l \leqslant n} \Phi(A_l)} \tag{7}$$

where $i = 1, 2, \ldots, n$, $\Phi(A_i) = \sum\limits_{l=1}^{n} \sum\limits_{j=1}^{m} \Phi_j(A_i, A_l)$,

$$\Phi_j(A_i, A_l) = \begin{cases} \sqrt{\dfrac{w_{jr}(f_{ij} - f_{lj})}{\sum\limits_{t=1}^{m} w_{tr}}} & \text{if } f_{ij} \geq f_{lj} \\ -\dfrac{1}{\beta}\sqrt{\dfrac{\left(\sum\limits_{t=1}^{m} w_{tr}\right)(f_{lj} - f_{ij})}{w_{jr}}} & \text{if } f_{ij} < f_{lj}, \end{cases} \tag{8}$$

where β is greater than zero and the relative weight $w_{jr} = w_j / w_r$, $w_r = \max\limits_{1 \leq t \leq m} w_t$ and $j = 1, 2, \ldots, m$.

The best alternative is the greatest value of $\xi(A_i)$ in (7) and the next one is the next smaller value of $\xi(A_i)$, i = 1, 2, $\ldots, n$. The ranking obtained from the TODIM method in both trapezoidal and triangular cases with $\beta = 1$ is compared with our solutions ranked by the values of Q with $\nu = 0.5$ in Table 29. By the two different methods and cases, it illustrates that A_4 stands for the proper alternative.

Table 29. The alternatives ranked by the extended VIKOR and TODIM approaches

	Ranking alternatives				
VIKOR (trapezoidal)	A_1, A_2, A_4	A_1, A_2, A_4	A_1, A_2, A_4	A_5	A_3
VIKOR (triangular)	A_4	A_1	A_2	A_3, A_5	A_3, A_5
TODIM (trapezoidal)	A_4	A_5	A_1	A_3	A_2
TODIM (triangular)	A_4	A_5	A_1	A_3	A_2

5. Conclusion

To select the best teaching assistant from the student applicants, the extended VIKOR method with trapezoidal and triangular linguistic variables are used in this research with five applicants and nine criteria. We conclude the work as follows.

- Trapezoidal and triangular fuzzy numbers and the weight of criteria from three committees and rating alternatives with respect to each criterion are provided.
- The crisp values of the aggregated fuzzy weight W of criteria and aggregated fuzzy rating of alternatives are calculated by using the centre of area defuzzification method.
- The best and worst values of all criterion ratings for the trapezoidal and triangular linguistic variables are illustrated to calculate the values of S, R and Q.
- With the trapezoidal linguistic variable, the best alternative is the set of A_1, A_2 and A_4 (they are in closeness) while for the case of the triangular linguistic variable, the alternative A_4 is the best solution when $\nu = 0.5$.
- With varied ν from zero to one, for both linguistic variables, the best alternatives are either A_1 or A_4 (including A_2 only $\nu = 0.5$)(Figures 3, 4).
- Comparing the two schemes provides almost the same best alternative. The trapezoidal approach gives three best alternatives A_1, A_2, A_4 while the using of a triangular linguistic variable, sharper than the trapezoidal one, provides only one alternative A_4, which is one of the best solutions of the trapezoidal scheme.
- Notice that the first three places of alternatives ranked by the values of Q when $\nu = 0.5$ with the triangular case have only one alternative for each place while with the trapezoidal variables, the first three positions have three alternatives in each place (Tables 14 and 26). This shows that the trapezoidal variables are proper for the problem with more ambiguity.
- The results from the extended VIKOR method are compared with the ranking attained from the TODIM approach. It shows that the alternative A_4 is the best alternative.
- This repeat calculation of different linguistic variable may help committees to decide and select the right teaching assistant.

Notice that although the extended VIKOR approach is a well-known method and provide vigorous results, it is not favoured for rank reversal phenomenon such as adding a new alternative to the initial list or eliminating some alternatives and then the rank is substantially modified. An example and a counter-example can be found in [20].

For future work, the triangular and trapezoidal fuzzy neutrosophic numbers in multiple criteria group decision-making may be employed to select the teaching assistants, see [10, 12–14, 29] for more details.

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