DOI: 10.53122/ISSN.0452-5027/2022.1.29

Measurement of Efficiency

Data Envelopment Analysis in Performance Auditing

One of the objectives of performance auditing is to examine and assess the efficiency of the audited processes, programmes and organisations. To this end, the ratio method is usually used (the relation between output and input). However, it is faulty and has several significant limitations. Simultaneously, more advanced analytical methods are often considered as too complicated. The author, apart from discussing the basic issues, including the definition of efficiency, focuses on the application of the so called non-parametric methods for assessing efficiency, especially the Data Envelopment Analysis (DEA) method. This method is known in academic circles, but it is also frequently applied in evaluation and benchmarking in business and in the public sector, and is recommended by various institutions and government agencies in many countries. Although the DEA is used by some audit institutions, it still seems to be not very common in auditing. In his article the author presents, in the most accessible manner possible, the idea of the DEA method, simple tools for its application and case studies.

WIESŁAW KARLIŃSKI

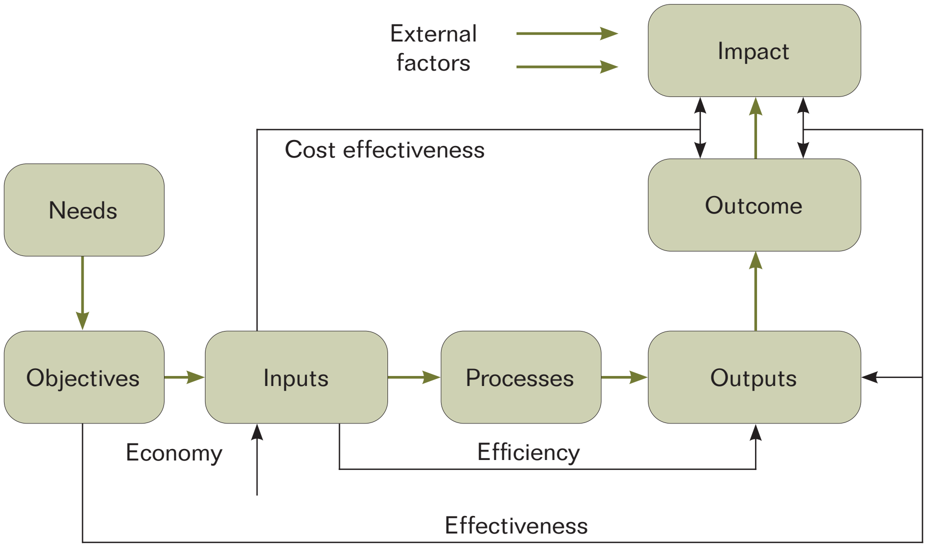
Efficiency in performance auditing

Performance audit, according to the INTOSAI standards[1] is the examination of whether government undertakings, systems, operations, programmes, activities or organisations operate in accordance with the principles of economy, efficiency and effectiveness, and whether there is room for improvement.

The understanding of the notions listed below will be easier if we relate them to

[1] ISSAI 300/9.

Picture 1. **Life cycle of an action (public intervention) and a model for its auditing**



Source: Own study, considering INTOSAI GUID 3910 (2019) and the Performance Audit Manual of the European Court of Auditors (2017)[2].

the typical life cycle of an action (intervention) that is going to be audited (see the picture above).

Usually, an action (intervention) is launched when needs arise, which in the case of a public intervention comprise social needs. The objectives of an intervention are set on the basis of those needs. Then, appropriate inputs (resources) are acquired[3] with defined costs incurred. Inputs are used to produce outputs of the process (products). Products are usually not the goal in itself, but rather a means to achieve outcomes, in accordance with the intervention objectives accepted. Frequently, more long-term expectations are linked to an intervention, related to a broader impact on the environment in which the intervention is made.

On the basis of the above elements the following notions (principles)[4] can be defined:

Economy, which means minimisation of costs designated for obtaining the input necessary to perform an intervention, assuming that the input is available on right

[2] In the Performance Audit Manual of the European Court of Auditors slightly different nomenclature is used for output elements of the process.

[3] The notion of resources seems to be more appropriate here, but in the literature on the topic more often the notion of input is used, understood not only in financial terms.

[4] Compare with INTOSAI *GUID 3910 Central Concepts for Performance Auditing*, <www.issai.org>.

time, in an appropriate quantity and of appropriate quality. Thus, economy refers to the stage of inputs (resources) acquisition.

Efficiency denotes the best possible use of inputs in order to create products in the desired quantity and of the desired quality, and on right time. Therefore, efficiency refers to the relation between products (outputs) and inputs incurred, and it may take two forms:

- output based efficiency – research question: "Are we getting the most output (quantity, quality, time) from our inputs?";
- input based efficiency – research question: "Could the same output have been achieved with less input?".

Effectiveness refers to the implementation of the objectives set and achieving the intended or desired outcomes of an intervention. Thus, effectiveness refers to the relation between objectives on one hand, and outcomes on the other hand. Most often effectiveness is measured in relation to outcome or impact, but in some cases it can be measured against products. When examining effectiveness, we have to answer the following questions:

- to what extent have the objectives been met?
- can outcome or impact be assigned to products (process results)?

while we do not consider the cost at which the outcome has been achieved.

Cost effectiveness combines the elements of economy, efficiency and effectiveness through an analysis of the connection between outcome and the cost of input acquisition. Cost effectiveness is expressed in qualitative categories (as the unit cost of outcome), or in quantitative categories (as the number of outcomes per a cost unit), and it is applied in Value for Money audit.

The model presented here is not the only one used in performance auditing. In American or Canadian studies slightly different definitions of effectiveness are applied. Effectiveness is understood as achievement of expected outcomes from the process outputs (products), so the principle of effectiveness is perceived as a relation between products and outcomes[5].

Efficiency that measures the relation between outputs and inputs can be presented in a relative manner and then it can be called productivity, or in a normative manner – and then it is called technical efficiency.

The notion of efficiency is commonly used in economic analyses, but economists usually precisely say what type of efficiency they mean: by using an appropriate adjective, or by indicating the author of the concept. In the classification of efficiency, the key are the notions introduced in 1957 by J. M. Farrell. He proposed to distinguish technical efficiency to measure the relation between inputs and outputs, meaning the creation of a product in accordance with the production function (without input waste), as well as price efficiency, which denotes acquisition of inputs at the most beneficial prices[6]. The product of the two elements is referred to as general or

[5] See: <https://www.caaf-fcar.ca/en/efficiency-concepts-and-context/efficiency-economy-and-effectiveness>.

[6] M. J. Farrell: "The Measurement of Productive Efficiency", *Journal of the Royal Statistical Society*, Series A, No 3/1957.

cost efficiency. The notions of technical efficiency and price efficiency are therefore close to the notions of efficiency and economy used in auditing. The difference between the efficiency defined in auditing standards and Farrell's technical efficiency mainly consists in efficiency being a relative measurement – so for its assessment it is necessary to have a reference level (to compare with other entities, or with another period), while Farrell's technical efficiency, in accordance with the definition, is an absolute measurement, taking the value from 0 to 1 (it is as if the proportion of actual efficiency to the efficiency that is optimal for a given technology). In practice, it is difficult to unambiguously indicate optimal efficiency, or to set absolute value of technical efficiency. Therefore, analogically to efficiency as understood in auditing, it is frequently measured in a relative manner, which is especially used in the DEA method to be discussed below.

Considering the above, we could approximately assume that efficiently, as understood in accordance with Farrell's general (cost) efficiency concept, means economically and productively. Such understanding of the notion of "efficient" seems to be compliant with the provisions of Article 68 of the Act on public finance, which states that the objective of management control is to ensure – in the first place – that an action is effective and efficient. However, it is often stated that – in the case of the public sector – we should not speak of efficiency if it does not coincide with effectiveness, and there is some reason in it. A solution for such an approach to the issue of efficiency would be the notion of cost effectiveness, presented in the intervention model (Picture 1, p. 47).

Within technical efficiency, the so called pure technical efficiency is distinguished (it measures the relation between output and input separately from returns to scale of the inputs used), and the scale efficiency, which is related to a non-linear relation between output and input that frequently exists. Returns to scale can be illustrated as follows: at a certain value of inputs their growth results in a growth in efficiency, at a certain stage a growth in inputs does not affect their efficiency, and once a certain value is exceeded – a growth in inputs may have a negative impact on efficiency. On the other hand, in the case of the input based model, a situation may arise when the scale of outputs is too small for the process to be efficient, and solutions should be sought, consisting in joint use of resources and processes by several entities.

INTOSAI GUID 3910, in the area of efficiency, lists a third element too, called allocative efficiency, referring to an appropriate allocation of the resources in order to achieve better output[7].

Further considerations will be related only to measuring efficiency understood as technical efficiency (pure technical

[7] In the literature, a slightly different understanding of the notion of allocative efficiency is used, i.e. equating it with Farrell's price efficiency. There are also studies in which allocative efficiency refers to the stage of transforming products into outcomes, which is a totally different approach to the issue, and will not be analysed here.

efficiency and scale efficiency). While as for the issue of resources allocation, it will be mentioned only when the DEA method is discussed.

Methods for measuring efficiency (technical efficiency) can be divided into three groups:

- indicator,
- non-parametric,
- parametric.

In the article, only the first two groups of methods will be presented. This is because the third group is rarely used in auditing, as it calls for knowing the functional relation between output and input, and distribution of these variables. While in econometric studies, various parametric methods are used, among other the Stochastic Frontier Approach (SFA), the Distribution Free Approach (DFA), and the Thick Frontier Approach (TFA).

Measuring efficiency with the indicator method

The indicator analysis is the simplest analytical method used in performance auditing. It consists in calculating the relation between values (usually total values) of two different features, or values of the same feature for different periods. In the case of using the indicator method for efficiency assessment E, the indicator will be the quotient of the value related to outputs (Y) and the value of inputs (X).

$$E = \frac{outputs}{inputs} = \frac{Y}{X} \qquad (1)$$

For the indicator calculated in this way to be the most precise measure of process efficiency, two conditions have to be fulfilled:

- It must comprise the most complete set of inputs and outputs related to the given process;
- It must comprise inputs that are meant to achieve the analysed outputs only.

These conditions are difficult to meet in practice, since we hardly ever analyse processes that can be described with one type of input and one type of output, and also rarely inputs are used in one process only.

Most frequently, various types of inputs are used in a process and/or more than one type of output is achieved. Efficiency could be then described with a quotient whose numerator contains a sum of various outputs while denominator – a sum of various inputs.

$$E = \frac{\sum_i u_i y_i}{\sum_j v_j x_j} \qquad (2)$$

However, individual components of sums are often related to elements that are difficult to compare, which hinders setting precise weights for individual types of outputs and inputs (ui and vj). So it would be yet more difficult to ensure a unified approach when comparing the indicator measured in this way for various entities. If, for example, we would like to compare how personnel resources of municipal guards are used, it seems justified to take into account the number of inhabitants, as well as the area the guards are responsible for. A dilemma arises how to consider such two different components in one indicator, and yet more – how to compare two municipalities with totally different population density and surface area.

Table 1. **Errors of an analysis with the use of aggregated data (Simpson's paradox)**

	Company A			Company B			Total		
	trained	passed		trained	passed		trained	passed	
K1	50	45	90%	450	300	67%	500	345	69%
K2	200	80	40%	50	5	10%	250	85	34%
Total	250	125	50%	500	305	61%			

Source: Own study.

Hence indicator methods are used usually to calculate proportions of individual components (one type of output and one type of input), so as to assess partial efficiency. Within one type, summing up (aggregation) of individual data takes place frequently. Still, even if data are summed up that may seem homogenous, i.e. possible to sum up, we can obtain an incorrect result of efficiency indicator.

Let us assume that we wish to evaluate the efficiency of two training companies (A and B) that conducted training at two types of courses (C1 and C2), concluded with an exam. Since both companies carried out training at the same type of courses, we can assume that it is enough to use aggregated data to obtain the efficiency indicator. Company A trained 250 persons, and 125 persons passed the exam, and company B trained 500 persons, and 305 persons passed the exam. Using a simple indicator analysis, we conclude that the efficiency of company B (61%) is much higher than that of company A (50%). However, when we compare detailed data (see the table below), we arrive at a totally different conclusion.

Firstly, the table shows that the pass rate within course C1 is by more than twice higher than the pass rate for course C2 (69% to 34%). Secondly, much more persons applied to company A for course C2 (more difficult) – and to company B just the opposite (more persons applied for course C1). As a result, although in company A the pass rate was higher for both course C1 (90% to 67%) and course C2 (40% to 10%), by using aggregated data we achieved the opposite result, which suggests that the pass rate is higher in company B.

The above example illustrates the phenomenon called Simpson's paradox, and the reader can find descriptions of numerous hypothetical and actual cases of its occurrence (e.g. the court case against the University of Berkley in 1973 related to discrimination of women in recruitment for MA studies, described by Carlberg[8]). Since in auditing, especially in performance auditing, we often make conclusions on the basis of aggregated data, it is necessary to verify whether aggregates do not comprise categories that significantly

[8] C. Carlberg: *Statistical Analysis: Microsoft Excel 2016*, Pearson Education Inc., 2018.

vary as for the indicator value, which can be a potential risk of Simpson's paradox.

As for the other condition of the accuracy of efficiency measurement (adjusting inputs and outputs), it seems to be easier, provided that we have precise data at our disposal. Frequently inputs (e.g. personnel resources) are used not only to produce the product that we measure, and we have to consider it when calculating efficiency indicators, even more when comparing indicators for different entities, or different periods.

Measuring efficiency with non-parametric methods

Non-parametric methods are methods that allow for comparing efficiency (technical efficiency) of processes characterised by numerous resources and/or numerous outputs, without knowing the functional relation between outputs and inputs, that is for solving the problem defined in the equation 2 (p. 50). Firstly, it calls for having data related to a higher number of objects (entities), and secondly – applying methods of so called linear programming, i.e. application of proper IT tools.

The most popular non-parametric method for calculating technical efficiency[9] is the Data Envelopment Analysis. In the case of examining efficiency dynamics (efficiency changes over time), the Malmquist productivity index is determined, based on the DEA method.

In the DEA method, technical efficiency is measured through analysing outputs and inputs of numerous similar objects (entities) and determining, on this basis, the best practice frontier against which the efficiency of individual objects is assessed. Best practice frontier is based (spread) on the objects with the highest efficiency, and not on some idealised standard. Inputs are usually marked with the letter x (x1, x2, …), and outputs with the letter y (y1, y2, …), while the objects analysed are called Decision Making Units (DMU).

The essence of the analysis relies on determining, for each of DMU_k, the efficiency index E_k that meets the following requirements:

$$max \frac{\sum_i u_i y_i}{\sum_j v_j x_j} \qquad (3)$$

assuming that:

$$E_k \leq 1 , u_i \geq 0, \ v_j \geq 0$$

For efficient objects, the value E_k=1, while for others it is below 1.

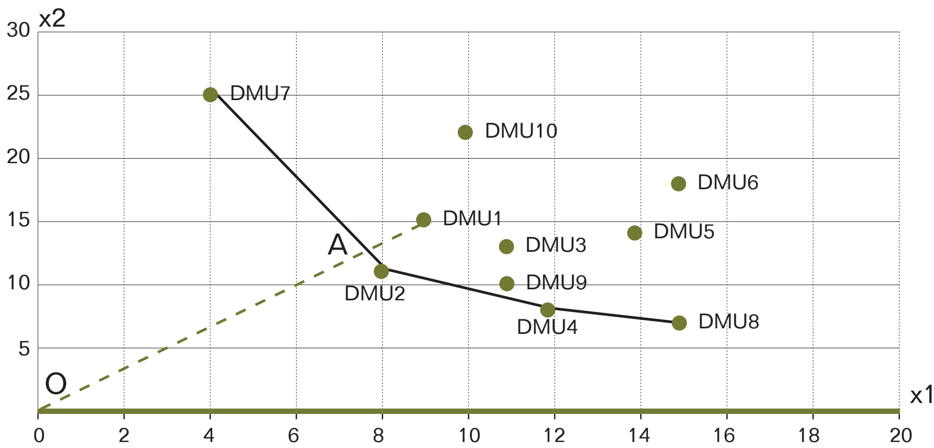
The graphic illustration of the idea of envelope has been presented in the pictures 2 and 3, pp. 53 and 54.

Picture 2 illustrates the data envelope for the DEA model focused on inputs (minimising inputs to achieve the given output), and it is related to the simplest variant: two inputs (x1 and x2) and one output (y1) of the same value for all DMUs.

Since the output (y) is constant, the most efficient objects (DMU7, DMU2, DMU4 and DMU8) are those which achieved the output at the lowest inputs x1 and x2. The envelope connects the

[9] In order to be consistent with the literature in the field, I will use the notion of technical effectiveness, and not efficiency or productivity, although in the case of the DEA they are relative, so actually equivalent.

Picture 2. **Data envelope for the input based model**



Source: Own study.

most efficient DMUs, while the objects above the envelope are less efficient. The measure of the efficiency of DMU1 is the quotient of the distance O-A and the distance O-DMU1, where A is the intersection of the O-DMU1 segment with the envelope.

While picture 3 illustrates the envelope in the DEA model based on output (achieving the best outputs with the given inputs), assuming the simplest case (one input identical for all DMUs and two outputs).

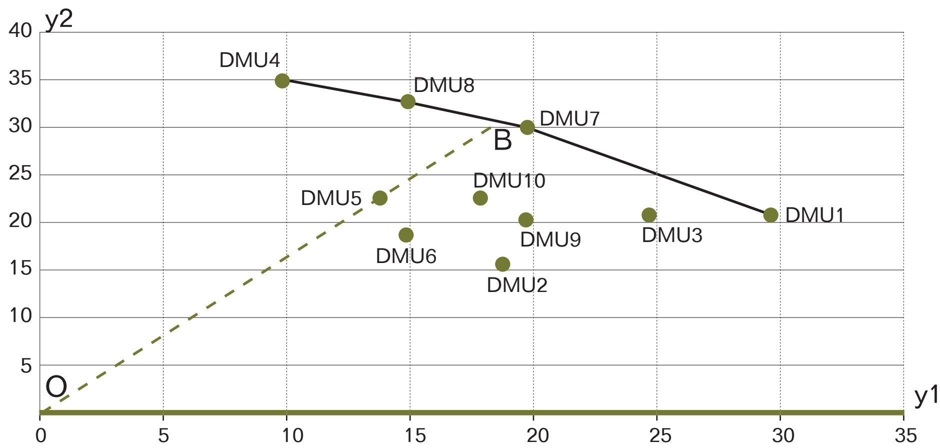
Since the value of input (x) is constant, the most efficient are the objects (DMU4, DMU8, DMU7 and DMU1) that achieve the highest set of y1 and y2 outputs at the given input, and they determine the envelope. The other objects, located below the envelope, are less efficient (technically efficient). The measure of object DMU5 is the ratio of the distance O-DMU5 to the distance O-B.

The above pictures are only illustrations, as it is hard to imagine a situation where all objects achieve the same output (y) or incur the same input (x). Moreover, in real studies the number of inputs and outputs is usually higher.

The application of the DEA method allows for:

- determining optimal (frontier) objects;
- setting the efficiency level of other objects against the best practice frontier (presented in decimal values or in %), which can be used in ranking;
- indicating model (reference) objects for all non-optimal objects;
- indicating change directions.

The basic classical version of the DEA method, known as the CCR, was developed in 1978 by A. Charnes, W.W. Cooper and E. Rhodes (the name of the method comes from the first letters of their surnames). The CCR model assumes the constant returns to scale (CRS), Picture 3. **Data envelope for the output based model**



Source: Own study.

i.e. a linear relation between outputs and inputs[10].

However, when we compare objects with very different scales of inputs, we can expect that the scale will have an impact on the values of efficiency indexes. In 1984 R.D. Banker, A. Charnes and W.W. Cooper proposed to modify the DEA method to allow for variable returns to scale (VRS), called the BCC – after their surnames[11].

The essence of constant and variable returns to scale is illustrated in the picture 4, p. 55.

The picture presents four different objects: A, B, C and D. The objects (points) A, B and C are located on the productivity frontier considering variable returns to scale (VRS), but only B is fully technically efficient (it has achieved the productivity frontier independent of scale). In the case of objects A and C, due to the production scale, the outputs are lower than they would be with constant returns to scale (CRS) – so with a linear relation between outputs and inputs. The object D is not technically efficient, and the measure of its relative technical efficiency depends on the method we adopt.

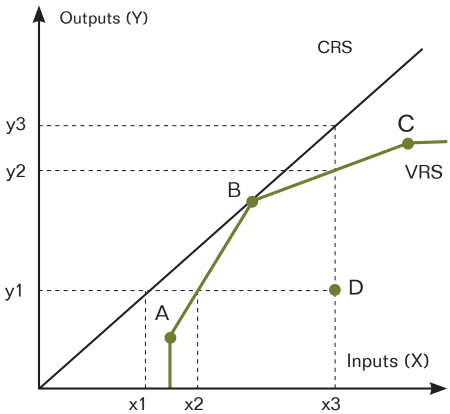
In the case of an analysis based on input (minimised inputs for the given y1 output), the index of technical efficiency (TE_I) can be defined as:

$$TE_I = \frac{x1}{x3} = \frac{x1}{x2}\frac{x2}{x3} = SE_I \; x \; PTE_I \qquad (4)$$

[10] A. Charnes, W.W. Cooper, E. Rhodes: "Measuring the efficiency of decision making units", *European Journal of Operational Research*, Vol.2, Issue 6, 1978

[11] R.D. Banker, A. Charnes, W.W. Cooper: "Some Models for Estimating Technical and Scale Inefficiencies in Data Envelopment Analysis", *Management Science* No 30(9)/1984.

Picture 4. **Constant and varying returns to scale in the DEA method**



Source: Own study.

While in the case of an analysis based on output (maximised outputs with the given inputs x3), technical efficiency index (TE_O) can be defined as:

$$TE_O = \frac{y1}{y3} = \frac{y1}{y2}\frac{y2}{y3} = PTE_O \; x \; SE_O \quad (5)$$

In the two formulas, the PTE denotes pure technical efficiency, measured with the use of the BCC and the SE – scale efficiency.

The above analysis shows that if variables of returns to scale are considered (the BCC method), the PTE indexes will be higher or equal than those achieved with the CCR method, and they will differ depending on whether it is an input based analysis (BCC-I), or an output based analysis (BCC-O). With a linear relation between outputs and inputs, efficiency indexes (TE) for CCR-I and CCR-O are identical, and small differences appear only at the stage of setting the so called lambda factors, called intensity weights or benchmarking factors, used to determine reference (model) objects for each non-optimal object.

The summary value of benchmarking factors (lambda) for every DMU is also used to determine the RTS index (Returns to Scale), which can serve to determine the impact of scale on efficiency, and it typically takes three values:

- increasing – a growth in scale will increase technical efficiency (objects to the left of point B in picture 4);
- constant – optimal scale (point B);
- decreasing – decreasing scale values (objects to the right of point B).

In 1978, the notion of slacks was additionally introduced to the DEA model. A slack is a possibility of additional decreasing of a single input, or increasing of a single output in the case when we have already proportionally decreased all inputs (or we have increased all outputs) in order to achieve the frontier value. Positive values of slacks in the case of inputs can call for changing input allocation, as understood in INTOSAI GUID 3910.

It is worth emphasising that the efficiency frontier (envelope) is determined by the best objects in the given set. If we change the set by adding or removing some objects, the efficiency frontier may change. This feature was used in the modification of the DEA method, called the Context Dependent DEA (CD-DEA). The essence of the CD-DEA is to make a step-by-step analysis:

- step one is to identify efficient (frontier) objects in the whole set of objects;
- next steps are to identify efficient objects in the set decreased by efficient objects identified during the previous step.

This procedure is used in social research and it allows for setting layers of objects with regard to their efficiency. As a result, we indicate more realistic reference (model) objects, so e.g. the model for efficient objects of step three would be the objects assumed as efficient during step two, and not during step one.

Another measure used in efficiency examination is the productivity index developed by S. Malmquist in 1953[12]. It is used to determine productivity changes over time. If productivity is expressed as y/x, the Malmquist index (M) will take the following form:

$$M = \frac{y_{t2}/x_{t2}}{y_{t1}/x_{t1}}, \qquad (6)$$

with t1 and t2 denoting two different periods of time.

Depending on whether M is below 1, above 1, or equal 1, we will refer to decreasing, increasing, or constant productivity over time.

The index defined in this way comprises all the drawbacks of the indicator measurement of productivity and efficiency described above. Therefore, it comes as no surprise that soon after the DEA method was developed (in the 1990s) a concept was elaborated to measure the index M with the use of the DEA model. In defining the index M with the use of the DEA, technical efficiency factors are considered for a given entity, calculated for a combination of four different possibilities, which can be described, in a simplified way, in the following manner:

- $TE^{t1}(x,y)_{t1}$ – technical efficiency over period t1;
- $TE^{t2}(x,y)_{t2}$ – technical efficiency over period t2;
- $TE^{t2}(x,y)_{t1}$ – technical efficiency: x and y of period t1 and technology of period t2;
- $TE^{t1}(x,y)_{t2}$ – technical efficiency: x and y of period t2 and technology of period t1;

with TE being calculated with the input-based or output-based model.

The Malmquist index is determined with the use of geometric mean (root of a product) of the indexes obtained with the use of technologies of period t1 and t2. Depending on the orientation (on inputs or on outputs) the index takes the form of:

$$M_I = \sqrt{\frac{TE^{t2}(x,y)_{t2}}{TE^{t2}(x,y)_{t1}} \frac{TE^{t1}(x,y)_{t2}}{TE^{t1}(x,y)_{t1}}}, \qquad (7)$$

$$M_O = \sqrt{\frac{TE^{t2}(x,y)_{t1}}{TE^{t2}(x,y)_{t2}} \frac{TE^{t1}(x,y)_{t1}}{TE^{t1}(x,y)_{t2}}}. \qquad (8)$$

When determining the index, it is often decomposed into two or more components. The simplest decomposition allows for identifying the component that measures technological change (TC), and the component that measures technical efficiency over time – efficiency change (EC), i.e. appropriate use of resources. As a result, the Malmquist index can be written as follows:

$$M = TC_{t1,t2}\, EC_{t1,t2} \qquad (9)$$

12 S. Malmquist: “Index Numbers and Indifference Surfaces”, *Trabajos de Estadistica* No 4/1953.

Simple formulas that illustrate the above decomposition can be found, among other, in the study by J. Jones[13].

Typically, the Malmquist index is determined with the application of constant returns to scale in the version focused either on inputs or on outputs. The values of the index and its components (TC and EC) for numerous different objects (e.g. entities) can be compared with one another.

Scope of application of the DEA and its implementation

The DEA method is broadly used and has been applied, among others, to examine financial, insurance, educational, cultural and sports institutions, as well as the healthcare sector and generally public services. In Poland, the method started being used only in the second half of the 1990s (the first publication is dated 1996), yet the scale of its application has increased astronomically, and currently there are hundreds of Polish scientific and analytical studies based on the DEA. It is related to the advantages of this method that include[14]:

- possibility to examine objects characterised by numerous inputs and numerous outputs;
- no need to have an in-depth knowledge of production functions and production parameters;
- possibility to define inputs and outputs as natural units, or even indexes, and not only as monetary units.

On the other hand, one has to bear in mind the drawbacks of the method, which include:

- need to have a relatively high number of objects to compare;
- sensitivity to untypical objects;
- instability in the case of a strong correlation between individual resources or outputs;
- cases of a large number of objects considered as efficient, especially with the use of the BCC method. As a result, we arrive at a group of objects of the same efficiency 1 (100%) and the ranking may apply only to the remaining objects;
- no possibility for making statistical conclusions.

While discussing the DEA, I will restrict myself to the above models, i.e. the CCR and the BCC with slacks, although in analytical studies numerous various DEA models are applied.

The first problem with the use of the DEA method is the selection of objects (DMU), i.e. the objects to be subject to comparative analysis. For obvious reasons, these should be objects with a similar objective of activity and factors affecting their functioning. Sometimes the number of entities is closed by principle (the number of company branches, the number of regions, etc.), and in other situations this number is selected in an arbitrary manner. The required number of DMUs depends on the number of outputs (i) and inputs (j) comprised in the analysis. Cooper and

[13] J. Jones: *Measuring efficiency*, International Handbook on the Economics of Education, 2004.

[14] B. Guzik, *Basic DEA models in examining economic and social efficiency*, University of Economy in Poznań, 2009.

others[15] propose that the number of DMU should be higher than 3 (i+j). The DEA method is not free from the risk of error due to data aggregation, as described in the previous section. That is why it is worth defining DMU in such a way as to minimise this risk (e.g. it is better to compare – as for technical efficiency – selected wards of various hospitals than whole hospitals, which frequently vary with regard to structure).

Another problem is related to the definition of inputs and outputs comprised in the model. The selection depends mainly on content related assumptions, but – in constructing analytical models – the aim is always to limit the number of variables. An increase in variables in the DEA leads to higher values of indexes, and at the same time – it decreases the selectivity of the method, and also increases the risk of co-linearity that is adverse to conclusions. J. Jones[16] proposes to select a set of inputs and outputs considering empirical premisses, experts' opinions and statistical relation between outputs and inputs, and in the case of a high number of outputs or inputs – to aim at eliminating individual variables. At the stage of elimination, she suggests verifying whether elimination of a variable from the model significantly decreases the values of efficiency indexes. If it does not decrease indexes, or if it is related to a small number of DMU, the given variable can be removed from the model. In the case of outputs, we will always strive for them to cover significant objectives of entities (objects) related to the analysed inputs. When variables are selected (especially when the objective of the examination is to make an evaluation), it also has to be analysed whether the evaluated object has an impact on the values of variables, i.e. whether it is a fully *decision making unit* (DMU). It is worth emphasising once again that both individual inputs and outputs can be expressed in different units, or they may be of the index nature. It also has been proven that[17] in the case of standard versions of the CCR and BCC models, a change in scale of input or output presentation has no impact on the values of efficiency indexes.

The latest decision problem that a researcher has to face is to choose the focus: on input or on output. More often, input based research is made, i.e. examining to what extent inputs have been used optimally in order to achieve the actual outcome.

The implementation of the DEA calls for applying appropriate IT tools, either commercial or non-commercial. A list of exemplary tools is given e.g. in the book by J. Paradi and others[18]. As for

15 W. Cooper, L. Seiford, K. Tone: *Data envelopment analysis: a comprehensive text with models, applications, references and DEA-solver software*, 2nd edition. Springer, New York, 2007.

16 J. Jones, op.cit.

17 A. Domagała: *Application of the Data Envelopment Analysis method in examining efficiency of European Stock Exchanges*, doctoral dissertation, University of Economy in Poznań, 2009.

18 J.C. Paradi, H.D. Sherman, F.K. Tam: *Data Envelopment Analysis in the Financial Services Industry,* <https://link.springer.com/content/pdf/bbm%3A978-3-319-69725-3%2F1.pdf>.

non-commercial tools, they are based mainly on the R CRAN environment, which – by nature – is an open platform. However, free of charge tools are available from Excel spreadsheets with the Solver tool, or on the basis of the MS Access programme.

According to A.K. Yadava[19], in 2020 there were 12 different R packages available that allowed for calculating with the use of the DEA methods. Out of this broad list, two packages seem to be especially important:

• Benchmarking, as it is the most popular package applied in implementation of various DEA methods;
• deaR, due to, among others, the availability of a detailed instruction that allows for applying the package by persons who are less cognizant of the R environment[20].

The two packages allow for both: application of various DEA models, and calculating the Malmquist productivity index. However, solutions based on the R platform call for basic knowledge of this environment. That is why they seem to be destined for analytical teams rather than for individual auditors.

An interesting offer are tools developed with the use of the R environment, yet available in an online version – so much easier to use. Is such a form, the said deaR package can be available, developed by a team of the Lausanne University, while for the audit environment it is worth recommending the pioneeR package developed by the team of the Supreme Audit Office of Norway (Riksrevisjonen). PioneeR is available in the public domain of the SAI of Norway[21].

For an "ordinary" user, more friendly than R packages are the applications that use Microsoft tools, including Excel Solver. Unfortunately, in this case non-commercial versions have a relatively limited scope of analytical capacities. Out of non-commercial tools based on Excel, two can be recommended:

• DEAFrontier Free Version[22] – a tool developed by Joe Zhu; the free version operates only for the CCR-I and CBB-I model, and it can be used for educational and non-commercial research purposes, the programme is installed as an Excel Add-in;
• DEA Spreadsheet Solver[23] – a tool developed in 2021 by G. Erdogan of the University of Bath. The tool is an Excel file with macros (xlsm) taking advantage of Solver, and it operates the CCR-I and BCC-I methods. Unlike the previous tool, this one is open (open VBA code) and there are no restrictions on its application. In agreement with the author, I have developed a Polish language version of the programme interface. I have elaborated the presentation of outputs in the way that, in my opinion, is useful for auditors.

19 <https://www.researchgate.net/publication/340930743_Available_Packages_in_R_for_Data_Envelopment_Analysis_DEA>
20 <https://www.uv.es/deaRshiny/deaR.html>
21 <https://github.com/Riksrevisjonen/pioneeR>
22 Available at <http://www.deafrontier.net/deafree.html>.
23 Available at <https://people.bath.ac.uk/ge277/dea-spreadsheet-solver/>.

An interesting and relatively simple to operate alternative to Excel is the MaxDEA application[24] based on MS Access. The MaxDEA Basic version is available for free, and it allows for calculating with the use of the CCR-I, CCR-O, BCC-I and BCC-O methods.

Later, I will focus on the DEA Spreadsheet Solver tool, which can be used for simple analyses conducted individually by auditors, and which does have legal restrictions on its application.

The DEA Spreadsheet Solver sheet is very simple, and it allows for defining up to 99 DMU objects, as well as up to 20 various inputs and 20 various outputs. Once these parameters have been determined, a special data spreadsheet is open, where input data (inputs and outputs of individual objects) have to be given or copied. Objects can be given abbreviated names, which makes it easier to interpret the results. Then, a result spreadsheet is prepared and – depending on the analysis model (CCR or BCC) – calculations are made. Some of them are made with the use of Solver Excel, and others are recorded in the form of an Excel function, which allows for their potential modification. The results can be presented in full detail, or they can be limited to efficiency indexes and reference objects. The limited version is simple and readable, but it can give a certain shortage of information – that is why I proposed an additional version of results presentation: expanding the list of reference objects with benchmarking factors, and adding information on the RTS index. Efficiency indexes sorted by values can be presented in the form of a bar box. All the above components will be discussed in the example below. In the expanded version of results, it may be additionally interesting to look at the values of slacks for individual inputs and outputs.

Unfortunately, the tool does not allow for determining the Malmquist index. For a single object you can, admittedly, calculate the index by indicating its four components, but in the case of calculating the index for every object, which is important in benchmarking, it would be too time consuming.

Exemplary uses of the DEA

In order to illustrate uses of the DEA method, I have employed the data related to the functioning of municipal guards in the capital cities of 16 Polish regions, discussed in the audit report No. NIK P/15/006/KAP *Financing of municipal guards*, published in March 2016, and available at the NIK website[25]. For my analysis, I have selected the following data:

- on the side of inputs: average expenditure in thousands PLN and employment in full time jobs;
- on the side of outputs: city surface in square km, number of population in thousands, and the number of interventions in thousands.

[24] Available at <http://www.maxdea.cn/>.

[25] <https://www.nik.gov.pl/kontrole/wyniki-kontroli-nik/pobierz,kap~p_15_006_201508071410581438956658~01,typ,k.pdf>

Table 2. **Functioning of municipal guards – entry data for a DEA method**

DMU	Expenditure (thousand)	Jobs	Surface (km)	Population (thousand)	Interventions (thousand)
Ww	129,798.3	1,522.7	517	1,725	3,010.4
Kr	29,736.8	374.7	327	760	687.2
Lo	28,211.5	408.0	293	712	515.7
Wr	20,757.0	279.2	293	633	550.0
Po	17,043.0	258.3	262	548	449.4
Gd	18,533.9	226.7	262	461	382.0
Sz	8,811.9	123.0	301	408	246.6
By	13,552.2	185.3	176	359	323.3
Lu	8,081.8	116.3	147	344	202.9
Ka	8,824.2	113.0	165	304	157.9
Bi	9,407.2	126.3	102	295	149.8
Ki	6,049.0	77.3	110	200	115.6
Rz	3,788.9	60.3	116	183	58.7
Ol	4,732.9	80.3	112	185	127.4
Op	4,778.3	59.0	97	120	100.5
Zg*	1,676.8	25.7	58	119	14.4

*) entity with a different organisational structure.
Source: Own study based on the NIK's audit.

Individual cities, which make DMU objects, have been indicated with symbolic names, and the data have been presented in the table above.

Since this is an object analysis only, all entities have been included, although the last one has a different organisational structure, which probably had an impact on the value of inputs, and at the same time – on the efficiency index. In the analysis, other measurements of outputs have not been included, such as the feeling of security of the citizens, or the number of offences.

The DEA analysis has been made with the use of the DEA Spreadsheet Solver and it has been verified with the use of the MaxDEA Basic and DEAFrontier Free Version, and the list of results has been presented in the table 3, p. 62.

The analysis performed with the CCR-I method (constant returns-to-scale) shows that technically efficient objects (value of TE=100%) are Sz and Zg*, and so they

Table 3. **Functioning of municipal guards – results of the analysis made with the DEA CCR-I and BCC-I methods**

DMU	TE	Refer.	RTS	Slacks expenditure	Slacks jobs	PTE	Reference	SE
Ww	99%	Sz	Dec	20,420.5	-	100%		99%
Kr	91%	Sz	Dec	2,646.1	-	100%		91%
Lo	65%	Sz	Dec	-	9.28	93%	Kr, Wr	70%
Wr	98%	Sz	Dec	741.5	-	100%		98%
Po	94%	Sz	Dec	-	19.23	99%	Wr, Sz	96%
Gd	84%	Sz	Dec	1,926.9	-	85%	Ww, Sz	99%
Sz	100%		Const	-	-	100%		100%
By	87%	Sz	Dec	241.1	-	88%	Ww, Wr, Sz	99%
Lu	91%	Sz, Zg	Inc	-	2.97	92%	Sz, Ol, Zg	99%
Ka	77%	Sz, Zg	Const'	636.9	-	78%	Sz, Zg	99%
Bi	66%	Sz, Zg	Const'	320.4	-	67%	Sz, Zg	99%
Ki	77%	Sz, Zg	Inc	407.1	-	87%	Sz, Op, Zg	89%
Rz	89%	Sz, Zg	Dec	-	4.09	89%	Sz, Zg	100%
Ol	96%	Sz	Inc	-	13.70	100%		96%
Op	85%	Sz	Inc	468.7	-	100%		85%
Zg*	100%		Const	-	-	100%		100%

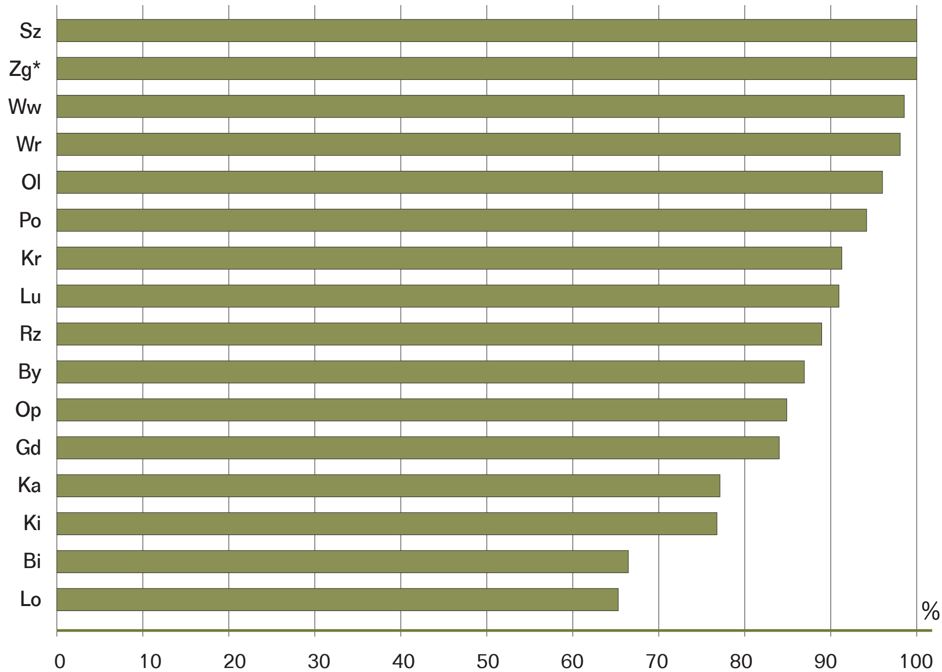
*) entity with a different organisational structure.
Source: Own calculations with the use of the DEA Spreadsheet Solver.

make reference objects. The lowest efficiency indicator is for Lo, Bi, Ki and Ka (below 80%). Some entities operate in the decreasing returns to scale conditions (RTS=Dec), and in four cases a growth in scale would improve technical efficiency (RTS=Inc). Positive values of slacks on the part of inputs indicate which input items could be decreased to improve efficiency.

The method for varying returns to scale (BCC-I) points at seven efficient objects, for which pure technical efficiency PTE=100%. This is due to the component related to scale efficiency (SE). Returns to scale are especially unfavourable for Lo. The objects with the lowest pure technical efficiency are Bi and Ka – in their case returns to scale virtually do not exist (RTS close to Const).

The above example is for illustration purposes only. In reality, we should consider whether it is possible to obtain additional data on outputs. Also, if object Zg

Picture 5. **Ranking of technical efficiency (TE)**



*) entity with a different organisational structure.
Source: Own calculations made with DEA Spreadsheet Solver.

was excluded from the analysis, frontier values would change.

Recommendations for the DEA method in auditing

The Data Envelopment Analysis (DEA) has for many years been used also outside academic circles or scientific studies, and has been successfully applied in practice by government agencies and institutions, as well as by businesses in many countries for evaluation and benchmarking. It comes as no surprise that it is recommended as a source of analytical evidence in performance auditing[26]. Unfortunately, there is no detailed guidance on how to use the method in the audit practice. When analysing the list of publications about the DEA method – and A. Emrouznejad[27] referred to 10,300 by 2016, including 1,100 in 2016

[26] INTOSAI *GUID 3920 The Performance Auditing Process*, par. 49.
[27] A. Emrouznejad, G. Yang: "A survey and analysis of the first 40 years of scholarly literature in DEA: 1978–2016", *Socio-Economic Planning Sciences*, 2018.

– I found only few publications related to auditing. It is worth emphasising, though, that this list is not complete, and audit institutions tend to present findings rather than to describe the methods they use. Nevertheless, I was little surprised to see that the use of the DEA method in auditing has been patented at some point[28]. Yet the patent is related to the application of the DEA as an analytical method in financial statements examination, and the authors of the patent recommend using the DEA:

- At the initial stage of an audit, in order to determine its scope and assess the initial risk level of the client,
- As an analytical procedure at the stage of general review, in order to detect anomalies and to assess the legitimacy of financial statements,
- In order to benchmark the client against other firms in the same sector, to provide consistent and reliable flags and references.

When it comes to using the DEA in performance audits conducted by Supreme Audit Institutions (SAI), the SAIs of the following countries are said to do it: USA, Denmark, Sweden, Portugal and Norway (the pioneeR IT tool already referred to). In Europe, the SAIs of Nordic countries are especially active in various applications of the DEA, especially the SAI of Sweden (Riksrevisjonen), which used the DEA method in the audit of labour offices (in 2006 and in 2012), higher education institutions (in 2011 and in 2019), courts (in 2017) and prisons (in 2020). However the information about the application of the method available from the respective audit reports (e.g. *Resource efficiency and productivity of Swedish higher education institutions in the Nordic countries*[29]) are rather scarce, due to the size of the reports.

The method needs to be popularised – as evidenced by the new project[30] implemented within Strategic Goal 2 of the EUROSAI Strategic Plan for 2017–2024 "Helping SAIs deal with new opportunities and challenges by supporting their institutional capacity development". The project is led by the SAI of Sweden, and it comprises workshops on using the DEA method in performance auditing. The kick off meeting of the project related to the DEA and other benchmarking methods was held on 26 April 2022[31].

The DEA method is not the panacea for all the problems that we may have with measuring efficiency. The results obtained with this method depend, to a large extent, on the appropriate selection of objects to benchmark, and on the selection of the sets of inputs and outputs, therefore they may be considered doubtful by auditees. The alternative in the form of the indicator method – although more understandable for the readers of audit reports – can be applied only in simple cases, or used to determine partial results only.

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[28] E. Feroz and others: *Application of Data Envelopment Analysis in auditing*, US Patent Application Publication, US 2005/0288980 A1.

[29] <https://www.riksrevisionen.se/download/18.151bf9df173c7975ca0b6ca9/1597663844637/RiR_2019_21_SAMF%20ENG_FINAL.pdf>

[30] <https://www.eurosai.org/en/ESP-2017-2024/ProjectGroups>

[31] <https://www.eurosai.org/en/calendar-and-news/calendar/index.html?calYear=2022&calMonth=3>

It needs to be remembered that the DEA, similarly as other advanced analytical methods, does not ensure the so called hard audit evidence, so results obtained with the use of this method are often only auxiliary. On the other hand, benchmarking – and the DEA is a natural tool for it – can be, in the public sector, the only trail that allows for evaluating the efficiency of a given entity. Substantial flexibility in defining inputs and outputs allows for extending the application of the DEA method to examine outcomes, and not only products – which is especially vital in performance auditing. It is worth quoting the objective of performance auditing: it is not only to evaluate, but also to show room for improvement and points of reference – and the DEA makes it possible.

When planning to use the DEA, auditors can take advantage of numerous publications dedicated to this method. There are also publications available that compare the application of the DEA in numerous various studies in the same sector, e.g. education, schooling, public services, healthcare, etc. They can be helpful in selecting the DEA method and in defining an optimal list of inputs and outputs. However, auditors are reluctant to read academic publications. For them it would be more interesting to read about how the method is applied in auditing, to discuss its advantages and disadvantages, and to get examples and practical guidance. It exceeds the scope of one publication, though.

We should hope that the said EUROSAI project will contribute to making the method more popular in the community of Supreme Audit Institutions. However, it is worth mentioning that the materials created as part of such projects should be cumulated, enabling a gradual transition to more advanced analytical models and presented as open access.

Eng. **WIESŁAW KARLIŃSKI,** PhD
specialist in the field of audit methodology and the application of analytical methods

Key words: performance auditing, performance measurement, non-parametric methods, benchmarking, DEA, data envelope method

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