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Impact of expenditure on social assistance on household income at the regional level in Poland

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Abstract. The main aim of the paper is a statistical analysis of changes in household income distribution at the regional level in Poland taking into account the impact of government spending on social assistance. Various linear models (incorporating relations for spline functions) and the vector autoregression models (VAR) were used in the research. The linear models formulated for voivodships (NUTS 2) contained a dichotomous variable with values dependent on the existence of social programmes introduced by the Polish government in 2016. An independent variable representing expenditure per capita on social assistance specified for the national level was also used. The results for these models were compared with the findings of both microsimulation studies obtained on the basis of the Household Budget Surveys (HBS) and the total assessment of the social programmes, and they indicate a significant influence of social assistance expenditure on the amounts of available income. The calculations were conducted using data from the Statistics Poland databases: Local Data Bank (and in particular, data from the Polish HBS for the years 2000–2018) and from the Macroeconomic Data Bank, and from the annual reports on the implementation of the state budget. They were performed by means of the R-project environment and R-commander overlay, using the lm function as well as the vars module for the R-project environment. The study also involved using the Gretl package.

Keywords: available income, econometric models, Vector Autoregression Model, R-project, Gretl, expenditures on social assistance, household income

JEL: C01, C21, C22, D31, E64, H53, H55

Wpływ wydatków na pomoc społeczną na dochód gospodarstw domowych według województw

Streszczenie. Głównym celem artykułu jest analiza statystyczna zmian rozkładu dochodów gospodarstw domowych w Polsce na poziomie regionalnym z uwzględnieniem wpływu wydatków rządowych na pomoc społeczną. W badaniu wykorzystano modele liniowe, które zawierają relacje wykorzystujące funkcje sklejane, oraz wektorowe modele autoregresyjne (VAR). Modele liniowe dla województw zawierały zmienną dychotomiczną o wartościach zależnych od funkcjonowania programów socjalnych wprowadzonych przez polski rząd w 2016 r. Wykorzystano również zmienną niezależną określającą wydatki na pomoc społeczną *per capita* na poziomie kraju. Wyniki dla tych modeli zostały porównane z podobnymi miarami wyznaczonymi w badaniach mikrosymulacyjnych na podstawie badania budżetów gospodarstw domowych oraz z łączną oceną programów społecznych. Wskazują one, że wydatki na pomoc społeczną mają znaczący wpływ na wartości dochodu rozporządzalnego. Do obliczeń wykorzystano dane z baz GUS: Banku Danych Lokalnych (w szczególności dane z badania budżetów gospodarstw

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domowych za lata 2000–2018) oraz Banku Danych Makroekonomicznych, a także dane z corocznych sprawozdań z wykonania budżetu państwa. Użyto środowiska R-project oraz nakładki R-commander z zastosowaniem funkcji lm, jak również modułu vars dla środowiska R-project. Posłużono się także programem Gretl.

Słowa kluczowe: dochód rozporządzalny, modele ekonometryczne, wektorowy model autoregresyjny, R-project, Gretl, wydatki na pomoc społeczną, dochody gospodarstwa domowego

1. Introduction

According to the Eurostat database,[1] child poverty (children at risk of poverty and social exclusion) afflicted 23.3% of the child population in Poland in 2016, while the following year, the rate of this phenomenon significantly decreased, to 16.8%, which placed Poland close to Germany (18%). The reason for this change is twofold, i.e. the reduced unemployment rate connected to notable economic growth during the last few years as well as pro-family policies of the government. The statistical analysis of changes in income inequality and poverty before and after launching the Family 500+[2] child benefit programme was recently discussed by Jędrzejczak and Pekasiewicz (2019). According to this paper, the programme seems to be having a noticeable impact on income distribution among Polish households, which results in the reduction of poverty and inequality, especially for lower income groups and families with many children. This shows that reducing poverty depends not only on the level of production or the unemployment rate, but also on the level of social expenditure, such as unemployment benefits and social assistance. Such relationships can be described using VAR (vector autoregression) models. The first applications of these models date back to the 1980s and can be found in the paper by Blanchard and Quah (1989), which interprets the fluctuations in GNP and the unemployment rate as due to two types of disturbances: those that have a permanent effect on the output and those that have not any such effect, which was demonstrated using the VAR model. A more extended approach was presented in Desaling Germay (2016), where the Granger causality test confirms the relationships between the unemployment rate, the GDP percentage change compared to the previous period, and industrial production in the years 1983–2010, based on the unemployment rate quarterly data obtained from the OECD. Relationships between GDP growth and the unemployment rate were also presented in Ekanem (2005), where VAR models with a more complex structure appear. In this work, the crucial role is played by the following indicators M1: the ratio of gross private domestic investment to GDP, the consumer confidence index on GDP, overall job growth, the growth of jobs in manufacturing and services, as well as on unemployment. Such relationships can also be useful in tracking technology limitations for particular national economies using VAR models. Kalinowska (2016) shows the role of the unemployment benefit system in stabilising the economy in Poland in 2008–2013. This indicates that social expenditure can

[1] People at risk of poverty or social exclusion by age and sex (dataset ilc_peps01), http://ec.europa.eu/eurostat/product?code=ilc_peps01&language=en&mode=view.

[2] See: https://www.gov.pl/web/rodzina/rodzina-500-plus.

play a role similar to the one presented in our paper. The work of Jappelli and Pistaferri (2010) explains in more detail the role of anticipated income changes, including positive and negative shocks, using an exhaustive literature review. The authors emphasise the importance of consumer expectations in shaping expenses and – indirectly – income, which is also demonstrated in our paper. Based on the estimated measures, presented in Jędrzejczak and Pekasiewicz (2019), it can be concluded that in 2016, significant changes were observed in both the average income and the scale of poverty among families with a different number of children (for various family types, the results of two-sample *t*-tests for means and proportions showed $p < 0.0001$). Particularly notable differences appeared in the sub-group of families with four or more children, where the proportion of households afflicted by poverty decreased from 38.9% to 16.7%. In total, the poverty rate decreased by 1.2 percentage points, which means that over 160,000 households ceased to be poor.

It would also be interesting to formulate a sufficient statistical model which can be used to better understand the changes in income distribution in Poland during the last few years and to assess the impact of different macroeconomic variables, including social assistance indicators, on the household income per capita.

Although a linear function is very convenient, it is extremely unlikely that the empirical personal or household incomes are linear and additive functions of various covariates. Therefore, due to the insufficient level of consistency of simple linear models employed to describe socio-economic phenomena, it may sometimes be justified to use a non-linear approach. The selection of a non-linear model was restricted to the different form of spline-type functions, including natural splines and B-splines (the overall number of the considered function forms was greater than 70), and the main selection criteria were values of the coefficient of determination (R^2), *F*-statistics for linear regression, and both Akaike and Bayesian Information Criterions. Such an approach helps not to miss a part of valuable non-linear information hidden in income data. However, this involves the need for a more careful selection of explanatory variables due to the sensitivity of non-linear models to changes in their parameters. Substantial changes in social policy in Poland, launched after 2015, indicate the possibility of taking into account qualitative changes, e.g. regarding income distribution models. However, one should bear in mind the ambiguous impact of social programmes on the amount of income. It also seems reasonable to ask about the impact of the decrease in the unemployment rate on GDP growth and thus on the economic situation of households. A quantitative assessment of the impact of both the social assistance expenditure and wage increases on household disposable income can therefore be a useful tool for measuring the effects of introducing qualitative changes in such expenditure in the state budget.

The analysis below is intended to perform a statistical analysis of changes in household income distribution at the regional level in Poland taking into account the impact of government spending on social assistance. Various linear models (incorporating relations for spline functions) and the vector autoregression models (VAR) were used to this effect.

2. Methodology

The results presented in the paper were obtained mainly on the basis of the Polish Household Budget Survey (HBS) data from the Polish Local Data Bank (LDB)[3] (and in particular, data from the Polish HBS for the years 2000–2018) and the Macroeconomic Data Bank (MDB).[4] Additionally, the study uses data from the State Budget Reporting for the years 2000–2018.[5]

The simplest spline or spline function S is a special function defined piecewise by polynomials, with each polynomial being a function of one variable. The S function takes values from the range $[a, b]$ and maps them to the set of real numbers, which can be expressed by the relationship

$$S : [a, b] \to \mathbb{R}.$$

Since S is defined as a piecewise function, it is possible, by selecting k, to indicate the division into ordered disjoint subintervals called 'pieces', of the range $[a, b]$:

$$[t_i, t_{i+1}] \quad \text{for} \quad I = 0, \dots, k - 1,$$

$$[a, b] = [t_0, t_1] \cup [t_1, t_2] \cup \dots \cup [t_{k-2}, t_{k-1}] \cup [t_{k-1}, t_k],$$

$$a = t_0 \le t_1 \le t_2 \le \dots \le t_{k-1} \le t_k = b.$$

Each of these subintervals is associated with a P_i polynomial

$$P_i : [t_i, t_{i+1}] \to \mathbb{R}.$$

For the i-th interval $[a, b]$, the spline function S is defined by means of the polynomials P_i in the following way:

$$S(t) = P_0(t), \quad t_0 \le t < t_1,$$

[3] See Local Data Bank, https://bdl.stat.gov.pl/BDL/start (use Category K3 – Population, Group G10 – Private households, Subgroup P1869 – Average monthly available income per capita).
[4] See Macroeconomic Data Bank, https://bdm.stat.gov.pl/ (e.g. the following annual categories: Living conditions of population, and Labour market).
[5] See the website archive of the Polish Ministry of Finance – Finanse publiczne, Budżet państwa, Wykonanie budżetu państwa – https://mf-arch2.mf.gov.pl/web/bip/ministerstwo-finansow/dzialalnosc/finanse-publiczne/budzet-panstwa/wykonanie-budzetu-panstwa/sprawozdanie-z-wykonania-budzetu-panstwa-roczne and https: //www.gov.pl/web/finanse/wykonanie-budzetu-panstwa.

$$S(t) = P_1(t), \quad t_1 \leq t < t_2,$$
$$\vdots$$
$$S(t) = P_{k-1}(t), \quad t_{k-1} \leq t < t_k.$$

The given $k - 1$ points t_j $(0 \leq j \leq k)$ are called knots.

Vector $\boldsymbol{t} = (t_0, \dots, t_k)$ is called a knot vector for the spline. If each of the polynomial pieces P_i has degree at most n, then the spline is said to be of degree $\leq n$ (or of order $n + 1$).

A common spline is constructed of piecewise third-order polynomials with continuity (i.e. functions of C^2 class) which pass through a set of control points. The second derivatives of the spline polynomials are the set equal to 0 at the endpoints of the interval of interpolation $[a, b]$, which gives the so-called natural spline. Thus, let the i-th piece of the spline be represented by

$$P_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3.$$

Given the set of $k + 1$ coordinates $(x_0, y_0), (x_1, y_1), \dots, (x_k, y_k)$ we wish to derive k splines $P_i(x)$, which satisfy the following equations for $1 \leq i \leq k - 1$:

$$P_0(x_0) = y_0,$$

$$P'_{i-1}(x_i) = P'_i\ (x_i),$$

$$P''_{i-1}(x_i) = P''_i\ (x_i),$$

$$P''_0\ (x_0) = P''_{k-1}(x_k) = 0.$$

Solving the equations for a_i, b_i, c_i and d_i gives the natural cubic splines.

When using the spline functions in a linear model, one can treat the values of the function specifying the natural knots as a matrix, resulting from an appropriate transformation, with the number of columns equal to the degrees of freedom (depending on the nodes) and with the number of rows equal to the number of observations. The model can be treated as the case of an additive model. More details on the possibilities of applying such a transformation can be found in the documentation for the splines package for the R-project.

The vector autoregression model (VAR) is a model of the stochastic process used to explain linear correlations for multidimensional time series. The VAR model is a generalisation of a one-dimensional autoregressive model (AR), thus allowing the analysis of more than one variable. The VAR model involves all the variables in the same way: each variable has a corresponding equation explaining its evolution based on the relationship of its lagged values, the lagged values of the other model variables, and a random component.

The structure of the VAR model makes it possible to describe the stability of interactions over time for time series $\boldsymbol{y}_t$ with n components through a multidimensional autoregressive model, which can be presented as follows:

$$\boldsymbol{y}_t = \mathbf{A}_1\boldsymbol{y}_{t-1} + \mathbf{A}_2\boldsymbol{y}_{t-2} + \cdots + \mathbf{A}_p\boldsymbol{y}_{t-p} + B\boldsymbol{x}_t + \boldsymbol{\epsilon}_t.$$

The number of lags p is defined as the order of the VAR model. The $\boldsymbol{x}_t$ vector (if included in the model) contains various exogenous variables, which comprise the free term and the occurrence of a time-dependent trend and seasonal components. Vector $\boldsymbol{\epsilon}_t$ is usually assumed to be in the form of the vector white noise with a covariance matrix $\boldsymbol{\Sigma}$.

The aforementioned equation can be written in the following concise form:

$$\mathbf{A}(L)\boldsymbol{y}_t = B\boldsymbol{x}_t + \boldsymbol{\epsilon}_t,$$

where $\mathbf{A}(L)$ is a matrix polynomial in the lag, or in the matrix form

$$\begin{bmatrix} \boldsymbol{y}_t \\ \boldsymbol{y}_{t-1} \\ \cdots \\ \boldsymbol{y}_{t-p-1} \end{bmatrix} = \mathbf{A} \begin{bmatrix} \boldsymbol{y}_{t-1} \\ \boldsymbol{y}_{t-2} \\ \cdots \\ \boldsymbol{y}_{t-p} \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ \cdots \\ 0 \end{bmatrix} x_t + \begin{bmatrix} \boldsymbol{\epsilon}_t \\ 0 \\ \cdots \\ 0 \end{bmatrix},$$

where $\mathbf{A}$ from the above equation is called a companion matrix to the matrix polynomial and takes the following form:

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 & \cdots & \mathbf{A}_P \\ I & 0 & \cdots & 0 \\ 0 & I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \end{bmatrix}.$$

The R-project system (R Core Team, 2018) was used in the calculations. More specifically, it was the lm function from the standard stats package, ns and bs functions from the splines package, VAR function (for obtaining the model estimates), normality.test (for the assessment of normality of residuals see Jarque & Bera, 1987) and causality (for Granger causality assessment see Kusideł, 2000; Lütkepohl, 2005) from the vars package (Pfaff, 2008a, 2008b). Some calculations (including the calculations for non-linear models) were performed using the R-commander extension (see Fox, 2017; Fox & Bouchet-Valat, 2018). Calculations for VAR models were also – independently – performed using the Gretl package (see Baiocchi & Distaso, 2003). This package was also used to determine the test values for autocorrelation (Ljung & Box, 1978) and the homoscedasticity of the residues of VAR-type models.

3. Results and discussion

In the analysis, several characteristics of personal income, in particular the characteristics of the available income per capita and other household income, were estimated as ratio estimators taking into consideration survey weights. A summary of the results obtained under the first of these models is presented in Table 1.

Table 1 shows that the model which utilises GDP per capita is statistically significant, as are its parameters. Also the determination coefficient, which equals 0.8044, is high. However, one could prepare a model with better statistical properties that would account for the variability of household income by voivodship. Such a model is also described in Table 1 and it utilises the average gross monthly wage as an explanatory variable. Due to the higher R^2 value (0.9487), and a higher p-value corresponding to the F-statistic, one can conclude that the use of the variable describing the average wage will be more appropriate in this case.

Table 1. Diagnostics of the regression model describing the average available income per capita in voivodships for the years 2002–2018 with GDP per capita and the average gross monthly wage

Explanatory variable	Parameter estimate	Standard error	*t*-statistic	*p*-value
GDP per capita				
Intercept	295.2472	25.6513	11.51	< 2e-16***
Gross domestic product per capita in current prices	0.0237	0.0007	33.32	< 2e-16***
Determination coefficient	R^2 = 0.8044, corrected R^2 = 0.8037			
F-statistic	F_{emp} = 1100, p-value < 2.2e-16			
Information criterion	Akaike = 3483.335, Bayesian = 3494.152			
The average gross monthly wage				
Intercept	–133.5667	18.0115	–7.42	0.000002***
Average monthly gross wages and salaries	0.3897	0.0055	70.64	< 2e-16***
Determination coefficient	R^2 = 0.9487, corrected R^2 = 0.9485			
F-statistic	F_{emp} = 4989, p-value < 2.2e-16			
Information criterion	Akaike = 3119.507, Bayesian = 3130.325			

Note. Significance levels: *** – [0, 0.001], ** – (0.001, 0.01], * – (0.1, 0.5].
Source: authors' calculations based on Statistics Poland's LDB and MDB, and State Budget Reporting.

As the aim of the publication was to show the impact of various socio-economic conditions that have occured within the last two years (including social programmes, especially the Family 500+ programme) on household income per capita, the models also include a dichotomous variable with the values 0 for 2002–2015 and 1 for 2016–2018. The linear regression model incorporating this dummy variable, despite the average gross monthly wage, is described in Table 2 and presented in Figure 1.

Table 2. Diagnostics of the regression model describing the average available income per capita in voivodships for the years 2002–2018 with the average gross monthly wage and the dichotomous variable

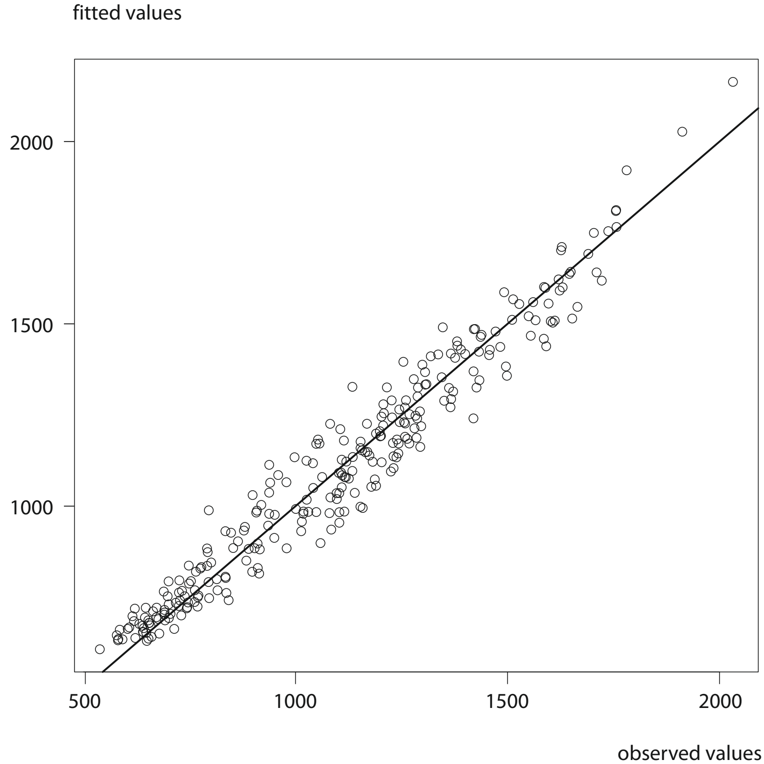
Explanatory variable	Parameter estimate	Standard error	*t*-statistic	*p*-value
Intercept	–94.3257	20.5024	–4.601	0.0000065***
Average monthly gross wages and salaries	0.3742	0.0068	55.058	< 2e-16***
Dichotomous variable	54.2959	14.5602	3.729	0.000234***
Determination coefficient	$R^2 = 0.9512$, corrected $R^2 = 0.9508$			
F-statistic	$F_{emp} = 2621$, *p*-value < 2.2e-16			
Information criterion	Akaike = 3107.798, Bayesian = 3122.221			

Note. As in Table 1.
Source: authors' calculations based on Statistics Poland's LDB and MDB, and State Budget Reporting.

From the model diagnostics described in Table 2 it follows that the impact of the additional variable is statistically significant. It corresponds (in terms of value) to the level of expenditure per capita in 2017, which, among other factors, was related to the Family 500+ programme. This is due to the fact that on average, every Pole paid annually PLN 619 to the programme, i.e. PLN 51.58 a month. Thus, the model described in Table 2 approximately reflects the impact of the Family 500+ programme on family disposable income. This variable can describe not only the changes caused by the above-mentioned programme, but also other dependencies resulting e.g. from the improvement in the economic situation and an increase in average wages. It is worth noting that the results are related to a statistical analysis based on a simulation study described in the work of Brzeziński and Najsztub (2017).

In the course of the further analysis, it was decided that the impact of the variables other than the value of GDP per capita and average gross wages would be taken into account, as well as it would be checked whether the used models were non-linear. A number of tests were carried out using the lm function which utilises non-linear functions as arguments of a linear model, including B-spline and spline functions.

Figure 1. Empirical versus theoretical values of the average available income per capita obtained for the linear model with the average gross wage and the dichotomous variable



Source: authors' calculations based on Statistics Poland's LDB and MDB, and State Budget Reporting.

Table 3. Diagnostics of non-linear B-spline and spline regressions describing the average available income per capita in voivodships for the years 2002–2018 versus GDP per capita

Explanatory variable	Parameter estimate	Standard error	*t*-statistic	*p*-value
		B-spline regression		
Intercept	621.98	47.84	13.00	< 2e-16***
bs(GDPPC,df=5)1	–37.98	80.77	–0.47	0.639
bs(GDPPC,df=5)2	326.84	48.42	6.75	9.26e-11***
bs(GDPPC,df=5)3	1090.91	84.69	12.88	< 2e-16***
bs(GDPPC,df=5)4	944.91	118.21	7.99	4.02e-14***
bs(GDPPC,df=5)5	1419.64	111.95	12.68	< 2e-16***
Determination coefficient	$R^2 = 0.8669$, corrected $R^2 = 0.8644$			
F-statistic	$F_{emp} = 346.6$, *p*-value < 2.2e-16			
Information criterion	Akaike = 3386.568, Bayesian = 3411.809			

Table 3. Diagnostics of non-linear B-spline and spline regressions describing the average available income per capita in voivodships for the years 2002–2018 versus GDP per capita (cont.)

Explanatory variable	Parameter estimate	Standard error	*t*-statistic	*p*-value
		Spline regression		
Intercept	590.89	37.83	15.62	<2e-16***
ns(GDPPC,df=5)1	414.56	40.94	10.13	<2e-16***
ns(GDPPC,df=5)2	671.48	46.54	14.43	<2e-16***
ns(GDPPC,df=5)3	925.78	50.99	18.16	<2e-16***
ns(GDPPC,df=5)4	1308.16	97.24	13.45	<2e-16***
ns(GDPPC,df=5)5	1325.61	75.30	17.60	<2e-16***
Determination coefficient	$R^2 = 0.8686$, corrected $R^2 = 0.8661$			
F-statistic	$F_{emp} = 351.7$, *p*-value < 2.2e-16			
Information criterion	Akaike = 3383.116, Bayesian = 3408.356			

Note. As in Table 1. GDPPC – GDP per capita.

Source: authors' calculations based on Statistics Poland's LDB and MDB, and State Budget Reporting.

Table 4. Diagnostics of the non-linear B-spline and spline regressions describing the average available income per capita in voivodships for the years 2002–2018 versus average gross wage

Explanatory variable	Parameter estimate	Standard error	*t*-statistic	*p*-value
		B-spline regression		
Intercept	575.49	27.99	20.558	<2e-16***
bs(WAGE,df=5)1	124.19	49.91	2.488	0.0134*
bs(WAGE,df=5)2	364.22	29.29	12.433	<2e-16***
bs(WAGE,df=5)3	857.31	49.73	17.238	<2e-16***
bs(WAGE,df=5)4	1327.70	56.81	23.369	<2e-16***
bs(WAGE,df=5)5	1396.37	70.27	19.872	<2e-16***
Determination coefficient	$R^2 = 0.9509$, corrected $R^2 = 0.9500$			
F-statistic	$F_{emp} = 1031$, *p*-value < 2.2e-16			
Information criterion	Akaike = 3115.218, Bayesian = 3140.459			
		Spline regression		
Intercept	583.44	22.22	26.26	<2e-16***
ns(WAGE,df=5)1	431.62	25.46	16.95	<2e-16***
ns(WAGE,df=5)2	578.59	28.95	19.98	<2e-16***
ns(WAGE,df=5)3	989.84	27.25	36.32	<2e-16***
ns(WAGE,df=5)4	1406.7	60.44	23.27	<2e-16***
ns(WAGE,df=5)5	1323.71	48.75	27.15	<2e-16***
Determination coefficient	$R^2 = 0.9516$, corrected $R^2 = 0.9507$			
F-statistic	$F_{emp} = 1046$, *p*-value < 2.2e-16			
Information criterion	Akaike = 3111.481, Bayesian = 3136.722			

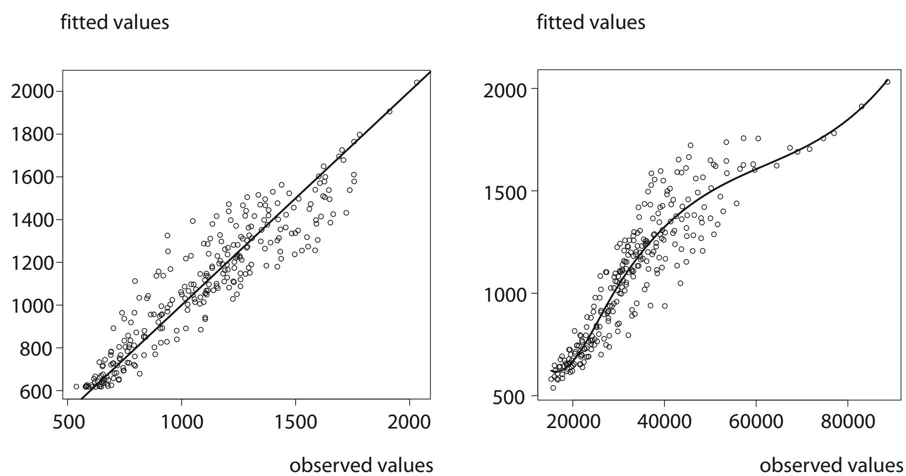
Note. As in Table 1. WAGE – average gross wage.

Source: authors' calculations based on Statistics Poland's LDB and MDB, and State Budget Reporting.

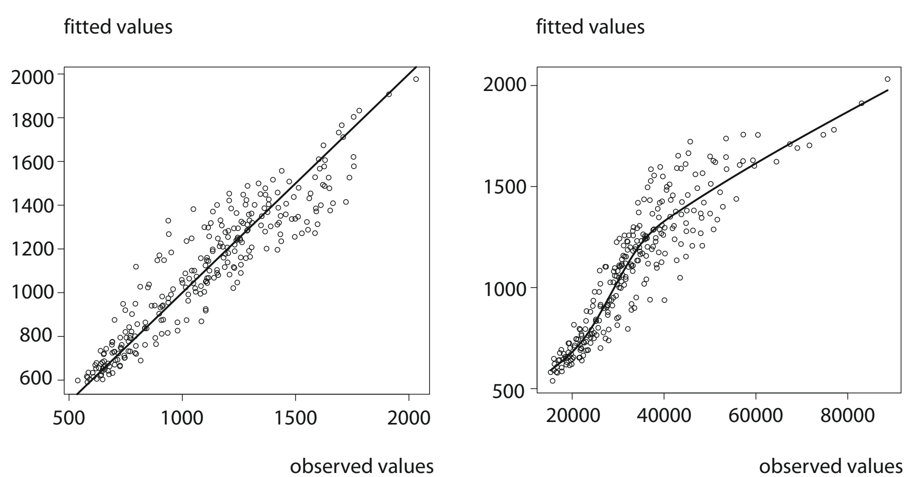
The results of these analyses are presented in Tables 3 and 4 and Figures 2 and 3. They show that auxiliary data describing levels of wages can be helpful in estimating the value of the available income per capita, also using a non-linear approach. The models shown in Table 4 demonstrate a very high level of consistency with the data with determination coefficients higher than 95%, when the explanatory variable describing average gross wages in voivodships has been added.

Figure 2. Plots for the non-linear B-spline and spline regression describing the average available income per capita versus average GDP per capita – empirical versus theoretical values (left), the scatter plot and regression line (right)

B-spline regression

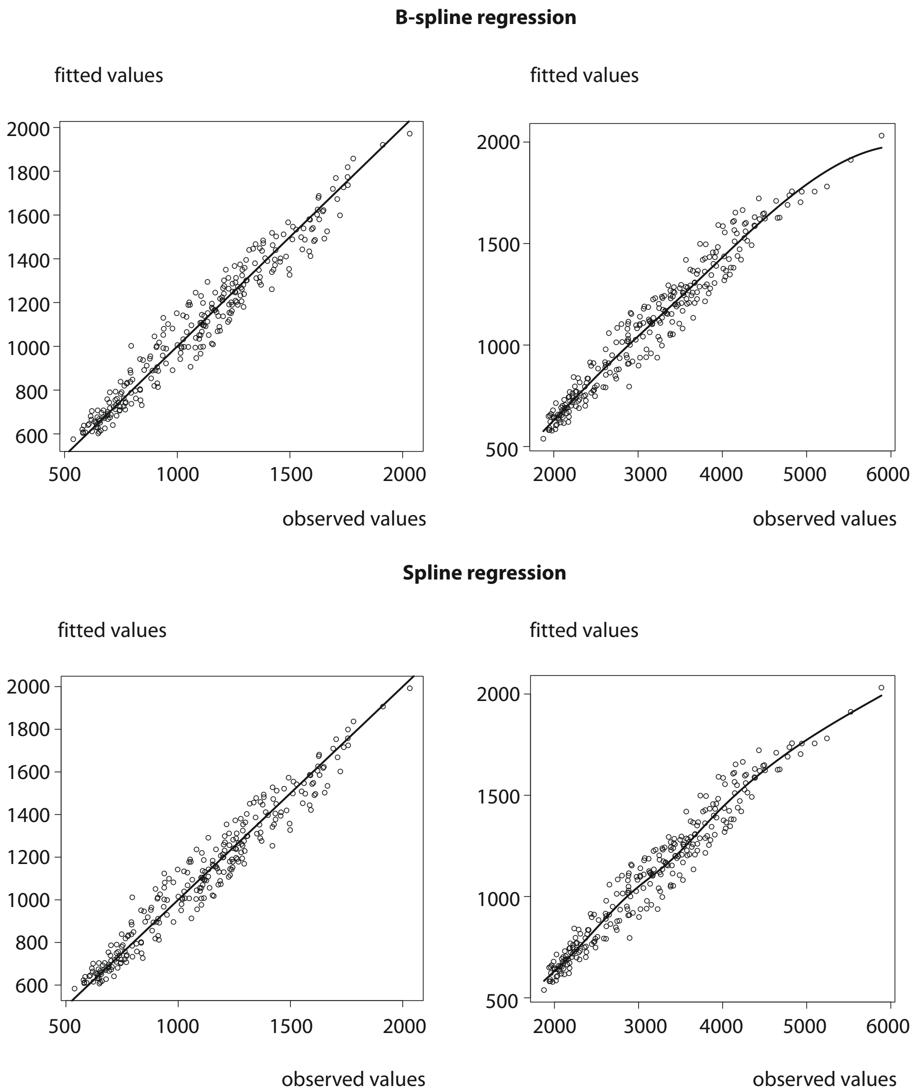


Spline regression



Source: authors' calculations based on Statistics Poland's LDB and MDB, and State Budget Reporting.

Figure 3. Plots for the non-linear B-spline and spline regression describing the average available income per capita versus the average gross wage – empirical versus theoretical values (left), the scatter plot and regression line (right)



Source: authors' calculations based on Statistics Poland's LDB and MDB, and State Budget Reporting.

This finding can also be confirmed by the calculations made by means of the non-linear approach and dichotomous variable (Table 5 and Figure 4), which in this case indicates a value slightly above PLN 70. In this model both quantitative variables, GDP per capita and average gross wages, have been incorporated. The goodness-of-fit was relatively accurate also in this case, i.e. $R^2 = 0.9468$.

This helped to obtain a more realistic estimate of the impact of social programmes, such as Family 500+ on the average available income per capita of Polish households. It would be interesting to compare the results obtained in this study with the values specified in the paper of Brzeziński and Najsztub (Table 1). This table shows the annual benefits obtained by households through the Family 500+ programme as classified in income decile groups. According to the values in the 3rd and 4th decile groups, the change in income (PLN/month) is 81.8 and 52.8, respectively. However, it should be stipulated that these values refer to household income calculated according to the OECD equivalence scale. Thus ordinary (not equivalised) income may be lower, in which case the difference between income estimated on the basis of the HBS (both taking and not taking into account the impact of social programmes) would become greater. What is also noteworthy is the fact that the values of the regression coefficient estimated for the dichotomous variable, determined for different model variants, change from about PLN 70 to slightly above PLN 80, which confirms the accuracy of the observations. It is also worth noting that the value of about PLN 75 usually applies to models including both GDP per capita and average monthly gross wages. The removal of one of the variables most often leads to inappropriate assessments of the coefficient value for the dichotomous variable, which may indicate a weakness of this approach. It should be emphasised that the value of this coefficient is better determined by models containing non-linear relations.

Table 5. Diagnostics of the non-linear spline regression describing the average available income per capita in voivodships for the years 2002–2018 versus average gross wage, GDP per capita and dichotomous variable (0 for the years 2002–2015 and 1 for the years 2016–2018)

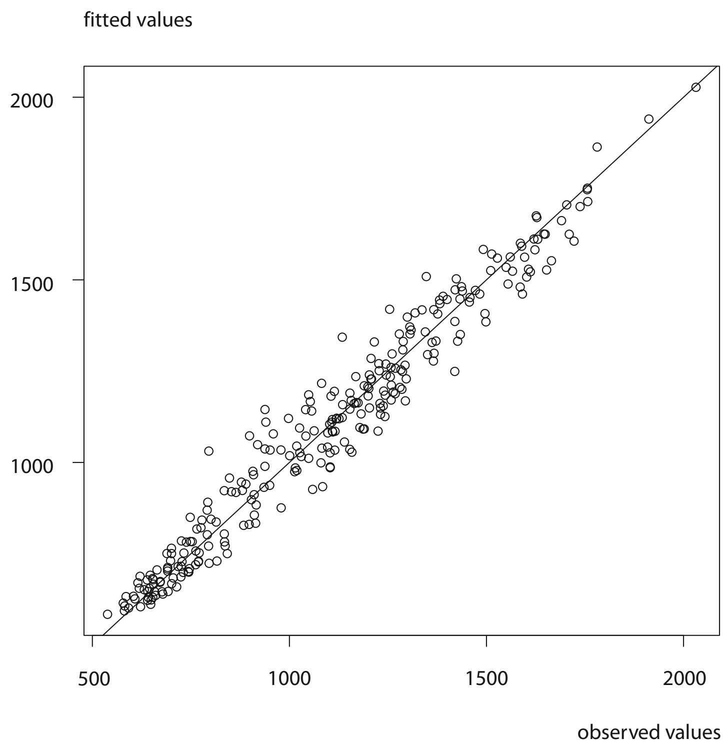
Explanatory variable	Parameter estimate	Standard error	*t*-statistic	*p*-value
Intercept	580.19	22.94	25.294	< 2e-16***
ns(GDPPC,df=5)1	121.33	53.46	2.269	0.0241*
ns(GDPPC,df=5)2	179.29	56.19	3.190	0.0016**
ns(GDPPC,df=5)3	122.87	58.73	2.092	0.0374*
ns(GDPPC,df=5)4	291.12	117.77	2.472	0.0141*
ns(GDPPC,df=5)5	269.38	132.16	2.038	0.0425*
ns(WAGE,df=5)1	311.46	52.60	5.921	1.01e-08***
ns(WAGE,df=5)2	418.55	58.08	7.206	6.22e-12***
ns(WAGE,df=5)3	784.74	59.96	13.088	<2e-16***
ns(WAGE,df=5)4	1040.34	130.89	7.948	5.78e-14***
ns(WAGE,df=5)5	1025.49	146.09	7.019	1.93e-11***
Dichotomous variable	75.17	18.68	4.023	7.54e-05***
Determination coefficient	$R^2 = 0.9468$, corrected $R^2 = 0.9444$			
F-statistic	$F_{emp} = 394.5$, *p*-value < 2.2e-16			
Information criterion	Akaike = 2909.267, Bayesian = 2955.355			

Note. Significance levels as in Table 1. Abbreviations as in Tables 3 and 4.

Source: authors' calculations based on Statistics Poland's LDB and MDB, and State Budget Reporting.

Because the above-mentioned models have the weakness of capturing the more cumulative impact of both wage growth and GDP growth per capita, as well as an increase in expenditure on social assistance, an additional analysis was carried out by means of a different approach. In this approach, in addition to other macroeconomic variables, expenditure on social assistance, derived from annual reports of state budget expenditure per capita, was used. These expenses relate to section 853 of the state budget (supplemented by expenses from section 855 – Family for 2017 and 2018), and are taken from line c – implementation. In addition, the values of two macro-economic variables, i.e. the registered unemployment rate and average monthly gross salary, have been taken into account. It should be noted that these variables can be referred to as (partial) determinants of the available income of Polish households in the period under consideration.

Figure 4. Empirical versus theoretical values of the average available income per capita obtained for the non-linear spline regression model with GDP per capita, the average gross wage and the dichotomous variable



Source: authors' calculations based on Statistics Poland's LDB and MDB, and State Budget Reporting.

Further analysis was carried out using a VAR model, which allows each variable to be explained by its own lagged values in addition to the current and past values of the remaining variables, assuming interaction with a delay according to the scheme for the geometric series. Table 6 demonstrates such relationships for one of the model equations that determines the impact of social assistance expenditures on the other variables. As Table 6 shows, the implied relationships reflect the multipliers of the considered variables in a relatively straightforward way. From that it follows that the expenditure on social assistance contributes to an increase in household available income, as does an increase in wages. There is an inverse relationship between this variable and the unemployment rate. The significance of the impact of social assistance expenditure on household available income is also visible. The value of the determination coefficient which is close to 1 ($R^2 = 0.9968$), and the significance of the regression model, both confirm the model quality. The satisfactory goodness-of-fit of the model with the available income data can also be observed in Figure 5. Similarly, Table 6 can be supplemented with a graph specifying the value of the impulse response function (Figure 6). It should also be noted here that in the period under consideration, a significant increase in available income of Polish households was observed, along with an increase in social assistance expenditure. Likewise, an increase in wages translated into an increase in available income, and the model additionally shows that a decline in unemployment could be expected in the years after 2017.

Table 6. Diagnostics of the VAR model describing the influence of lagged variables on the average available income per capita versus average gross wage, unemployment rate, average monthly gross salary and social assistance expenditure for the years 2000–2018

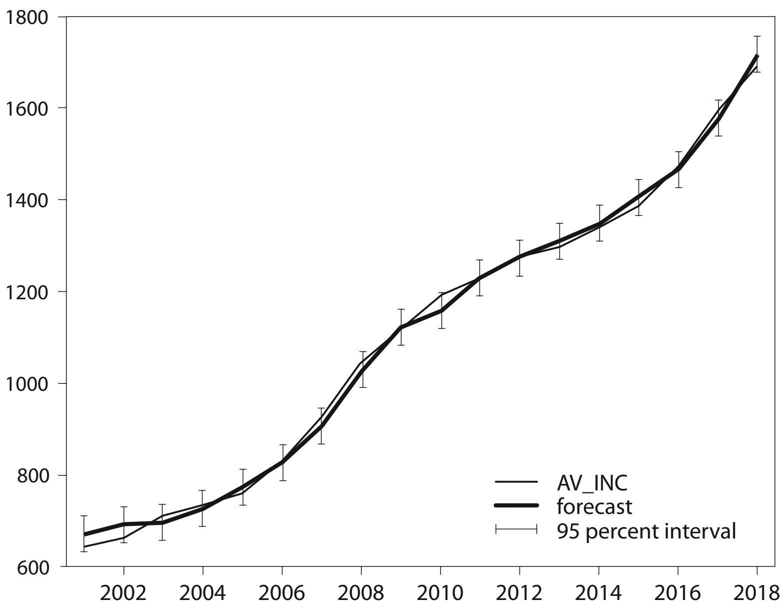
Explanatory variable	Parameter estimate	Standard error	*t*-statistic	*p*-value
Intercept	92.434	81.790	1.130	0.2788
SOC_ASS_1	0.059	0.029	2.000	0.0669*
UNEMP_1	−8.137	2.547	−3.195	0.0070***
SALARY_1	0.218	0.108	2.016	0.0650*
AV_INC_1	0.387	0.273	1.416	0.1804
Arithmetic mean of dependent variable	1107.283			
Standard error of dependent variable	332.0128			
Residual sum of squares	5899.289			
Standard error of residuals	21.30238			
Determination coefficient	$R^2 = 0.9967$, corrected $R^2 = 0.9959$			
F-statistic	$F_{emp}(4, 13) = 1029.137$, *p*-value for *F*-statistic = 4.08e-16			
Autocorrelation of residuals – rho1	−0.0062			
Durbin-Watson statistic	1.7894			

Note. Significance levels: *** – [0, 0.001], ** – (0.01, 0.05], * – (0.05, 0.10]. SOC_ASS – social assistance expenditures, UNEMP – unemployment rate, SALARY – average monthly gross salary, AV_INC – average available income per capita.

Source: authors' calculations based on Statistics Poland's LDB and MDB, and State Budget Reporting.

Having said that though, it should be noted that the conclusions made on the basis of the VAR model may be ambiguous due to the short length of time series in question. Therefore, the statistics and diagnostic tests available for this model, which can provide additional information about the quality of the results, were analysed and presented in Tables 7–14. They allow checking whether the estimation method used is consistent with the statistical assumptions made for the VAR models (see e.g. Domański et al., 2014). In particular, it may be helpful to test for the autocorrelation of residuals, the normality of residuals and to verify the hypothesis on the lack of heteroscedasticity. In Table 7, one can observe the lack of first-order autocorrelation and homoscedasticity of residuals. It is also worth noting that the normality of residuals was rejected only in the case of the model of social assistance expenditures. This result might be partially caused by a large change in the value of social assistance expenditure in 2016.

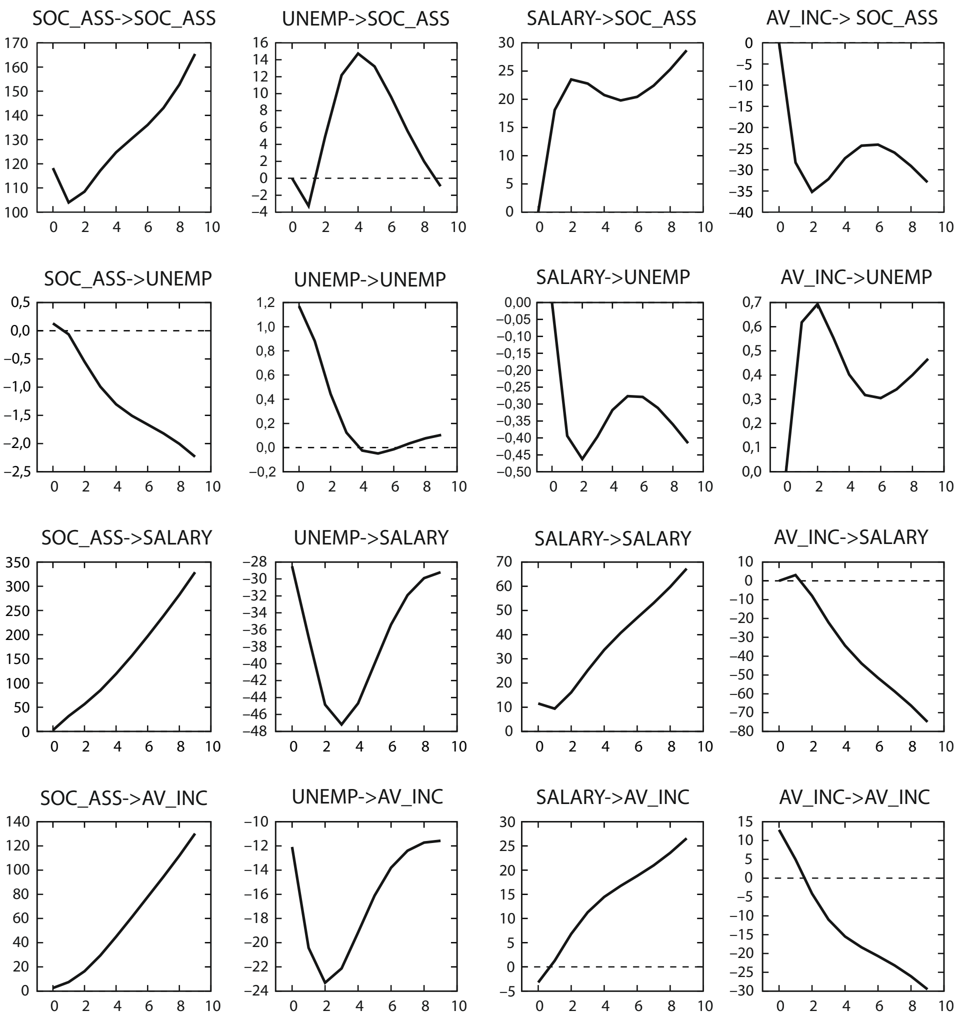
Figure 5. Expected versus observed values of the average available income per capita obtained for the VAR model (Table 6)



Source: authors' calculations based on Statistics Poland's LDB and MDB, and State Budget Reporting.

At this point it seems reasonable to test the impact of this variable on the other variables incorporated in the model, i.e. the unemployment rate, average wages and available income. Such an analysis was performed using the Granger causality test. The Granger causality test applied for the case under consideration (see Lütkepohl, 2005, pp. 102–104, please note the discussion on the number of degrees of freedom for F distribution) is subject to the asymptotic F(3.52) distribution with a critical value of about 2.790. As we are testing the null hypothesis of no causality, the values of F statistics above the critical value would be desired in order to reject it (see Lütkepohl, 2005, pp. 103–104, example 3.6.2). It follows from Table 8 that the variable denoting expenditure on social assistance shows significant impact on the other variables, i.e. the unemployment rate, wages and available income per capita. Note that the Granger causality is also preserved for the other variables (e.g. average available income per capita) for each of the functions analysed, with the most pronounced impact observed for the social assistance expenditure. It can be added, however, that the satisfactory p-values obtained for social assistance expenditure may also result from the large variability of this factor starting from the year 2016.

Figure 6. Impulse response analysis for the model describing interrelationships including the average available income per capita, the average monthly gross salary, registered unemployment rate and social assistance expenditure, using the VAR method



Note. Abbreviations as in Table 6.
Source: authors' calculations based on Statistics Poland's LDB and MDB, and State Budget Reporting.

Table 7. Test statistics for the Ljung-Box, Jarque-Bera and LM tests for residuals of the VAR model describing the influence of lagged variables on the average available income per capita versus average gross wage, unemployment rate, average monthly gross salary and social assistance expenditure for the years 2000–2018

Selected tests for residuals – H_0	SOC_ASS	UNEMP	SALARY	AV_INC
Residuals do not show autocorrelation of the order of 1 (Ljung-Box[a])	0.3487	1.8963	0.0363	0.0007
	(0.5550)	(0.1680)	(0.8490)	(0.9790)
Distribution of residuals is normal (Jarque-Bera[b])	8.6651	0.2361	3.3165	0.8128
	(0.0131)	(0.8886)	(0.1905)	(0.6661)
Residuals are homoscedastic (LM test[c])	0.5990	0.0554	0.2236	0.0571
	(0.4389)	(0.8139)	(0.6363)	(0.8112)

a The Ljung-Box statistic follows an asymptotic χ^2 distribution with a critical value for the considered case χ^2 (0.05, 1) = 3.841. Since we assume no autocorrelation, the Ljung-Box statistic should be less than the critical value (at a given significance level). b The Jarque-Bera statistic follows an asymptotic χ^2 distribution with a critical value for the considered case χ^2 (0.05, 2) = 5.991. In order not to reject H_0 which assumes normality, the Jarque-Bera statistic should be less smaller than the critical value (at a given significance level). c Lagrange Multiplier statistic follows an asymptotic χ^2 distribution with a critical value for the considered case χ^2 (0.05, 1) = 3.841. Since we assume the homoscedasticity of residuals, the empirical test statistic should be less smaller than the critical value (at a given significance level).

Note. *p*-value in the parenthesis. Abbreviations as in Table 6.

Source: authors' calculations based on Statistics Poland's LDB and MDB, and State Budget Reporting.

Table 8. *F*-statistics for the Granger causality test under the VAR model describing the influence of lagged variables on the average available income per capita from the HBS versus average gross wage, unemployment rate, average monthly gross salary and social assistance expenditure for the years 2000–2018

Interactions between variables	*F*-statistic	*p*-value
SOC_ASS → UNEMP, SALARY, AV_INC	$F_{emp}(3, 52) = 34.579$	1.976e-12
UNEMP → SOC_ASS, SALARY, AV_INC	$F_{emp}(3, 52) = 3.437$	0.02340
SALARY → SOC_ASS, UNEMP, AV_INC	$F_{emp}(3, 52) = 3.821$	0.01505
AV_INC → SOC_ASS, UNEMP, SALARY	$F_{emp}(3, 52) = 28.581$	4.659e-11

Note. Abbreviations as in Table 6.

Source: authors' calculations based on Statistics Poland's LDB and MDB, and State Budget Reporting.

Due to the fact that the main aim of the study was to assess the impact of the economic situation on the average available income of households, it was decided to verify the adequacy of an alternative approach. It was based on the model obtained after removing the average monthly gross salary variable and taking into account the indicators of the current and leading consumer confidence, synthetically describing the current trends in individual consumption. These indicators come from the Consumer Tendency Survey (GUS, 2004–2018) and are included in the given models in the annual version. Both these variables turned out to have a significant impact on the value of the household available income in Poland, which, however, was not observed for the variable representing social assistance. According to the tests for random components which were carried out for both the considered autoregressive models, and due to the good quality of the models determining available income (high R^2 and a satisfactory value of F-statistic for the regression equation), it can be assumed that this approach can also be justified (see Tables 9–14).

Table 9. Diagnostics of the VAR model describing the influence of lagged variables on the average available income per capita versus social assistance expenditure, unemployment rate and current consumer confidence indicator for the years 2000–2018

Explanatory variable	Parameter estimate	Standard error	t-statistic	p-value
Intercept	138.099	64.893	2.128	0.0530*
SOC_ASS_1	0.021	0.033	0.642	0.5319
UNEMP_1	0.096	3.767	0.026	0.9800
AV_INC	0.966	0.032	29.826	< 0.0001***
CCCI_1	2.614	1.044	2.504	0.0264**
Arithmetic mean of dependent variable	1107.283			
Standard error of dependent variable	332.013			
Residual sum of squares	5223.365			
Standard error of residuals	20.045			
Determination coefficient	$R^2 = 0.997$, corrected $R^2 = 0.996$			
F-statistic	$F_{emp}(4, 13) = 1162.731$, p-value for F-statistic = 1.85e-16			
Autocorrelation of residuals – rho1	−0.590			
Durbin-Watson statistic	2.977			

Note. Significance levels as in Table 1. Abbreviations as in Tables 4 and 6. CCCI – current consumer confidence indicator.

Source: authors' calculations based on Statistics Poland's LDB and MDB, and State Budget Reporting.

From the Granger causality analysis (Tables 8, 11 and 14) it seems obvious that the indicators of consumer confidence have a more significant impact on the remaining variables than the variable describing social assistance expenditure. The values of F-statistics are visibly smaller for the variable representing social assistance expenditure under these models (see Tables 11 and 14) than the corresponding values for the previous model (see Table 8). In practice, this means that the impact of the governmental expenditure on social assistance may turn out ambiguous. Moreover, the analysis for these models is slightly hampered by the fact that the residuals for the social assistance variable are not normally distributed. Therefore, it can be assumed that the results presented here may be useful in the analysis of the effects of social policy, including the level of expenditure on social assistance, but they do not guarantee the maintenance of an adequate level of available income forecast, e.g. due to the impact of the economic situation. What is more, the analysis of the forecast of the value of the available income carried out using the model for the current consumer confidence index indicates a slower wage growth resulting from this model. It may also indicate that caution is advisable when using this type of forecasts.

Table 10. Test statistics for the Ljung-Box, Jarque-Bera and LM tests for residuals of the VAR model describing the influence of lagged variables on the average available income per capita versus social assistance expenditures, unemployment rate and current consumer confidence indicator for the years 2000–2018

Selected tests for residuals – H_0	SOC_ASS	UNEMP	AV_INC	CCCI
Residuals do not show autocorrelation of the order of 1 (Ljung-Box[a])	0.7748	0.0239	6.0823	0.0293
	(0.3790)	(0.8770)	(0.0140)	(0.8641)
Distribution of residuals is normal (Jarque-Bera[b])	29.643	0.5540	1.0572	1.5387
	(3.66e-07)	(0.7580)	(0.5901)	(0.4630)
Residuals are homoscedastic (LM test[c])	0.0162	0.3764	3.0360	0.5857
	(0.8986)	(0.5395)	(0.0814)	(0.4441)

a–c See footnotes in the Table 7.

Note. *p*-value in the parenthesis. Abbreviations as in Tables 6 and 9.

Source: authors' calculations based on Statistics Poland's LDB and MDB, and State Budget Reporting.

Table 11. *F*-statistics for the Granger causality test under the VAR model describing the influence of lagged variables on the average available income per capita based on the HBS data versus social assistance expenditure, unemployment rate and current consumer confidence indicator for the years 2000–2018

Interactions between variables	*F*-statistic	*p*-value
SOC_ASS → UNEMP, AV_INC, CCCI	$F_{emp}(3, 52) = 4.591$	0.00633
UNEMP → SOC_ASS, AV_INC, CCCI	$F_{emp}(3, 52) = 2.059$	0.11700
AV_INC → SOC_ASS, UNEMP, CCCI	$F_{emp}(3, 52) = 6.991$	0.00049
CCCI → SOC_ASS, UNEMP, AV_INC	$F_{emp}(3, 52) = 6.012$	0.00135

Note. Abbreviations as in Tables 4, 6 and 9.

Source: authors' calculations based on Statistics Poland's LDB and MDB, and State Budget Reporting.

Table 12. Diagnostics of the VAR model describing the influence of lagged variables on the average available income per capita versus social assistance expenditure, unemployment rate and leading consumer confidence indicator for the years 2000–2018

Explanatory variable	Parameter estimate	Standard error	*t*-statistic	*p*-value
Intercept	160.392	63.871	2.511	0.0260**
SOC_ASS_1	0.030	0.032	0.934	0.3676
UNEMP_1	−1.772	3.356	−0.528	0.6064
AV_INC_1	0.957	0.032	30.027	< 0.0001***
LCCI_1	1.728	0.736	2.348	0.0354**

Table 12. Diagnostics of the VAR model describing the influence of lagged variables on the average available income per capita versus social assistance expenditure, unemployment rate and leading consumer confidence indicator for the years 2000–2018 (cont.)

Explanatory variable	Parameter estimate	Standard error	*t*-statistic	*p*-value
Arithmetic mean of dependent variable	1107.283			
Standard error of dependent variable	332.013			
Residual sum of squares	5438.097			
Standard error of residuals	20.453			
Determination coefficient	R^2 =0.997, corrected R^2 = 0.996			
F-statistic	$F_{emp}(4, 13) = 1116.691$, *p*-value for *F*-statistic = 2.41e-16			
Autocorrelation of residuals – rho1	−0.523			
Durbin-Watson statistic	2.867			

Note. Significance levels as in Table 1. Abbreviations as in Tables 4 and 6. LCCI – leading consumer confidence indicator.

Source: authors' calculations based on Statistics Poland's LDB and MDB, and State Budget Reporting.

Table 13. Test statistics for Ljung-Box, Jarque-Bera and LM tests for residuals of the VAR model describing the influence of lagged variables on the average available income per capita versus social assistance expenditure, unemployment rate and leading consumer confidence indicator for the years 2000–2018

Selected tests for residuals – H_0	SOC_ASS	UNEMP	AV_INC	LCCI
Residuals do not show autocorrelation of the order of 1 (Ljung-Box[a])	0.7746	0.1901	4.8480	0.2039
	(0.3790)	(0.6630)	(0.0280)	(0.6520)
Distribution of residuals is normal (Jarque-Bera[b])	30.2260	0.4619	1.1651	0.5759
	(2.733e-07)	(0.7938)	(0.5585)	(0.7498)
Residuals are homoscedastic (LM test[c])	0.0147	0.2619	2.0482	0.9426
	(0.9031)	(0.6090)	(0.1520)	(0.3320)

a–c See footnotes in the Table 7.

Note. *p*-value in the parenthesis. Abbreviations as in Tables 6 and 12.

Source: authors' calculations based on Statistics Poland's LDB and MDB, and State Budget Reporting.

Table 14. *F*-statistics for the Granger causality test under the VAR model describing the influence of lagged variables on the average available income per capita based on the HBS data versus social assistance expenditure, unemployment rate and leading consumer confidence indicator for the years 2000–2018

Interactions between variables	*F*-statistic	*p*-value
SOC_ASS → UNEMP, WAGE, LCCI	$F_{emp}(3, 52) = 5.7694$	0.00175
UNEMP → SOC_ASS, WAGE, LCCI	$F_{emp}(3, 52) = 3.4580$	0.02283
WAGE → SOC_ASS, UNEMP, LCCI	$F_{emp}(3, 52) = 6.3193$	0.00098
LCCI → SOC_ASS, UNEMP, WAGE	$F_{emp}(3, 52) = 6.1804$	0.00113

Note. Abbreviations as in Tables 4, 6 and 12.

Source: authors' calculations based on Statistics Poland's LDB and MDB, and State Budget Reporting.

4. Conclusions

The analysis above demonstrates that the expenditure on social assistance has a significant impact on the amount of available income as evaluated on the basis of the Household Budget Survey. The analysis provides conclusions on qualitative changes in the amount of disposable income in 2016–2017. They are partly consistent with other assessments, regarding for example the impact of the Family 500+ programme on the income of households. However, it should be emphasised that the impact of the Family 500+ programme is probably not the only factor contributing to the growth of income of Polish households.

It may be interesting to examine whether the observed trends will continue in the coming years. Taking into account the current situation related to the coronavirus pandemic and the existence of many factors limiting economic growth (in particular the GDP per capita growth), the results of the presented approach should be treated with special caution. Due to the high dynamics of the currently observed economic phenomena (see Wójcik, 2014), the assessment of such changes on the basis of yearly data may be insufficient.

As the HBS is conducted on a relatively large sample, the preliminary assessment of the income situation of Polish households could be carried out in a shorter time horizon, i.e. a monthly period. Such data should be available through the Statistical Bulletin published monthly by Statistics Poland, which provides data for a country level, and the monthly Consumer Tendency Survey. In our paper, we have demonstrated that it is possible to combine the results of income estimates and consumer trends, which made our approach relatively accurate for the period of the analysis.

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