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Extended Enders and Siklos test for threshold cointegration

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Abstract. In our previous studies, we modified the Enders and Siklos test for threshold error correction to a version allowing the individual threshold variable to be responsible for the asymmetric mechanism of the system. The idea was to learn about the threshold mechanism both in the long and short run. In this paper, we tested for the asymmetry of the adjustment of the error correction mechanism towards the long-run path. The subsamples within regimes differ in size with respect to the threshold value. The novelty lies in the division of both short and long-run variables according to a threshold variable with a given threshold value (assumed or estimated). We named the test extended Enders and Siklos test (exE-S). The present study focuses on the power and size of the modified procedure. A simulation study was designed, computed and conducted. The results are favourable for the proposed approach, although they strongly depend on the difference in values between the adjustment parameters in the regimes.

Keywords: threshold error correction test, power, size, Monte Carlo, economic growth
JEL: C22, O47

1. Introduction

The paper aims to evaluate the size and power of a novel extended Enders and Siklos test for threshold cointegration and compare it with the size and power of the original one. The test was first described and applied in Boehlke et al. (2017, 2018), and Gałecki and Osińska (2019). However, its size and power were not examined in the previous publications. This paper fills the gap which arose in that area.

Enders and Siklos (2001) defined their test in the context of a threshold cointegration. A threshold cointegration, as the opposite of a linear one, assumes asymmetry in the short-run speed of adjustment to the steady-state, mainly when the bottom-up and top-down adjustment directions are considered. As the previous literature, this paper considers a threshold cointegration (1,1).

The concept of threshold cointegration refers to both cointegration and nonlinearity of the threshold type. The literature on this issue relates to approaches involving a single equation and a system of equations. Balke and Fomby (1997) applied the idea of nonlinear threshold modelling developed by Chan (1993) and Tong (1990) and joined it with the concept of cointegration. Enders and Siklos (2001) developed the testing scheme for nonlinear cointegration and asymmetry, assuming that the

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system's reaction is asymmetric around lagged ECM or lagged momentum ECM. Since the ECM is stationary, the threshold value is supposed to be zero. Stigler (2010) provided a broad overview of the different methods related to Threshold Error Correction Modelling (TECM), including both univariate and multivariate models.

The original Enders and Siklos test was extended by Kapetanios et al. (2006), who considered two cases referred to as threshold variables. In the first one, a threshold variable enters the cointegrating vector and in the second case, it is not present in the long-run equation. In the former case, the threshold variable is responsible for both long-run and short-run dynamics. In the latter one – the cointegrating vector remains independent of the threshold, since it works in the short run only. The approach that fits the idea above was proposed by Kapetanios et al. (2006), who applied it in the smooth transition regression model framework. The threshold model can be considered as a special case of a smooth transition model. Bruzda (2007) fitted the Kapetanios et al. test to the threshold cointegration case.

Tsay (1998) further developed testing for threshold cointegration and examined whether the variable of interest is generated by a linear or nonlinear process. The null hypothesis assumes that Y_t is generated by a linear data generating process, while the alternative hypothesis assumes that it follows the multivariate threshold process. Hansen and Seo (2002) proposed a new test for threshold cointegration, where the test statistic depends on the covariance structure of the processes under examination. The starting point for the test is the linear vector error correction model (VECM). They assumed that each process is integrated of order one. There is only one cointegrating vector in the model. The null hypothesis carries a linear model with cointegration, whereas the alternative one a threshold model with cointegration. In the present paper, the Tsay, and Hansen and Seo tests were used to check the robustness of the proposed procedure in empirical studies.

Many authors found that economic and financial processes often exhibited non-equal reactions to positive or negative stimulus. Granger and Lee (1989), using a threshold model with a sign function revealed asymmetries in sales, production and inventories in the United States. The most frequent asymmetric relationships are those related to price transmission. Frey and Manera (2007) provided a broad overview of the existing literature on asymmetries in price transmission, finding that a threshold-type asymmetry is quite common in a wide range of markets, mainly financial – Martens et al. (1998), fuel – Ghassan and Banerjee (2015), Leszkiewicz-Kędzior and Welfe (2014), Gosińska et al. (2020), as well as the wheat market – Hassouneh et al. (2017). Piłatowska and Włodarczyk (2017) showed a threshold error correction relationship between CO_2 emissions and economic growth. Boehlke et al. (2019) found a vast array of applications related to economic growth modelling.

This study describes the extended Enders and Siklos test and provides a series of simulations showing its size and power. The paper's novelty is that it shows evidence that threshold cointegration can be led not only by the $ECM_{t-1}/\Delta ECM_{t-1}$ term but also individual variables responsible for the threshold mechanism.

The paper is organised as follows: Section 2 briefly describes the procedure of Enders and Siklos and its extensions and discusses a modified testing approach using a TECM basis, in Section 3 the simulation results are presented, while the empirical example is shown in Section 4. The conclusions are presented in the last part of the paper.

2. Extended Enders and Siklos test

Enders and Siklos (2001) assumed no cointegration in the null hypothesis, whereas nonlinearity is assumed under the alternative hypothesis applying a two-regime threshold model. The threshold variable is defined as a SETAR variable, which is either lagged error correction term ECM_{t-1} or the M-TAR variable, i.e. momentum error correction variable ΔECM_{t-1}. The value of a threshold can be estimated or assumed to be constant. The authors adopted zero as the natural threshold value for the mentioned variables in the original paper. The consequences of the Enders and Siklos test are related to the following cases: threshold cointegration and no threshold cointegration, which implies a linear cointegration, a stationary TAR model, or a partial cointegration.

Enders and Siklos' (2001) procedure consists of the stages listed below.

1. It is assumed that a linear cointegrating equation exists under the conditions defined in Engle and Granger (1987):

$$Y_t = \alpha_0 + \sum_{i=1}^{k} \alpha_i X_{it} + u_t, \tag{1}$$

2. The testing regression is estimated as:

$$\Delta\hat{u}_t = I_t \rho_1 \hat{u}_{t-1} + (1 - I_t)\rho_2 \hat{u}_{t-1} + \sum_{i=1}^{p} \beta_i \Delta\hat{u}_{t-i} + \varepsilon_t, \tag{2}$$

where

$$\hat{u}_t = Y_t - \hat{Y}_t = ECM_t.$$

$$I_t = \begin{cases} 1 & \text{for} \quad \hat{u}_{t-1} \geq \gamma \\ 0 & \text{for} \quad \hat{u}_{t-1} < \gamma \end{cases} \quad \text{or} \quad I_t = \begin{cases} 1 & \text{for} \quad \Delta\hat{u}_{t-1} \geq \gamma \\ 0 & \text{for} \quad \Delta\hat{u}_{t-1} < \gamma \end{cases}$$

and $\gamma = 0$.

It is assumed that the threshold in Equation (2) is defined in terms of the error correction mechanism: (ECM) $\hat{u}_{t-1}$ or $\Delta\hat{u}_{t-1}$.

3. The set of two null hypotheses to be tested takes the following form:

$$H_0^1 : \rho_1 = \rho_2 = 0, \tag{3}$$

$$H_0^2 : \rho_1 - \rho_2 = 0. \tag{4}$$

H_0^1 is for the case of no threshold cointegration; consequently, the Engle-Granger linear cointegration is confirmed, H_0^2 assumes a symmetric reaction, being the argument for linear cointegration. If both hypotheses are rejected, the Enders and Siklos procedure indicates threshold cointegration around the long-run equilibrium. The short-run adjustment is asymmetric with respect to positive and negative changes. A precise interpretation of the set of hypotheses to be tested (3-4) was provided by Balke and Fomby (1997). This interpretation is presented in Table 1.

Table 1. Possible models under the no-TECM hypotheses

System characteristics	Linearity	Nonlinearity
No cointegration	H_0^1: No linear cointegration	H_1^2: No cointegration. Nonlinear residual process
Cointegration	H_1^2: Linear cointegration	H_1^2: Nonlinear cointegration

Source: based on Balke and Fomby (1997).

Stigler (2010) emphasised that testing for threshold cointegration involves two issues that must be solved simultaneously: cointegration and nonlinearity. Hence, the following cases are possible: cointegration and threshold effects, cointegration and no threshold effects, no cointegration and no threshold effects and, finally, no cointegration and threshold effects.

The results of the Enders and Siklos approach allow the identification of asymmetric reactions around the entire cointegrating vector (which can be unknown). Still, they do not indicate individual threshold variables responsible for the asymmetric mechanism of the system. In many cases, single variables can diversify the mechanism of a short-run adjustment. Two possible cases are considered: the first, when a threshold variable enters the cointegrating vector and the second, where it is not present in the long run. The threshold variable is responsible for both long-run and short-run dynamics in the first case. In contrast, in the second case, the cointegrating vector remains independent of the threshold since it only works in the short run.

The approach that fits the idea above was partially proposed by Kapetanios et al. (2006) and modified by Bruzda (2007). Having (1) unchanged, the testing of Equation (2) is a matter of the re-formulation into the following form:

$$\Delta Y_t = I_t \, \rho_1 \hat{u}_{t-1} + (1 - I_t)\rho_2 \hat{u}_{t-1} + \omega \Delta X_t + \sum_{j=1}^{p} \psi_j \Delta Z_{t-j} + \varepsilon_t, \tag{5}$$

where indicator functions I_t remain the same as defined above and $\gamma = 0$. This test can be extended by allowing for other than $\hat{u}_{t-1} = 0$ and $\Delta\hat{u}_{t-1} = 0$ threshold variables. The set of possible threshold variables is defined in vector Z_t:

$$Z_t = (Y_t, X_{1t}, X_{2t}, \ldots, X_{kt})' .$$

Then the threshold value (empirical level of γ) is a subject of estimation, where

$$I_t = \begin{cases} 1 & \text{for} \quad Z_{t-i} \geq \hat{\gamma} \\ 0 & \text{for} \quad Z_{t-i} < \hat{\gamma} \end{cases} \tag{6}$$

or

$$I_t = \begin{cases} 1 & \text{for} \quad \Delta Z_{t-i} \geq \hat{\gamma} \\ 0 & \text{for} \quad \Delta Z_{t-i} < \hat{\gamma} \end{cases} \tag{7}$$

and

$$-\infty < \hat{\gamma} < \infty ; \quad \hat{\gamma} = arg \min_{\gamma} AIC(\gamma). \tag{8}$$

This approach allows the identification of asymmetric reactions in the long run, although it is possible for individual variables to be the threshold. In this approach, the number of observations in the short run remains equal in both regimes.

Boehlke et al. (2018) proposed a new testing procedure based on the entire set of variables available in long-run and short-run equations. This procedure extends the set of possible thresholds and determines the way they impact the identification of the periods of intense economic growth within the observed sample. Long-run equation (1) remains the same. The testing equation is modified to the form:

$$\begin{aligned} \Delta Y_t = {} & I_t \, \rho_1 \hat{u}_{t-1} + (1 - I_t)\rho_2 \hat{u}_{t-1} + \sum_{s=1}^{p_{II}} I_t \beta_{s1} \Delta Y_{t-s} + \\ & + \sum_{s=1}^{p_I} (1 - I_t)\, \beta_{s2}\, \Delta Y_{t-s} + \sum_{i=0}^{k} I_t \alpha_i \Delta X_{it} + \sum_{i=0}^{k} (1 - I_t) \alpha_i \Delta X_{it} + \\ & + \sum_{j=1}^{q_{II}} \sum_{i=1}^{k} I_t \gamma_{j1} \Delta X_{it-j} + \sum_{j=1}^{q_I} \sum_{i=1}^{k} (1 - I_t) \gamma_{j2} \Delta X_{it-j} + \varepsilon_t, \end{aligned} \tag{9}$$

where

$$I_t = \begin{cases} 1 & \text{for} \quad Z_{t-i} \geq \hat{\gamma} \\ 0 & \text{for} \quad Z_{t-i} < \hat{\gamma} \end{cases} \quad \text{or} \quad I_t = \begin{cases} 1 & \text{for} \quad \Delta Z_{t-i} \geq \hat{\gamma} \\ 0 & \text{for} \quad \Delta Z_{t-i} < \hat{\gamma} \end{cases}$$

$$Z_t = (Y_t, X_{1t}, X_{2t}, \dots, X_{kt})'$$

and

$$-\infty < \hat{\gamma} < \infty \, ; \; \hat{\gamma} = arg \min_{\gamma} AIC(\gamma).$$

In the proposed model, the short-term equations differ between the regimes in terms of the following: a vector of explanatory variables, number of observations and parameters estimate. The approach seems fairly complex, because it shows asymmetries around the long-run and in the short-run dynamics. The advantage of such an approach is that in the final TECM different sets of variables can act in different regimes having the long-run relationship unchanged. However, its limitation is related to the number of observations; if the time series of interest is short, some results may remain unverified.

Three approaches to the TECM specification described above should be considered as nested – the last one nests the second approach, and the second nests the first one. The sequence of testing from the simplest to the broadest course validates the results. If they can be confirmed by Enders and Siklos, Kapetanios et al. and the extended Enders and Siklos approach, the nonlinear mechanism underlying the relationship in question becomes very likely.

3. Simulation results

3.1. The experiment

In the experiment, thresholds ECM_{t-1} and ΔECM_{t-1} were used to ensure the comparability of the original and extended Enders and Siklos test results. The experiment was based on the Monte Carlo method. The simulations included the following steps:

1. generating time series Y_t and X_t (both 1(1)), with the error terms defined as: $\varepsilon_t \sim N(0,1), \eta_t \sim N(0,1)$;
2. generating long-run relationship $Y_t = 0.5 - 0.2X_t + u_t$, where $\hat{u}_t = ECM_t$;
3. checking the stationarity of the residuals ($\hat{u}_t \sim I(0)$);
4. calculating threshold variables $\hat{u}_{t-1}$ and $\Delta\hat{u}_{t-1}$;

5. determining the threshold values to satisfy the following sample proportion between the regimes:
   a) 50%–50%;
   b) 60%–40%;
   c) 80%–20%.

   To estimate the threshold values from $\hat{u}_t$, the following rules were applied: (a) at the median level, (b) at decile_6 level, and (c) at decile_8 level.

   The assumed sample sizes including 50; 100; 500; 1,000 and 2,000 correspond to different situations observed in practice. Typical economic time series observed monthly, quarterly or at an annual frequency consist of 50 or 100 observations. The numbers 500; 1,000 and 2,000 enable the verification whether the longer time series increase the power of the test.
6. performing the Enders and Siklos test (E-S) based on the equation:

$$\Delta\hat{u}_t = I_t\,\rho_1\,\hat{u}_{t-1} + (1 - I_t)\rho_2\,\hat{u}_{t-1} + \varepsilon_t,$$

   where $\varepsilon_t \sim N(0,1)$ and I_t were defined in Equation (2);
7. performing the extended test (exE-S) based on the short-run model of the form:

$$\Delta Y_t = I_t\,\rho_1\hat{u}_{t-1} + (1 - I_t)\rho_2\hat{u}_{t-1} + I_t\,a_1\,\Delta x_t + (1 - I_t)a_2\,\Delta x_t + I_t\,b_1\,\Delta Y_{t-1} + (1 - I_t)b_2\Delta Y_{t-1} + \varepsilon_t.$$

In the experiment, parameters ρ_1 and ρ_2 were assumed to change in the range of [–0.99; –0.09] with 0.1 steps. The parameters in the short-run equation (a_1, a_2, b_1, b_2) are defined as follows:

- symmetric negative (–0.3; –0.3; –0.3; –0.3);
- asymmetric negative (–0.3; –0.3; –0.6; –0.3);
- symmetric positive (0.3; 0.3; 0.3; 0.3);
- asymmetric positive (0.3; 0.3; 0.6; 0.3);
- symmetric mixed (–0.3; 0.3; –0.3; 0.3);
- asymmetric mixed (0.3; –0.3; 0.6; –0.3).

A total of 10,000 replications were carried out and the simulation procedure was performed in the Gretl package. Threshold variables ECM_{t-1} and ΔECM_{t-1} were taken from the long-run regression. The threshold value was presumably known and equal to zero. After each sampling, the observations were assigned to one of the two regimes, and models were tested for parameters significance. Insignificant variables were excluded from the model.

In the case of the exE-S test, H_0^1 is tested using the Wald test, like in the case of the E-S. When H_0^2 is considered, the significance of the difference between parameters of error correction mechanism, i.e. ρ_1 and ρ_2 is subject to testing. Two null hypotheses are defined in (3) and (4). The distribution of the Wald test is typically analysed in the form of a chi-squared test or F test, whereas the latter is appropriate for small samples. It is proven that if $X \sim F(n_1, n_2)$, the limiting distribution of n_1X as $n_2 \to \infty$ is the chi-square distribution with n_1 degrees of freedom (Hogg et al., 2005). Taking into account a large sample, the exE-S test was compared to the chi-squared distribution with n_1 degrees of freedom, where n_1 is the number of restrictions (for H_0^1 $n_1 = 2$, and for H_0^2 $n_1 = 1$). Using the Kolmogorov-Smirnov goodness of fit test, the exE-S test did not allow the rejecting of the null hypothesis assuming that its distribution fits the chi-squared distribution at the significance level of 1%.

3.2. Power of the extended and original Enders and Siklos test

Power is an essential characteristic of the statistical test. Power refers to the probability of rejecting H_0 when it is false. On the other hand, size is defined as the probability of rejecting the null when it is true. The standard approach of Neyman and Pearson is to maximise the power while limiting the size by a pre-specified significance level of α (Lloyd, 2006).

In the study, the power and size of the original and extended tests were checked for sensitivity according to:

a) the sample size;
b) the number of observations in the regimes;
c) the values of parameters ρ_1 and ρ_2 and their difference;
d) the parameter values and asymmetry in the short run;
e) the significance levels.

These imply that the number of the results of the simulations is vast. Therefore, only the crucial ones are presented in the paper. First of all, the power of all tests for H_0^1 is equal to 1 for all cases; the refore it is not presented here.[1] The power results for H_0^2 are presented in Figures 1 and 2, and Table 2. The simulated values are shown at the median level calculated over 10,000 replications. Figure 1 refers to the 0.1 difference between the values of parameters ρ_1 and ρ_2, while Figure 2 shows the results when the difference is 0.2. For more considerable differences, the power approaches 1. The figures presenting power results include the *a–d* characteristics mentioned above. The significance level was assumed to take the value of 1%.

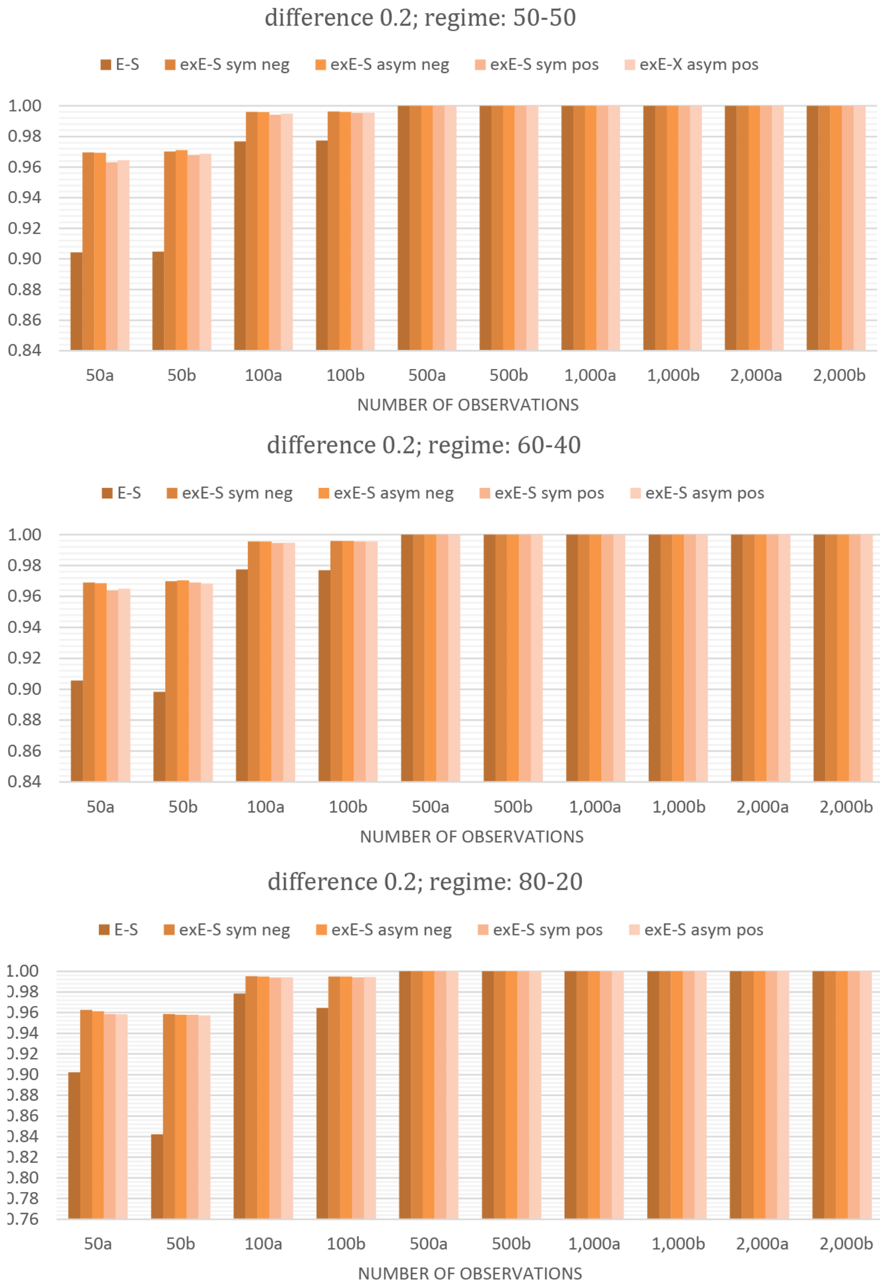
[1] All results are available upon request.

Figure 1. Power of the exE-S and E-S test, difference: 0.1

Note. Difference=$|\rho_1$-$\rho_2|$; a – ECM_{t-1}; b – ΔECM_{t-1}; sym neg – parameters in the short-run equation equal in regimes, negative; asym neg – parameters in the short-run equation non-equal in regimes, negative; sym pos – parameters in the short-run equation equal in regimes, positive; asym pos – parameters in the short-run equation non-equal in regimes, positive. Significance level: 1%.

Source: authors' work.

Figure 2. Power of the exE-S and E-S, difference: 0.2



Note. Difference=$|\rho_1 - \rho_2|$; a – ECM_{t-1}; b – ΔECM_{t-1}; sym neg – parameters in the short-run equation equal in regimes, negative; asym neg – parameters in the short-run equation non-equal in regimes, negative; sym pos – parameters in the short-run equation equal in regimes, positive; asym pos – parameters in the short-run equation non-equal in regimes, positive. Significance level: 1%.

Source: authors' work.

Table 2. Power of the exE-S and E-S tests

Obs. no.	Z_t	Regime	ρ_1–ρ_2	H_0^1: E-S	H_0^2: E-S	H_0^1:exE-S[1]	H_0^2:exE-S[1]	H_0^1:exE-S[2]	H_0^2:exE-S[2]
100	ECM_{t-1}	50–50	0.1	1.00	0.4157	1.00	0.6919	1.00	0.7077
100	ΔECM_{t-1}	50–50	0.1	1.00	0.4134	1.00	0.6943	1.00	0.7072
100	ECM_{t-1}	60–40	0.1	1.00	0.4174	1.00	0.6872	1.00	0.7041
100	ΔECM_{t-1}	60–40	0.1	1.00	0.4197	1.00	0.6946	1.00	0.7040
100	ECM_{t-1}	80–20	0.1	1.00	0.4228	1.00	0.6851	1.00	0.7026
100	ΔECM_{t-1}	80–20	0.1	1.00	0.3846	1.00	0.7241	1.00	0.7341
1,000	ECM_{t-1}	50–50	0.1	1.00	1.00	1.00	1.00	1.00	1.00
1,000	ΔECM_{t-1}	50–50	0.1	1.00	1.00	1.00	1.00	1.00	1.00
1,000	ECM_{t-1}	60–40	0.1	1.00	1.00	1.00	1.00	1.00	1.00
1,000	ΔECM_{t-1}	60–40	0.1	1.00	1.00	1.00	1.00	1.00	1.00
1,000	ECM_{t-1}	80–20	0.1	1.00	1.00	1.00	1.00	1.00	1.00
1,000	ΔECM_{t-1}	80–20	0.1	1.00	1.00	1.00	1.00	1.00	1.00
100	ECM_{t-1}	50–50	0.2	1.00	0.9768	1.00	0.9945	1.00	0.9958
100	ΔECM_{t-1}	50–50	0.2	1.00	0.9772	1.00	0.9950	1.00	0.9957
100	ECM_{t-1}	60–40	0.2	1.00	0.9775	1.00	0.9944	1.00	0.9958
100	ΔECM_{t-1}	60–40	0.2	1.00	0.9770	1.00	0.9951	1.00	0.9959
100	ECM_{t-1}	80–20	0.2	1.00	0.9783	1.00	0.9921	1.00	0.9949
100	ΔECM_{t-1}	80–20	0.2	1.00	0.9645	1.00	0.9930	1.00	0.9949
1,000	ECM_{t-1}	50–50	0.2	1.00	1.00	1.00	1.00	1.00	1.00
1,000	ΔECM_{t-1}	50–50	0.2	1.00	1.00	1.00	1.00	1.00	1.00
1,000	ECM_{t-1}	60–40	0.2	1.00	1.00	1.00	1.00	1.00	1.00
1,000	ΔECM_{t-1}	60–40	0.2	1.00	1.00	1.00	1.00	1.00	1.00
1,000	ECM_{t-1}	80–20	0.2	1.00	1.00	1.00	1.00	1.00	1.00
1,000	ΔECM_{t-1}	80–20	0.2	1.00	1.00	1.00	1.00	1.00	1.00
100	ECM_{t-1}	50–50	0.5	1.00	1.00	1.00	1.00	1.00	1.00
100	ΔECM_{t-1}	50–50	0.5	1.00	1.00	1.00	1.00	1.00	1.00
100	ECM_{t-1}	50–50	0.5	1.00	1.00	1.00	1.00	1.00	1.00
100	ΔECM_{t-1}	50–50	0.5	1.00	1.00	1.00	1.00	1.00	1.00
100	ECM_{t-1}	60–40	0.5	1.00	1.00	1.00	1.00	1.00	1.00
100	ΔECM_{t-1}	60–40	0.5	1.00	1.00	1.00	1.00	1.00	1.00
1,000	ECM_{t-1}	60–40	0.5	1.00	1.00	1.00	1.00	1.00	1.00
1,000	ΔECM_{t-1}	60–40	0.5	1.00	1.00	1.00	1.00	1.00	1.00
1,000	ECM_{t-1}	80–20	0.5	1.00	1.00	1.00	1.00	1.00	1.00
1,000	ΔECM_{t-1}	80–20	0.5	1.00	1.00	1.00	1.00	1.00	1.00
1,000	ECM_{t-1}	80–20	0.5	1.00	1.00	1.00	1.00	1.00	1.00
1,000	ΔECM_{t-1}	80–20	0.5	1.00	1.00	1.00	1.00	1.00	1.00
100	ECM_{t-1}	50–50	0.9	1.00	1.00	1.00	1.00	1.00	1.00
100	ΔECM_{t-1}	50–50	0.9	1.00	1.00	1.00	1.00	1.00	1.00
100	ECM_{t-1}	50–50	0.9	1.00	1.00	1.00	1.00	1.00	1.00
100	ΔECM_{t-1}	50–50	0.9	1.00	1.00	1.00	1.00	1.00	1.00
100	ECM_{t-1}	60–40	0.9	1.00	1.00	1.00	1.00	1.00	1.00
100	ΔECM_{t-1}	60–40	0.9	1.00	1.00	1.00	1.00	1.00	1.00
1,000	ECM_{t-1}	60–40	0.9	1.00	1.00	1.00	1.00	1.00	1.00
1,000	ΔECM_{t-1}	60–40	0.9	1.00	1.00	1.00	1.00	1.00	1.00
1,000	ECM_{t-1}	80–20	0.9	1.00	1.00	1.00	1.00	1.00	1.00
1,000	ΔECM_{t-1}	80–20	0.9	1.00	1.00	1.00	1.00	1.00	1.00
1,000	ECM_{t-1}	80–20	0.9	1.00	1.00	1.00	1.00	1.00	1.00
1,000	ΔECM_{t-1}	80–20	0.9	1.00	1.00	1.00	1.00	1.00	1.00

Note. The symbols: H_0^1:E-S, H_0^2:E-S refer to the E-S test, and H_0^1:exE-S[1], H_0^2:exE-S[1], H_0^1:exE-S[2], H_0^2:exE-S[2] to the exE-S test with short-run parameters (1) a={–0.3; 0.3} and b={–0.3; –0.3}, and (2) a={0.3; –0.3}, and b={0.6; –0.3}, respectively. Parameters in the short term equation are asymmetric, both positive and negative (1) a={–0.3; 0.3} and b={–0.3; –0.3}, and (2) a={0.3; –0.3}, and b={0.6; –0.3}. Significance level: 1%.

Source: authors' calculations.

The results presented above show that the power of the exE-S test strongly depends on the difference between ρ_1 and ρ_2. If $\rho_1 - \rho_2 = 0.1$, the results presented in Figures 1–2 exhibit insufficient power, particularly for 50 and 100 observations. It is due to the weak asymmetry effect between the regimes. Also, the E-S test loses its power in such a case. The results align with the power of the threshold cointegration tests presented in Bruzda (2007, pp. 326–327). She considered the Kapetanios et al. test in the form presented in Equation (5), particularly the case when the difference varied between 0.0 and 0.4 and the number of observations was 100. The power of the test when the threshold value was known and equal to 0 for the 5% significance level was between 0.062 and 0.684 in the case of ECM_{t-1} and between 0.01 and 0.996 in the case of ΔECM_{t-1}. In the case of the test based on Equations (5–7), the power for 0.1 asymmetries is much higher, so it is for higher asymmetry.

The results presented in Table 2 show that the difference in power when ρ_1- ρ_2 change from 0.1 to 0.2 is substantial. Suppose the difference increases to 0.5, the power of both tests is entirely satisfactory. Table 2 contains results for 100 and 1,000 observations when the parameters in the short run change their signs and values. These have no impact on the examined test's power. Also, the division between regimes related to the number of observations in the regimes does not influence the results.

3.3. Size of the extended and original Enders and Siklos tests

Results of the power are reliable only when the size of the test is kept across various assumptions. In simulations, the size was assumed to take the following values: 0.01; 0.05; 0.1. Figures 3 and 4 show empirical sizes for H_0^1 and H_0^2, respectively. The number of observations was 100 and 1,000.

Figure 3. Size of exE-S and E-S for H_0^1

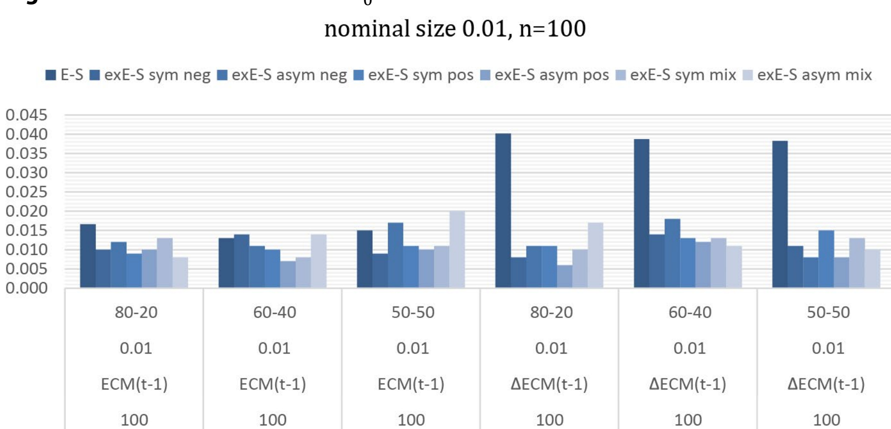
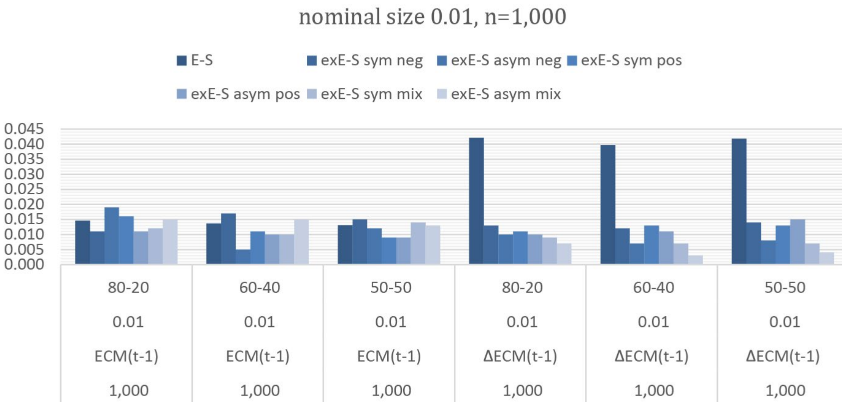
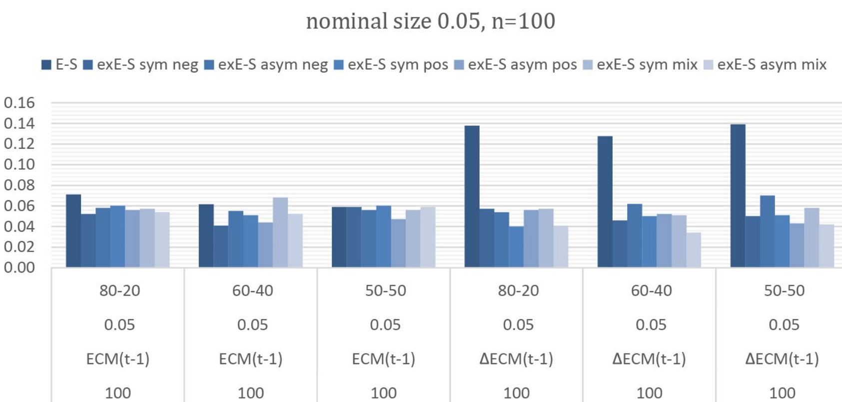
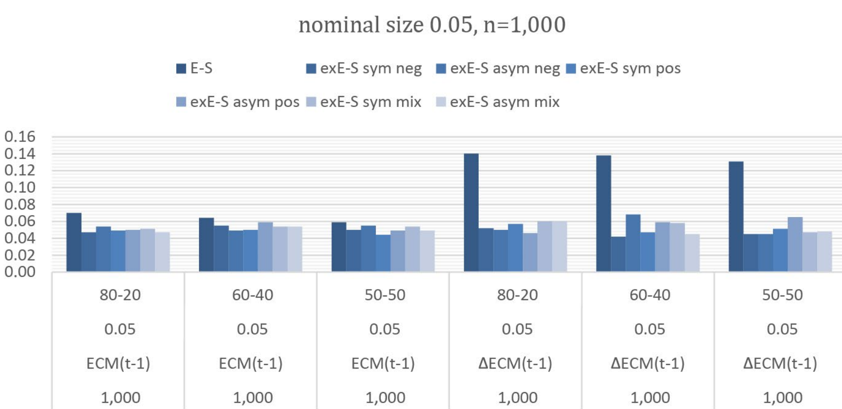


Figure 3. Size of exE-S and E-S for H_0^1 (cont.)







Note. sym neg – parameters in the short-run equation equal in regimes, negative; asym neg – parameters in the short-run equation non-equal in regimes, negative; sym pos – parameters in the short-run equation equal in regimes, positive; asym pos – parameters in the short-run equation non-equal in regimes, positive; sym mix – parameters in the short run equal in modulus, opposite signs; asym mix – parameters in the short run non-equal in modulus, opposite signs.

Source: authors' work.

Figure 4. Size of exE-S and E-S for H_0^2

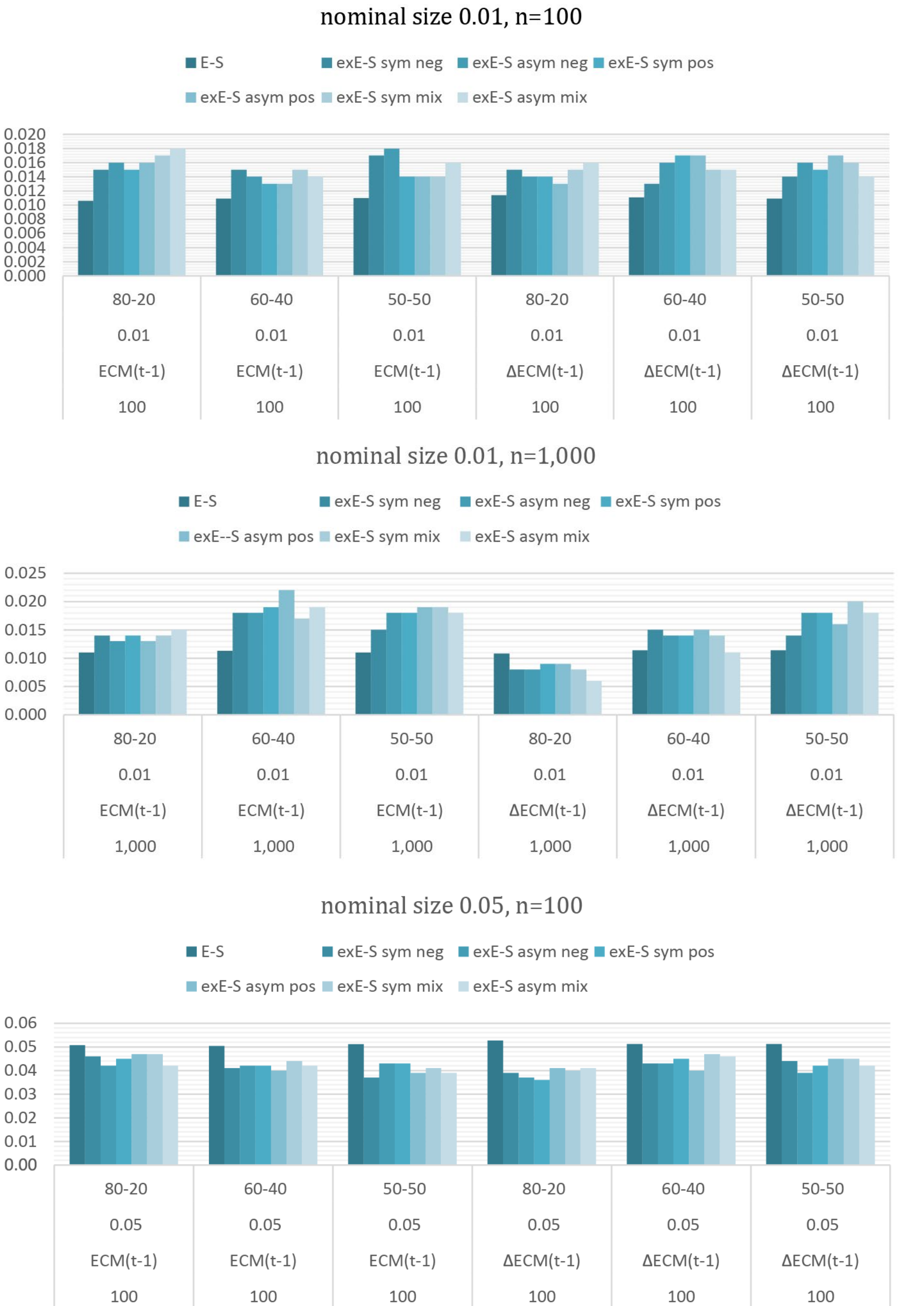
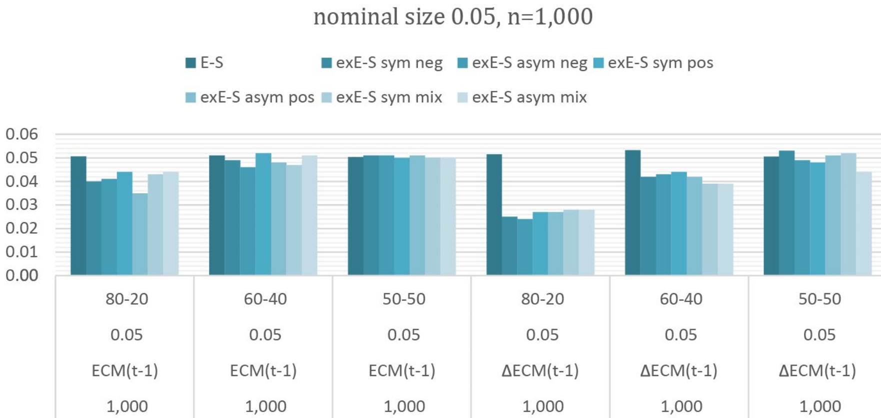


Figure 4. Size of exE-S and E-S for H_0^2 (cont.)



Note. sym neg – parameters in the short-run equation equal in regimes, negative; asym neg – parameters in the short-run equation non-equal in regimes, negative; sym pos – parameters in the short-run equation equal in regimes, positive; asym pos – parameters in the short-run equation non-equal in regimes, positive; sym mix – parameters in the short run equal in modulus, opposite signs; asym mix – parameters in the short run non-equal in modulus, opposite signs.

Source: authors' work.

Figure 3 presents the size for H_0^1. The E-S test size is higher than the nominal one {0.01; 0.05}, but close to it when the threshold variable is assumed to be ECT_{t-1}. In case the threshold variable is ΔECT_{t-1}, the size is much larger than the assumed one. In the case of 0.01, the estimated size is more than three times higher than the assumed one, while for 0.05, the estimated size is over twice as large. The exE-S test keeps its nominal size more stable. The observed differences concern the number of observations in regimes and short-run parameters. The test was insensitive to the threshold variable.

Figure 4 shows the empirical size for H_0^2, which distinguishes asymmetric effects between the regimes. In this case, the E-S test size is very close to the nominal one, disregarding the assumptions. The extended test gave the best results when the nominal size was 0.05. In the case of 0.01, the size of the extended test was larger than the nominal one. The number of observations in both regimes mattered when the proportion was 80%–20% in the respective regimes. In that instance the size was smaller than the nominal one. It is evident if the threshold variable was ΔECM_{t-1}.

4. Empirical example

The successful applications presented in Boehlke et al. (2019) implied further interest in using the exE-S test in the area of economic growth. In the empirical illustration, the example of the Israeli economy is presented. The economy of Israel was the subject of numerous analyses of the factors of its success. Trajtenberg (2001) characterised R&D expenditures, Chorev and Anderson (2006) analysed success in Israeli high-tech start-ups, and Aharoni (2014) provided an in-depth insight into the Israeli economic processes. The paper uses annual data for the years 1980–2017 to uncover the signs of threshold cointegration while GDP *per capita* is considered an endogenous variable. The data were downloaded from OECD (http://stats.oecd.org/), the Federal Reserve Bank of St. Louis (https://fred.stlouisfed.org/), and the Central Bureau of Statistics in Israel (https://www.cbs.gov.il/EN/pages/default.aspx).

Figure 5 presents the Israeli GDP *per capita* expressed in US dollars in constant prices of 2010 and transformed into logarithms. One can notice a structural break around 2002. The Quandt (1958) test results confirmed it with a value of 29.5 (p-value 0.0013). The structural break was strongly related to the dot-com bubble, which significantly affected the Israeli start-ups (Zilberfarb, 2006).

Figure 5. GDP *per capita* in Israel in 1980–2017

Note. Israeli GDP *per capita* expressed in US dollars in constant prices as of 2010, then transformed into logs.
Source: authors' calculations.

As threshold variables, the following were tested: R&D expenditures (R&D), short interest rate (IRs), military expenditures (MilExp), the exchange rate of Israeli shekel to USD (EXR), and savings (Sav). All the potential thresholds are lagged. Above these, the standard threshold variables, i.e. ECM_{t-1} and ΔECM_{t-1} were tested using both E-S and exE-S tests. The Tsay test and Hansen and Seo tests validated the results. The results are presented in Table 3.

Table 3. Results of testing for threshold cointegration for GDP *per capita* using E-S, exE-S, Tsay and Hansen and Seo tests

Threshold variable	Test	H_0^1: (ρ_1=ρ_2=0)	H_0^2: (ρ_1–ρ_2=0)	Tsay $H_0: \psi = 0$	H-S $H_0: A_1 = A_2$
ECM(*t*–1)	E-S	0.0002	0.6230	NA	NA
	exE-S	0.3712	NA		
ΔECM(*t*–1)	E-S	0.0003	0.6938	0.0037	0.9312
	exE-S	0.0021	0.0000		
R&D(*t* –2)	exE-S	0.0071	0.0000	0.0000	0.2461
ΔIRs(*t* –2)	exE-S	0.0110	0.0000	0.4825	0.0001
MilExp(*t* –3)	exE-S	0.0084	0.0000	0.0083	0.0110
EXR(*t* –4)	exE-S	0.0009	0.0000	0.0000	0.9999
Sav(*t* –4)	exE-S	0.0552	0.0035	0.3629	0.0001

Note. Only *p*-values are presented in the table. E-S – original Enders and Siklos test, exE-S – extended Enders and Siklos test, Tsay – Tsay test, H-S – Hansen and Seo test. NA – not available. The results indicating threshold cointegration are shadowed. Significance level: 5%.

Source: authors' calculations.

The results indicate that three out of four tests did not confirm threshold cointegration taking ECM_{t-1} and ΔECM_{t-1} with a zero threshold value. Only the Tsay test showed threshold cointegration for ΔECM_{t-1}. The proposed exE-S test displayed three possible threshold variables: $R\&D_{t-2}$, ΔIRs_{t-2}, and Sav_{t-4}. The values of the threshold were estimated and set at the following levels: 1.0300, 0.0002 and 3.0887, respectively. The first threshold variable was additionally confirmed by the Tsay test and the two other by the Hansen and Seo test. Empirical results should be confronted with economic facts and foundations. As the Israeli economy is based on innovations, both variables, R&D expenditures and savings, are reasonable. The short-term interest rate also refers to savings and investments. It is worth noting that R&D investment is closely related to government contracts and therefore they are also economically worthwhile (Lichtenberg, 1995).

5. Conclusions

In the paper, the power and size of the exE-S were analysed. A simulation experiment was conducted in order to present the advantages and limitations of the test. Moreover, an empirical example was provided. The results of both the simulation and empirical analysis are promising and allow formulating several conclusions. The power of the exE-S test is satisfactory for all parameter values, nevertheless, it depends on the difference between the ρ_1 and ρ_2 parameters. If the number of observations is relatively small (i.e. 50 and 100), the power is lower when the difference is 0.1. It corresponds to a weak asymmetry effect in the regimes and is similar to the E-S test results. However, the power of the exE-S is bigger than that of the E-S test. For greater values of $\rho_1 - \rho_2$ differences, the power of both tests is high.

The signs of short-run parameters in TECMs do not influence the results. The number of observations in each regime is not meaningful for power, however, it is important for the TECM model construction. The simulation results for size are slightly different. In the case of H_0^1, both the E-S and exE-S tests have their size close to the nominal one. The E-S test performs worse if the threshold variable is ΔECM_{t-1}. In the case of H_0^2, the E-S test preserves its size despite the parameters change. The exE-S test has its size higher than the assumed 1% and identical for the 5% significance level. In the case of the size of the extended test, the number of observations in regimes in the 80%–20% proportion decreases the size. The empirical example concerning economic growth in Israel indicates that the testing results using the exE-S test give an in-depth insight into threshold variables for the TECM model in comparison to the E-S test. The results are either supported by the Tsay or the Hansen and Seo test.

Using statistical tests in an empirical study is uncertain due to a low number of observations, differences between the model and the original data generating process, and many other circumstances. Therefore, it is recommended to apply a hierarchical procedure, i.e. to start with the E-S test first to recognise whether a threshold error cointegration around ECM_{t-1} (or ΔECM_{t-1}) exists. Then, one should search deeply for individual thresholds using the exE-S test. When the sample size is relatively small, extra caution in statistical inference is advised. The validation of the results with the use of other tests (i.e. Tsay test, Hansen and Seo test) concludes the process.

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