The use of the bootstrap method for the assessment of investment effectiveness and risk – the case of confidence intervals estimation for the Sharpe ratio and TailVaR

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ABSTRACT

This paper is aimed at presenting application of bootstrap interval estimation methods to the assessment of financial investment's effectiveness and risk. At first, we give an overview of various methods of bootstrap confidence interval estimation, i.e. bootstrap-t interval, percentile interval and BCa interval. Then, bootstrap confidence interval estimation methods are used to estimate confidence intervals for the Sharpe ratio and TailVaR of the Warsaw Stock Exchange sectoral indices. The results show that the bootstrap confidence intervals of different types are quite similarly positioned for each of the analysed index and measure. Taking into the account the locations of confidence intervals for both the Sharpe ratio and TailVaR, the real estate sector tends to be the most advantageous from the investor's viewpoint.

JEL Classification: C130, C150, G110

Keywords: Bootstrap, confidence intervals, Sharpe ratio, TailVaR, stock market index

1. INTRODUCTION

The assessment of investment effectiveness and risk is an essential step in evaluating a particular investment opportunity. However, selecting a right measure of either effectiveness or risk and the way to compute it possesses a problem for both academics and practitioners. The problem is getting worse when a researcher not only wants to calculate a point estimation

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of a particular measure, but he or she is also interested in finding a confidence interval for the measure.

In general, it is historical values that are usually used to compute confidence intervals for different effectiveness and risk measures. Two main approaches can be distinguished here. One assumes that rates of return are independently and identically distributed (hereinafter: i.i.d.) random variables. The methods based on this assumption are easy to apply, however results may be biased (because of a serial correlation and/or deviations from the normal distribution). The other approach relaxes the assumption of i.i.d. rates of return and utilises the ARMA-GARCH models. The methods here are much more computationally advanced but, in return, they can give better results.

The aim of this paper is to present how to use an alternative approach, i.e., the bootstrap method in evaluating investment effectiveness and risk based on confidence intervals for the Sharpe ratio and TailVaR. We focus on three types of bootstrap confidence intervals, i.e. the bootstrap-t confidence interval, the percentile bootstrap confidence interval and the BCa bootstrap confidence interval. The paper demonstrates how to use them to estimate the Sharpe ratio and TailVaR and how the results should be interpreted.

This paper is organised as follows. The first section provides a literature review on attempts to use the bootstrap method for point and confidence interval estimation for Sharpe ratio and VaR-family risk measures. The next section gives an overview of three types of bootstrap confidence intervals that are used in our analysis. The third section introduces the methodology of empirical research, and the fourth section presents its results. The final section concludes.

This paper is based on the diploma thesis by K. Jarno (2019).

2. LITERATURE REVIEW

The Sharpe ratio is frequently used to assess investment effectiveness. The ratio measures the excess rate of return (the rate of return above the risk-free rate) per unit of a total risk measured by the standard deviation of the rate of return in line with the following formula (Sharpe 1966)

$$SR = \frac{\mathbb{E}(R_t) - R_f}{\sqrt{\mathrm{Var}(R_t)}}, \tag{1}$$

where R_t denotes the rate of return in period t while R_f is the risk-free rate.

The basic Sharpe ratio estimator (the so-called ex post Sharpe ratio) is calculated by subtracting the proxy for the risk-free rate from the sample's average rate of return and dividing the result by the sample's standard deviation of the rate of return (this estimator is used in many empirical studies, c.f. Pilotte and Sterbenz 2006, Hodges, Taylor, and Yoder 1997, Auer and Schuhmacher 2013).

Assuming that returns are i.i.d., the confidence interval can be computed based on the quantiles of Student's t-distribution. Since the distribution of the basic Sharpe ratio estimator approaches the normal distribution when sample size approaches infinity, another solution is to compute an approximate confidence interval (Lo 2002, pp. 37–39).

Although bootstrap methods are listed among the possible approaches used to estimate the confidence interval for the Sharpe ratio (Riondato 2018), they are not popular. A study by Scherer (2004) is a rare example. It involved the computation of percentile bootstrap confidence intervals for the Sharpe ratio and the Sortino ratio of hedge funds while assuming that returns are i.i.d. followed by an introduction of methods that allow giving up this assumption. In turn, Ledoit and Wolf (2008) developed a statistical test for comparing the Sharpe ratios of two investments.

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The test does not assume any particular distribution of returns and utilizes the bootstrap method. Finally, in their study, Chaudhry and Johnson (2008) opted for bootstrap percentile confidence intervals to estimate the 95% confidence interval for the skewness of differential rates of return (rates of return above a specified benchmark).

The skewness and kurtosis of returns are of particular importance when it comes to any portfolio analysis because the findings of many empirical studies suggest that distributions of returns deviate from the normal distribution (c.f. Mandelbrot 1963, Fama 1965, Cont 2001, Rachev, Menn, and Fabozzi 2005). Heavy tails of distributions pose a problem especially when assessing the VaR-family (value at risk) risk measures since they aim at capturing the properties of distribution tails. One of the representatives of this group of risk measures is TailVaR (also called *tail conditional expectation*) which is defined as follows (Artzner et al. 1999, p. 223)

$$TailVaR_{\alpha}(R_t) = -\mathbb{E}\left[R_t \mid R_t \leq -VaR_{\alpha}(R_t)\right], \tag{2}$$

where $VaR_{\alpha}(R_t)$ denotes VaR which is computed as α-quantile of returns (R_t) distribution with the reversed sign.

TailVaR quantifies the expected value of the loss given that the rate of return is equal to or smaller than $-VaR_{\alpha}(R_t)$.

Although the Monte Carlo methods are frequently used when computing VaR-family risk measures (c.f. Hull 2009), bootstrap methods are rarely applied. Lin, Wang, and Fuh (2006) introduced a novel approach to estimate the loss probability and developed an algorithm for evaluating VaR that utilizes the bootstrap method. In turn, Mancini and Trojani (2011), applied the bootstrap approach combined with GARCH models estimation.

3. BOOTSTRAP CONFIDENCE INTERVALS

In this section, we shall discuss the basic bootstrap procedure to approximate the distribution of a random variable and present different bootstrap methods to construct confidence intervals.

Let us start with introducing the concept of an empirical distribution function which is crucial for our further discussion. The empirical distribution function (Krzyśko 2004, p. 13) for a sample $\mathbf{X} = (X_1, X_2, \ldots, X_n)^\top$ is a function $F_n: \mathbb{R} \times \mathbb{R}^n \to [0,1]$ defined as

$$F_n(t, \mathbf{X}) = \frac{\#\{1 \leq j \leq n : X_j \leq t\}}{n}, \tag{3}$$

where $t \in \mathbb{R}$, $\mathbf{X} \in \mathbb{R}^n$, and # denotes the number of elements of the set.

Efron (1979, p. 2) introduced the bootstrap method to statistics by applying it to the problem of the approximation of the distribution of a random variable. Let $R(\mathbf{X},F)$ be a random variable depending on a random sample $\mathbf{X} = (X_1, X_2, \ldots, X_n)^\top$ from a population, where F is the distribution function for population. The bootstrap sample is the random sample $\mathbf{X}^*$ of size n from the distribution of the fixed sample $\mathbf{X}$. The problem is how to approximate the distribution of R using the realizations of sample $\mathbf{X} = (X_1, X_2, \ldots, X_n)^\top$. To solve this problem Efron (1979, pp. 3–4) proposes the following bootstrap procedure:

1. Based on the realization of random sample $\mathbf{X} = (X_1, X_2, \ldots, X_n)^\top$ construct empirical distribution function F_n.
2. Select B independent bootstrap samples $\mathbf{X}^{*1}, \mathbf{X}^{*2}, \ldots, \mathbf{X}^{*B}$ from $\mathbf{X}$. B is a big number, e.g., $B = 1000$.
3. For each bootstrap sample and the empirical distribution function F_n, compute the value of random variable $R(\mathbf{X},F)$. Let $R_1^*, R_2^*, \ldots, R_B^*$ denote the values obtained.

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4. Approximate the distribution of random variable $R(\mathbf{X},F)$ using the bootstrap distribution based on the values $R_1^*, R_2^*,\ldots, R_B^*$.

Let us now focus on different constructions of confidence intervals based on the bootstrap methods. Let θ denote a parameter of population distribution, which we would like to estimate.

3.1. The bootstrap-t confidence interval

The construction of the bootstrap-t confidence interval (Efron and Tibshirani 1998, pp. 153–162) is similar to the constructions of confidence intervals for the expected value based on random variables with normal or Student's t-distributions under the normality assumption. However, we do not make any specific assumption about the distribution of population in this bootstrap method. Given B bootstrap samples, we compute

$$Z_b^* = \frac{\widehat{\theta}_b^* - \widehat{\theta}}{\widehat{se}_b^*}, \tag{4}$$

where $\widehat{\theta}_b^* = s\left(x^{*b}\right)$, $b = 1, 2,\ldots, B$ is the value of estimator $\widehat{\theta}$ of θ for a given bootstrap sample x^{*b}, and $\widehat{se}_b^*$ is a standard error of this estimator under the bootstrap sample. The distribution of statistic Z_b^* is approximated based on the data by the procedure described above. The α-th quantile $\hat{t}^{(\alpha)}$ of this distribution is defined as

$$\frac{\#\left\{Z_b^* \leq \hat{t}^{(\alpha)}\right\}}{B} = \alpha. \tag{5}$$

The bootstrap-t confidence interval with confidence level $1 - 2\alpha$ is as follows

$$\left(\widehat{\theta} - \hat{t}^{(1-\alpha)} \times \widehat{se}, \widehat{\theta} - \hat{t}^{(\alpha)} \times \widehat{se}\right). \tag{6}$$

Note that in the denominator of (4), there is $\widehat{se}_b^*$, which requires an iterated bootstrap to be obtained (Shao and Tu 1995, pp. 131–132), which we shall describe in Section 4. To compute the quantiles, we first have to rank the values Z_b^* in the non-decreasing order. For example, when $B = 10000$, then the 0.05-th quantile is the 500-th ordered value of Z_b^*, while the 0.95-th quantile is the 9500-th ordered value of Z_b^*. When $B \times \alpha$ is not an integer number, computation is a little more complicated. Efron and Tibshirani (1998, pp. 160–161) propose the following method to obtain the quantiles:

1. Compute the largest integer number k such that $k \leq (B + 1) \times \alpha$.
2. The α-th and $(1 - \alpha)$-th quantiles are k-th and $(B = 1 - k)$-th ordered values of Z_b^*.

When $B = 9999$, $k = 500$, since $(9999 + 1) \times 0{,}05 = 500$. Again the 0.05-th quantile is the 500-th ordered value of Z_b^*, and the 0.95-th quantile is the 9500-th ordered value of Z_b^*.

3.2. The bootstrap percentile confidence interval

The bootstrap percentile confidence interval is based on the quantiles of the bootstrap distribution of an estimator (Efron and Tibshirani 1998, pp. 170–176). Like in the case of the bootstrap-t confidence interval, we consider the semi-parametric model. The distribution of estimator $\widehat{\theta}$ is approximated using the Efron procedure described above. Let $\widehat{\theta}_B^{*(\alpha)}$ be α-th quantile of this bootstrap distribution. The bootstrap percentile confidence interval with confidence level $1 - 2\alpha$ is as follows

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$$\left(\widehat{\theta}_B^{*(\alpha)}, \widehat{\theta}_B^{*(1-\alpha)}\right). \tag{7}$$

The bootstrap percentile confidence intervals have some advantageous properties such as range preserving and transformation invariance (c.f. Efron and Tibshirani 1998, pp. 173–176). In practice, bootstrap percentile confidence intervals are determined using the method discussed earlier for the bootstrap-t confidence interval.

3.3. The BC_a bootstrap confidence interval

Like the percentile confidence interval, the BC_a bootstrap confidence interval (Efron and Tibshirani 1998, pp. 184–188) is also based on the quantiles of the bootstrap distribution of estimator, but these quantiles are not the α-th and $(1 - \alpha)$-th quantiles. The name BC_a is the abbreviation of bias-corrected and accelerated.

The BC_a bootstrap confidence interval with confidence level $1 - 2\alpha$ is as follows:

$$\left(\widehat{\theta}_B^{*(\alpha_1)}, \widehat{\theta}_B^{*(\alpha_2)}\right), \tag{8}$$

where

$$\alpha_1 = \Phi\left(\hat{z}_0 + \frac{\hat{z}_0 + z^{(\alpha)}}{1 - \hat{a}\left(\hat{z}_0 + z^{(\alpha)}\right)}\right), \tag{9}$$

$$\alpha_2 = \Phi\left(\hat{z}_0 + \frac{\hat{z}_0 + z^{(1-\alpha)}}{1 - \hat{a}\left(\hat{z}_0 + z^{(1-\alpha)}\right)}\right), \tag{10}$$

$\Phi(\cdot)$ denotes the distribution function of the standard normal distribution $N(0, 1)$, and $z^{(\alpha)}$ is the α-th quantile of this distribution. Moreover, $\hat{z}_0$ is a bias-correction of the form

$$\hat{z}_0 = \Phi^{-1}\left(\frac{\#\left\{\widehat{\theta}_b^* < \widehat{\theta}\right\}}{B}\right), \tag{11}$$

where $\Phi^{-1}(\cdot)$ is the inverse function of the distribution function of the standard normal distribution. $\hat{z}_0$ measures median bias of $\widehat{\theta}^*$, i.e., the difference between the median of $\widehat{\theta}_b^*$ and $\widehat{\theta}$. On the other hand, the acceleration $\hat{a}$ is defined in the following way

$$\hat{a} = \frac{\sum_{i=1}^{n}\left(\widehat{\theta}_{(\cdot)} - \widehat{\theta}_{(i)}\right)^3}{6\left[\sum_{i=1}^{n}\left(\widehat{\theta}_{(\cdot)} - \widehat{\theta}_{(i)}\right)^2\right]^{3/2}}, \tag{12}$$

where $\widehat{\theta}_{(\cdot)} = \sum_{i=1}^{n}\frac{\widehat{\theta}_{(i)}}{n}$ and $\widehat{\theta}_{(i)} = s\left(x_{(i)}\right)$, and $x_{(i)}$ is the original sample without i-th observation.

From among the presented bootstrap confidence intervals, the BC_a bootstrap confidence interval has the best properties, i.e., it automatically adjusts the endpoints of the interval due to its transformation-respecting property and is characterized by second-order accuracy, i.e., its error tends to zero at the rate of $1/n$. For comparison, the bootstrap percentile confidence interval also has the first-mentioned property, but it is characterized by first-order accuracy, i.e., its error tends to zero at the rate of $1/\sqrt{n}$. In turn, the bootstrap-t confidence interval is characterized by

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second-order accuracy, but it is not automatic due to transformations (Efron and Tibshirani 1998, pp. 187–188). For more on the accuracy of individual types of bootstrap confidence intervals, see Davison and Hinkley (1997, pp. 211–220).

4. METHODS AND DATA

A stock sectoral index is a theoretical portfolio that reflects changes in market valuation of companies that operate in a particular sector of the economy and their shares are traded in the public market. A comparison of the performance of different sectoral indices can form a basis for conclusions on the effectiveness and risk of various sectoral investment strategies. The study includes 14 sectoral indices of the Warsaw Stock Exchange, i.e., WIG-automobiles&parts, WIG-banking, WIG-chemical, WIG-clothes, WIG-construction, WIG-energy, WIG-food, WIG-IT, WIG-media, WIG-mining, WIG-oil&gas, WIG-pharmaceuticals, WIG-real estate, and WIG-telecom.

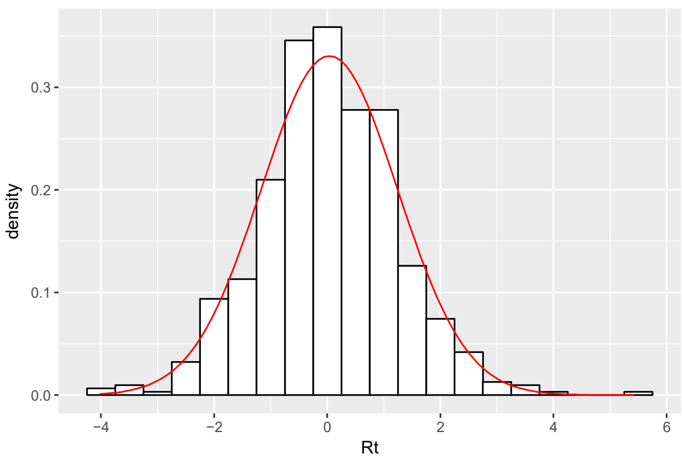
The data was sourced from the Stooq.pl website. Based on daily closing prices for the time period ranging from 2 January 2017 to 28 June 2019 (620 observations in total for each index) logarithmic rates of return were computed according to the following formula

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right), \tag{13}$$

where P_t is the closing price on day t and P_{t-1} is the closing price on the previous day. We assumed that returns were i.d.d.

The histograms of return for WIG-banking, WIG-construction, and WIG-chemical along with the density function of the normal distribution are presented in Figures 1–3. Empirical distributions tend to have kurtosis larger than the normal distribution. What is more, some asymmetries can be observed.

Figure 1
Histogram of WIG-banking returns



Source: Authors' calculations based on data provided by Stooq.pl

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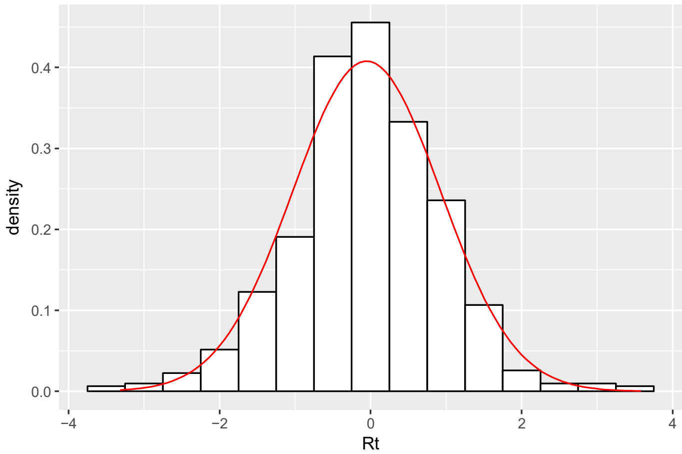
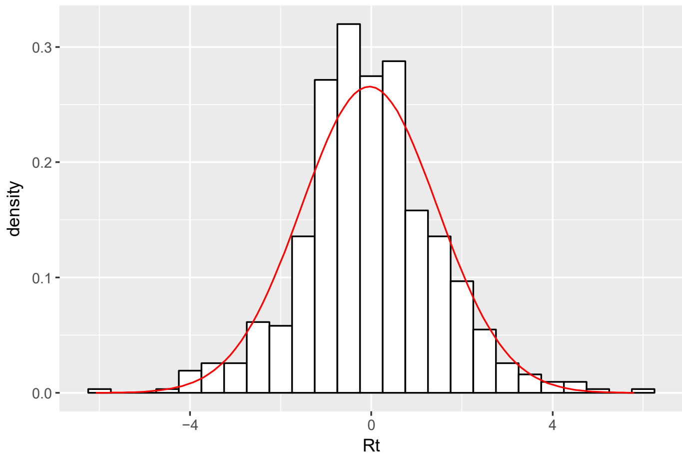
Figure 2
Histogram of WIG-construction returns



Source: Authors' calculations based on data provided by Stooq.pl

Figure 3
Histogram of WIG-chemical returns



Source: Authors' calculations based on data provided by Stooq.pl

The next step of the study was to compute the 95% bootstrap confidence intervals discussed in the previous section for the Sharpe ratio and TailVaR for all the indices.

1/365 of the reference rate of the National Bank of Poland, which remained unchanged (1.5%) during the whole period of the analysis, was used as a proxy for the risk-free rate, i.e., 0.0041%.

$VaR_{\alpha}(R_t)$ was computed for probability $\alpha = 5\%$.

With a view to estimating bootstrap confidence intervals for the Sharpe ratio, 2000 bootstrap samples were performed for all the said types of these intervals. As regards the bootstrap-t confidence interval, the number of "nested" replications was also set to 2000. Due to the fact that TailVaR is a parameter the value of which is situated on the left tail of the distribution, 5000 bootstrap samples were used to construct bootstrap confidence intervals (there again 5000 nested bootstrap samples in the bootstrap-t confidence interval). Standard errors were estimated according to the procedure described below.

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Let $x = (x_1, x_2,\dots, x_n)$ be the realization of the random sample $\mathbf{X} = (X_1, X_2, \dots, X_n)^\top$ from a certain distribution with distribution function F. Based on x, the value of estimator $\hat{\theta} = s(x)$ for $\theta = t(F)$ was computed. To estimate the standard error for the bootstrap estimator, we use (iterated) bootstrap method as mentioned in Section 3. Namely, we first select (with replacement) B independent bootstrap samples $x^{*1}, x^{*2},\dots, x^{*B}$ from original sample x. Then for each bootstrap sample we compute the value of estimator, i.e.,

$$\hat{\theta}_b^* = s(x^{*b}),\ b = 1, 2,\dots, B \tag{14}$$

and estimate the standard error $se_F(\hat{\theta})$ by using the standard deviation computed based on B bootstrap samples:

$$se_B = \sqrt{\frac{\sum_{b=1}^{B}\left(\hat{\theta}_b^* - \hat{\theta}^*\right)^2}{B-1}}, \tag{15}$$

where $\hat{\theta}^* = \frac{\sum_{b=1}^{B}\hat{\theta}_b^*}{B}$.

5. RESULTS

Table 1 presents the estimates of bootstrap confidence intervals for the Sharpe ratios of 14 sectoral indices created by the Warsaw Stock Exchange. The bounds of different bootstrap confidence intervals are similar for a given index and any pattern of mutual position of confidence intervals of different types is not visible. All the intervals for Sharpe ratios include zero and negative values. This fact is not favourable from the viewpoint of a potential investor. The lower bounds of bootstrap-t confidence intervals range from the level of -0.1448 for WIG-construction up to -0.04 for WIG-real estate. The corresponding numbers for percentile confidence intervals are -0.1343 (WIG-construction) and -0.0393 (WIG-real estate), while for BC_a confidence intervals the range is from -0.1378 (WIG-construction) to -0.0403 (WIG-real estate). The upper bounds of bootstrap-t confidence intervals range from 0.0214 for WIG-construction up to 0.1216 for WIG-real estate. The range for percentile confidence intervals is 0.0263–0.1193, while for BC_a confidence intervals it is 0.0225–0.1186 (in both cases the minimal result refers to WIG-construction and the maximal result is due to WIG-real estate).

In general, the intervals for the following indices have the most advantageous location (i.e., the lower bound is least negative and the upper bound is most positive): WIG-banking, WIG-media, WIG-real estate (the leader), and WIG-telecom. At the other end are confidence intervals for WIG-automobiles&parts, WIG-chemical, WIG-construction, and WIG-food which tend to be worst located.

Table 2 presents the estimates of bootstrap confidence intervals for TailVaR of 14 sectoral indices created by the Warsaw Stock Exchange. Like in the case of the Sharpe ratios, the bounds of different bootstrap confidence intervals are similar for a given index. However, a clear pattern of the mutual position of bootstrap confidence intervals of different types is visible here, i.e. for a given index the percentile confidence interval has the lowest both lower and upper bounds, while the bootstrap-t confidence interval has the largest upper bound. This regularity is observed in all the cases.

The lower bounds of bootstrap-t confidence intervals range from 1.2966 for WIG-real estate up to 3.3013 for WIG-mining. The corresponding numbers for percentile confidence intervals are 1.2741 (WIG-real estate) and 3.2458 (WIG-mining) while for BC_a confidence intervals the

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range is from 1.2956 (WIG-real estate) to 3.2937 (WIG-mining). The upper bounds of bootstrap-t confidence intervals range from 1.663 for WIG-real estate up to 4.1422 for WIG-oil&gas. The range for percentile confidence intervals is 1.6049–3.9687, while for BC_a confidence intervals the range is 1.6388–4.066 (in both cases the minimal result refers to WIG-real estate and the maximal result is due to WIG-oil&gas).

From the investor's point of view, the location of the confidence intervals for TailVaR of WIG-real estate seems to be the most favourable, followed by WIG-banking, WIG-construction, WIG-IT, WIG-media, WIG-food, and WIG-telecom. In turn, at the bottom of the ranking are: WIG-chemical, WIG-energy, WIG-mining, WIG-clothes, and WIG-oil&gas.

The results suggest that the real estate sector tends to be the most advantageous from the viewpoint of an investor opting for sectoral investment strategies and focusing on the Sharpe ratio and TailVaR in his or her investment decisions. The portfolio of WIG-real estate index comprises companies that are leading Polish residential and commercial real estate developers.

Table 1
Bootstrap confidence intervals for the Sharpe ratios

Index	Interval type		
	Bootstrap-t	Percentile	BC_a
WIG- automobiles&parts	(-0.1244, 0.0364)	(-0.1278, 0.0300)	(-0.1245, 0.0344)
WIG-banking	(-0.0516, 0.1043)	(-0.0510, 0.1048)	(-0.0506, 0.1055)
WIG-chemical	(-0.1065, 0.0415)	(-0.1085, 0.0445)	(-0.1077, 0.0462)
WIG-clothes	(-0.0705, 0.0885)	(-0.0739, 0.0860)	(-0.0689, 0.0884)
WIG-construction	(-0.1448, 0.0214)	(-0.1343, 0.0263)	(-0.1378, 0.0225)
WIG-energy	(-0.0936, 0.0643)	(-0.0993, 0.0591)	(-0.0951, 0.0627)
WIG-food	(-0.1231, 0.0388)	(-0.1270, 0.0358)	(-0.1271, 0.0352)
WIG-IT	(-0.0683, 0.0894)	(-0.0657, 0.0914)	(-0.0655, 0.0917)
WIG-media	(-0.0596, 0.1039)	(-0.0621, 0.1009)	(-0.0580, 0.1041)
WIG-mining	(-0.0786, 0.0812)	(-0.0805, 0.0784)	(-0.0793, 0.0796)
WIG-oil&gas	(-0.0643, 0.0959)	(-0.0635, 0.0941)	(-0.0621, 0.0957)
WIG- pharmaceuticals	(-0.0758, 0.0797)	(-0.0778, 0.0774)	(-0.0812, 0.0760)
WIG-real estate	(-0.0400, 0.1216)	(-0.0393, 0.1193)	(-0.0403, 0.1186)
WIG-telecom	(-0.0546, 0.0980)	(-0.0590, 0.0943)	(-0.0599, 0.0929)

Source: Authors' calculations based on data provided by Stooq.pl

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Table 2
Bootstrap confidence intervals for the TailVaR (in %)

Index	Interval type		
	Bootstrap-t	Percentile	BC_a
WIG-automobiles&parts	(2.3443, 3.7028)	(2.2700, 3.3284)	(2.3716, 3.6442)
WIG-banking	(2.2398, 2.8714)	(2.2019, 2.7603)	(2.2498, 2.8217)
WIG-chemical	(3.0226, 3.8796)	(2.9956, 3.7765)	(3.0496, 3.8501)
WIG-clothes	(3.0152, 3.6273)	(2.9645, 3.5496)	(3.0159, 3.6052)
WIG-construction	(2.0220, 2.5556)	(1.9822, 2.4642)	(2.0237, 2.5164)
WIG-energy	(3.1658, 4.0965)	(3.0961, 3.9571)	(3.1633, 4.0541)
WIG-food	(1.8576, 2.4104)	(1.8395, 2.3396)	(1.8699, 2.3845)
WIG-IT	(2.0403, 2.6405)	(1.9959, 2.5603)	(2.0477, 2.6145)
WIG-media	(1.9022, 2.7181)	(1.8626, 2.5051)	(1.9174, 2.6335)
WIG-mining	(3.3013, 3.9116)	(3.2458, 3.8214)	(3.2937, 3.8949)
WIG-oil&gas	(3.2130, 4.1422)	(3.1552, 3.9687)	(3.2223, 4.0660)
WIG- pharmaceuticals	(2.7954, 3.5016)	(2.7257, 3.3880)	(2.7789, 3.4405)
WIG-real estate	(1.2966, 1.6630)	(1.2741, 1.6049)	(1.2956, 1.6388)
WIG-telecom	(2.2661, 2.9437)	(2.2256, 2.8545)	(2.2770, 2.9072)

Source: Authors' calculations based on data provided by Stooq.pl

6. CONCLUSIONS

The main goal of this paper was to demonstrate how to use an alternative approach, i.e., bootstrap methods, to evaluate investment effectiveness and risk based on confidence intervals for the Sharpe ratio and TailVaR. We assumed that returns are i.i.d. The aim of the paper was achieved over two stages. First, the construction and properties of three types of bootstrap confidence intervals were discussed. Then, we demonstrated how to use them in practice for assessing an investment opportunity.

The results of the research can serve as a reference for other researchers investigating methods of estimating various investment effectiveness and risk measures. What is more, the results of the analysis of the bootstrap confidence intervals for the Sharpe ratios and TailVaR of the Warsaw Stock Exchange sectoral indices deliver a clear assessment of their performance, which can be useful for all the researchers and practitioners interested in the Polish capital market performance and investment strategies.

To sum up, the implicit advantage of using bootstrap confidence intervals (non-parametric bootstrap) is that it does not require making any assumptions about the shape of the distribution, and thus the universality of the approach.

On the other hand, the approach presented in this paper requires i.i.d. assumption. Moreover, in general the bootstrap method is computationally expensive.

The considerations presented in this paper can serve as a starting point for further analysis on the theoretical basis and practical possibilities of using bootstrap methods in assessing the effectiveness and risk of investment opportunities. The analysis can by expanded to other types of effectiveness and risk measures, as well as point estimation.

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