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# THE CAPM AND FAMA-FRENCH MODELS IN POLAND

## 1. INTRODUCTION

The cross-sectional differences in asset expected returns have attracted considerable attention in finance literature. The Single-Factor Capital Asset Pricing Model (CAPM) and the Fama-French Three–Factor (FF) models occupy an essential place among models of expected returns.

The CAPM model of William Sharpe [28] and John Lintner [21], although simple and logical, is based on certain restrictive assumptions about the functioning of the market. By the end of 70's and early 80's, the empirical evidence against the CAPM became very strong ([2], [3]). It was noticed, that return differences of some assets grouped according to the firms' financial characteristics were not captured by the betas. Some attempts to explain these anomalies were made, among others, by Banz [2] and Reinganum [25] who investigated the size effect and found a return premium on some small stocks. The book-to-market effect was researched by Rosenberg et al. [26] and confirmed later by Blume and Stambaugh [6]. They found the premium in the case of stocks with high ratios. Fama and French [10] had investigated the explanatory power of returns on some factors associated with the companies' characteristics, such as size, book value to market value ratio, leverage stock's price earnings ratio. As a result of their efforts, they proposed a three factors model to explain the stock returns: the excess of return in relation to the market (market factor), the difference between the returns of portfolio with the large and small capitalization (size factor, SMB) and the difference between the returns on portfolios of high and low book value to market value (HML). The inclusion of those factors improved the model fitting to the US empirical data. The investigations conducted for other markets (Australian, Canadian, German, French, Japan, and UK) showed that the size and BE/ME effects might have an international character. The asset pricing model on Polish stocks has been examined and tested, among others, by Bołt and Miłobędzki [7], Fiszeder [12], Kowerski [18], Urbański [30], Zarzecki [31].

In this study, we investigate the ability of the Fama-French three-factor model to explain cross-sectional differences in asset returns for the period December 2002 to January 2010. The research is conducted on the Warsaw Stock Exchange (WSE). The financial data originate from the CEDULA[1] which is a daily official bulletin providing information from WSE. The systematic risk and the risk premium estimations

[1] www.gpw.com.pl

are estimated using the Generalized Method of Moments (GMM) which takes into account autocorrelation and heteroscedasticity in time series. Procedures for testing whether the intercepts are jointly null is carried out by test proposed by MacKinlay and Richardson [22]. All calculations presented in this study have been performed using application developed and dedicated for the purposes of this paper.

The study has the following structure. In Section 2, the econometric model and the methodology for estimating model parameters are presented. The empirical study is discussed in Section 3. The paper ends with concluding remarks.

## 2. THEORY AND METHODOLOGY

According to a $K$ factor asset pricing model, the expected return on a portfolio in excess of the risk-free rate is explained by its sensitivity of its return to $K$ common factors. The expected excess returns satisfy the linear relations which can be written in a matrix form as:

$$E(\boldsymbol{R}_t) = \gamma_0 + \gamma_1\beta_1 + \ldots + \gamma_K\beta_K, \tag{1}$$

where $\boldsymbol{R}_t$ is an $N \times 1$ vector of portfolios' excess returns at time $t$

$$\boldsymbol{R}_t = \alpha + \beta_1 f_{1t} + \ldots + \beta_K f_{Kt} + \varepsilon_t,\ t = 1, \ldots, T, \tag{2}$$

$\gamma_1, \ldots, \gamma_K$ are risk premiums, $\alpha, \beta_1, \ldots, \beta_K$ are $N \times 1$ vectors of factor sensitivities or loadings, $f_{1t}, \ldots, f_{Kt}$ are common factors at time $t$, and $\epsilon_t$ is an $N \times 1$ vector of error terms at time $t$.

### 2.1. THE SYSTEMATIC RISK ESTIMATION AND THE INTERCEPT TEST

The estimation of the systematic risk parameters is performed using the Generalized Method of Moments [15] approach which is robust to both conditional heteroscedasticity and serial correlation in the return residuals as well as in the factors.

The GMM estimator of the vector of parameters $\varphi = (\alpha^T, \beta_1^T, \ldots, \beta_T^T)^T$ must fulfill the following conditions:

$$E\left[g_t(\varphi)\right] = E\left[\varepsilon_t \otimes \begin{pmatrix} 1 \\ \boldsymbol{F}_t \end{pmatrix}\right] = E\begin{bmatrix} \boldsymbol{R}_t - \alpha - \beta \boldsymbol{F}_t \\ (\boldsymbol{R}_t - \alpha - \beta \boldsymbol{F}_t) \otimes \boldsymbol{F}_t \end{bmatrix} = \mathbf{0} \tag{3}$$

where $\boldsymbol{F}_t = (f_{1t}, \ldots, f_{Kt})^T$ is the $K \times 1$ vector of factors at time $t$, and $\beta = [\beta_1, \ldots, \beta_K]$ is the $N \times K$ matrix of unknown parameters. If the system is exactly identified, i.e. the number of equations is equal to the number of unknowns, the GMM procedure is equivalent to the Ordinary Least Square (OLS) regression.

Hansen [15] shows that the GMM estimator $\hat{\varphi}$ has an asymptotic normal distribution with the mean equal to $\varphi$ and asymptotic variance matrix

$$var[\hat{\varphi}] = \left(\boldsymbol{D}^T \boldsymbol{S}_0^{-1} \boldsymbol{D}\right)^{-1} \tag{4}$$

where $\boldsymbol{D}$ is the matrix of the derivatives of $g_t(\varphi)$ with respect to all parameters, and

$$\boldsymbol{S}_0 = \sum_{j=-\infty}^{\infty} E\left[g_t(\varphi) g_{t-j}^T(\varphi)\right].$$

To account for heteroscedasticity and autocorrelation, the consistent estimator $\boldsymbol{S}_T$ of $\boldsymbol{S}_0$ can be defined as

$$\boldsymbol{S}_T = \boldsymbol{\Omega}_0 + \sum_{j=1}^{m} w(j, m)\left[\boldsymbol{\Omega}_j + \boldsymbol{\Omega}_j^T\right],$$

where $w(j, m)$ is some weighting function, and

$$\boldsymbol{\Omega}_j = \frac{1}{T} \sum_{t=j+1}^{T} g_t(\hat{\varphi}) g_{t-j}^T(\hat{\varphi}),\ j = 0, \ldots, m.$$

(assuming that $E(f_i \varepsilon_i) = 0$ and $m = 0$, variances of $\hat{\varphi}$ are equal to variances obtained using the separate OLS regressions for each portfolio).

The multifactor model can generate efficient portfolios if all intercepts are jointly equal to zero. An ordinary (GRS) test for that implication was suggested by Gibbons, Ross and Shanken [13]. It works correctly, when the disturbances are temporally independent and jointly normal, with the zero mean. This test, however, does not adjust for autocorrelated series, which is a feature often found in financial series. The test based on GMM approach, proposed by MacKinlay and Richardson [22], can be employed to test null hypothesis that the intercept vector is zero. In this case, the test statistic can take the following form:

$$\chi_N^2 = \hat{\alpha}' \, var[\hat{\alpha}]^{-1} \hat{\alpha}, \tag{5}$$

where $var[\hat{\alpha}]$ is the upper left corner of matrix (4). In case the null hypothesis is true, this statistic is distributed as $\chi^2$ with $N$ degrees of freedom. To improve the finite sample behavior of this test, this statistic should be scaled by $\left(\frac{T-N-K}{T}\right)$ ([22]).

### 2.2. THE RISK PREMIUM ESTIMATION

The risk-return relationship can be estimated in two stages. At the first stage, beta estimates are obtained from the separate time-series regressions for each asset (2).

Then, these estimates are used in the second-pass cross-sectional regression (CSR) (1). Since the independent variable in the CSR is measured with an error, the second-pass estimate is a subject to an errors-in-variables problem. To omit this problem, all unknown parameters $\varphi = \left(\alpha^T, \beta_1^T, \ldots, \beta_K^T, \gamma_0, \ldots, \gamma_K\right)^T$ in (1) and (2) should be estimated simultaneously. The GMM approach can be used for this purpose since it is robust to both the conditional heteroscedasticity and serial correlation in the return residuals as well as in the factors. However, it is difficult to obtain the GMM estimators. This is due to a large number of parameters to estimate coupled with the nonlinearity of the model [27].

In this work we consider the two step GMM approach suggested by Harvey and Kirby [16]. The model is estimated sequentially. First, the parameter vector $\varphi = \left(\alpha^T, \beta_1^T, \ldots, \beta_K^T, \gamma_0, \ldots, \gamma_K\right)^T$ is partitioned into two sub-vectors $\varphi = (\varphi_1^T, \varphi_2^T)^T$, where $\varphi_1 = (\alpha^T, \beta_1^T, \ldots, \beta_K^T)^T$ and $\varphi_2 = (\gamma_0, \ldots, \gamma_K)^T$. Then, the sample moments can be expressed as:

$$g_t(\varphi) = \left[\varepsilon_t(\varphi_1) \otimes F_t)^T, \left(R_t - [1_N, \beta]\varphi_2\right)^T\right]^T = \left[g_{1t}(\varphi_1), g_{2t}(\varphi)\right]^T$$

and the GMM conditions are:

$$E\left[g_{1t}(\varphi_1), g_{2t}(\varphi)\right]^T = \mathbf{0}. \tag{6}$$

The first part of the GMM condition is exactly identified. Hence, the estimator $\hat{\varphi}_1$ is equal to the time-series OLS estimator obtained from equation (2). The two step strategy of the GMM method uses the estimated parameters $\hat{\varphi}_1$ in the second part of condition (6). Hence, the problem boils down to find a minimum

$$\min_{\varphi_2} g_{2T}(\hat{\varphi}_1, \varphi_2)^T \boldsymbol{W}_{2T} g_{2T}(\hat{\varphi}_1, \varphi_2),$$

where $\boldsymbol{W}_{2T}$ is a weighting matrix. Following Ogaki [23], the optimal weighting matrix can be chosen as

$$W_{2T} = \left[\left[-D_{21}D_{11}^{-1}I_N\right]S_T\left[\left[-D_{21}D_{11}^{-1}I_N\right]^T\right]^{-1} \tag{7}$$

where $\boldsymbol{D}_{ij}$, $i, j = 1, 2$, is the $(i, j)$ block of the $N(K+2) \times (N+1)(K+1)$ matrix of the derivatives of $g_T(\varphi)$ with respect to all the parameters, and $\boldsymbol{S}_T$ is a consistent estimator of the covariance matrix of moment conditions (6). Since $\boldsymbol{D}_{21}$ in (7) is a zero matrix, thus the weighting $\boldsymbol{W}_{2T}$ matrix reduces to

$$W_{2T} = \left[[0\ I_N]S_T\left[[0\ I_N]^T\right]^{-1}.$$

The above discussion leads to the conclusion, that the consistent GMM estimator $\hat{\varphi}_2$ satisfies the minimum of the function:

$$\left(\overline{\boldsymbol{R}} - \left[\boldsymbol{1}_N, \hat{\beta}\right]\varphi_2\right)^T \boldsymbol{S}_{2T}^{-1} \left(\overline{\boldsymbol{R}} - \left[\boldsymbol{1}_N, \hat{\beta}\right]\varphi_2\right),$$

where $\overline{\boldsymbol{R}}$ is a vector of the averages of the excess returns of the portfolios, and the covariance matrix $\boldsymbol{S}_{2T}$ is estimated following Andrews [1]:

$$\boldsymbol{S}_{2T} = \boldsymbol{\Omega}_0 + \sum_{j=1}^{m} w(j, m)\left[\boldsymbol{\Omega}_j + \boldsymbol{\Omega}_j^T\right],$$

with $w(j, m)$ being Parzen or Bartlett kernel and

$$\boldsymbol{\Omega}_0 = \frac{1}{T}\sum_{t=j+1}^{T} \boldsymbol{h}_t \boldsymbol{h}_{t-j}^T, \ \boldsymbol{h}_t = \hat{\beta}(\boldsymbol{F}_t - \overline{\boldsymbol{F}}) + \varepsilon_t,$$

where $\overline{\boldsymbol{F}}$ is a vector of factor averages.

The obtained $\hat{\varphi}_2$ estimator is used as a starting point for the iteration to attain the efficiency bound to the GMM estimator. In this work, the second stage is iterated with the inverse of the long run covariance matrix until the convergence condition is fulfilled.

The advantage of the two-step GMM procedure is that all estimations are linear, which makes it easier to find out the solution.

## 3. EMPIRICAL ANALYSIS

### 3.1. DATA SET AND PORTFOLIO CONSTRUCTION

This study uses the data set containing stock prices traded in the Warsaw Stock Exchange (the number of stocks varies from 120 to 400 during the sample period). The sample used spans from December 2002 to January 2010. All prices are closing prices adjusted for splits and dividends. The analysis is restricted to the companies with available returns. The risk free rate ($R_f$) is calculated from the 52-weeks treasury bills.

The sample is divided into 12 portfolios on the basis of the selected characteristics: size and BE/ME ratio. The portfolios are formed as follows. At the beginning of each month stocks are allocated to the four groups based on their sorted sizes. Next, the stocks in each group are allocated in an independent sort to three book-to-market equity (BE/ME) groups. The portfolios are updated at the beginning of each month. The averages and standard deviations of the excess return for the 12 size-BE/ME relative portfolios are reported in Table 1. The statistics show that the portfolios with a small capitalization and high BE/ME earn the highest average excess return but consist of the firms of the highest risk. This portfolio has the biggest average monthly excess return equal to 2.2% with the standard deviation of 11%, whereas the portfolio of big firms with low BE/ME has the average excess return of −1% with the smallest standard deviation equal to 7.4%.

Table 1

Descriptive statistics for 12 size-BE/ME portfolios for the sample period December 2002 to January 2010

| Size | Summary statistics | | | | | |
|---|---|---|---|---|---|---|
| | Low | 2 | High | Low | 2 | High |
| | $R_i$ | | | | | |
| | Average | | | Standard deviation | | |
| Small | –0.013 | 0.012 | 0.022 | 0.109 | 0.111 | 0.110 |
| 2 | –0.008 | –0.001 | –0.002 | 0.080 | 0.082 | 0.101 |
| 3 | –0.006 | –0.006 | 0.000 | 0.080 | 0.074 | 0.091 |
| Big | –0.010 | –0.015 | 0.000 | 0.074 | 0.092 | 0.081 |
| | ln (ME) | | | | | |
| | Average | | | Standard deviation | | |
| Small | 3.426 | 3.350 | 3.237 | 0.518 | 0.501 | 0.627 |
| 2 | 4.549 | 4.521 | 4.528 | 0.425 | 0.455 | 0.485 |
| 3 | 5.776 | 5.677 | 5.640 | 0.409 | 0.415 | 0.421 |
| Big | 7.519 | 7.874 | 7.731 | 0.292 | 0.415 | 0.441 |
| | ln (BE/ME) | | | | | |
| | Average | | | Standard deviations | | |
| Small | –1.158 | –0.176 | 0.446 | 0.733 | 0.617 | 0.577 |
| 2 | –1.201 | –0.443 | 0.190 | 0.492 | 0.471 | 0.458 |
| 3 | –1.422 | –0.701 | –0.009 | 0.386 | 0.436 | 0.437 |
| Big | –1.573 | –0.851 | –0.200 | 0.349 | 0.389 | 0.320 |

The monthly returns of the two selected portfolios are presented in Fig. 1. Portfolio 3 consists of small firms with the high BE/ME ration, whereas Portfolio 10 consists of big firms with the low BE/ME. One can notice that monthly returns of portfolio 3 are higher than monthly returns of portfolio 10 in contrary to the deviations of the excess returns.

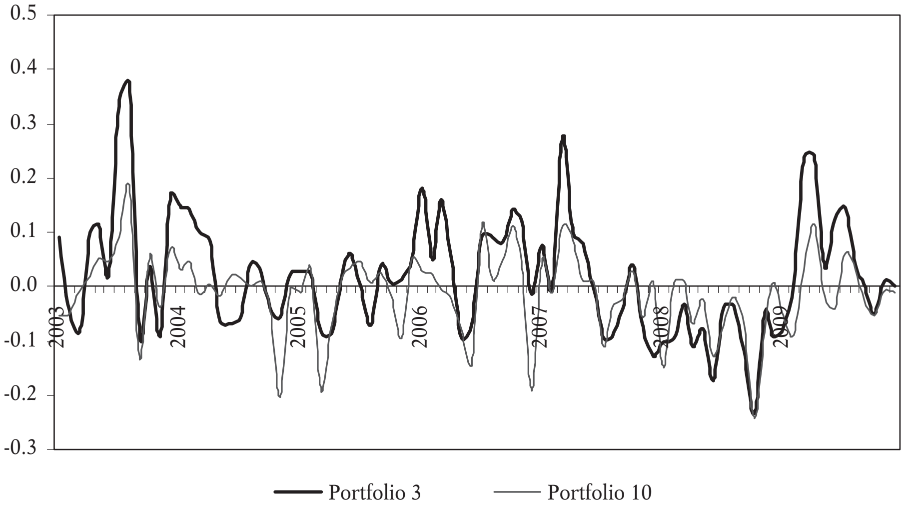



Figure 1. Monthly returns of the selected portfolios

The explanatory returns SMB, and HML are formed similarly to the 12 size-BE/ME relative portfolios. At the beginning of each month stocks are allocated to the two groups (S small or B big) based on the fact whether their market equity is below or above the median. Next, the stocks in each group are allocated in an independent sort to three book-to-market equity (BE/ME) groups (L low, M medium, or H high) based on the breakpoints for the bottom 30 percent, middle 40 percent, and top 30 percent of the values of BE/ME for WSE stocks. The SMB is the difference, each month, between the average of the returns on the three small-stock portfolios (S/L, S/M, and S/H) and the average of the returns on the three big-stock portfolios (B/L, B/M, and B/H). The HML is the difference between the average of the returns on the two high-BE/ME portfolios (S/H and B/H) and the average of the returns on the two low-BE/ME portfolios (S/L and B/L). The third factor ($R_M$) is the return of the market portfolio (WIG) in the excess of the risk-free rate. The time series of the factors are depicted in Figure 2. The sensitive similarity of the market and the HML factors can be observed.

Table 2

Summary statistics of the factors

| | $R_M$ | SMB | HML |
|---|---|---|---|
| Average | 0.005 | 0.007 | 0.016 |
| Standard deviation | 0.072 | 0.037 | 0.046 |

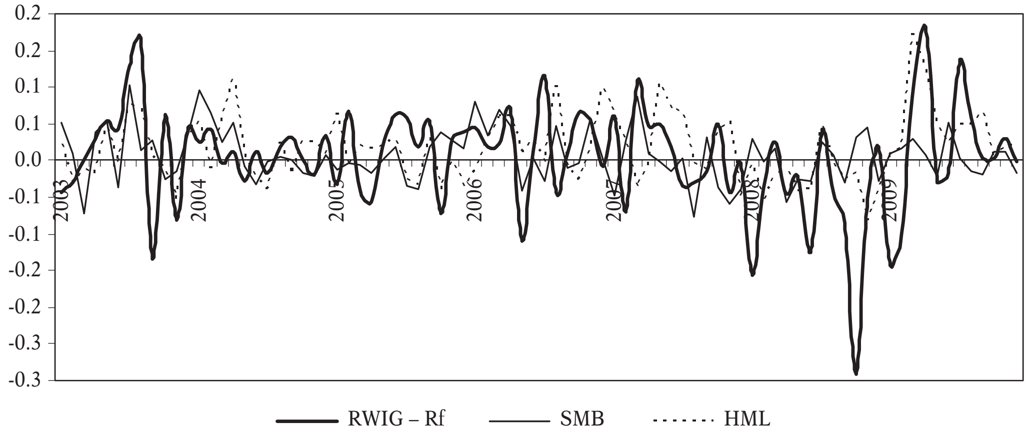



Figure 2. Time series of the factors

Table 2 reports the basic statistics of the factors. The mean of $R_M$ is statistically insignificantly positive ($|t| = 0.69$), whereas the means of the SMB and HML are significantly positive $|t_{\mathrm{SBM}}| = 1.65$ and $|t_{\mathrm{HML}}| = 3.25$. This means that small size portfolios with high BE/ME gain bigger profits. These calculations confirm the basic model assumption, that the rate of the return depends on the risk connected with the investments in stocks of small firms and in the stocks of firms with high ratio of book value to market value which are underpriced by market.

Table 3

The correlation between factors; the corresponding *p-values* are given in parentheses

| | $R_M$ | SMB | HML |
|---|---|---|---|
| $R_M$ | 1 | | |
| SMB | 0.136<br>(0.190) | 1 | |
| HML | 0.331<br>(0.000) | 0.192<br>(0.056) | 1 |

It is desirable that the factors are not correlated. Table 3 shows that the SMB and HML factors are uncorrelated, however, a small (0.331) but significant ($p = 0.000$) correlation between the $R_M$ and the HML factors can be noticed.

### 3.2. SYSTEMATIC RISK COMPONENTS

This section reports the estimators of the factor loadings calculated by regression (2) and the tests of the significance of the constant. These estimators are obtained using the method described in Section 2.1. The variances of unknown parameters are obtained from (4) and calculated with the different bandwidths. The point estimators of unknown parameters in (2) are independent on the bandwidths in contrary to the variances. However, only a small change of the significance can be noticed. Therefore, sole zero bandwidth results are reported in Table 4.

Table 4

Time Series Regression of Portfolios

| Regression: $R_{it} = \alpha_i + \beta_{R_M;i} R_{Mt} + \beta_{SMB;i} SMB_t + \beta_{HML;i} HML_t + \varepsilon_{it}$ | | | | | | |
|---|---|---|---|---|---|---|
| Size | Low | 2 | High | Low | 2 | High |
| | $\hat{\alpha}$ | | | $t(\hat{\alpha})$ | | |
| Small | –0.019 | –0.005 | 0.000 | –3.068 | –0.731 | 0.063 |
| 2 | –0.011 | –0.013 | –0.021 | –2.623 | –2.375 | –3.375 |
| 3 | –0.007 | –0.014 | –0.015 | –1.204 | –2.822 | –2.801 |
| Big | –0.011 | –0.020 | –0.009 | –2.379 | –3.303 | –2.692 |
| | $\hat{\beta}_{R_M}$ | | | $t\left(\hat{\beta}_{R_M}\right)$ | | |
| Small | 1.010 | 0.970 | 0.964 | 8.344 | 10.435 | 11.801 |
| 2 | 0.858 | 0.787 | 0.930 | 16.278 | 11.323 | 16.952 |
| 3 | 0.911 | 0.767 | 0.948 | 13.612 | 13.051 | 13.182 |
| Big | 0.858 | 1.069 | 0.968 | 13.561 | 16.795 | 14.150 |
| | $\hat{\beta}_{SMB}$ | | | $t\left(\hat{\beta}_{SMB}\right)$ | | |
| Small | 1.570 | 1.460 | 1.323 | 8.810 | 8.644 | 10.797 |
| 2 | 0.885 | 0.744 | 0.909 | 7.045 | 5.878 | 4.986 |
| 3 | 0.136 | 0.365 | 0.309 | 0.745 | 3.061 | 2.453 |
| Big | 0.115 | 0.170 | –0.079 | 1.295 | 0.847 | –0.901 |
| | $\hat{\beta}_{HML}$ | | | $t\left(\hat{\beta}_{HML}\right)$ | | |
| Small | –0.578 | 0.114 | 0.435 | –3.329 | 0.645 | 3.841 |
| 2 | –0.426 | 0.140 | 0.444 | –3.175 | 1.005 | 3.363 |
| 3 | –0.310 | 0.094 | 0.482 | –2.273 | 0.767 | 3.073 |
| Big | –0.311 | –0.127 | 0.288 | –2.510 | –0.894 | 3.178 |

The results of the estimation of the systematic risk vector are similar to Fama-French study of the American market in the years 1963-2001 (see [24]). We can notice that the HML loadings are significantly negative for the portfolios with low BE/ME values and increase monotonically in the significantly positive values for portfolios with high BE/ME values. The increase is independent on the size of the portfolios. These changes occur for all portfolios sorted by the firm size. The SMB loadings are significant for the small size portfolios and insignificant for big portfolios. We can also notice that this values monotonically decrease from portfolios with low BE/ME to portfolios with high BE/ME values. This regularity is noticed only for small portfolios. We can notice that the systematic risk connected with the market factor is independent on the method of constructing the portfolios. Hence, this factor shows a weak ability to explain the rate of returns.

Next, the tests whether the intercepts of equation (2) are jointly equal to zero, are carried out. The $\chi^2$ statistic (5) is calculated for this purpose. Table 5 presents results for the one factor model (CAPM) and FF three-factor model for the same sample period. One can observe that the Fama-French three-factor model with additional loadings, the SMB and HML factors, is consistently better than the CAPM.

Table 5

Intercept tests: GRS and $\chi^2$ and the corresponding *p-values*

| CAPM | | FF | |
|---|---|---|---|
| GRS | $\chi^2$ | GRS | $\chi^2$ |
| 3.38<br>(0.001) | 46.67<br>(0.000) | 2.27<br>(0.017) | 29.00<br>(0.004) |

(Assuming that $E(f_i \varepsilon_i) = 0$, the GRS test and $\chi^2$ test are asymptotically comparable.) The results indicate that FF model is better than the CAPM. However, one may argue that the three-factor model does not describe satisfactorily the rate of portfolios. Using the GRS test, we have obtained the GRS-statistic equal to 2.27 (for comparison, 2.91 were obtained for Fama-French 25 portfolios of US market [24]). This means that the Fama-French factors do not account for all factors describing the excess returns. The problem of the intercept test needs additional discussion which is out of scope of this study and will be the subject of the future work.

### 3.4. RISK PREMIUM COMPONENTS

This section presents the estimates of equation (1) obtained by the GMM method described in Section 2. The choice of the bandwidth is more critical than the choice of the weighting matrix [14]. Therefore, we estimate the model with three different bandwidths: zero lag, one lag and four lags. The latter is suggested by Hall [14] ($T^{1/3} \approx 4$)

because this minimizes the mean square error of the covariance estimator. We also set the bandwidth equal to one because the analysis of the series has shown that for most portfolios we can notice only one lad autocorrelation.

Table 6 presents the risk premium estimation and *t*-statistics for the twelve size-BE/ME portfolios in the FF three-factor model. The result of the CAPM approach are presented for the comparison purposes. It has been done only for GMM 0 lag.

Table 6

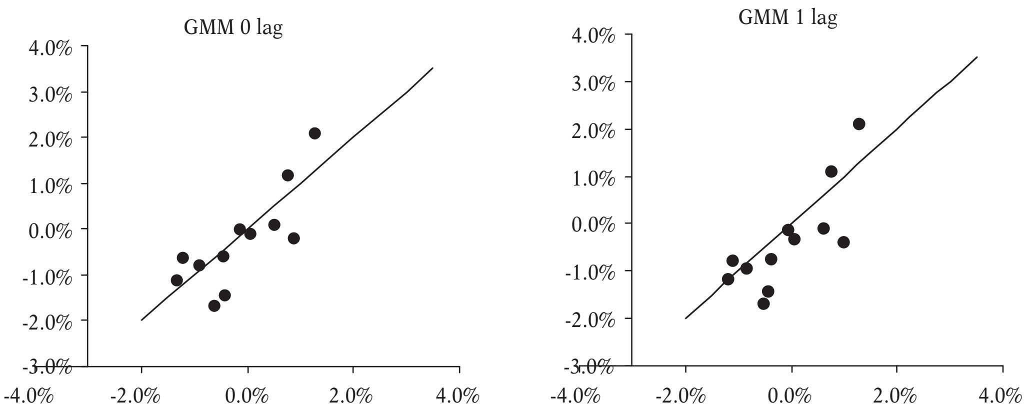
The risk premium estimation and *t*-statistics for portfolios based on ME and BE/ME criteria in the CAPM and FF three-factor model

| Method | kernel | $\hat{\gamma}_0$ | $t(\hat{\gamma}_0)$ | $\hat{\gamma}_{R_M}$ | $t(\hat{\gamma}_{R_M})$ | $\hat{\gamma}_{SMB}$ | $t(\hat{\gamma}_{SMB})$ | $\hat{\gamma}_{HML}$ | $t(\hat{\gamma}_{HML})$ |
|---|---|---|---|---|---|---|---|---|---|
| GMM 0 lag | | **–0.042** | –3.155 | **0.039** | 2.349 | | | | |
| | | –0.017 | –1.130 | 0.011 | 0.594 | **0.007** | 1.853 | **0.018** | 3.421 |
| GMM 1 lags | Parzen | –0.019 | –1.267 | 0.014 | 0.781 | **0.007** | 1.805 | **0.019** | 3.858 |
| | Bartlett | –0.020 | –1.364 | 0.019 | 1.035 | **0.007** | 1.810 | **0.021** | 4.228 |
| GMM 4 lags | Parzen | –0.012 | –0.839 | 0.014 | 0.783 | **0.009** | 2.369 | **0.022** | 4.441 |
| | Bartlett | –0.007 | –0.456 | 0.009 | 0.482 | **0.010** | 2.490 | **0.022** | 4.367 |

We have obtained significantly positive risk premium for SMB and HML factors. The risk premium connected with HML factor equals 2% and is higher than estimated risk premium connected with the firm size factor. More attention should be paid to the fact that the methods proved the insignificance of the constant. This seems to contradict with significantly different from zero the overall value of constants estimated in the first step. This problem requires further investigations. Moreover, some positive although insignificant value of the market risk factor can be noticed. This remarks implies that market beta has little or no ability in explaining the cross-sectional mean returns and that firm size (SMB) and book-to-market equity (HML) effects seem to describe it in a meaningful manner.

To evaluate the goodness of fit of this models, the cross-sectional $R^2$ measure employed by Jagannathan and Wang [17] and Lettau and Ludvigson [20] is used. The $R^2$ measures the fraction of cross-sectional variation in average returns that is explained by the model. In the discussed case, $R^2 = 65.1\%$ (this is close to the result ($R^2 = 71\%$) obtained for the US data [24]).

Figure 3 presents adjustment between the observed means of monthly returns (*y*-axis) and the expected monthly returns (*x*-axis) predicted by the model.




a b

Figure 3. Average Monthly Returns ($y$-axis) × Predicted Monthly Returns ($x$-axis)

## 4. CONCLUDING REMARKS

The empirical study was carried out on data from December 2002 to January 2010. To account for heteroscedasticity and autocorrelation the unknown parameters of model were estimated using the GMM method. The results obtained from the study confirm the hypothesis that the Fama-French SMB and HML factors quite satisfactorily describe changes of portfolios returns in the sample period. The rate of return depends on the risk connected with the investments in stocks of small firms and in stocks of firms underpriced by the market with high BE/ME ratio. The risk premiums are significant in the case of the SMB and HML factors. The GMM point estimator indicates a premium close to 2% in the case of the HML and close to 0.7% in the case of the SMB. The risk premium connected with the market factor seems insignificant, however it cannot be excluded. The goodness of fit of the three-factor model evaluated by the cross-sectional $R^2$ measure is greater than sixty five percent.

Presented results should be considered as an introduction to a wider study. Investigations on whether portfolio constructions have an influence on the estimation results and the discussion on the choice of proper statistical tools will be the subject of the future work.

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REFERENCES

[1] Andrews D.W.K., [1991], *Heteroscedasticity and Autocorrelation Consistent Covariance Matrix Estimation*, „Econometrica" 59, 817-858.
[2] Banz R.W., [1981], *The Relationship Between Return and the Market Value of Common Stocks*, „Journal of Financial and Quantitative Analysis", 14, 421-441.
[3] Basu S., [1977], *Investment Performance of Common Stocks in Relation to their Price-Earnings Ratios: A Test of the Efficient Market Hypothesis*, „The Journal of Finance" 32, 663-682.
[4] Black F., Jensen M., Scholes M., [1972], *The Capital Asset Pricing Model: Some Empirical Tests*, in Studies in the Theory of Capital Markets, Michael C. Jensen, ed. New York: Praeger, pp. 79-121.
[5] Blume M.E., Friend I., [1975], *The Asset Structure of Individual Portfolios with Some Implications for Utility Functions*, „Journal of Finance" 30, pp. 585-604.
[6] Blume M.E., Stambaugh R.F., [1983], *Biases in Computed Returns: An Application to the Size Effect*, „Journal of Financial Economics" 12, 387-404.
[7] Bołt T.W., Miłobędzki P., [2002], *Weryfikacja modelu CAPM dla giełdy warszawskiej*, Prace Naukowe Akademii Ekonomicznej im. Oskara Langego we Wrocławiu, Nr 952, 89-95.
[8] Fama E.F., MacBeth J., [1973], Risk, Return and Equilibrium: Empirical Tests, „Journal of Political Economy" 81, 607-636.
[9] Fama E.F., French K.R., [1992], *The Cross-Section of Expected Stock Returns*, „Journal of Finance" 47(2), 427-465.
[10] Fama E.F., French K.R., [1993], *Common Risk Factors in the Returns on Stocks and Bonds*, „Journal of Financial Economics" 33, 3-56.
[11] Fama E.F., French K.R., [1996], *Multifactor Explanations of Asset Pricing Anomalies*, „Journal of Finance" 51(1), 55-84.
[12] Fiszeder P., [2009], *Modele klasy GARCH w empirycznych badaniach finansowych*, Wydawnictwo Naukowe Uniwersytety Mikołaja Kopernika, Toruń.
[13] Gibbons M.R., Ross S.A., Shanken J., [1989], *A Test of the Efficiency of a Given Portfolio*, „Econometrica" 57(5), 1121-1152.
[14] Hall A.R., [2005], *Generalized Method of Moments*, Oxford University Press, NewYork.
[15] Hansen L.P., [1982], *Large Sample Properties of Generalized Method of Moments Estimators*, „Econometrica" 50(4), 1029-1054.
[16] Harvey C., Kirby C., [1995], *Analytic Tests of Factor Pricing Models*, Working Paper (Duke University, Durham).
[17] Jagannathan R., Wang Z., [1998], *An asymptotic theory for estimating beta-pricing models using cross-sectional regression*, „Journal of Finance" 57, 2337-2367.
[18] Kowerski M., [????], *Trójczynnikowy model Famy i Frencha dla Giełdy Papierów Wartościowych w Warszawie*, „Przegląd Statystyczny" 55(4), 131-148.
[19] Lakonishok J., Shapiro A.C., [1983], *Systematic risk, total risk and the size as determinants of stock market returns*, „Journal of Banking and Finance" 10, 115-132.
[20] Lettau M., Ludvigsson S., [2001], *Consumption, aggregate wealth, and expected stock returns*, „Journal of Finance" 56, 815-850.
[21] Lintner J., [1965], *The Valuation of Risky Assets and the Selection of Risky Investmentin Stock Portfolios and Capital Budgets*, „Review of Economics and Statistics" 47, 13-37.
[22] MacKinlay A.C., Richardson M.P., [1991], *Using Generalized Method of Moments to Test Mean-Variance Efficiency*, „Journal of Finance", 46(2), 511-27.
[23] Ogaki M., [1993], *Generalized Method of Moments: Econometric Applications*, Handbook of Statistics 11, 455-485.
[24] Petkova R., [2006], *Do the Fama–French Factors Proxy for Innovations in Predictive Variables?*, The Journal of Finance, 2, 581-612.
[25] Reinganum M.R., [1981], *Misspecification of Capital Asset Pricing: Empirical Anomalies Based on Earnings' Yields and Market Values*, „Journal of Financial Economics" 9, 19-46.
[26] Rosenberg B., Reid K., Lanstein R., [1985], *Persuasive Evidence of Market Inefficiency*, „The Journal of Portfolio Management" 13, 9-17.

[27] Shanken J., Zhou G., [2007], *Estimating and Testing Beta Pricing Models: Alternative Methods and their Performance in Simulations*, „Journal of Financial Economics" 84, 40-86.
[28] Sharpe W.F., [1964], *Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk*, „Journal of Finance" 19, 425-442.
[29] Schmidt P., [1976], *Econometrics*, Marcel Dekker, New York.
[30] Urbański S., [2007], *Time-Cross-Section Factors of Rates of Return Changes on Warsaw Stock Exchange*, „Przegląd Statystyczny" 54(2), 94-121.
[31] Zarzecki D., [1999], Metody wyceny przedsiębiorstw. Zarys teorii a praktyka, Eds.: Accountancy Development Foundation in Poland, Warsaw, ISBN: 83-7241-099-2.
[32] Zellner A., [1962], *An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias*, „Journal of the American Statistical Association" 57, 348-368.

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## MODEL CAPM ORAZ MODEL FAMY I FRENCHA NA WARSZAWSKIEJ GIEŁDZIE PAPIERÓW WARTOŚCIOWYCH

### Streszczenie

Tematem prezentowanej pracy jest weryfikacja trójczynnikowego modelu Famy Frencza dla danych z Warszawskiej Giełdy Papierów Wartościowych. Okres badania obejmuje lata 2002-2010. Do estymacji nieznanych parametrów modelu zastosowano uogólnioną metodę momentów (GMM), Przyjęto założenie istnienia heteroskedastyczności i autokorelacji szeregów czasowych biorących udział w badaniu. Ponadto dopuszczono możliwość istnienia korelacji czynników objaśniających z błędami losowymi występującymi w modelu regresji. Uzyskane wyniki potwierdziły tezę, że trójczynnikowy model Famy Frencza zadowalająco opisuje zmiany stóp zwrotu na rynku polskim w badanym okresie. Wynik tego badania należy jednak traktować jako wstęp do bardziej wnikliwych analiz.

**Słowa kluczowe:** trójczynnikowy model Famy-Frencha, Uogólniona Metoda Momentów, ryzyko systematyczne, premia za ryzyko

## THE CAPM AND FAMA-FRENCH MODELS IN POLAND

### Summary

The main objective of this paper is to verify the performance of the Fama-French model for the Polish market. The estimates for individual stock returns are obtained using the monthly data from the Warsaw Stock Exchange for the period December 2002 to January 2010. The Generalized Method of Moments is used to test hypotheses that lead to the validation of the Fama-French model. We find that the cross-sectional mean returns are explained by exposures to the three factors, and not by the market factor alone. These results are consistent with previous studies of developed markets.

**Key words:** Fama–French three-factor model, Generalized Method of Moments, risk premium