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Special Relativity: from metric to philosophy

Abstract
The Theory of Relativity from Einstein starts from some everyday system of Cartesian space-coordinates plus a linear time-parameter, read from a good old grand-father's clock. The theory then contemplates a group of certain homogeneous, linear transformations with constant coefficients, transformations which involve all four coordinates and which there is reason to interpret as going over the another inertial systems, that moves with constant translational velocity with respect to the first.
In fact the very backbone of the theory is that all laws of nature shall be the same for every frame reached in this way, including the original one from which we started, there should be no difference or distinction in principle between all these inertial frames, any one of which can be reached from any other one by a transformation of that group a so called Lorentz-transformation. In short, all laws of nature are assumed to be invariant to Lorentz-transformation.

Key words: **Special relativity, nature laws, Einstein, and Lorentz, and philosophical foundations.**

INTRODUCTION

The transformation in physical concepts, which was brought about by the theory of relativity, had been in preparation for a long time. As long ago as 1887, in a paper still written from the point of view of the elastic-solid theory of light, Voigt mentioned that it was mathematically convenient to introduce a local time t' into a moving reference system. The origin of t' was taken to be a linear function of the space coordinates, while the time scale was assumed to be unchanged.

Essentially physical results were now obtained, in addition to the purely formal recognition that it was mathematically convenient to introduce a local time t' in a moving coordinate system. It was shown that all experimentally observed effects of first order in v/c (ratio of the translational velocity of the medium to the velocity of light) could be explained quantitatively by the theory when the motion of the electrons embedded in the aether was taken into account.[1]

The theory gave an explanation for the fact that a common velocity of medium and observer relative to the aether has no influence on the phenomena, as far as quantities of first order are concerned. However, the negative result of Michelson's interferometer experiment, concerned as it was with an effect of second order in v/c, created great difficulties for the theory. To remove these, Lorentz and, in dependently, Fitzgerald put forward the hypothesis that all bodies change their dimensions when moving with a translational velocity v. This change of dimension would be governed by a factor $k \cdot \sqrt{\left(1 - v^2/c^2\right)}$ in the direction of motion, with k as the corresponding factor for the transverse direction; k itself remains undetermined. Lorentz justified this hypothesis by pointing out that the molecular forces might well be changed by the translational motion. He added to this the assumption that the molecular rest in a position of equilibrium and that their interaction is purely electrostatic. It would then follow from the theory that a state of equilibrium exists in the moving system, provided all dimensions in the direction of motion are shortened by a factor $\sqrt{1 - v^2/c^2}$, with the transverse dimensions unaltered.

It now remained to incorporate this Lorentz contraction in the theory, as well as to interpret the other experiments which had not succeded in showing the influence of the earth's motion on the phenomena in question. There was first of all Lamor who, as early as 1900, set up the formulae now generally known as the Lorentz transformation, and who thus considered a change also in the time scale.

Lorentz's review article, completed towards the end of 1903, contained several brief allusions which later proved very fruitful. Meanwhile, he conjectured that if the idea of a variable electromagnetic mass was extended to all ponderable matter, the theory could account for the fact that the translational motion would produce only the abovementioned contraction, and no other effects, even in the presence of molecular motion. In addition, he raised the important question whether the size of the electrons might be changed by the motion.[2]

However, in the introduction to his article, Lorentz still maintained the principle that the phenomena depended not only on the relative motion of the bodies, but also on the motion of the aether. We now come to the discussion of the three contributions, by Lorentz, Poincaré and Einstein, which contain the line of reasoning and the developments that form the basis of the theory of relativity. Chronologically, Lorentz's paper came first.[3]

He proved, above all, that Maxwell's equations are invariant under the coordinate transformation

1 Cf. K. O. FRIEDRICHS – *From Pythagoras to Einstein*, Singer Company, New York, 1965, 31-36.
2 Cf. W. H. McCREA – *Relativity Physics*, Wiley and Sons, Inc., London, 1954, 8-10.
3 Cf. A. EINSTEIN – *The Meaning of Relativity*, Chapman and Hall, London, 1973, 29-31.

$$x' = k \cdot \frac{x - vt}{\sqrt{(1-\beta^2)}}; \qquad y' = k \cdot y;$$

$$z' = k \cdot z; \qquad t' = k \cdot \frac{t - \left(v/c^2\right)x}{\sqrt{\left(1-\beta^2\right)}}; \qquad (\beta = v/c);$$

provide the field intensities in the primed system are suitably chosen. This, however, he proved rigorously only for Maxwell's equations in charge free space. The terms which contain the charge density and current are, in Lorentz's treatment, not the same in the primed and the moving systems, because he did not transform these quantities quite correctly.

He therefore regarded the two systems as not completely, but only very approximately, equivalent.

By assuming that the electrons too, could be deformed by the translational motion and that all masses and forces have the same dependence on the velocity as purely electromagnetic masses and forces, Lorentz was able to derive the existence of a contraction affecting all bodies. He could also explain why all experiments known had falled to show any influence of the earth's motion on optical phenomena: a less immediate consequence of his theory is that one has to put $k = 1$. This means that the transverse dimensions remain unchanged during the motion, if indeed this explanation is at all possible. We would like to stress that even in this paper the relativity principle was not al all apparent to Lorentz. And in contrast to Einstein, he tried to understand the contraction in a causal way.[4]

The formal gaps left by Lorentz's work were filled by Poincaré. He stated the relativity principle to be generally and rigorously valid. Since he, in common wish the previously discussed authors, assumed Maxwell's equations to hold for the vacuum, this amounted to the requirement that all laws of nature must be covariant with respect to the Lorentz transformation. The invariance of the transverse dimensions during the motion is derived in a natural way from the postulate that the transformations which effect the transition from a stationary to a uniformly moving system must form a group which contains as a subgroup the ordinary displacements of the coordinate system.[5]

Poincaré further corrected Lorentz's formula for the transformations of charge density and current and so derived the complete covariance of the field equations of electron theory. It was Einstein, finally, who in a way completed the basic formulation of the special relativity. His paper of 1905 was submitted at almost the same time as Poincaré's article and had been written without previous knowledge of Lorentz's paper of 1904. And, finally, I make up the philosophical formulations of Special Relativity.[6]

4 Cf. *Ibidem*, 31-34.
5 Cf. *Ibidem*, 27-29.
6 Cf. S. J. PROKHOVNIK – *The Logic of Special Relativity*, At the University Press, Cambridge, 1967, 1-11.

THE POSTULATES OF SPECIAL RELATIVITY: FROM METRIC TO PHILOSOPHY

The failure of the many attempts to measure terrestrially any effects of the earth's motion on physical phenomena allows us to come to the highly probable, if not certain, conclusion that the phenomena in a given reference system are, in principle, independent of the translational motion of the system as a whole. To put it more precisely there exists a triply infinite set of reference systems moving rectilinearly and uniformly relative to one another, in which the phenomena occur in an identical manner. We shall follow Einstein in calling them Galilean reference systems so named because the Galilean law of inertia holds in them.[7]

It is unsatisfactory that one can not regard all systems as completely equivalent or at least give a logical reason for selecting a particular set of them.

Once the postulate of relativity is stated, the concept of the aether as a substance is thereby removed from the physical theories. For there is no point in discussing a state of rest or of motion relative to the aether when these quantities can not be observed experimentally. Nowadays this is all the less surprising as attempts to derive the elastic properties of matter from electrical forces are beginning to show success. It would, therefore, be quite inconsistent to try, in turn, to explain electromagnetic phenomena in terms of the elastic properties of some hypothetical medium. Actually, the mechanistic concept of an aether had already come to be superfluous and something of a hindrance when the elastic-solid theory of light was superseded by the electromagnetic theory of light. In this letter the aether substance had always remained a foreign element. Einstein has recently suggested an extension of the notion of an aether.[8]

It should no longer be regarded as a substance but simply as the totality of those physical quantities which are to be associated with matterfree space.

In this wider sense there does, of course, exist an aether; only one has to bear in mind that it does not possess any mechanical properties. In other words, the physical quantities of matterfree space have no space coordinates or velocities associated with them.[9] It might seem that the postulate of relativity is immediately obvious, once the concept of an aether has been abandoned. Naturally, we cannot subject the whole universe to a translational motion and then investigate whether the phenomena are thereby altered. Our statement will, therefore, only be of heuristic value and physically meaning feel when we regard it as valid for any and every closed system. But when is a system a closed system? Would it be sufficient to stipulate that all masses should be far enough removed? Experience tells us that this is sufficient for uniform motion, but not for a more general motion. The postulate of relativity implies that a uniform motion of the centre of mass of the

7 Cf. *Ibidem*, 27-51.
8 Cf. R. B. LINDSAY; H. M. ARGENAU – *Foundations of Physics*, Dover Publications, New York, 1957, 330-350.
9 Cf. P. T. LANDSBERG – *The Enigma of Time*, Adam Hilger Ltd., Bristol, 1982, 37-39.

universe relative to a closed system will be without influence on the phenomena in such a system.

The relativity theory obeys to these postulates, and there are now forever the opportunity to formulate the generalization from Newtonian mechanic and to define where is the extension and unification of all classical theories of Physics.[10]

With the Special Theory of Relativity we have a new workshop to write the equations of classical theories, and carries out one bridge to the Quantum Mechanics. The two relativistic postulates are the basis of the new mathematical equations adaptations from the classical to the new physics. Although we will see that he also helped to initiate the development of Quantum Mechanics, it evolved in a way that he found unsatisfying. Modern quantum mechanics holds that only the relative likelihood of various outcomes from an experiment may be predicted and not the result of a specific measurement.

Einstein played an important role as a consultant and sympathetic protagonist in the development of quantum mechanics, in his later years Einstein's research concentrated on attempts at a unified theory of gravitation and electromagnetism.

His philosophy and work took him out of the mainstream of physics, although his only contributions continued to play an important role in the theory of modern physics. Einstein's reputation was greater and more lasting than that of any other scientist of this century.[11]

According to the Special Theory of Relativity, the invariance of c is put forward before any procedure for synchronizing clocks, i.e., for defining coordinate-time, it proposed. Further more only in the light of the postulated invariance of c, does the convention of clock-synchronism make sense, i.e., prove to be convenient. The following except demonstrates that theoretical considerations – totally unrelated to sense – experience preceded and founded Einstein's operational definition of distant simultaneity. Note that the facts adduced by Einstein – which might incidentally include the Michelson-Morley results – do not involve any distant simultaneities but only round-trip velocities, so no convention about clock-synchronisation need he invoked. As for reducing the General Theory of Relativity to a system of relations between sense-data, it was neither undertaken nor even seriously envisaged by Einstein, for he admitted that the connection between the coordinates and the results of measurement had been made problematic by the new theory.[12]

It was really Hume who convinced Einstein that no causal laws could be directly induced from the facts, whether the latter he presumed to be objective or purely perceptual causal connection do not inhere in the phenomena as they present

10 Cf. C. M. WILL – *Theory and Experiment in Gravitational Physics*, University Press, Cambridge, 1993, 15-35.
11 Cf. R. G. NEWTON – *Thinking about Physics*, Princeton University Press, Oxford, 4, 13, 28, 29, 32, 34, 37, 59-60, 136, 167.
12 Cf. P. SUPPES (edited) – *Space, time and geometry*, D. Reidel Publishing Company, Dordrecht, 1973, 178-180.

themselves to us. They are added by the mind's own operation to the results of observation.

Meanwhile, we find in Einstein's thinking, on the one hand, an insistence on the epistemological primacy of concepts directly connected with sense-experience; and on the other, the recognition that science operates with notions which, though ultimately linked in sense-data, remain logically independent of the latter.[13]

Einstein was not driven to scepticism of the inductive method. For a time, he was drawn to Kant's theory that the possibility of science to guaranted by the so called synthetic *a priori* principles: these are imposed, with absolute necessity, by the mind on the material provided by the senses.[14]

The way in which this revolutionary modification had to be carried out was, however, determined, not by any phenomenalist analysis of experience, but by examining the transformational properties of Maxwell's equation, and hence the possibility of turning c into an invariant.

THE AXIOMATIC NATURE OF LORENTZ TRANSFORMATION: METRIC AND PHILOSOPHY

At first sight it appears as if the two postulates were incompatible. For, let us take a light source L with moves relative to an observer A with velocity v and consider a second observer B at rest with respect to L. Both observers must then see as wave fronts spheres whose centres are at rest relative to A and B, respectively. In other words, they see different spheres. This contradiction disappears, however, if one admits that space points which are reached by the light simultaneously for A, are not reached simultaneously for B.

This brings us directly to the relativity of simultaneity.[15]

Here, it will first of all be necessary to say what is meant by the synchronization of two clocks at different places. The following definition was chosen by Einstein.

A light ray is emitted from point P at time t_p, is reflected at Q at time t_Q, and returns to P at time t'_p. The clock at Q is then considered synchronized with that at P if $t_Q = (t_p + t_p)/2$ Einstein uses light for regulating the clocks because the two postulates enable us to make definite statements about the mode of propagation of the light signals. Naturally, one could think of other ways of comparing the clocks, such as transporting them, or using mechanical or elastic couplings, etc. Only it must be stipulated that no such method should lead to a contradiction with the optical regulation method.

13 Cf. W. B. BONOR *et alii* – *Classical General Relativity*, University Press, Cambridge, 121-128.
14 Cf. J. M. JAUCH – *Foundations of Quantum Mechanics*, Addison-Wesley Publishing Company, London, 1977, 67-110.
15 Cf. H. A. LORENTZ; A. EINSTEIN ; H. MINKOWSKI – *O Princípio da Relatividade*, Volume I, tradução do alemão, Fundação Calouste Gulbenkian, Lisboa, 1972, 49-55.

We can now derived the transformation formulas which connected the coordinates $(x, y, z, t, and\ x', y', z', t')$ of two reference systems k and k' in uniform relative motion. The x-axis is chosen to lie along the direction of motion in such a way that k' moves relative to k with velocity v in the positive x-direction. All writers start with the requirement that the transformation formulae should be linear. This can be justified by the statements that a uniform rectilinear motion in k must also be uniform and rectilinear in k'. Furthermore it is to be taken for granted that finite coordinates in k remain finite in k'. This also implies the validity of Euclidean geometry and the homogeneous nature of space and time.

It follows from the two postulates that the equation:

$$x^2 + y^2 + z^2 - c^2 t^2 = 0$$

entails the corresponding equation:

$$x'^2 + y'^2 + z'^2 - c^2 t^2 = 0 .$$[16]

Since the transformation is to be a linear one, this is only possible if:

$$x'^2 + y'^2 + z'^2 - c^2 t^2 = k\left(x^2 + y^2 + z^2 - c^2 t^2\right)$$

where k is a constant depending on v. If one also bears in mind that any motion parallel to the x-axis must remain so after the transformation formulae will be seen to follow immediately. It is, however, still necessary to show that k can be put equal to 1. Einstein's procedure consists in applying transformation once more, with the velocity in the opposite direction:

$$x^{ij} = k(-v) \cdot \frac{x' + vt'}{\sqrt{(1-\beta^2)}} ;$$

$$y^{ij} = k(-v) y' ; \qquad z^{ij} = k(-v) z' ;$$

$$t^{ij} = k(-v) \frac{t' + (v / c^2) x'}{\sqrt{(1-\beta^2)}} .$$

Then:

$$x^{ij} = k(v) \cdot k(-v) x ;$$

$$y^{ij} = k(v) \cdot k(-v) y ;$$

$$z^{ij} = k(v) \cdot k(-v) z ;$$

$$t^{ij} = k(v) \cdot k(-v) t .$$

Since k^{ij} is at rest relative to k, it must be identical with it. Hence,

$$k(v) \cdot k(-v) = 1 .$$

16 Cf. A. EINSTEIN – *The Meaning of Relativity*, Chapman and Hall, London, 1973, 26-30.

The above relation then gives $k(v)=1$, since k must be positive. Poincaré arrives at this conclusion in a similar way. He considers the totality of all linear transformations which transform:

$$x^2+y^2+z^2-c^2t^2=0$$

into itself and demands that it should contain as subgroups:

- The one-parameter group of translations parallel to the x-axis;
- The ordinary displacements of the coordinate axes;
- Once again $k=1$ follows, since Einstein's symmetry requirement $k(v)=k(-v)$.

We have thus obtained the definite result that:

$$x'=\frac{x-vt}{\sqrt{(1-\beta^2)}}\,;\qquad y'=y\,;\qquad z'=z\,;$$

$$t'=\frac{t-\left(v/c^2\right)x}{\sqrt{\left(1-\beta^2\right)}}\,.$$

The simple structure of the anterior formulae makes one wonder. Whether they could not have been derived from general group-theoretical considerations, without having to assume the invariance of the last equations.[17]

From the group-theoretical assumption it is only possible to derive the general form of the transformation formulae, but not their physical content. Incidentally, it is to be noted that contains the transformation formulae of ordinary mechanics:

$$x'=x-vt\,;\qquad t'=t$$

which can be obtained by putting $\alpha=0$.

The rod is, therefore, contracted in the ratio $\sqrt{1-\beta^2}:1$, as was already assumed by Lorentz. Since the transverse dimensions of a body remain unaltered, the same formula applies to the contraction of its volume:

$$V=V_0\sqrt{\left(1-\beta^2\right)}.$$

We have seen that this contraction is connected with the relativity of simultaneity, and for this reason the argument has been put forward that it is only an apparent contraction, in other words, that it is only simulated by our space-time measurements.[18]

If a stable is called real only when it can be determined in the same way in all Galilean reference systems, then the Lorentz contraction is indeed only apparent, since an observer at rest in k' will see the rod without contraction.

17 Cf. *Ibidem*, 28-29.
18 Cf. D. TER HAAR – *Elements of Hamiltonian Mechanics*, second edition, Pergamon Press, Oxford, 1971, 1-21.

But we do not consider such a point of view as appropriate, and in any case the Lorentz contraction is in principle observable. It shows that the determination of the simultaneity of spatially separated events, which is necessary for the observation of the Lorentz contraction, can be carried out entirely with the help of measuring rods, without the use of clocks.

For let us think of using two rods A_1B_1 and A_2B_2 of the same rest length l_0 which move relative to E with equal and opposite velocity v. We mark the points A^* and B^* in k at which the points A_1 and A_2 and B_1 and B_2, respectively, overlap. The distance A^*B^*, as measured by rode at rest in k, will then have the value:

$$l = l_0\sqrt{\left(1-\beta^2\right)}.$$

It, therefore, follows that the Lorentz contraction is not a property of a single measuring rod taken by itself, but is a reciprocal relation between single measuring rod taken by itself, but is a reciprocal relation between two rods moving relatively to each other, and this relation is in principle observable.[19]

Analogously, the time scale is changed by the motion. Let us again consider a clock which is at rest in k'. The time t' which it indicates in k' is its proper time τ and we can put the coordinate x' equal to zero. It then follows that:

$$t = \frac{\tau}{\sqrt{\left(1-\beta^2\right)}}; \qquad \tau = \sqrt{\left(1-\beta^2\right)}t$$

Measured in the time scale of k, therefore, a clock moving with velocity v will lag behind one at rest in k in the ratio:

$$\sqrt{\left(1-\beta^2\right)} : 1.$$

While this consequence of the Lorentz transformation was already implicitly contained in Lorentz's and Poincaré's results; it received its first clear statement only by Einstein.[20] It is obvious that experiments which are intended to show the effect of the motion of the coordinate systems as a whole on phenomena within it must, according to the theory of relativity, show a negative result. It is nevertheless instructive to investigate how such experiments are seen from a system k which is at rest. For this purpose we shall discuss the Michelson interferometer experiment. Let l_1 be the length, measured in K_1 of the interferometer arm parallel to the direction of motion, and l_2 that of the arm at right angles to it. The time t_2 taken by the hight to traverse the arms are then given by:

$$ct_1 = \frac{2l}{1-\beta^2}; \qquad ct_2 = \frac{2l}{\sqrt{(1-\beta^2)}}$$

19 Cf. M. A. TONNELAT – *Les Principes de la Théorie Electromagnétique et de la Relativité*, Masson et Cie, Paris, 1959, 107-109.
20 Cf. V. FOCK – *The Theory of Space, Time and Gravitation*, Pergamon Press, London, 1959, 12-16.

Now, because of the Lorentz contraction, we have:

$$l_z = l_0 \cdot \sqrt{\left(1-\beta^2\right)}; \qquad t_2 = t_0;$$

so that

$$ct_1 = ct_2 = \frac{2l_p}{\sqrt{(1-\beta^2)}}$$

It would therefore seem that an observer travelling with k' measures a velocity of light:

$$c' = c\sqrt{\left(1-\beta^2\right)}$$

different from that measured by an observer in k. This is the point of view put forward by Abraham. According to Einstein, however, the time-dilatation:

$$t' = t\sqrt{\left(1-\beta^2\right)}$$

has still to be taken into account, so that:

$$ct_1' = ct_2' = 2l_0,$$

and thus velocity of light is the same in k' as in k. According to Abraham there is to time dilatation. Abraham's point of view is consistent with Michelson's experiment, but it contradicts the postulate of relativity, since it would in principle admit of experiments which would allow one to measure the absolute motion of a system.[21]

Let us discuss the difference between Einstein's and Lorentz's points of view still further. Einstein showed in particular that the distinction between local and true time disappears with a more profound formation of the concept pf time. Lorentz's local time is shown to be simply the time in the moving system k'.

There are as many times and spaces as there are Galilean reference systems. It is also of great value that Einstein rendered the theory independent of any special assumptions about the constitution of matter.

The epistemological basis of the theory of relativity has recently been undergoing a close examination from the side of philosophy. In this connection the opinion has been expressed that the theory of relativity has thrown overboard the concept of causality. We take the view that it is perfectly satisfactory from the standpoint of the theory of knowledge in to say that the relative motion is the cause of the contraction, since this latter is not the property of a single measuring rod, but a relation between two such rods. Also it is unnecessary to refer, as Holst dues, to all matter present in the universe, in order to satisfy the causality condition.[22]

21 Cf. R. C. TOLMAN – *Relativity, Thermodynamics and Cosmology*, At the Clarendon Press, Oxford, 32-35.
22 Cf. P. C. W. DAVIES – *Space and Time in the modern Universe*, University Press, Cambridge, 1977, 29-56.

THE FOUR-DIMENSIONAL SPACE-TIME WORLD: METRIC AND PHILOSOPHY

The two postulates of relativity and of the constancy of the velocity of light can be combined into the single requirement that all physical laws should be invariant under the Lorentz transformation. From now on we shall take to mean by the Lorentz transformation the totality of all linear transformation which satisfy the identity. Each such transformation can be made up of rotation of the coordinate system and the special Lorentz transformation of the type. Mathematically speaking, therefore, the special theory of relativity is the theory of invariants of the Lorentz group.

The work of Minkowsky has been fundamental for the development of the theory. He managed to give the theory an extraordinarily elegant form by making consistent use of two facts. If instead of the ordinary time t, the imaginary quantity $u = ict$ is introduced, the behaviour of the space and time-coordinates. It is completely equivalent in the Lorentz group, and thus also in the physical laws which are invariant with respect to this group. In fact, the invariant which is characteristic for the Lorentz transformation:

$$x^2 + y^2 + z^2 - c^2t^2$$

goes over into:

$$x^2 + y^2 + z^2 + u^2$$

It is therefore expedient from the legitimity not be separete space and time, but to consider the four-dimensional space-time manifold.

We shall follow Minkowski by calling it, in short, "world".[23]

Since expression is invariant under the Lorentz transformation and is also quadratic in the coordinates, it would seem natural to define it as the square of the distance of the world point $P(x, y, z, u)$ from the origin, in analogy to the corresponding square of the distance:

$$x^2 + y^2 + z^2$$

in ordinary space. With this a world geometry (metric) is determined which is closely related to Euclidean geometry.

The two geometries are not completely identical because of the imaginary character of one of the coordinates. The latter property implies, for instances, that two world points whose distance from each other is zero do not necessarily coincide, such matter. Notwithstanding these geometrical differences, we can regard the Lorentz transformations as orthogonal linear transformations of the world coordinates and as rotations of the world coordinate exes, in analogy with the rotations of coordinate system in R_s. Moreover, just as the ordinary vector and tensor cal-

23 Cf. J. L. SYNGE – *Relativity: the special theory*, North-Holland Publishing Company, Amsterdam, 1958, 56-57.

culus can be looked upon as an invariant theory of the orthogonal linear coordinate transformations in R_s, so the invariant theory of the Lorentz group takes the form of a four-dimensional vector and tensor calculus.[24] Therefore, we can express the second aspect which is essential to Minkowski's representation of the theory, in the following way: because the Lorentz group leaves a quadratic form of the four world coordinates invariant, the invariant theory of this group can be represented geometrically and it then appears as a natural generalization of the ordinary vector and tensor calculus for a four-dimensional manifold. But the linear transformations of special relativity involve all four-coordinates x_1, x_2, x_3, x_4 , you can identify the same world-point after the transformation, but there is no good meaning in speaking of the same point in space after the transformation unless you also refer to the moment of time in which it is contemplated neither is there a meaning in speaking of the same moment of time after the transformation without reference to the point in space where it is contemplated.

What in one frame is the same point in space, envisaged at different moments of time, well in general turn out to be two different points in space in the other form envisaged at two different moments.[25]

Again, what in one frame is the same moment at two different points, will in general be mapped in the other frame as different moments referring to different points in space. It is this state of affairs which has given birth to all the much discussed paradoxes in the Special Theory of Relativity so difficult to explain to the non-mathematician, while the mathematician is prepared to encounter some clashes with customary views from the mere fact that all four-coordinates are involved in the transformation.

From what has been said it is to be anticipated that neither the "distance" between two points in space nor the time interval between the happening of the two events are invariant to Lorentz transformation; either of them may even vanish in one frame, but not vanish in another frame. If we take for convenience one of the two events to happen at the origin at time zero, the other one at the point x_1, x_2, x_3 at time x_4 the square of their distance in that frame will be given by the Pythagorean theorem thus:

$$x_1^2 + x_2^2 + x_3^3$$

and their time interval by x_4.[26]

Since all frames are to be of equal right, the same expressions will hold in any other frame, only with the x_k''s for the x_k. But only neither is invariant. We shall have in general:

24 Cf. H. REICHENBACH – *Space and Time*, translated from german, Dover Publications, Inc, New York, 1958, 109-111.

25 Cf. *Ibidem*, 58-101.

26 Cf. R. MILLS – *Space, Time and Quanta*, W. H. Freeman and Company, New York, 1994, 81-106.

$$x_1'^2 + x_2'^2 + x_3'^2 \neq x_1^2 + x_2^2 + x_3^3$$
$$x_4' \neq x_4$$

However – and this is the cardinal point – the Lorentz transformation is characterized by the fact that the following expression is "invariant":

$$-x_1^2 - x_2^2 - x_3^2 + x_4^2 = -x_1'^2 - x_2'^2 - x_3'^2 + x_4'^2$$

I said the transformations are characterized by this "invariance". Indeed it is well-might an exhaustive definition distinguishing Lorentz transformations among all possible homogeneous linear transformations of the four-coordinates. The state of affairs bears formal analogy to the case of orthogonal transformations in there dimensions, which are characterized among all linear transformations by the invariance of the distance: $x_1^2 + x_2^2 + x_3^2$.[27]

A formal description of the injunction can be given in the following terms. If you write down the four linear transformation formulae and transcribe every term containing x_4 or x_4' thus:

$$ax_4 = (-ia)\cdot(ix_4); \qquad x_4' = (-i)\cdot(ix_4')$$

in other words if you regard it as a transformation between the variables x_1, x_2, x_3, x_4 and x_1', x_2', x_3', x_4' then, this is an orthogonal transformation in four dimensions but with some relevant injunction on the coefficients as to being either real or purely imaginary.

The invariant of two world-points or point-events, one of which was for simplicity taken to happen at the origin of the four-dimensional frame, is the square of the time-interval minus the square of the distance.[28]

The invariant of two world-points or point-events, one of which was be simplicity taken to happen at the origin of the four-dimensional frame, is the square of the time-interval minus the square of the distance. In the first case x_4^1 can never vanish, and thus, in virtue of our conventions about the determinant and the coefficient $\partial x_4^1 / \partial x_4$ it cannot change its sign on Lorentz transformation. The second event coordinates is then called later or earlier with respect to the first according to whether $x_4 < 0$.[29]

The Lorentz transformation is correctly the dictionary of the metric from inertial systems, and defines a new epistemological situation to the relationship on the effects of verificability to the frame articles of relativistic metric in the spacetime connection.

27 Cf. R. U. SEXL; H. K. URBANTKE - *Relativity, Groups, Particles*, translated from the german, Springer-Verlag, New York, 2000, 49-59.
28 Cf. D. KRAMER; H. STEPHANI *et alli* – *Exact Solutions of Einstein's Field Equations*, University Press, Cambridge, 1979, 27-41.
29 Cf. J. L. SYNGE – *Relativity: the general theory*, North-Holland Publishing Company, Amsterdam, 1960, 1-41.

I think so that the Lorentz transformations are the *a posteriori* sentences to the spacetime metric, because they show the sense of metric stationary in the space-time relationship. These predicates refer the transcendental and universal opportunity.

Only the theory of special relativity shall be taken in account here. The main point of importance is the fusion of time and space to four-dimensional space-time, as formulated by Minkowski. However, the meaning of space-time can be demonstrated considering as example the simplest equation of the Lorentz transformations for inertial systems. It can be written as fallows:

$$x = x' \cosh\varphi + t' \sinh\varphi \,;$$

$$t = x' \sinh\varphi + t' \cosh\varphi$$

$$\text{with } \tanh\varphi = v/c \,.$$

It is assumed that the coordinates of the systems are orientated parallel to each other, that the relative movement takes place along the x-axis and that the systems have the same origin at time $t = t' = 0$. Also v represents the relative speed of the coordinate systems and c is the speed of light.[30]

This form of the equations was chosen here because of its analogy to the classical transformation of rotated systems:

$$x = x' \cos\alpha - y' \sin\alpha$$

$$y = x' \sin\alpha + y' \cos\alpha$$

when the rotation of angle α takes place around the z-axis.

It is well known that according to the theory of relativity the flow of time is different in a system that is moving. This difference is given by the above equation of the Lorentz transformation. The analogy between the two equations can be interpreted to show that this difference flow of time is analogous to the rotation of coordinate systems, which correspond to view the world from different perspectives. Thus time and space are fused into a relativistic unity. This fusion of time with the space leads to the concept of space-time in the theory of special relativity. This concept, however, leads to important role to play simplifications. Many cases of this can be found in physics, especially where relativistic effects enter the game. In quantum field theory, e.q., integrations are in general performed on the four-dimensional space-time. Also space and time coordinates are not treated completely and necessarily identical, however the difference is not easily noticeable.

Consciousness at one time can reach only one three-dimensional plane of this four-dimensional space-time. This plate corresponds to the present moment. But this presence is an arbitrary point of reference, arbitrary in the same way as a

30 Cf. B. HOFFMANN – *Perspectives in Geometry and Relativity*, Indiana University Press, London, 1966, 58-62.

certain point in space.[31] The relation of events at different times is in principle not different from the relation of events at the same time but at different places. This is usually taken to imply that there is a symmetry between past and present just as well as left and right are symmetrical to each other.

CONCLUSION

For the description of a physical event, one must specify four numbers, say x, y, z, and t, of which the first three refer to the site where the event takes place while the fourth refers to the time of its occurrence. For another observer, the same event would be specified by a different set of numbers, say x', y', z', and t'.

The mathematical connection between these two sets of numbers would depend upon the characteristic features of the transitive from one system of reference to the other. Since there is a large variety of systems of reference, there is a large variety of transformations one has to deal with. One may, therefore, regard the complete set of four members as the coordinates of a world point, representing the event in question in a four-dimensional world of x, y, z and t, rather than regard x, y and z separately as the spatial coordinates of the event in the three-dimensional physical space and t separately as the time of its occurrence in a one dimensional continuum of time.[32]

The foregoing scheme was first suggested by Minkowski in 1908 and has since been employed extensively. According to this scheme, a physical event is represented by a world points in the four-dimensional world of events – usually referred to as the "Minkowski world" or the space-time continuum – which may be defined as the totality of representation points of all possible physical events. The position of the representative point, corresponding to a particular event, in the Minkowski world is determined by four coordinates x, y, z and t, as referred to a set of four axes the so called coordinates axed. The development of an event is depicted by a trajectory, usually referred to as the world line, along which the corresponding world point evolves. The relative position of one event with respect to another is denoted by a vector – the line element – that joint the two events in the Minkowski worlds and so forth.[33]

In this representation the study of relativistic theory reduces to a study of the geometry of the four-dimensional continuum. In consequence, the formation of the special relativity theory assumes so elegant a form that the importance and usefulness of Minkowski's contribution can hardly be overestimated.

As regards transformations, their classification would now be based on the fact whether they involve one or both of the following features:

- A rotation of the coordinate axes in the x, y, z, t continuum, with six degrees of freedom, and;

31 Cf. A. CICHNEROWICZ – *Théories Relativistes de la Gravitation et de l'Electromagnétisme* ; Masson et Cie, Éditeurs, Paris, 1955, 3-26.
32 Cf. S. W. HAWKING; G. F. R. ELLIS – *The Large Scale Structure of Space-time*, At the University Press, Cambridge, 1980, 118-120.
33 Cf. C. MOLLER – *The theory of Relativity*, At the Clarendon Press, Oxford, 1966, 31-62.

- A displacement of the origin of the coordinate system, with four degrees of freedom.[34]

We have seen that the fundamental invariant of the homogeneous Lorentz transformations is:

$$s^2 = x^2 + y^2 + z^2 - c^2t^2 = x'^2 + y'^2 + z'^2 - c^2t'^2 = s'^2$$

If we employ the set of coordinates:

$$x_1' = x\,;\quad x_2' = y'\,;\quad x_3' = z'\,;\quad x_4' = \dot{\imath}c\,t'$$

The anterior equations are remained in this:

$$s^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2 = x_1'^2 + x_2'^2 + x_4'^2 = s^2$$

Written in this form, the analogy with the three-dimensional world becomes complete, except for the fact that the fourth square in the sum here is really negative.[35]

However, because of the imaginary character of the four-coordinate employed, the geometry of this continuum is only formally identical with Euclidean geometry; that is why it is usually referred to as quasi-Euclidean.

The immediate advantage of the transition from the anterior equation is that new the rotation of the axes of an inertial observer would not lead to an oblique coordinate system. On the other hand, these rotations would now involve only rectangular coordinate systems Lorentz transformations would, therefore, become linear orthogonal. Confining to the case of homogeneous transformations, wiz the ones that do not involve a displacement of the origin, we can write:

$$x_t' = \sum_1^4 x_{ik} \cdot x_k$$

where x_{ik} 's are the coefficient of transformation. Geometrically, the coefficient x_{ik} may be understood as the cosine of the angle between the x_i axis and the x_k axis. In view of the specific nature of the coordinates and the coefficient x_{sp} and x_{4t} would be real while the coefficient x_{4t} and x_{4s} would be purely imaginary.[36]

From now onward we adopt the so called "summation convention", i.e., if an index occurs twice in a term, a summation over all possible values of that index is automatically implied. The convention applies to all the equations that appear in the sequel, unless a statement is made to the contrary.For a continuous transformation – the one obtainable from the identity transformation $(S \to S)$ by a continuous rotation of the axes – the value of the determinant must be +1. Such transformations are usually referred to as proper transformation.We can have transformations which involve reflections of the coordinates axes and may, therefore, be discontinuous. Such transformations are necessarily discontinuous; they are usually

34 Cf. R. d'INVERNO – *Introducing Einstein's Relativity*, At the Clarendon Press, Oxford, 1992, 33-40.
35 Cf. G. L. NABER – *The Geometry of Minkowski Spacetime*, Springer-Verlag, Berlin, 1992, 7-87.
36 Cf. I. CHAVEL – *Riemannian Geometry: a modern introduction*, At the University Press, Cambridge, 1993, 49-66.

referred to as improper transformation. Of course, transformations that involve an even number of reflections must again have: $\alpha = +1$.

Also the Special Theory of Relativity is play a very important role in yours applications, and defines a progress significantly to the Physics.[37]

Epistemologically, we are not, however, sufficiently advanced in our knowledge of Nature's elementary laws to adopt this more perfect method without going out of our depth. At the close of our considerations we shall see that in the most recent studies there is an attempt.It more clearly follows from the above what is implied by preferred states of motion. They are preferred as regards the laws of Nature. States of motion are preferred when, relative to the formulations of the laws of Nature, coordinate systems within them are distinguished in that with respect to them those laws assume a form preferred by simplicity. According to classical mechanics the states of motion of the inertial frames in this sense are physically preferred.[38]Classical mechanics permits a distinction to be made between unaccelerated and accelerated motions; it also claims that velocities have only a relative existence, dependent on the selection of the inertial frame, while accelerations and rotations have an absolute existence. This state of affairs can be expressed thus: According to classic mechanics velocity relativity exists, but not acceleration relativity. After these preliminary consideration we can pass to the actual topic of one contemplations, the relativity theory, by characterizing its development so far in terms of principles.[39]

The special theory of relativity is an adaptation of physical principles to Maxwell-Lorentz electrodynamics. From earlier physics it takes the assumption that Euclidean geometry is valid for the laws governing the situation of rigid bodies, the inertial frame and the law of inertia. The postulate of equivalence of inertial frames for the formulation of the laws of Nature is assumed to be valid for the whole of physics as special relativity principle. From Maxwell-Lorentz electrodynamics it takes the postulate of invariance of the velocity of light in a vacuum according to the light principle.[40]To harmonize the relativity principle with the light principle, the assumption that an absolute time exists, had to be abandoned. Thus the hypothesis is abandoned that arbitrarily moved and suitably set identical clocks function in such a way that the times shown by two of them, which meet, agree. A specific time is assigned to each inertial frame; the state of motion and the time of the inertial frame are defined, in accordance with the stipulation of meaning, by the requirement that the light principle should apply to it. The existence of the inertial frame thus defined and the validity of the law of inertia with respect to it are assumed. The time for each inertial frame is measured by identical clocks that are rationary relative to the frame.[41]The laws of transformation for space-coordinates and time for the transition from one inertial frame to another,

37 Cf. T MAUDLIN – *Quantum non-locality and Relativity*, Blackwell, London, 2002, 29-36.
38 Cf. R. M. ROSENBERG – *Analytical Dynamics*, Plenum Press, New York, 1977, 7-17.
39 Cf. G. E. O. GIACAGLIA – *Mecânica Geral*, Editora Campus, Rio de Janeiro, 1982, 131-144.
40 Cf. M. S. LONGAIR – *Theoretical concepts in physics*, At the University Press, Cambridge, 1984, 37-60.
41 Cf. B. RUSSELL – *ABC of Relativity*, Unwin Paperbacks, London, 1977, 68-79.

the Lorentz transformations as they are termed, are unequivocally established by these definitions and the hypotheses concealed in the assumption that they are free from contradiction. Their immediate physical significance lies in the effect of the motion relative to the used inertial frame on the form of rigid bodies – Lorentz contradiction – and on the rate of the clocks. According to the special relativity principle the laws of Nature must be covariant relative to Lorentz transformations; the theory thus provides a criterion for general laws of Nature. It leads in particular to a modification of the Newtonian print motion law in which the velocity of light in a vacuum is considered the limiting velocity, and it also leads to the realization that energy and inertial mass are of like nature.[42]

The special relativity theory resulted in appreciable advances. It reconciled mechanics and electrodynamics. It reduced the number of logically independent hypotheses regarding the latter. It enforced the need for a clarification of the fundamental concepts in epistemological terms. It united the momentum and energy principle, and demonstrated the like nature of mass and energy. Yet it was not entirely satisfactory quite apart from the quantum problems, which all theory so far has been incapable of really solving. In common with classical mechanics the special relativity theory favours, certain states of motion to all other states of motion.

This was actually more difficult to tolerate that the preference for a single state of motion as in the case of the theory of light with a stationary ether, for this imagined, i.e., the light ether. A theory which from the outset prefers to state of motion should appear more satisfactory. Moreover the previously mentioned vagueness in the definition of the inertial frame or in the formulation of the law of inertia raise doubts which obtain their decisive importance, owing to the empirical principle for the equality of the inertial and heavy mass, in the light of the following consideration.

Let k be an inertial frame without a gravitational field, k' a system of coordinates accelerated uniformly relative to k. The behaviour of material points relative to k' is the same as if k' were an inertial frame in respect of which a homogeneous gravitational field exists. On the basis of the empirically known properties of the gravitational field, the definition of the inertial frame thus proves to be weak. The conclusion is obvious that any arbitrarily moved frame of reference is equivalent to any other for the formulation of the laws of nature, that there are thus no physically perfected states of motion at all in respected of regions of finite extension.

The implementation of this concept necessitates an even more profound modification of the geometric-kinematical principles than the special relativity theory.[43]

The Lorentz contraction, which is derived from the latter, leads to the conclusions that with regard to a system k' arbitrarily moved relative to an inertial frame k, the laws of Euclidean geometry governing the position of rigid bodies do not apply. We arrive at the formal description of the field by the following consideration. For each infinitesimal point environment in an arbitrary gravitational field a local

42 Cf. J. G. TAYLOR – *Special Relativity*, At the Clarendon, Oxford, 1975, 1-12.
43 Cf. R. K. PATHRIA – *The Theory of Relativity*, second edition, Pergamon Press, Oxford, 15-48.

frame of coordinates can be given for such a state of motion that relative to the local frame no gravitational field exists a local inertial frame.

In terms of this inertial frame we way regard the results of the special relativity theory as correct to a first approximation for the infinitesimally small region. There are an infinite number of such local inertial frames at any space-time point; they are associated by Lorentz transformations. These latter are characterised in that they leave invariant the distance ds of two infinitely adjacent point events defined by the equation:

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$$

which distance can be measured by means of scales and clocks. For $x, y, z, t,$ represent coordinates and time measured with reference to a local inertial frame.[44]

To describe space-time regions of infinite extent arbitrary point coordinates in four dimensions are required which serve no other purpose than to provide an unambiguous designation of the space-time points by four numbers each x_1, x_2, x_3 and x_4, which takes account of the continuity of the four-dimensional manifold by Gaussian coordinates. The mathematical expression of the general relativity principle is then, that the systems of equations expressing the general laws of Nature are equal for all such systems of coordinates.[45] Since the coordinate differentials of the local inertial frame are expressed linearly by the differential dx_{ik} of a Gaussian system of coordinates, when the latter is used for the distance ds of two events an expression of the form:

$$ds^2 = g_{ik} \cdot dx_i \cdot dx_k$$

is obtained.

The g_{ik}, which are continuous functions of x_{ik}, determine the metric in the four-dimensional manifold where $|ds|$ is defined as an absolute parameter measurable by means of rigid scales and clocks.

These same parameters g_{ik}, however, also describe with reference to the Gaussian system of coordinates the gravitational field which we have previously found to be identical with the physical cause of the metric.

The care as to the validity of the special relativity theory for finite regions is characterised in that when the system of coordinates is suitably chosen, the values of g_{ik} for finite regions are independent of g_{ik}.

In accordance with the general theory of relativity the law of point motion in the pure gravitational field is expressed by the equation for the "geodesic line".[46]

44 Cf. W. G. DIXON – *Special Relativity, the foundations of macroscopic physics*, At the University Press, Cambridge, 1978, 25-30.
45 Cf. Ch. W. MISNER; K. S. THORNE; J. A. WHEELER – *Gravitation*, W. H. Freeman and Company, San Francisco, 1973, 385-431.
46 Cf. Th. FRANKEL – *Gravitational Curvature*, W. H. Freeman and Company, Sam Francisco, 1979, 35-45.

Actually the geodetic line is the simplest mathematically which in the special case of constant g_{ik} becomes rectilinear. Here, therefore, we are confronted with the transfer of Galileo's law of inertia to the general theory of relativity.[47] In mathematical terms the catch for the field equations amounts to ascertaining the simplest generally covariant differential equations to which the gravitational potentials g_{ik} can be subjected. By definition these equations should not contain higher derivatives of g_{ik} with respect to x_{ik} than the second and these only linearly, which condition reveals these equations to be a logical transfer of the Poisson field equation of the Newtonian theory of gravity to the general theory of relativity. And that exhausts the direct consequences of the relativity principle. I shall turn to those problems which are related to the development which I have traced. Already, Newton recognized that the law of inertia is unsatisfactory in a context so far unmentioned in the exposition, namely that it gives no real cause for the special physical position, of the states of motion of the inertial frames relative to all other states of motion.[48] It makes the observable material bodies responsible for the gravitational behaviour of a material point, yet indicates in material cause for the material point but devise the cause for it (absolute space).

47 Cf. H. A. ATWATER – *Introduction to General Relativity*, Pergamon Press, Oxford, 1974, 136-159.

48 Cf. B. HOFFMANN – *Relativity and its Roots* – W. H. Freeman and Company, New York, 1983, 81-103.