The theory of ideas and Plato's philosophy of mathematics

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Abstract

In this article I analyze the issue of many levels of reality that are studied by natural sciences. Particularly interesting is the level of mathematics and the question of the relationship between mathematics and the structure of the real world. The mathematical nature of the world has been considered since ancient times and is the subject of ongoing research for philosophers of science to this day. One of the viewpoints in this field is mathematical Platonism.

In contemporary philosophy it is widely accepted that according to Plato mathematics is the domain of ideal beings (ideas) that are eternal and unalterable and exist independently from the subject's beliefs and decisions. Two issues seem to be important here. The first issue concerns the question: was Plato really a proponent of present-day mathematical Platonism? The second one is of greater importance: how mathematics influences our understanding of the nature of the world on its many ontological levels?

In the article I consider three issues: the Platonic theory of "two worlds", the method of building a mathematical structure, and the ontology of mathematics.

Keywords

mathematical Platonism, ontology, Platonic Academy.

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Evolution has provided us with the ability to describe the world appropriately, at least on our scale. However, from the point of view of evolution, there is no reason for us to be able to describe it on other scales. Science and philosophy convince us that these different scales really exist. The problem is, however, that we are forced to perceive the world on many scales from the perspective of one particular scale. The questions thus arise: what does the world look like on our scale, and what does it look like on other scales? If we focus our attention on natural sciences, I think that one can agree with the statement that the most accurate description of reality at various scales is provided by the language of mathematics. If so, then the image of the world depends essentially on the description that is made in this language. This means, however, that mathematics determines what the world looks like, especially at those levels that are not directly accessible for us. But is mathematics the same as the structure of the world, or is it only its subjective image? Already in Antiquity attempts were made to answer these questions. Particular attention was paid to them by Plato and members of his Academy who initiated the program of mathematical natural sciences—a program that has become the basis for the development of the technical civilization. In this way, the Academy has become not only the first university in the history of the world, but also the place where the most important project crucial for the development of science and civilization was created. Its creators, apart from Plato, were such eminent thinkers as Eudoxos of Knidos, Theaetetus, Menaichmos, Theudius, Leon, Speusippus, Xenocrates, Heraklides of Pontus.

It is widely accepted in the contemporary philosophy of mathematics that according to Plato mathematics is the domain of ideal beings (ideas) that are eternal, unchanging and exist independently of the subject's decisions (Brown, 2008, pp.12–16). Two issues seem to be important. The first one concerns the question: Was Plato really thinking that way? The second one is more important: How does mathematics influence our understanding of the nature of the world on its many levels of being?

Three things need consideration. The first one is related to the Platonic theory of "two worlds." The second one—to the method of building a mathematical structure. The third one concerns the ontology of mathematics.

A popular belief is that in Plato's philosophy we have two worlds separate from each other, one of which is the world of phenomena, and the other is the world of ideas (Dembiński, 2007). The conclusion derived from this assumption is that mathematics belongs to the Platonic world of ideas, which is eternal and unchanging. In this world, mathematical objects obtain the status of ideal beings and their existence is independent from the subject and phenomena. One can only acquire knowledge about this self-existing world of mathematical objects through the intellect. Unfortunately, this popular belief is in fact inconsistent with Plato's philosophy and his understanding of mathematics. Nevertheless, it has persevered in the history of the interpretation of Platonic philosophy and has been transferred to the philosophy of mathematics. The reason lies in the adaptation of a simplified vision of Platonism, which assumes the existence of only two levels of reality, the level of ideas and the level of phenomena, with the mathematical objects situated among the former. Meanwhile, in his analysis of mathematical objects' mode of existence, Plato concluded that such objects are related neither to the world of ideas nor to the world of phenomena. He claimed that they occupy an intermediate position and do not belong to any of them.[1] Instead, they are merely the product of our mind. This position requires a further explanation, because it is of crucial importance for Plato's understanding of mathematics, as well as on the contemporary discussions about mathematical Platonism.

Plato tries to explain his position, first and foremost, in books VI and VII of the *Republic* and in *Letter VII*. The original starting point is the conviction that the first stage of cognition (including mathematical cognition) emerges from the observation of nature in its phenomenal form. Then the sensory imaginations arise, which provide only the images of the phenomena (*eikasia*). We see, says Plato, the reflections of reality that are born in our senses. They are imperfect and often illusive.[2] At the next stage, we try to make these images credible (*pistis*). We try to confirm the data of sensual experience by examining and observing certain states of things, from as many perspectives as possible (Plato, 1955, *Republic*, 509d-511e). Today, such behavior would correspond to empirical tests. The most important element of this study has given Plato the ability to discern patterns present in phenomena. These patterns indicate the order according to which the phenomena are organized, as well as the existence of regularities that this order defines. Plato discusses movement patterns, harmony patterns, or (in relation to human activities) ethical and aesthetic patterns. In the context of the Platonic philosophy of mathematics, the most important role is played by the movement patterns

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[1] "Further, he states that besides sensible things and the Forms there exists an intermediate class, the objects of mathematics, which differ from sensible things in being eternal and immutable, and from the Forms in that there are many similar objects of mathematics, whereas each Form is itself unique." (Aristotle, 1924, p.987b).

[2] "first, shadows, and then reflections in water and on surfaces of dense, smooth and bright texture, and everything of that kind, if you apprehend." (Plato, 1955, *Republic*, 510a).

of the celestial bodies, based on the man's ability to recognize them. Plato considers this ability to be the supreme gift of the gods. Observation of patterns in nature: rhythms, motifs, harmony, symmetry or proportion, directs the subject's attention towards their source (Plato, 1955, *Timaeus*, 47a-e). But the source itself is no longer available for sensual cognition. It is available only for intellectual cognition. On the border between sensual and intellectual cognition, there is a kind of intuition that Plato describes as "the suspicion of truth." The point is that from the sensory data, basing on the perceived regularities, patterns and proportions, one is able to formulate hypotheses regarding their source. Plato calls these hypotheses "true opinions (*alēthēs doxa*)" (Plato, 1955, *Meno*, 85c.98a.97c). To confirm their value, one is asked to verify them. This is the task of yet another higher cognitive power, which is the reason (*dianoia*). To put it simply, the task of the reason is to conduct logical chains of inference and to analyze causal relationships, what Plato calls "causal splicing (*symploke*)" (Plato, 1955, *Meno*, 97e-98a). Reason is the authority of the subject, who plays an essential role in the Platonic philosophy of mathematics. His role boils down to creating intellectual models of sensually given states of affairs and patterns perceived in nature. These models are the representation of phenomena and patterns at the level of intellect, based on the abstraction skills (*aphairesis*) and they constitute the product of activities confirming the permanent occurrence of a certain set of features in a certain class of objects.[3] Abstraction concerns the regularities observed in nature, which in turn

[3] „For I think you are aware that students of geometry and reckoning and such subjects first postulate the odd and the even and the various figures and the three kinds of angles and other things akin to these in each branch of science, regard them as known, and, treating them as absolute assumptions, do not deign to render any further account of them to themselves or others, taking it for granted that they are obvious to everybody." (Plato, 1955, *Republic*, 510c).

were derived at the levels of *eikasia* and *pistis* (*doxa*). Such models are also mathematical objects, according to Plato. The examples are numbers. Euclid, who was educated at the Plato Academy, complies with Plato's intuition by defining the number as "a multitude made up of monads" (*Arithmos de, to ek monadōn sygkeimenon plēthos*). A number is defined by a monad (multiple monads). But what is a monad? It is a model created at the level of intellect, allowing to describe the infinite multitudes appearing at the level of the senses (things and their reflections). In contrast to sensual images, which are always different, a monad is "always equal to every other, and no different from any other, and has no part in it" (Plato, 1955, *Republic*, 526a). It is a model created by the intellect, presenting structural features of every sensual multitude. The model understood in this way has a completely different status than counted items and, most importantly, it is the creation of the subject. In this sense too, the numbers are the objects of the subject. The same applies to geometrical objects. A line is a length without a width, a surface possesses only length and width, and a circle is a plane figure contained by one line comprised of points equally distant from the circle's center. None of the sensory objects has such properties. One can consider the internal structure of the model, one can also analyze the relationship of a given model to other models, one can finally examine which models are possible, which are necessary and which are completely excluded. The analysis of these models is the subject of the work of mathematicians. However, a mathematician, let alone a philosopher, cannot avoid asking questions concerning the legitimacy of creating such models. The question arises: What makes mathematical objects, which are human creations, not arbitrary?

Searching for the answer, Plato appealed to the concept of ideas-measures. He claimed that mathematics can neither derive its own justification from the phenomena, which are variable and temporary, nor from the arbitrary decisions of the subject. However, there must be something that guarantees the functioning of the cosmic order, and also ensures the correctness of the constructed mathematical models. In this context, Plato proposes the adoption of the eternal model of the organization of the world, which is created according to unchanging regularities that define the order of the Cosmos. It were these norms, these measures establishing the model of the cosmic order, which he called ideas. Today, their equivalent would be the laws of nature and the laws of physics. It is not difficult to notice that these laws exist differently than phenomena do. A physical law and its implementation have different ontological statuses. As far as we know, the former is immutable, has the feature of unity (there are no two identical laws), and does not depend on the decision of the subject. We can only say about such laws that they are, and that they are always as they are. To this way of existence Plato attributed the name of being, and he called their mode of being "really real" (*ontōs on*). He claimed that beings, ideas inhabit a separate reality which constitutes the (eternal) organizational model of the Cosmos, and is the essence of existence of all phenomenal structures and processes. This model manifests itself in the form of symmetry, proportions, various types of harmony, which can be understood as defining the essence and behavior of phenomenal structures. The goal of philosophy and science, says Plato, is to reach the idea-measures, that condition a particular kind of order, regardless of whether it is cosmic, ethical or aesthetic. Of course, this also applies to the mathematical order. That is why Plato also postulates the existence of mathematical ideas, that form the basis and cause of mathematical order. However, the most important is, and what must be always remembered, that these ideas are not mathematical objects themselves. Mathematical ideas constitute a completely autonomous world, existing outside the world of our mathematics, just as the laws of physics exist outside the world of physical theories we create. As Aristotle comments, no mathematical operations can be performed on ideas (Aristotle, 1924, *Metaphysics*, 1081a). One can only examine the relationships that exist between them. However, for such a study the mathematical method with its axiomatic approach is inapplicable. It is rather the dialectical method, whose purpose is precisely to study the relationship between the ideas, which is appropriate. The confusion concerning mathematical Platonism stems from the unawareness of the difference between mathematical ideas and mathematical objects. Plato tries to explain precisely that issue in *Letter VII*.[4]

Plato considers a simple mathematical object—a circle. We can assign a name to it. It could be changed, because—as he argues—"none of the objects, we affirm, has any fixed name, [...] nor is there anything to prevent forms which are now called 'round' from being called 'straight,' and the 'straight' – 'round'; and men will find the names no less firmly fixed when they have shifted them and apply them in an opposite sense." (Plato, 1955, *Letters*, 343b). Next, we try to formulate a fairly precise definition of a circle. It should cover everything that is round and circular. Most often, according to Plato, an imperfect definition is formulated, based on specific wording, which "inasmuch as it is compounded of names and verbs, it is in no case fixed with sufficient firmness." (Plato, 1955, *Letters*, 343b). In the further course of the procedure, an attempt may be made to build a model or a schema, corresponding to what has been defined. We can do this by creating thoughtful constructions, presenting drawings or spatial visualizations. Later, the analysis of thus obtained model and its relationship to other models (mathematical objects) is developed

[4] For an in-depth discussion of this issue, see: (Dembiński, 2003, pp.55–110).

into a special theory, which includes all previous stages. Theory is the highest degree of cognition. That level can be attained by a cognizing subject thanks to his abilities, i.e. sensual perception, abstraction, and logical analysis. Plato, however, strongly believes that regardless of the degree of precision available at the above-mentioned levels of cognition, one should be aware that "their inaccuracy is an endless topic" (Plato, 1955, *Letters*, 343b), how much arbitrariness and uncertainty is associated with them, and how much they depend on the cognizing subject and its limitations. Meanwhile, mathematical cognition is required to be certain; to be characterized by necessity, universality and truthfulness. Therefore, there must be some basis, upon which we could justify and validate the four existing procedures of cognition (name, definition, model and theory). We find it, according to Plato, in the idea of the circle, "the circle as such" (*autos o kyklos*). Such an idea must be called a real being (*alethés òn*), the essence of a thing (*tode ti*). Plato describes it as "the Fifth (*to pempton*)" (Plato, 1955, *Letters*, 342a–343d). The circle "in itself" exists differently than the one which is the intellectual model or the circle we draw, which the wheelwright creates, or which we observe in phenomena. The circle "in itself" is the highest, unchanging and only measure of all circularity, a condition for the possibility of creating theories, models, definitions and names involving the circular. The circle "in itself", as a regularity, as the measure of the specificity of everything that is circular, is unique, unchangeable and independent of the subject's beliefs. The same applies to numbers. If we take a numerical idea, for example the ideal number two, then with its help we are able to determine the essence of each mathematical two. Using the descriptive language, one can say that the ideal number two defines the structural features of each mathematical two. And the mathematical two is only an intellectual model created by the subject. If we add two plus two, we are not in the world of ideas, but at the level of our mathematics. A good description of this situation was presented by Michael Heller (2006). He accepts the distinction between mathematics with a lowercase "m", and Mathematics with a capital "M". The former is the mathematics created by man. The latter is the mathematics which is inherent to nature, and to which we have no direct access. Indirect access is provided only by means of representation, which is our mathematics, the mathematics with a lowercase "m". The Mathematics with a capital "M" corresponds to the level of Platonic, mathematical ideas. This situation is explained by Plato in the "Metaphor of the Cave". We humans are only able to see the world of shadows (Plato, 1955, *Republic*, 514–518d). We know, however, that these shadows are shadows of something we do not directly see (real figures, fire). Therefore, we are forced to create images, models of what we do not see. These models come with all the disadvantages and limitations that arise in the subjective process of cognition. However, according to Plato, there are moments (being a gift of the gods), when we are temporarily given a somewhat vague, intuitive "seeing" of the outlines of regularity, organizing the order of nature. These are the moments when someone in the cave suddenly "senses" that there is "something" which is the cause and condition of the existence of shadows. Plato describes this moment as the "conversion of the soul" (*periagoge tes psyches*). In such a "foresight", however, we are not able to stay for long, instead we return quickly to our shadows, and again continue the arduous inference from the representation. We return to our mathematical world of shadows, to our mathematics named with the lowercase "m".

The proponents of mathematical Platonism claim that—according to Plato—mathematical objects exist independently of the subject. We still need to answer the question: Where did this conviction originate? The answer is simple. Such a belief was born not in the thoughts of Plato but in the thoughts of his successors, Speusippus and Xenocrates (see Dembiński, 2010; Dillon, 2003). Speusippus decided that the Platonic world of ideas should be inhabited by mathematical objects.[5] He assigned to them all the attributes of an idea: separate existence, eternity, immutability and independence from the subject. He decided that it is unnecessary to double the worlds and postulated the existence of something beyond just mathematics. In this way, he put mathematics at the top of the world of beings. Mathematics took the place of the Platonic world of ideas. In this way, he expected to eliminate the difficulties associated with explaining the relationship between ideas (ideal numbers, ideal figures) and the objects of mathematics. He thought that it is sufficient to recognize the objects of mathematics themselves as ideals and that there is no need to introduce difficult notions of ideal numbers and figures. Aristotle did not consider such a solution as a good one. Probably because he thought that Speusippus wanted to replace in this way philosophical cognition of the world—and even its very existence—with mathematics. Aristotle attributed to Speusippus the claim that the entire philosophy of his time can be reduced to mathematics (Aristotle, 1924, *Methaphysics*, 992a30). Speusippus stand was strengthened by Xenocrates, another Academy scholar, who decided to replace ontology with mathematics. He claimed that mathematics is the only acceptable ontology, because the world is in fact created and constituted according to mathematical patterns and structures.

[5] "Those who recognize only the objects of mathematics as existing besides sensible things, abandoned Ideal number and posited mathematical number because they perceived the difficulty and artificiality of the Ideal theory." (Aristotle, 1924, *Methaphysics*, 1086a).

Ideas are, according to Xenocrates, identical with mathematical numbers. Their geometrical form constitutes geometric ideas. Aristotle considered this solution to be the worst one. Perhaps he thought so because he accepted Plato's conviction that ideas are independently existing entities on which no mathematical operations can be carried out. As he believed, treating the objects of mathematics as ideas would exclude the possibility of the existence of mathematics. However, the proposal of Xenocrates could have a different meaning. Recognizing mathematical objects as ideas, Xenocrates wanted to draw attention to the deep relationship between ontology and mathematics, where mathematics is understood as the only acceptable ontology. If the world were ultimately created according to mathematical structures and patterns, mathematics would be its proper ontology. A similar viewpoint is adopted, for example, by R. Penrose and many mathematicians admitting mathematical Platonism. Thus, modern advocates of mathematical Platonism must remember that by adopting Platonism, they essentially adopt the position of Speusippus and Xenocrates, not of Plato himself. This, of course, is still Platonism in a broad sense. Nevertheless, it is not a Platonism understood as Plato's position.

Above considerations lead to the following conclusions: First of all, Plato's concept of mathematics is not the one that is usually referred to by the adherents of mathematical Platonism. Secondly, the mathematics we use is always our human mathematics created by us for the sake of representing nature, whereas nature itself uses different Mathematics, the outline of which we can see only fragmentarily through intellectual intuition. We will never fully see the Mathematics of nature, because it is directly inaccessible for us. We know, however, that this "capital-M" Mathematics is there, and that it justifies the existence of our mathematics, the "lowercase-M" mathematics of the shadow world. This also applies to logic that organizes the world on its many levels. We, the people who see only shadows, want to describe and understand other levels of reality with this view. But these levels exist differently, have a completely different structure, and different logic. Getting to know them requires different methods. Plato suggested that this situation should be taken into account, without confusing the modes of existence at various levels. The most common type of fallacy we fall into is the attempt to describe other levels using the methods valid at our own level. Yet ideas, mathematical objects, and other time-space phenomena exist differently. Today, we begin to understand that the logic valid at the microscale is different from the one at our scale, and different from the logic at the macroscale. Perhaps it is worth to resort to the intuition of the ancient thinkers who, as the poets say: are closer to the gods, and see better than us.

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