Filozofia Nauki
Rok XXII, 2014, Nr 1(85)

Tomasz Bigaj

Quantum Dispositions and the Notion of Measurement

1. PRELIMINARIES

The idea that the ontological foundations of quantum mechanics may be naturally reconstructed in terms of irreducible dispositional properties has been floating around for quite a while.[1] Ian Thompson (1988) observes that although in classical physics we already encounter dispositional properties that have to do with interactions between objects, only in quantum mechanics 'structural' properties such as position and velocity behave like dispositional properties due to the fact that only rarely do they admit a definite value. Thus it may be tempting to understand quantum properties as probabilistic dispositions to acquire certain precise values (such dispositions are often called 'propensities'). Mauro Dorato (2006, 2007) is convinced that the majority of interpretations of quantum mechanics commit us to the existence of irreducible quantum dispositions, although he interprets this fact as a sign that our understanding of the ontological underpinnings of quantum mechanics is far from satisfactory. Dorato associates quantum dispositions with non-definite properties and stresses their contextual and relational character. He points out that each interpretation of quantum mechanics (the Bohm theory, the many worlds interpretation, the spontaneous localization theory, and the like) gives a slightly different perspective on quantum dispositions and their characteristics.

Quantum dispositions (propensities) play an important role in an interesting attempt to solve the perennial measurement problem that has been made by Mauricio

[1] An outline of early dispositional accounts of quantum properties by Margenau, Heisenberg, and Maxwell can be found in (Suárez 2007).

Suárez (2004a, 2004b, 2007). Generalizing Arthur Fine's selective interaction approach, Suárez argues that quantum systems possess irreducible dispositional properties corresponding to particular observables, and that these dispositions are best represented by appropriate mixed states which give the same probability distributions of possible outcomes as the initial state of the system (which is generally not a mixture). Given the assumption that during measurements of an observable *A* the measuring apparatus interacts only with the selected *A*-propensities of the system, it may be argued that the linear Schrödinger evolution should be applied to the mixed *A*-state representing only the selected part of the initial state, and as a result the final state of the compound system will be a mixture of the pointer states rather than a superposition.[2] Regardless of one's appraisal of this particular solution to the measurement problem, it has to be noted that the dispositional approach clearly shows a potential to give an insight into some of the most recalcitrant conceptual problems of quantum mechanics.

To my knowledge, the clearest and most comprehensive exposition of the potential benefits of the dispositional approach to quantum mechanics has been given in a paper by Martin Thomson-Jones (unpublished). He holds a view that the connection between the metaphysics of dispositions and the interpretive issues pertaining to quantum mechanics is bi-directional: not only can the metaphysics of dispositions help us solve some interpretive problems in quantum mechanics, but also our philosophical understanding of the notion of dispositions can be enhanced by considering examples from quantum theory. As an illustration, Thomson-Jones lists several questions that can be posed and discussed at the interface between the metaphysics of dispositions and quantum mechanics, including the question of whether quantum mechanics can shed some light on the problem of the reducibility of dispositions to their causal bases, and the question of whether the problems that beset the view that all properties are dispositional can affect our interpretations of quantum mechanics. In the second part of his paper, he methodically lays down the foundations for the notion of quantum dispositions that could be used to solve one particular interpretive problem in Bohm's version of quantum theory.

Thomson-Jones's cautious and systematic approach to quantum dispositions provides a ready-made model for the analysis given in this paper, and the main issue that I am going to consider here has been prompted by his remarks about dispositional properties of quantum systems. In short, I am going to discuss a relatively neglected and yet to my mind quite serious problem that arises as a result of defining quantum dispositions in terms of measurement interactions. The problem, as I see it, is that this natural way of defining quantum dispositions leads to circularity if we accept the

---

[2] The weakest point of this argument as I see it is the assumption of the selective character of measurements. It is hard to accept that a measurement of an observable *A* interacts only with the *A*-dispositions of the system, since *A*-measurements clearly affect dispositions related to other observables, incompatible with *A*. For instance, by measuring the *z*-spin of an electron we inexorably change the probability distribution of values of its *x*-spin.

standard quantum-mechanical description of measurement interactions. After having stated the problem, I will move on to discuss some strategies of overcoming it, using a little-known theorem about quantum measurements due to Peter Mittelstaedt. Although this paper is primarily focused on the difficulties with the dispositional approach to quantum mechanics (and on some ways to avoid them), in section 6 I will briefly mention a few potential benefits of this approach that to my knowledge have escaped the attention of the experts. My general position is that despite the troubles affecting the dispositional approach (and one has to remember that no interpretation of quantum mechanics is entirely free from conceptual conundrums), the benefits should outweigh the difficulties in the long run.

2. TYPES OF QUANTUM DISPOSITIONS

The most complete information about a quantum system that is available in standard quantum mechanics is encompassed in the state vector φ (strictly speaking the most general mathematical way of representing quantum states is with the help of a density operator W, but I'll ignore this for the sake of simplicity). The vector φ contains two types of information regarding the state of the system at a given moment t. One is the characterization of all the properties that the system possesses at the moment t. This component of the state, often called the value state (after Bas van Fraassen, cf. van Fraassen 1991: 275), is usually presented in terms of the probabilities of outcomes for measurements of observables that are applicable to the system. But φ also encodes the information about possible future behaviour of the system, given interactions with its environment. Thus, if we know the exact form of the Hamiltonian governing the current interactions between the system and its environment, the future evolution of the system is given by applying the unitary evolution operator U to φ: $\varphi(t') = U(t, t')\, \varphi(t)$. In van Fraassen's terminology, this part of the overall state is referred to as the dynamic state.[3]

Both value and dynamic states can be naturally presented in terms of dispositions of the system to exhibit certain behaviour in certain circumstances. In the case of the value state, we can speak about various dispositions of the system to give rise to macroscopically observed outcomes upon appropriate measurements. Let us focus on the part of the value state pertaining to a given observable A. First, we must distinguish two cases: one in which the state φ of the system is an eigenstate for A, and the other in which it is not. In the first case, there is a particular value a_i that is certain to be revealed upon measurement of A. Hence, the A-related disposition that the system possesses is of a 'sure-fire' (deterministic) type: if the system undergoes a measurement-type interaction with some macroscopic apparatus, pertaining to observable A, then the apparatus is bound to record value a_i. On the other hand, if φ is not an

[3] The use of van Fraassen's terminology does not imply that I endorse here his modal interpretation of quantum mechanics.

eigenstate of A, the A-related dispositions are indeterministic, meaning that for each possible value a_i of A there is an objective chance that it will be recorded upon measurement.

Dispositions related to the dynamic state are of a slightly different sort. These are, roughly speaking, dispositions of the system to acquire new states as a result of interactions with its environment. Hence, the stimulus in this case is not a measurement-type interaction, but any interaction represented by a particular Hamiltonian (we should keep in mind that measurements constitute a special subcategory of the category of all interactions that a particular system can participate in). More interestingly, the manifestations of dynamic dispositions are not macroscopically observed states of affairs, but rather the events of the system's entering new states as prescribed by the Schrödinger evolution. And, of course, these new states in turn can be interpreted in terms of both value dispositions and further dynamic dispositions. Thus the manifestations of dynamic dispositions are actually new dispositions (or, more precisely, events of acquiring new dispositions). This fact may be seen as a drawback, as it arguably leads to a regress (or circularity). Loosely speaking, it may be argued that dynamical dispositions will never properly actualize, as their actualization involves new dispositions, some of which can only be 'actualized' as yet new dispositions, and so on.[4]

However, this situation is not unique to the case of quantum dispositions. Philosophers who subscribe to the view that all properties have dispositional character (so-called dispositional monists) encounter the same difficulty. Any attempt to define properties in terms of their stimulus and manifestation leads either to an infinite regress or to circularity, if we assume that all properties are dispositions. Alexander Bird (2007: 138-146) has given a convincing argument in support of the claim that the regress/circularity does not necessarily imply that the identities of dispositional properties are impossible to establish.[5] If the entire web of properties interconnected by the stimulus/manifestation relation satisfies a particular structural condition (called the asymmetry condition), then it can be claimed that each property is given its unique identification in terms of its place in the entire structure. Whether this strategy could be applied to the case of quantum dynamic dispositions remains to be seen, but in principle I see no fundamental obstacles to that.

In this paper, I would like to focus exclusively on value dispositions, leaving proper analysis of dynamic dispositions for another occasion. As we have stated, the

[4] One way of circumventing this difficulty is to stipulate that the manifestations of dynamic dispositions should be limited to value dispositions only. Because value dispositions manifest themselves as non-dispositional outcomes of measurements, the regress is blocked in the second step. In this approach, dynamic dispositions would be dispositions to acquire new dispositions to reveal particular values in measurements. However, the trouble with this solution is that seemingly we lose a vital piece of information about the possible ways the system can behave dynamically after acquiring a new set of value-dispositions.

[5] I have discussed Bird's proposal at length in (Bigaj 2010a).

stimuli (the conditions of manifestation) of these dispositions are measurement interactions. But in order for this characterization to be of any use, we have to answer the question of what differentiates measurement interactions from any other interactions that we encounter in quantum mechanics. Thomson-Jones has suggested that we should rely on the general, quantum-mechanical characteristic of measurement processes as is typically given in the literature. The point is that without going into the details of particular interactions which occur during measurements of various physical quantities (spin, momentum, etc.) and which depend on specific experimental setups, we can formulate general requirements that have to be satisfied by any interaction that aspires to be a measurement of a given observable A. However, in what follows I will argue that this approach leads to circularity, and therefore it does not offer a good explication of what value dispositions are. Later I will consider some possible strategies of avoiding the problem.

3. THE QUANTUM THEORY OF MEASUREMENT

Let us start with presenting some general characteristics of quantum measurements which I announced in the previous section. As van Fraassen puts it, we would like to be able to answer the following question:

> Given systems X and Y, and observables A pertaining to X and B pertaining to Y, what must be minimally required of these in order to measure A on X by means of Y (chosen as apparatus) with B designated as pointer-observable? (van Fraassen 1991: 210)

An answer to this question is usually presented in the form of several conditions that are imposed upon measurement processes. In my exposition I will follow the approach adopted by Mittelstaedt (1998: 19-40), which I take to be the standard one. Let S denote the system under measurement and M the measuring apparatus. Mittelstaedt assumes that measuring processes consist of three steps: the preparation, the premeasurement, and the objectification and reading. In the first step the system S is prepared in an initial state $\varphi(S)$ while the measuring apparatus is in its 'ground' state $\Phi(M)$. The premeasurement involves an interaction between the system S and the apparatus M that leads to a unitary evolution of the compound system $S + M$. In the final stage of the measurement process the measuring apparatus should indicate some objective value that is interpreted as the measurement outcome. Let us stipulate that the measured observable pertaining to system S is represented by an operator A with the set of eigenvectors φ^{a_i} and eigenvalues a_i, and that the pointer observable Z defined in the state space of system M has eigenvectors Φ_i and eigenvalues Z_i that correspond to a_i via the pointer function f: $a_i = f(Z_i)$. Mittelstaedt accepts that measurements have to satisfy three conditions, the first of which is the calibration postulate:

(1) *The calibration postulate*. If the initial state of S is one of the eigenstates φ^{a_i}, and the measuring apparatus is in its ground state $\Phi(M)$, then after the premeasurement interaction the system $S + M$ should be in the joint state $\varphi^{a_i} \otimes \Phi_i$, where Φ_i corresponds to the value Z_i correlated with a_i: $a_i = f(Z_i)$.

It should be clear that what the calibration postulate effectively says is that a proper A-measurement always reveals the value a_i as its outcome if (but not *only* if) the initial state of the measured system is an eigenstate corresponding to this value. A generalization of this condition for an arbitrary initial state φ of system S takes the form of the probability reproducibility condition.

(2) *The probability reproducibility condition*. If the initial state φ of S is an arbitrary superposition of eigenstates φ^{a_i} ($\varphi = \Sigma_{j=1}^{n} c(\varphi, a_j)\varphi^{a_j}$), then each formal probability $p(\varphi, a_i) = |c(\varphi, a_i)|^2$ should match the relative frequencies of the cases in which the apparatus records the value $Z_i = f^{-1}(a_i)$ in a series of N measurements, where N is arbitrarily large.

Condition (2) ensures that the fundamental interpretive principle of quantum theory known as the Born rule is indeed satisfied in proper measurements. The quantum-mechanical algorithm for calculating probabilities of particular outcomes should be applicable to the statistics obtained in real experiments, provided that the ensemble consisting of identical particles is sufficiently large. Notice that it can be claimed that (1) follows from (2) as its special case. If $\varphi = \varphi^{a_i}$, the probability $p(\varphi, a_i)$ equals one, and according to (2) the outcome a_i is bound to be revealed in all N repetitions of the measurement. Thus it can be concluded that the state of the measuring apparatus after the interaction should be the eigenstate Φ_i, as prescribed by (1). But for our purposes it will be convenient to keep the two postulates separate.

Mittelstaedt imposes a third condition on measurements which he refers to as the pointer objectification postulate. Roughly, this postulate ensures that measurements always have definite results. Mittelstaedt expresses the pointer objectification postulate in the form of the requirement that the reduced mixed state which characterizes the apparatus M after the premeasurement interaction should admit the ignorance interpretation. It is a well-known fact that this postulate leads to an inconsistency with the linearity of quantum evolution.[6] For the purpose of this paper the pointer objectification postulate is irrelevant, hence I will omit it. Thus we can conclude that the conditions (1) and (2) imposed so far constitute a necessary (but not sufficient) criterion for a given interaction to be of the measurement type. That much should be enough to show that the dispositional interpretation of value states indeed ends up in a vicious circle.

[6] For a general exposition of the measurement problem, see Krips 2008.

4. THE CIRCULARITY PROBLEM

At the beginning of the paper, I made a suggestion that the value state of a given system can be interpreted in terms of dispositions, both deterministic and probabilistic. Let us now focus on the deterministic dispositions, associated with eigenstates of measured observables. If system S is in an eigenstate φ^{a_i} of observable A, we can attribute to it the deterministic disposition to reveal outcome a_i in all A-measurements. But now we recall that all interactions which purport to be measurements have to satisfy the calibration postulate (1) as a matter of conceptual necessity. Moreover, it is clear that postulate (1) is supposed to give a non-trivial characteristic of measurements, i.e. a characteristic such that some physical interactions should fail to satisfy it. But when we couple condition (1) with the dispositional interpretation of eigenstates, we face an immediate problem. Let us first suppose that the dispositional interpretation of eigenstates is to be understood as the stipulation that the term 'the eigenstate corresponding to value a_i' should mean 'the deterministic disposition to reveal value a_i upon measurement'. In that case the definition is clearly circular, as the meaning of the word 'measurement' in turn is given with the help of the term 'eigenstate'.

The appearance of circularity in the above definition can be eliminated if we elect to get rid of the notion of an eigenstate altogether and present the calibration postulate directly in terms of dispositions. The proposed reformulation of the postulate would be as follows: if at the beginning of an interaction with the measuring apparatus the measured system possesses the (sure-fire) disposition to reveal value a_i in this interaction, and the initial state of the measuring apparatus is $\Phi(M)$, then the state of the apparatus after the interaction will be Φ_i (the apparatus will record value a_i as the outcome). Unfortunately, the suggested reformulation hardly imposes any restriction at all on the set of interactions between the system S and the apparatus M. To see this, let us suppose that we want to show that a given interaction does not satisfy the dispositional version of the condition (1). In order to do that, we would have to find a system that possesses the disposition to give rise to an outcome a_i, but in spite of that, at the end of the interaction with apparatus M, the state of M is different than Φ_i. But in that case we could simply deny that S had the required disposition to begin with. Thus virtually any interaction between systems S and M can pass the test.[7]

One possible strategy to avert the circularity/triviality problem in this case could be to abandon the assumption that we should *reduce* the notion of an eigenstate to

[7] It may be also pointed out that there is another difficulty associated with this approach. If we insist on interpreting all quantum states as dispositions, then the requirement that the measuring apparatus after the interaction assumes the state Φ_i has to be cashed out in terms of possible measurements done on the measuring apparatus, and this clearly leads to a regress (I am grateful to James Ladyman for alerting me to this problem). One way of avoiding this complication may be to assume that for macroscopic objects their dispositions can be manifested directly, without the mediation of any further measuring devices.

that of dispositions. Perhaps we could instead adopt the view according to which quantum systems are equipped with both quantum states (including eigenstates) *and* dispositions. Apart from the fact that this approach clearly infringes the principle of ontological parsimony (why postulate two separate types of entities if one type can do the explanatory job?), it has another unpalatable consequence that we should be aware of. Notice that thanks to the calibration postulate the relation between being in an eigenstate and possessing an appropriate disposition becomes that of *analytic* entailment. But how can two separate entities be connected by a conceptually necessary link (as opposed to a merely nomically necessary one)? How can it be that in all possible worlds, including worlds with radically different laws of nature, it remains true that all objects with one property have the other one? It seems to me that there are only two acceptable explanations of this fact: either the two connected properties are in fact the same property (in which case we are back to the problem of circularity), or the alleged disposition is not a property in the strict sense of the word but rather some sort of a Rylean 'inference ticket' (which implies abandoning the programme of the dispositional reinterpretation of quantum mechanics).

The problem of circularity is not limited to the case of deterministic dispositions. It is straightforward to see that an analogous argument can be applied to probabilistic dispositions in the context of the probability reproducibility condition (2). According to the suggested interpretive rule, being in state φ which is an arbitrary combination of eigenstates φ^{a_i} implies that the system possesses a set of probabilistic dispositions D_i to give rise to outcome a_i with the objective chance being precisely equal to $p(\varphi, a_i)$. If we assume, as seems appropriate, that objective chances are revealed as relative frequencies when the number of repeated experimental runs goes to infinity, then it is clear that we will end up with the same conceptual difficulty as in the previous case. For now the condition (2) can be reinterpreted as stating that if the system possesses the disposition to give rise to outcome a_i with probability $p(\varphi, a_i)$ as a result of interaction with the measuring apparatus, then the relative frequency of a_i in a long run of measurements will match $p(\varphi, a_i)$. And again, this stipulation can be either interpreted as circular (if we define appropriate dispositions with explicit reference to the notion of measurement) or tautologous (we are effectively saying that *any interaction* with the measurement apparatus that gives rise to some well-defined frequency of outcome a_i is a sign that the system possesses the corresponding propensity).

5. PROBABILITY-FREE INTERPRETATION OF QUANTUM MEASUREMENT

Despite appearances, it turns out that there is an important difference between the circularity problem that affects deterministic dispositions and the similar problem impacting probabilistic dispositions. This has to do with the relations between the calibration postulate and the probability reproducibility conditions. I mentioned earlier that condition (1) can be seen as a special case of condition (2). Surprisingly, it can be shown that quite the opposite is also the case: in a sense the probabilistic postulate encompassed in the condition (2) can be derived from the probability-free condition (1) with the help of some quantum-mechanical formalism. This result, due to Mittelstaedt, can give us a glimmer of hope that the circularity problem can be overcome at least with regard to probabilistic dispositions. If the only condition that any measurement interaction has to satisfy (apart from the controversial pointer objectification requirement) is the probability-free calibration postulate, then the direct threat of circularity affecting probabilistic dispositions seems to be averted. Because Mittelstaedt's theorem is an important and surprising result that is relevant not only to the dispositional approach to quantum theory but potentially to other interpretational approaches as well (and yet, to my knowledge, its consequences has not been widely discussed in the literature), it may be worthwhile to present it in greater detail.

Let us consider N identically prepared systems, each of which is in the same initial state $\varphi^{(i)}$. The state of the entire system φ^N will be of course presented as the tensor product $\varphi^{(1)} \otimes \varphi^{(2)} \otimes \ldots \otimes \varphi^{(N)}$. We suppose that each system undergoes a measurement of an observable A, hence a full measurement consisting of N single measurements will reveal an N-tuple of numbers as its outcome. The initial individual states of the systems are supposed to be non-trivial superpositions of the eigenstates of the observable A, thus they can be presented as follows: $\varphi^{(i)} = \Sigma^n_{j=1}\, c(\varphi, a_j)\varphi^{a_j}$. The squared norm of the coefficients c defines, of course, the Born probabilities, which we can call 'formal probabilities', because for now we are not allowed to assume that they can be interpreted as real probabilities (frequencies) of particular outcomes (this is what we intend to prove). Let's symbolize those formal probabilities as $p(\varphi, a_i) = |c(\varphi, a_j)|^2$.

In the next step we should focus on the measuring apparatus, considered as a complex consisting of N individual apparatuses. The states of the apparatus are vectors in an N-fold tensor space. By Φ_i we will symbolize the state of a singular apparatus that corresponds to its pointer value Z_i, and therefore to the value a_i of measured observable A. Hence the state of the entire N-tuple of apparatuses after successful completion of measurement should be given by the vector $\Phi_l^N = \Phi_{l_1} \otimes \Phi_{l_1} \otimes \ldots \otimes \Phi_{l_N}$ corresponding to the following sequence of outcomes $(a_{l_1}, a_{l_1}, \ldots a_{l_N})$. The ingenious method of introducing a quantum-mechanical equivalent of probabilities that Mittelstaedt follows in his approach relies on a new operator F_k^N whose 'intuitive' role is to measure the relative frequency of the outcome a_k in a given sequence of N outcomes. Formally we will stipulate that F_k^N operates in the state space of the total measuring apparatus, and its eigenvectors are vectors Φ_l^N with corresponding eigenvalues $f^N(k, l)$, each of which is equal to the number of occurrences of the outcome a_k in the sequence of outcomes corresponding to Φ_l^N divided by N. Hence F_k^N can be defined as the following sum of the projection operators (note that this is a case of degeneracy, as distinct eigenvectors differing only by permutations of outcomes a_k will correspond to the same eigenvalue of F_k^N):

$$F_k^N = \sum_l f^N(k,l) P[\Phi_l^N]$$

From the calibration postulate and the assumption of linearity of quantum-mechanical evolution it can be derived in the standard fashion that after the measurement interaction each single measuring apparatus will be in the mixed state given as follows:

$$W'_M = \sum_i p(\varphi, a_i) P[\Phi_i]$$

and the state of the entire measuring device will be the following mixture:

$$W'^N_M = \sum_l p_l P[\Phi_l^N]$$

where $p_l = p(\varphi, a_{l_1}) \ldots p(\varphi, a_{l_N})$. Now Mittelstaedt formulates and proves the following mathematical theorem ('probability theorem' in his terminology, see Mittelstaedt 1998: 48, 125-127):

(PT) $$\{W'^N_M (F_k^N - p(\varphi, a_k))^2\} = 0$$

He interprets this formal result as showing that in the limit when N is infinite, the value of the operator F_k^N after the measurement interaction approaches the formal probability $p(\varphi, a_k)$ arbitrarily close. If that is the correct interpretation of (PT), then indeed we can claim that effectively the probability reproducibility condition has been derived from the 'probability-free' interpretation of quantum measurement that includes the calibration postulate as the only restriction imposed on measurement interactions. For certainly eigenvalues of F_k^N represent relative frequencies of post-measurement outcomes a_k (that is how we have defined this operator), so if these frequencies as obtained in real experiments tend to approach numbers $p(\varphi, a_k)$ derived from the quantum-mechanical formalism, this shows that the numbers $p(\varphi, a_k)$ are not merely *formal* probabilities, but full-fledged chances of outcomes obtained at the end of the measurement interaction. However, we have to be certain that the step from the formal theorem (PT) to its subsequent interpretation does not contain any hidden premises that go beyond the probability-free interpretation of quantum-mechanical formalism.

At first sight, Mittelstaedt's reading of (PT) seems to follow immediately from the textbook interpretation of the trace operator and its function in quantum mechanics. It is part of the standard reading of the quantum-mechanical formalism that the expression Tr(WA), where W is a density operator and A any Hermitean (self-adjoined) operator, represents the expectation value of A when the system is in the state described by W (this can be seen immediately when we apply the definition of trace as the sum of all diagonal elements of a given operator: $\mathrm{Tr}\{W'^N_M(F_k^N - p(\varphi, a_k))^2\} = \Sigma_l \langle \Phi_l^N | W'^N_M(F_k^N - p(\varphi, a_k))^2 \Phi_l^N \rangle = \Sigma_l p_l (f^N(k, l) - p(\varphi, a_k))^2$.) Given this interpretation of trace, (PT) can be expressed as stating that the expectation value of the operator measuring the difference between the relative frequency operator F_k^N and the formal probability

$p(\varphi, a_k)$ goes to zero for arbitrary large numbers N when the measuring apparatus is in the state W'^N_M. However, the very notion of the expectation value presupposes that there is a well defined distribution of probabilities over possible values of A (in our case represented by numbers p_l), so this can hardly be called a 'probability-free' interpretation. Hence, we have to delve deeper into the mathematical and interpretative significance of (PT).

Mittelstaedt (48) suggests an alternative way of looking at (PT), as is clear from the following fragment:

> In the limit of large N the post-measurement product state W'^N_M becomes an eigenstate of the relative-frequency observable F^N_k, and the relative frequency of the pointer value a_k approaches the probability $p(\varphi, a_k)$.

Can this conclusion be proven on the basis of the formal result (PT)? Mittelstaedt is silent about the details of how to do this, but we can try to supply the missing elements. To prove that W'^N_M is indeed an eigenstate of F^N_k with the corresponding value $p(\varphi, a_k)$, we have to show that the equation $F^N_k W'^N_M = p(\varphi, a_k)W'^N_M$ holds true when N approaches infinity. Abbreviating the operator $F^N_k - p(\varphi, a_k)$ as Ω we can present the aforementioned eigenequation in a simplified way: $\Omega W'^N_M = 0$. But observe that in this notation (PT) can be presented equivalently as $\mathrm{Tr}(\Omega^2 W'^N_M) = 0$. Can it be inferred from the fact that the trace of a given operator equals zero that the operator itself must be zero? Generally such an inference is invalid, however in our case we can appeal to the fact that the operator $\Omega^2 W'^N_M$ is positive, as it is the product of two positive operators (density matrices being positive in virtue of their definition). More specifically, we can diagonalize the matrix $\Omega^2 W'^N_M$ in the basis Φ^N_l, and in that way we will obtain a matrix whose diagonal elements will be non-negative real numbers $(f^N(k, l) - p(\varphi, a_k))^2 p_l$. As (PT) implies that the sum of all diagonal elements has to be zero (by the definition of trace), we can infer that the elements themselves have to be zero too, and because all the non-diagonal elements of the diagonalized matrix are already null, we conclude that the matrix $\Omega^2 W'^N_M$ approaches zero when N goes to infinity. From this it easily follows that $\Omega W'^N_M = 0$ in the limit, and hence W'^N_M indeed becomes an eigenstate for F^N_k when N is arbitrarily large.

Let us take stock of the last paragraphs. Mittelstaedt's result shows that if the calibration postulate is satisfied by a class of interactions between the system S and the measuring device M, then the frequencies of particular outcomes revealed as a result of these interactions will (in the limit) match the numbers derived from the initial state of S with the help of the usual quantum-mechanical algorithm. Thus no separate probabilistic postulate is necessary to characterize measurements. If we agree that the meaning of the concept of probability is completely exhausted by the relative frequency interpretation, then we can also conclude that there is no need to include any probabilistic interpretational rule (such as the Born rule) in the quantum-mechanical formalism whatsoever! However, it is important to keep in mind that in order to derive the probabilistic postulate from the probability-free interpretation of measurements we had to rely on premises that go beyond definitional characteristics of measurements. The most important of these premises is the eigenstate-eigenvalue (e/e) link that enabled us to make inference from the mathematical fact that W'^{N}_{M} is an eigenstate of the operator F^{N}_{k} with a particular value $p(\varphi, a_k)$ to the empirical conclusion that the actual observed frequency of a given outcome will be equal to $p(\varphi, a_k)$.[8] While the e/e link does not mention probabilities, it is still a factual claim that cannot be treated as true in virtue of conceptual (or mathematical) necessity.

6. THE CIRCULARITY REVISITED

How does the above-mentioned result bear on the problem of circularity that threatens the dispositional interpretation of quantum value states? As I have already tentatively suggested, if we are able to eliminate the probability reproducibility condition from the set of rules that jointly define the meaning of the term ‘measurement’, then the probabilistic dispositions seem not to be afflicted by circularity, in contrast to deterministic ones. But one may raise the following objection to this conclusion. The argument from the last section proves that postulate (2) is derivable from (1), hence although technically we can eliminate its presence, logically it will still be ‘there’. It is irrelevant — the objection goes — whether we decide to write down the probability reproducibility condition as a separate postulate, or whether, more economically, we elect to eliminate the redundancy and narrow down the postulates to the calibration condition only. In either case the condition (2) will remain satisfied by the class of measurement interactions, so how can we claim that the probabilistic dispositions will not be affected by the circularity problem?

The answer to this objection is based on the fact which I have already emphasized: postulate (1) itself does not imply (2) logically or mathematically but only in tandem with some extra factual assumptions.[9] Consequently, although (2) remains true about quantum measurements (as long as the premises required for its derivation are true), its truth is no longer a matter of conceptual necessity. Measurements do not *have* to satisfy (2) but only *happen* to satisfy it. Thus meeting condition (2) is not part of the meaning of the word ‘measurement’, and as a result the way we characterize probabilistic dispositions by reference to measurements does not lead to a vicious circle, for we can correctly pin down the meaning of the term ‘measurement’ without mentioning any probabilities or frequencies. To put it differently and perhaps more emphatically, postulates (1) and (2) taken together define a notion of measurement that is coextensional with the notion defined by (1) only, but these two notions have different intensions.

---

[8] Another extra premise is the linearity assumption that we mentioned earlier in the derivation.

[9] Note that contrary to this case, the implication from (2) to (1) that we have acknowledged earlier is of the logical type, for (1) is just a special case of the condition (2) if the latter is interpreted broadly as covering cases when φ is just one of the eigenvectors of A.

But the question remains what to do with deterministic dispositions which are still causing problems. A radical remedy would be to eliminate them altogether and stick to the probabilistic ones. This is actually the approach advocated by some proponents of the dispositional interpretation of quantum mechanics. The suggestion is to make the following distinction: dispositional properties characterize a system only with respect of observables for which the system is not in an eigenstate, whereas eigenstates are interpreted with the help of categorical properties of possessing a particular value of the observable. This approach to the notion of dispositions in quantum mechanics is motivated by the desire to extend the usual eigenstate-eigenvalue link, which provides a 'metaphysical' interpretation of eigenstates in terms of possessing categorical properties, while being silent about how to interpret situations in which the system is not in an eigenstate (other than implying that in this case the system *does not* possess any definite value; note that some interpretations of quantum mechanics, such as the Bohmian theory or modal interpretations, question just that implication). But now we can give a more specific answer to the question of how to interpret states that are non-trivial superpositions of eigenstates: if a system S is not in an eigenstate of observable A, it possesses a set of probabilistic A-dispositions (propensities) to reveal particular values with appropriate chances defined by its quantum state.[10] Mittellstaedt's result seems to support this approach to dispositions in quantum mechanics, for probabilistic dispositions can escape the circularity problem while deterministic ones apparently cannot.

In spite of this, I believe there are some reasons for hanging on to the notion of deterministic value dispositions. One of them (rather less important) may be the fact that it seems somewhat arbitrary to draw a sharp metaphysical distinction between categorical and dispositional properties on the basis of the mathematical difference that can be made infinitely small. For clearly non-eigenstates can approximate eigenstates arbitrarily closely: the superposition of two non-degenerate eigenstates $\varphi = \sqrt{\frac{1}{n}}\varphi_1 + \sqrt{\frac{n-1}{n}}\varphi_2$ is technically not an eigenstate for any arbitrarily large n, and yet it approaches φ_2 infinitesimally close when n goes to infinity. But why should we insist that when the probability of obtaining a particular outcome a equals 0.99999, the system possesses the propensity to reveal value a, while when the probability reaches 1, the disposition disappears, replaced by a firm categorical property?

[10] Dorato in (2006) makes it clear that in his approach the division between dispositional and categorical properties coincides with the division between indefinite and definite values of observables. On the other hand, the way Thomson-Jones proposes to amend the eigenstate-eigenvalue link is more flexible. He assumes that if a system is not in an eigenstate for a given observable A, then it possesses a set of A-related dispositions, but he leaves it open whether A-related properties of a system that *is* in a corresponding eigenstate should be interpreted as categorical or dispositional (Thomson-Jones, unpublished manuscript, p. 14). In personal communication Thomson-Jones admitted that he is inclined to interpret some definite quantum properties (such as spin) as deterministic dispositions, but he has reasons to reject the general claim that all such properties are dispositional.

But there is a more robust argument showing that in some cases it is beneficial to speak about dispositions of a system even though the system is in an eigenstate of an appropriate observable. Consider a compound system of two particles whose relative position is fixed, although each particle taken separately is not properly localized (this is obviously the famous entangled state that was introduced in the original EPR argument). This setup implies that the state of the combined system is one of the eigenstates of the operator $D = X_1 - X_2$ representing the difference between the particles' positions. According to the standard reading of the e/e link, we have to infer from this that the system possesses the following categorical property: the particles *are* precisely d meters apart. But the last locution apparently implies the statement 'Particle 1 is located d meters away from particle 2', which in turn entails that particle 1 is located *somewhere*, and this contradicts the fact that the separate states of the particles are not eigenstates of their position operators. This case is just one of many examples of the general situation in which a compound system is in an eigenstate of an operator defined as a function of some observables pertaining to separate components, while none of the components is in an eigenstate of their respective observables (another well-known example is the singlet-spin state of two spin-1/2 particles, where the total spin equals zero while separate spin-components remain indeterminate).

I believe that the dispositional interpretation of eigenstates can come in handy here, because it eliminates the appearance of inconsistency. Instead of maintaining that the EPR particles are *definitely d* meters apart, we can describe the situation by saying that they have the deterministic disposition to localize at a fixed spatial separation equal to d. This is perfectly compatible with the fact that the dispositions of separate particles to localize at a given region are irreducibly probabilistic. Similarly, we can insist that the total spin in the singlet-spin state is not actually equal to zero before measurement (which would suggest that the separate spins which contribute to the total value are already determinate), but rather that the system possesses the sure-fire disposition to disclose the separate spins as anticorrelated no matter what the outcomes of their measurements are. Notice that we have a clear case of non-supervenience (holism) here: the probabilistic dispositions pertaining to separate observables do not determine the sure-fire disposition of the entire system. But at least we can exploit a clear distinction between possessing a property and actualizing a disposition. No actualization of the 'global' disposition of an entangled system is possible without a prior actualization of separate 'local' dispositions, and consequently no categorical property characterizes the system before measurement, even though the system as a whole is in an eigenstate.

Finally, if we associate categorical A-properties with every eigenstate of A, we have to accept that quantum mechanics is strongly non-local, i.e. that it is possible to immediately change a physical property of a distant system by merely selecting a local observable for measurement (this is akin to so-called parameter dependence). Henry P. Stapp has shown that when two spin-half particles are prepared in a certain state known as the Hardy state, the probability of obtaining a particular outcome on one particle is 1 given that the other particle undergoes measurement of a certain observable (regardless of its outcome), while when an alternative selection is made, the probability drops to less than 1.[11] Under the categorical interpretation of eigenstates it looks as though a mere selection of a parameter to measure can create a new categorical property of a distant system. On the other hand, the dispositional account of eigenstates may be used to blunt the threat of non-locality thanks to the fact that the notion of a dispositional property is more flexible than that of a categorical property (it may be claimed, for instance, that quantum dispositions of systems do not have to be localized entirely where the system is located). The details of such an approach have to be worked out on a different occasion.

7. ALTERNATIVE SOLUTIONS

If we are convinced by these arguments, we have to face the problem of circularity again. How can we characterize the concept of measurement which figures in the definitional characteristic of deterministic value dispositions without referring back to the notion of disposition itself? For now I can only offer some tentative suggestions of solutions that are far from being satisfactory. It may be proposed, for instance, that instead of giving a universal quantum-mechanical characteristic of measurements we should rely on low-level, particular descriptions of experimental setups that are used in real laboratories to measure physical quantities, such as spin. For example, we may insist that a measurement of the spin of an electron in a given direction is just a physical interaction of this electron with a Stern-Gerlach magnet appropriately oriented in space. An obvious disadvantage of this approach is the fact that there can be more than one way of measuring one and the same quantity (for instance, instead of checking the deflection of the trajectory of an electron in a magnetic field we may want to detect the electromagnetic radiation that is emitted by the electron when its spin aligns itself with the direction of the field, and in that way record the direction of the spin along a given axis: if there is no radiation, the direction is 'up', if the radiation occurs, the direction is 'down'). Consequently, we would have more than one disposition associated with the same physical quantity, and we

[11] (Stapp 1997) contains the original formulation of the proof, while its most recent version can be found in (Stapp 2004). Stapp's argument is based on the logic of counterfactual conditionals, where the truth of the statement 'If observable A were measured, the outcome would be a' is taken as a counterfactual interpretation of the fact that the probability of the outcome a is one. Stapp claims that his argument proves conclusively that quantum theory is strongly non-local. In my extensive analysis of his claim, I show that the argument goes through only if we assume that there is a categorical property of the system which underlies the truth of the aforementioned counterfactual (Bigaj 2010b).

would have to face the pressing question why all these dispositions happen to agree with one another.

Another possibility is to enlist help from some non-standard interpretations of quantum mechanics in general and the notion of measurement in particular. Let me use an example: recently a lot of attention has been given to the interpretation known as the spontaneous localization theory (the GRW theory).[12] This interpretation is based on the conjecture (as of yet experimentally unconfirmed) that each quantum particle is characterized by a minute chance of 'jumping' from its unlocalized state to a state with some precise position. The chance of this event is so small that the evolution of a single quantum object is for all practical purposes as predicted by the standard Schrödinger equation, but if a sufficiently big number of quantum objects get correlated with one another, the probability that one of them will undergo a spontaneous localization and 'drag' the remaining particles with it is virtually one.

The GRW theory interprets measurements, in a traditional way, as interactions of microscopic quantum systems with macroscopic objects, and the recorded outcome is explained with the help of the spontaneous localization of a huge number of individual particles constituting the measuring device. Thus it may be theoretically possible to characterize measurements of particular observables without using postulates (1) and (2). Of course, certain prerequisites have to be satisfied in order for this to happen. As the fundamental assumption of the GRW theory is that position is the only quantum parameter that can spontaneously acquire an almost precise value, we have to find a way to correlate all other measurable properties with the position of some macroscopic objects. But it is possible to devise a measurement whose outcomes are differentiated only by a small displacement of a couple of photons, and yet due to the extreme sensitivity of the human retina such different outcomes can be discerned by an experimenter.[13] In order to deal with such cases, the physiological details of the perception process have to be taken into account. As is the case with virtually all attempts to solve the perennial problem of quantum measurement, the GRW theory has its share of internal difficulties, including the famous problem with the 'tails' of the wavefunction and the trouble with the proper relativization of the theory.

Personally, I am attracted to a different approach which could solve the circularity problem with value dispositions in a radical way. Unfortunately, it is also an approach that is no more than a promissory note rather than a fully developed theory. Let us first notice that from a metaphysical point of view there is something deeply unsatisfactory about value dispositions in that they are presented with the help of a notion which has clear observer-related connotations. The standard notion of measurement is definitely biased towards human observers and their sensory capacities, because an essential feature of measurements is that they should enable observers to

[12] For an extensive exposition of this theory prepared by one of its creators, see Ghirardi 2008. A more popular presentation can be found in (Ghirardi 2005: 404-436).

[13] See Albert, Vaidman 1989, Albert 1992.

gather information about the physical state of the system. Thus dispositions whose stimuli conditions involve measurements are unlikely candidates for fundamental properties of physical systems. After all, in possible universes in which there are no humans there is nothing that could privilege measurements over any other interactions. We may even wonder if measurements constitute a natural kind of physical interactions between systems. Taking this into account, we may agree that the spin property of a system is connected (even as a matter of physical necessity) with the disposition of the system to produce certain outcomes in spin-measurement interactions, but this disposition does not seem to constitute *the essence* of the property.[14] But this observation leaves us with the pressing question of what is the real dispositional essence of spin. Unfortunately, I am unable to give a fully satisfactory dispositional account of quantum properties — I can only suggest what such an account should look like. As we remember, dynamic dispositions of quantum systems are characterized in terms of interactions with the environment and the subsequent evolution leading to the future states. Analogously, we could think of value properties as dispositions whose stimuli are interactions with the environment, but whose manifestations are the events of acquiring new states not by the system that possesses the property, but by the objects that the system interacts with. Thus dynamic dispositions would be *passive*, while value dispositions *active*.

The main difference between the currently considered approach and the measurement-based approach to value dispositions is that we do not require the object which the property-bearer interacts with to be macroscopic, or to have anything that could play the role of a 'pointer' indicating particular values. Value dispositions are just interpreted as powers to change other systems' properties, no matter what these objects and properties are. As an illustration of this difference, let us consider a two-slit experiment with electrons. The property of an electron of being located near one of the two slits can be interpreted as the set of dispositions to change some properties of other particles near this slit. Suppose that another electron happens to sit next to the first slit and that the electron that passes through this slit inevitably flips the spin of the other electron. Strictly speaking, this interaction does not constitute a measurement, for an observer is incapable of telling what the spin of the 'detecting' electron is after the interaction. And yet we may say that the fact that the first electron passed through the first slit gets 'recorded' in the final state of the other electron. Thus the disposition to flip the spin of the second electron can count as a representative of the position of the first one. Interestingly, the interaction with the 'detecting' electron next to one of the slits has an observable effect on the behaviour of the

[14] Bird (2007: 157-158) notes that although the existence of a property may imply a wide range of counterfactual conditionals, not all of them can represent the essence (nature) of that property. Referring to the famous example of triangularity used by Hugh Mellor, Bird observes that it is unintuitive to assume that the disposition expressed in the conditional 'If someone were to count the corners of *x*, the result would be three' reveals the nature of triangularity, for even in possible worlds in which there are no agents capable of counting there would still be triangular objects.

electrons passing through the slits; namely, it destroys the interference pattern created by adding two probability amplitudes associated with the electrons passing through either one or the other slit (this fact is usually interpreted as indicating that a process broadly analogous to measurement is taking place here).

An obvious disadvantage of this new dispositional interpretation of value ascriptions is that there will be multitudes of distinct dispositions corresponding to a given measurable property. Clearly, there are numerous ways an object in a certain state can interact with its environment, depending on the particular characteristics of the environment and its relations with the object. Thus properties such as spin, momentum, position, etc. would turn out to be multi-track dispositions with different stimuli and manifestations, and as such would not count as fundamental physical properties. But at least we could interpret them as *clusters* of the fundamental dispositions to induce changes in the state of external objects. It remains to be seen whether all dispositions associated with a given observable can be shown to have some unique characteristic in common which differentiates them from other dispositions and justifies using a single umbrella term to cover them all. If achieving this goal was viable, we could free ourselves from the unfortunate consequences of including the observer-dependent notion of measurement in the description of fundamental properties of the physical world.

BIBLIOGRAPHY

Albert D. (1992), *Quantum Mechanics and Experience*, Cambridge (MA): Harvard University Press.

Albert D., Vaidman L. (1989), *On a Proposed Postulate of State Reduction*, „Physics Letters A" 139(1), 1-4.

Bigaj T. (2010a), *Dispositional Monism and the Circularity Objection*, „Metaphysica" 11, 39-47.

Bigaj T. (2010b), *How to (Properly) Strenthten Bell's Theorem Using Counterfactuals*, „Studies in History and Philosophy of Modern Physics" 41, 58-66.

Bird A. (2007), *Nature's Metaphysics*, Oxford: Clarendon Press.

Dorato M. (2006), *Properties and Dispositions. Some Metaphysical Remarks on Quantum Ontology* [in:] *Are There Quantum Jumps? On the Present Status of Quantum Mechanics*, A. Bassi, D. Durr, T. Weber, N. Zanghi (eds.), Melville (NY): American Institute of Physics, 139-157.

Dorato M. (2007), *Dispositions, Relational Properties and the Quantum World* [in:] *Dispositions and Causal Powers*, M. Kistler, B. Gnassonou (eds.), Burlington (VT): Ashgate, 249-270.

Ghirardi G. (2005), *Sneaking a Look at* God's Cards, Princeton–Oxford: Princeton University Press.

Ghirardi G. (2008), *Collapse Theories* [in:] *The Stanford Encyclopedia of Philosophy (Fall 2008 Edition)*, E. Zalta (ed.), URL = <http://plato.stanford.edu/archives/fall2008/entries/qm-collapse/>.

Krips H. (2008), *Measurement in Quantum Theory* [in:] *The Stanford Encyclopedia of Philosophy (Fall 2008 Edition)*, E. Zalta (ed.), URL = <http://plato.stanford.edu/archives/fall2008/entries/qt-measurement/>.

Mittelstaedt P. (1998), *The Interpretation of Quantum Mechanics and the Measurement Process*, Cambridge: Cambridge University Press.

Stapp H. P. (1997), *Nonlocal Character of Quantum Theory*, „American Journal of Physics" 65, 300-304.

Stapp H. P. (2004), *A Bell-Type Theorem without Hidden Variables*, „American Journal of Physics" 72, 30-33.

Suárez M. (2004a), *Quantum Selections, Propensities and the Problem of Measurement*, „The British Journal for the Philosophy of Science" 55(2), 219-255.

Suárez M. (2004b), *On Quantum Propensities. Two Arguments Revisited*, „Erkenntnis" 61(1), 1-16.

Suárez M. (2007), *Quantum Propensities*, „Studies in History and Philosophy of Modern Physics" 38, 418-438.

Thompson I. J. (1988), *Real Dispositions in the Physical World*, „The British Journal for the Philosophy of Science" 39(1), 67-79.

Thomson-Jones, M. (unpublished manuscript), *Dispositions and Quantum Mechanics*.

Van Fraassen B. (1991), *Quantum Mechanics. An Empiricist View*, Oxford: Clarendon Press.