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Natural capital in economic models[3]

1. INTRODUCTION

The problem of economic sustainability primarily covers issues of intergenerational equity (i.e. the concern for the well-being of future generations), the preservation of the capacity of natural capital to provide benefits important for social welfare, as well as the possibility of substituting natural capital with other forms of capital (Toman at al., 1995, p. 140). The role of natural capital, as the factor of growth and economic development, unfortunately is not emphasized strong enough in the mainstream economics. Sustainability concerns make the role of natural capital in the process of creating the social welfare more vivid. Many of interdisciplinary research indicate the crucial role of biodiversity for the ability of ecosystems to provide the ecosystem services (Cardinale at al., 2012, p. 59–67). There exists a fundamental discrepancy on the theoretical and methodological level between the mainstream and ecological economics. The most important issue is the substitutability of natural capital by another forms of capital (e.g. manufactured, human, etc.)

In the area of sustainability economics a lot of research was done in both, theoretical and empirical aspects (Pezzey, Toman, 2002). However, still the main approach is to use the perspective of neoclassical economics. It seems reasonable to introduce some aspects of ecological economics to the mainstream economics, in particular – to the analysis of the growth in the long run.

The goal of our paper is to make a critical analysis of selected growth models that use the notion of natural capital and construct the alternative model. In particular we treat the natural capital as a renewable resource and we use CES production function, weakening the assumption of substitutability of natural capital with other forms of capital. Therefore, our approach follows the main postulate of ecological economics, i.e. limited substitutability of natural resources.

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2. THE BASIC ASPECTS OF ECOLOGICAL ECONOMICS – CONCLUSIONS FOR MODELLING

The natural capital is a quite new concept, being developed from the beginning of 90s. According to Constanza, Daly (1992) the natural capital is the extension of economic concept of capital as "a stock that yields a flow of valuable goods or services into the future" on environmental goods and services. The natural capital is understood differently in the mainstream economics and ecological economics. The mainstream economics focuses the attention on the role of natural resources (in particular – fossil fuels), while the ecological economics emphasizes those elements of natural capital, which creates ecosystems. The natural capital in the form of ecosystems provides many diverse ecological services for both, production and consumption, as well as for the maintenance of the life on the Earth. It may be said that the ecological services represent the stream of benefits, gained by humans from natural capital[4].

One of the first complete classifications of ecological services was the one proposed by Constanza at al. (1998, p. 253–260). The classification, which is most often referred to in the recent literature, is the one presented in the Millenium Ecosystem Assessment – thirty one ecological services were identified and grouped into four categories: supporting, provisioning, regulational and cultural (see Millenium Ecosystem Assessment, 2005, p. 40).

According to England (1998, p. 8) the starting point for defining the natural capital should be the theory of production by Georgescu-Roegen, which recognizes two main elements of production: funds elements, which represent the agents of production process, and flow elements, which are used and transformed by agents.

One of the most important problems that are considered in ecological economics is the issue of substitutability of natural capital by another form of capital (material production factors, knowledge, etc.). Ecological economics follows the rule of limited substitutability resulting in the idea of strong sustainability (Hediger, 2006).

Moreover, ecological economics postulates the existence of some limiting boundaries for usage of nature. Passing them makes a serious danger for ecosystems sustainability in the local and global scales. The attempt of estimation of those values was undertaken in the international research project (Rockström at al., 2009). According to critics setting the limits of growth is quite arbitrary if one takes into account that six of the mentioned limits, i.e. changes in the land use, loss of biodiversity, nitrogen level, consumption of the drinking water, chemical and aerosol pollutions, have local character, not global. Therefore there is no

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[4] The links between biodiversity and the provision of services by ecosystems are important arguments for the protection of biodiversity and against the disappearance of species. However, as noted by Wilson (2002): "*The loss of the ivory-billed woodpecker has had no discernible effect on American prosperity. A rare flower or moss could vanish from the Catskill forest without diminishing the region's filtration capacity. But so what? To evaluate individual species by their known practical value at the present time is business accounting in the service of barbarism.*"

limit, that after passing it those processes start functioning in a fundamentally different way. There is too little evidence to state that breaking the limits in any of the mentioned areas would have negative influence on the social welfare (Nordhaus at al., 2012). The notion of critical natural capital is also developed within the ecological economics (Ekins, 2003; Chiesura, De Groot, 2003).

Taking into account the discussed issues we claim that the ecological growth models should meet the following assumptions:

1. The natural capital and other forms of capital have limited substitutability. This fact may be modelled in two nonexcluding ways – choosing a proper production function (CES or Leontieff) or taking some additional assumptions.
2. The natural capital has significant influence on the social welfare via economic sphere (resource function) as well as via direct influence on the well-being (regulatory and cultural functions). This should be captured by utility function, in particular in those models, where analysis is done from the social planner point of view.
3. The natural capital in the form of ecosystems is characterized by its internal dynamics and regeneration ability. At the same time the economic activity has negative effect on the resources usage and deterioration of natural environment, what influences the rate of regeneration and the availability of ecological services. Therefore the direction of evolution of natural capital is the result of several opposing factors.
4. The existence of the critical thresholds is postulated, so that the evolution of natural capital should be limited from below.

Ultimately, the adoption of a ”capital” definition of sustainability leads to a change in the research perspective. Instead of models describing the growth process of aggregate output (standard growth models), one should rather focus on modeling the process of shaping the resources of all types of capital, which are considered crucial for social well-being. Apart from the assumptions concerning the dynamics of particular types of capital, the assumptions as to the nature of mutual relations between them are also important.

3. SELECTED APPROACHES TO MODELLING NATURAL CAPITAL IN ECONOMIC PROCESSES

Literature on modelling the natural resource usage is abundant. However, the works in which authors directly use the term *natural capital* and try to describe its dynamics are less frequent. Another criterion for the selection of quoted results was that they present some characteristic and interesting ways of capturing the idea of natural capital in economic models - either using a particular form of production function (Kraev, 2002), considering a two-sector model (Commoli, 2006), a combination with the idea of material consumption (Rodrigues at al., 2005) or directly modeling the interaction between the four kinds of capitals (Roseta-Palma at al., 2010). Undoubtedly, in no way does our selection exhaust the rich literature of the subject.

The basic principles of ecological economics and the issues of formal description are discussed by Kraev (2002). Assuming strong complementarity between anthropogenic (H) and natural (N) capitals (what was expressed by the choice of Leontieff production function $Y = \min(AH, CN)$ [5]), an unbounded economic growth is impossible. However, the economy reacts differently in the long run depending whether the natural capital is treated as the stock or the flow (denoted by n) in the production function. The consequences of depletion of the flow are much more drastic – they lead to the zero production almost immediately (see Kraev, 2002, p. 281). As an attempt to weaken the assumption about strong complementarity Kraev considers a particular case of CES function:

$$Y = ((AH)^{-p} + (Cn)^{-p})^{-\frac{1}{p}}, \quad 0 < p < \infty, \tag{1}$$

where p is a parameter, characterizing the admissible rate of substitution[6].

The production function is characterized by weak complementarity, and simultaneously the main conclusion remains the same. If $AH \gg Cn$, then $Y \approx Cn$. Analogically to the production sector, the author extends the rules of ecological economics to consumption system. He assumes that the social welfare depends also on the ecological services (like clean air, water, the landscape), which are complementarities for the produced goods (market goods), usually considered in microeconomics. The utility function may have different form, nevertheless it should take into account the fact of limited substitutability.

Commoli (2006) considers two-sector economy, in which the produced capital K may be used for production of either, intermediate or final goods, i.e. $K = K_x + K_y$. The intermediate goods[7] are produced with use of capital K_x and natural resources X according to the production technology $M = H(K_x, X)$, in which the substitution is allowed. The final goods are produced with use of M and capital K_y, which are complementarities. Therefore:

$$M = H(K_x, X) = X^{\alpha} K_x^{1-\alpha} \;\; and \;\; F(M, K_y) = \min\left(M, \frac{K_y}{c}\right), \tag{2}$$

where $0 < \alpha < 1$ and $c > 0$ (parameters of model). The natural capital is considered in two forms: as the stock (X) and as the flow (M). Moreover, natural and man-made capitals are substitutes in intermediate goods sector and complementarities in the production of final goods.

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[5] A, C are efficiency factors.

[6] Leontieff function is the limiting case for Y when $p \to \infty$.

[7] The intermediate goods may be understood as extracted resources.

Commoli assumes that the stock is renewable, i.e. $X = g(X) - M$, where g is strictly concave biological recruitment function. Most common assumption is that $g(X) = \gamma\, X\left(1 - \frac{X}{S}\right)$ with some parameter $\gamma > 0$ being intrinsic growth rate of the renewable resources and $S > 0$ being a parameter describing the environmental carrying capacity.

Under the standard assumptions on accumulation of produced capital, the author proves that there exists a stable stationary point satisfying $K > 0$ and $X > 0$ iff the condition: $\gamma > \left(\frac{s}{\delta} - c\right)^{\frac{1-\alpha}{\alpha}}$ is fulfilled (see Commoli, 2006, p. 159). If the economy is in the stationary state (i.e. $\dot{K} = 0$ and $\dot{X} = 0$), then the condition above takes form: $\gamma > \left(\frac{K_x}{X}\right)^{1-\alpha}$. As the right-hand side of the last inequality is not greater than $\frac{K_x}{X}$, the sufficient condition for existence of stationary point can be interpreted as follows: *the intrinsic growth rate of the renewable resource be at least as great as the long-run equilibrium ratio of manufactured to natural capital in the raw materials sector of economy* (Commoli, 2006, p. 159).

Rodrigues at al. (2005) propose a combination of neoclassical growth theory with the concept of allocation of natural capital and economy's dematerialization (the concept developed among others by Bringezu, 2003). They assume that anthropogenic impact depends on the degree of material intensity of the economy. Natural capital usage influences negatively the endogenous dynamics of ecosystems, reducing the volume of available ecological services. The authors show that under some conditions an unbounded growth is possible, keeping the natural capital at some constant level. In this paper the natural capital is divided between a production (fraction u) and "free" natural capital (fraction $1 - u$), directly influencing the social welfare. The natural capital has a character of renewable stock, but it also depends on carrying capacity CC), which evolves[8]. The dynamics of natural capital is (r is the growth parameter of N):

$$\frac{dN}{dt} = rN(CC - N) - P. \tag{3}$$

An increase of free natural capital increases CC, which grows with the growth of the free part of natural capital. This is why the dynamics of CC is described by (see Rodriguez at al., 2005, p. 385):

$$\frac{dCC}{dt} = \frac{l}{(1-u)N}\frac{d(1-u)N}{dt} = \frac{l}{N}\frac{dN}{dt} - \frac{l}{1-u}\frac{du}{dt}. \tag{4}$$

where $l > 0$ is a constant parameter determining the growth of CC. The dynamics of natural capital is affected by structural influences (via dependence on u)

[8] Assuming a constant CC implies that despite the damage in ecosystem it may always renew to the equilibrium value defined by CC. Notice that CC is the upper bound of natural capital stock.

and is characterized by endogenous dynamics of CC. Anthropogenic pressure P depends on amount of materials used in production and consumption. If material intensity is denoted by m, then $P = mY$. Notice that m is a quantity dependent on the technology A and the production Y. The authors assume that the structure of economy changes in result of the growth of production, what influences the quantity m. Therefore:

$$P = m_0 A^{-a} Y^n, \qquad m = m_0 A^{-a} Y^{n-1}, \tag{5}$$

where a, n, m_0 are constant parameters[9].

The production and capital accumulation are assumed in form:

$$\dot{K} = Y - C - \delta\, K, \tag{6}$$

$$Y = AK^{\alpha}\,(uN)^{1-\alpha}. \tag{7}$$

In addition, the dependence between growth rates of technology and capital are assumed:

$$\frac{\dot{A}}{A} = g\left(\frac{\dot{K}}{K}\right), \tag{8}$$

with some increasing, concave and bounded function g, for which $g(\cdot) \equiv 0$ for negative arguments. The utility function accounts the benefits from ”free” natural capital part:

$$U = \ln C + \phi \ln\big((1-u)N\big). \tag{9}$$

An important characteristic of this function is that consumption is independent of environmental conditions. In biophysical steady-state $\dot{N} = \dot{CC} = 0$, what implies constant natural capital stock. The authors show that the economic growth is possible with constant level of natural capital, if the ratio of parameters a/n is bounded[10], i.e.:

$$1 + \frac{a}{g_0'} \leq \frac{a}{n} \leq \frac{1}{1-\alpha}. \tag{10}$$

Moreover, if the constraints (dependent on N^* and CC^*) on initial values of A and K are met, then it is possible to maintain increasing consumption (see Rodriguez at al., 2005, p.393).

[9] m_0 is scale parameter. If $n < 1$, then structural changes lead to a decrease in m. For example, the increase in share of service sector leads to smaller material intensity of the economy.

[10] Here $g_0' = g'(0)$ is maximal value of g'.

In many works, the role of human and social capital in creating prosperity is highlighted. Attempts to combine four capitals (i.e. produced, human, social and natural) into one model were undertaken by Roseta-Palma at al. (2010), thus referring to the "capital" definition of sustainable development proposed by Pearce (see Pearce, Atkinson, 1998, p. 9).

It is also worth noting that attempts were also made to empirically evaluate the stock of individual capital types (World Bank, 2006, 2011), one of the conclusions of which was to indicate the specific role of human and social capital for the development of countries. It seems that the advantage of the proposed model is an attempt to take account of the relationships between different kinds of capitals. By denoting K_P, K_H, K_S, K_N respectively manufactured, human, social and natural capital, the evolution of each and the interactions are defined as follows (Roseta-Palma at al., 2010, p. 604):

$$\dot{K}_P = Y - C - \delta_P K_P. \tag{11}$$

Human capital can be used in the production, education, accumulation of social capital and environmental protection (research and development, emission reduction, etc.). Thus, respectively, $K_H = H_Y + H_H + H_S + H_N$ and:

$$\dot{K}_H = \xi H_H + \alpha K_S - \delta_H K_H, \tag{12}$$

where $\xi > 0, \alpha \geq 0$ are efficiency parameters.

Notice that human and social capitals are substitutable in the creation of human capital. Social capital is "produced" with the use of human capital (via the creation of appropriate institutions and regulations), but at the same time its dynamics at all times depends on the size of the social capital, i.e.:

$$\dot{K}_S = \omega\, H_S + \Omega\, K_S, \tag{13}$$

where ω is again the efficiency parameter, and Ω may be positive or negative. The natural capital is again renewable resource, i.e.:

$$\dot{K}_N = R(K_N) - N_Y + P, \tag{14}$$

where $R(K_N)$ is natural regeneration rate, defined similarly to Rodrigues at al. (2005). N_Y denotes the stream of natural resources used in production, and P represents the positive effect of environmental protection. P depends positively on social capital K_S and the human capital H_N engaged in environmental protection, while negatively on manufactured capital K_P. This dependence is described in the form:

$$P = m_0 \frac{H_N^{\epsilon} K_S^{\kappa}}{K_P^{\varphi}}, \tag{15}$$

where m_0 is the scale parameter, and $\epsilon, \kappa, \varphi$ are respective elasticities. Production is aggregated via Cobb-Douglas technology, i.e. the substitutability between capitals is allowed:

$$Y = K_P^{\beta} N_Y^{\nu} H_Y^{\eta} K_S^{\sigma}, \qquad \beta + \nu + \eta = 1. \tag{16}$$

The essential feature of the model is the broader definition of social welfare, which depends not only on consumption, but also on the state of the environment (natural capital stock) and the level of trust and cooperation in society (social capital):

$$U(C, K_N, K_S) = \frac{\tau}{\tau - 1} \int_0^{\infty} (C K_N^{\Phi} K_S^{\Psi})^{\frac{\tau-1}{\tau}} e^{-\rho t} \, dt, \tag{17}$$

where τ is the coefficient of elasticity of intertemporal substitution, whereas Φ, Ψ are the parameters of the preferences of natural capital (nature) and social capital respectively. Solving the problem of dynamic optimization, the authors derive the constant growth rates of Y and K_H in the steady-state.

4. CRITICAL REMARKS AND ALTERNATIVE MODEL

These approaches, despite a number of simplifying assumptions, allow for a holistic consideration of natural capital in the processes of wealth creation. Natural capital occurs both as a renewable resource (being an argument in utility function) and as a flow (a resource used in production). Understanding the anthropogenic pressure in line with the material flow concept has the advantage that it does not reduce the problem only to the stream of pollutants emitted or used energy carriers. It seems that the indicators of material requirements are the best measures of the consumption of natural capital by individual countries, while the amount of materials consumed by the economy is currently being estimated by Eurostat. The significant disadvantage of the majority of models is the use of Cobb-Douglas production functions, i.e. allowing for substitution between individual capitals. In the light of the theory of ecological economy this is very unlikely. Creating a realistic model requires limited factor substitutions, but without establishing strong complementarity. Using CES can be a reasonable compromise.

The most difficult issue is to consider the role of technology. There are many approaches in the literature that model the process of technology development emphasizing category of knowledge accumulation (Romer, 2012), innovation process (Howitt, Aghion, 1999) or human capital (Lucas, 1988). However, they are substantially different. In the context of this work, it is legitimate to use the category of human capital. Taking into account the chosen assumptions of the models discussed in part 3 and the differences in human capital approach, we propose the following specifications for the growth model. Let K, H, N denote manufactured, human and natural capital, respectively. Human capital H is interpreted in the sense of Mankiw at al. (1992), i.e. it is generated by investing part of the economic output Y. We exclude social capital in this case in order to simplify the model. Aggregated product is produced using individual capitals, but aggregation is made using the CES function[11] (cf. Kraev, 2002) with $p \in (0, \infty)$:

$$Y = f(K, H, N) = (\alpha\, K^{-p} + \beta\, H^{-p} + \gamma\, N^{-p})^{-\frac{1}{p}}. \tag{18}$$

The production is divided between consumption, investment in produced capital and investment in human capital (education):

$$Y = C + I + E. \tag{19}$$

The dynamics of produced and human capital is:

$$\dot{K} = s_K Y - \delta K = Y - C - E - \delta K, \tag{20}$$

$$\dot{H} = s_H Y = E, \tag{21}$$

where s_K and s_H are respectively the investment rates in produced and human capitals[12]. It is possible to set them constant or use dynamic programming approach, when C and E are control variables. In what follows we consider the optimal control problem. The natural capital is a renewable resource, diminished by environmental pressure[13]:

$$\dot{N} = rN - P. \tag{22}$$

where the pressure and material intensity are defined as (Rodrigues at al., 2005):

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[11] CES function assumes a limited substitution between factors, including human and produced capital. Note, however, that we do not prejudge the degree of substitution, which depends on the parameter p. Taking into account the different degree of substitution between the factors requires the use of a nested CES function. The use of the nested production function seems to be a more realistic description of the economy, but we have abandoned this to simplify the model.

[12] Unlike Mankiw at al. (1992), we have abandoned the depreciation of human capital in order to simplify the model.

[13] Chapter 2 discusses the approach in which CC is variable, as does K_N is allocated between economic use and "free" part, according to u. This leads to the dynamics in the form $\dot{N} = rN(CC - N) - P$. However, the assumption of fixed natural capital at stationary state implies constant CC and u, therefore we simplify the model, choosing the form of eq. (22).

$$m = m_0 H^{-a} Y^{n-1}, \qquad P = mY \equiv m_0 H^{-a} Y^n. \tag{23}$$

The social welfare is defined as the value of functional:

$$W = \int_0^\infty e^{-\rho t}\, U(C,N)\, dt, \tag{24}$$

with the utility function $U(C,N) = \ln C + \ln N$ [14]. Our goal is to maximize it and study the quantitative properties of solutions.

In our considerations we assume all the functions to be differentiable in their domains. It implies immediately that all the differential equations of the model have unique solutions. The control variables are consumption C and investment in human capital E, while the state variables are all types of capital: K, H, N. Notice that U is concave and due to economic meaning of controls we may assume them to be uniformly bounded. Therefore the functional W attains maximum for some values $C = C^*$ and $E = E^*$. The corresponding capital paths are denoted by K^*, H^* and N^* respectively. The current value hamiltonian is:

$$\mathcal{H}(C,E;\ K,H,N) = U(C,N) + \lambda_K(Y - C - E - \delta K) + \lambda_H E + \lambda_N(rN - P). \tag{25}$$

4.1. Conclusions from the structure of the model

The standard way to start the analysis of the model is to assume that there exists a steady-state, when all the variables have constants rates of growth (we follow the standard notation for rate: g_X for variable X). However, as in (Rodrigues at al., 2005), we have to make additional assumption that our economy is in the biophysical equilibrium, i.e. $\dot{N} = 0$, because it is impossible for natural capital to grow without bounds. Then $rN = P$ and $P = m_0 H^{-a} Y^n$ are constant. We derive from here:

$$g_Y = \frac{a}{n} g_H. \tag{26}$$

Additionally we have $mY = const$, what gives:

$$g_m = -g_Y. \tag{27}$$

[14] The utility function has been simplified to make the model and later calculations more legible. In general, you must enter the parameters differentiating the marginal utility of consumption and natural capital. For the needs of the model, we can assume that the units of measure of both goods are chosen to make unit utilities of both goods equal.

We have to consider two possibilities. If N is constant and $K \gg N$, $H \gg N$, it is impossible for Y to grow with constant rate. The main obstacle is the production function with limited sustainability. To see this, recall Kraev (2002, p. 283). With constant N, and K and H approaching infinity we have:

$$\lim_{K,H\to\infty} Y = \lim_{K,H\to\infty} (\alpha\, K^{-p} + \beta\, H^{-p} + \gamma\, N^{-p})^{-\frac{1}{p}} = \gamma^{-\frac{1}{p}} N, \tag{28}$$

so g_Y approaches zero, as K and H grows with constant N. Taking into account (26), we obtain that $g_Y = g_H = 0$. For the proposed production function:

$$g_Y = \alpha \left(\frac{K}{Y}\right)^{-p} g_K + \beta \left(\frac{H}{Y}\right)^{-p} g_H + \gamma \left(\frac{N}{Y}\right)^{-p} g_N, \tag{29}$$

Ultimately, we have zero growth rates for all variables in the long-run. In other words, with the assumption of limited substitutability and constant natural capital stock, unbounded steady state economic growth is not possible. Obviously, some kind of technological progress either in the form of an increase in the efficiency of natural capital use in the production function, or in the form of increasing total factor productivity is the only way to overcome this obstacle. The question about the nature of this progress, i.e. exogenous or endogenous, is still under consideration. It appears that this conclusion is in line with the concept of steady state economy by Daly (1980) with constant capital stock and a constant population size. We hope to take this into consideration in the further work.

If N is abundant, i.e. $N \gg H$ and $N \gg K$, we can assume the possibility of steady-state growth. In this case biophysical equilibrium boils down to:

$$g_Y = \alpha \left(\frac{K}{Y}\right)^{-p} g_K + \beta \left(\frac{H}{Y}\right)^{-p} g_H. \tag{30}$$

Taking into account (26), we obtain:

$$\frac{a}{n} g_H = \alpha \left(\frac{K}{Y}\right)^{-p} g_K + \beta \left(\frac{H}{Y}\right)^{-p} g_H, \tag{31}$$

and finally:

$$\left(\frac{a}{n} - \beta \left(\frac{H}{Y}\right)^{-p}\right) g_H = \alpha \left(\frac{K}{Y}\right)^{-p} g_K. \tag{32}$$

The rates g_H and g_K are constant by assumption, so differentiating the last identity with respect to time t we get:

$$\left(\frac{K}{H}\right)^p = \frac{\alpha g_K(g_K - g_Y)}{\beta g_H(g_Y - g_H)}. \tag{33}$$

We conclude that the ratio $\frac{K}{H}$ is constant and:

$$g_K = g_H = \frac{n}{a} g_Y. \tag{34}$$

Equation (21) implies that in the steady-state $\frac{E}{H} = const$, so:

$$g_E = g_H. \tag{35}$$

Therefore by (20) we get:

$$g_K = \frac{Y - C}{K} - \frac{E}{K} - \delta, \tag{36}$$

which leads to the observation that the ratio $\frac{Y-C}{K}$ is constant. This may happen if and only if:

$$g_Y = g_C = g_K. \tag{37}$$

Combining (34) and (37) we get the necessary condition of the steady state: $a = n$, i.e. P elasticities of technology and output are equal. In particular, the conclusion about equal growth rates boils down (30) to:

$$\alpha\left(\frac{Y}{K}\right)^p + \beta\left(\frac{Y}{H}\right)^p = 1. \tag{38}$$

However, it is worth noting that the condition $a = n$ is very unlikely, so in the given model the steady state occurs with a probability close to zero. At the same time, if $a = n$ then by (34) and (37) we have $g_Y = g_C = g_K = 0$. This leads to conclusions about the inability of long-term growth under steady state assuming constant natural capital.

4.2. Optimization conditions

Despite the impossibility of unlimited economic growth in the long-run, we still can analyze the conditions maximizing social welfare. Pontriagin Maximum Principle provides the following necessary conditions for optimal controls and paths:

$$\begin{cases} \dfrac{\partial \mathcal{H}}{\partial C} = 0 \\ \dfrac{\partial \mathcal{H}}{\partial E} = 0 \\ \rho\lambda_K - \dot{\lambda}_K = \dfrac{\partial \mathcal{H}}{\partial K} \\ \rho\lambda_H - \dot{\lambda}_H = \dfrac{\partial \mathcal{H}}{\partial H} \\ \rho\lambda_N - \dot{\lambda}_N = \dfrac{\partial \mathcal{H}}{\partial N} \end{cases} \Leftrightarrow \begin{cases} \lambda_K = \dfrac{1}{C}, & (39.1) \\ \lambda_K = \lambda_H, & (39.2) \\ \rho\lambda_K - \dot{\lambda}_K = \lambda_K\left(\dfrac{\partial Y}{\partial K} - \delta\right) - \lambda_N \dfrac{\partial P}{\partial Y}\dfrac{\partial Y}{\partial K}, & (39.3) \\ \rho\lambda_H - \dot{\lambda}_H = \lambda_K \dfrac{\partial Y}{\partial H} - \lambda_N\left[\dfrac{\partial P}{\partial H} + \dfrac{\partial P}{\partial Y}\dfrac{\partial Y}{\partial H}\right], & (39.4) \\ \rho\lambda_N - \dot{\lambda}_N = \dfrac{\partial U}{\partial N} + \lambda_K \dfrac{\partial Y}{\partial N} + \lambda_N r - \lambda_N \dfrac{\partial P}{\partial Y}\dfrac{\partial Y}{\partial N}, & (39.5) \end{cases}$$

Transversality conditions take the form:

$$\lim_{t\to+\infty} e^{-\rho t}\lambda_K(t)K(t) = 0, \quad \lim_{t\to+\infty} e^{-\rho t}\lambda_H(t)H(t) = 0, \quad \lim_{t\to+\infty} e^{-\rho t}\lambda_N(t)N(t) = 0. \quad (40)$$

Basing on (18) and (23) we derive the formulas for partial derivatives:

$$\frac{\partial Y}{\partial H} = \beta\left(\frac{Y}{H}\right)^{p+1}, \quad \frac{\partial Y}{\partial K} = \alpha\left(\frac{Y}{K}\right)^{p+1}, \quad \frac{\partial Y}{\partial N} = \gamma\left(\frac{Y}{N}\right)^{p+1}, \quad (41)$$

$$\frac{\partial P}{\partial Y} = \frac{nP}{Y}, \quad \frac{\partial P}{\partial H} = -\frac{aP}{H}. \quad (42)$$

From (39.1) – (39.2) we immediately have $\lambda_{K^*} = \lambda_{H^*} = \frac{1}{C^*}$ and consequently:

$$g_{\lambda_K^*} = g_{\lambda_H^*} = -g_{C^*}. \quad (43)$$

In the standard way (see Kamien, Schwartz, 2012, p. 138) we may calculate the shadow price of natural capital:

$$\lambda_{N^*}(t) = \frac{\partial W}{\partial N}(t) = \int_t^{+\infty} \frac{\partial U}{\partial N} e^{-r(s-t)} ds = \frac{1}{rN^*}. \quad (44)$$

Therefore $g_{\lambda_{N^*}} = -g_{N^*}$. Now we turn our attention to conditions (39.3) and (39.5). We divide the first equation by λ_K, the second one by λ_N. Using (43) and (44) we conclude:

$$g_{N^*} = 2r - \rho + \gamma\,\frac{nP^*}{Y^*}\left(\frac{Y^*}{N^*}\right)^{p+1} \frac{\rho + \delta + g_{C^*}}{\alpha\left(\frac{Y^*}{K^*}\right)^{p+1} - (\rho + \delta + g_{C^*})}. \quad (45)$$

$$N^* = C^*\,\frac{1}{r}\,\frac{nP^*}{Y^*}\,\frac{\alpha\left(\frac{Y^*}{K^*}\right)^{p+1}}{\alpha\left(\frac{Y^*}{K^*}\right)^{p+1} - (\rho + \delta + g_{C^*})}. \quad (46)$$

Next, from equations (39.3) and (39.4), we derive the dependencies:

$$g_{C^*} = \alpha \left(\frac{Y^*}{K^*}\right)^{p+1} \left(1 - \frac{\lambda_{N^*}}{\lambda_{K^*}} \frac{nP^*}{Y^*}\right) - \delta - \rho, \tag{47}$$

$$g_{C^*} = \beta \left(\frac{Y^*}{H^*}\right)^{p+1} \left(1 - \frac{\lambda_{N^*}}{\lambda_{K^*}} \frac{nP^*}{Y^*}\right) + \frac{\lambda_{N^*}}{\lambda_{K^*}} \frac{aP^*}{H^*} - \rho. \tag{48}$$

Eliminating the ratio $\frac{\lambda_N}{\lambda_K}$ from those equations, we express the growth rate of consumption in terms of average productivities of manufactured and human capitals:

$$g_{C^*} = \left[1 + \frac{\rho}{\frac{Y^*}{H^*}}\right] \frac{\alpha \left(\frac{Y^*}{K^*}\right)^{p+1}}{\beta \left(\frac{Y^*}{H^*}\right)^{p} - 1} - (\delta + \rho). \tag{49}$$

The conclusions that we conduct from (45) and (49) are:

a. $\frac{\partial g_{C^*}}{\partial (Y/K)} > 0$, i.e. raising the average productivity of manufactured capital leads to greater consumption rate. Similar result for the growth rate of natural capital holds under the following condition on g_{C^*}:

$$\frac{\partial g_{N^*}}{\partial (Y/K)} > 0 \quad \Leftrightarrow \quad g_{C^*} < \frac{\alpha}{\beta} \frac{\frac{Y^*}{H^*} + \rho}{\frac{Y^*}{H^*}\left[\left(\frac{Y^*}{H^*}\right)^{p} - 1\right]} \left(\frac{Y^*}{K^*}\right)^{p+1} - \rho - \delta. \tag{50}$$

b. $\frac{\partial g_{C^*}}{\partial (Y/H)} < 0$, i.e. (surprisingly) raising the average productivity of human capital leads to smaller consumption rate. Positive impact of this raise on the rate of growth for natural capital is possible, if the consumption rate satisfies:

$$\frac{\partial g_{N^*}}{\partial (Y/H)} > 0 \quad \Leftrightarrow \quad g_{C^*} > \frac{\alpha}{2} \left(\frac{Y^*}{K^*}\right)^{p+1} - \rho - \delta. \tag{51}$$

c. $\frac{\partial g_{C^*}}{\partial (Y/N)} = 0$, while

$$\frac{\partial g_{N^*}}{\partial (Y/N)} > 0 \quad \Leftrightarrow \quad g_{C^*} < \alpha \left(\frac{Y^*}{K^*}\right)^{p+1} - \rho - \delta. \tag{52}$$

d. Greater depreciation rate δ results in smaller consumption rate, while $\frac{\partial g_{N^*}}{\partial \delta} = 0$.

e. Increase in p (so equivalently: decrease in substitutability) causes decrease in g_{C^*} provided:

$$\begin{cases} \ln\dfrac{Y^*}{K^*} < \dfrac{\beta\left(\frac{Y^*}{H^*}\right)^p}{\beta\left(\frac{Y^*}{H^*}\right)^p - 1}\ln\dfrac{Y^*}{H^*}, & if \quad \left(\dfrac{Y^*}{H^*}\right)^p > \dfrac{1}{\beta}, \\ \ln\dfrac{Y^*}{K^*} > \dfrac{\beta\left(\frac{Y^*}{H^*}\right)^p}{\beta\left(\frac{Y^*}{H^*}\right)^p - 1}\ln\dfrac{Y^*}{H^*}, & if \quad \left(\dfrac{Y^*}{H^*}\right)^p < \dfrac{1}{\beta}. \end{cases} \tag{53}$$

Increase in p has negative influence on g_{N^*} if:

$$\ln\frac{Y^*}{N^*} < \ln\frac{Y^*}{H^*}\,\frac{\beta\left(\frac{Y^*}{H^*}\right)^{p+1}}{\beta\left(\frac{Y^*}{H^*}\right)^{p+1} - 2\frac{Y^*}{H^*} - \rho}. \tag{54}$$

Assuming constant rate of consumption on optimal path $g_{C^*} = const$, we may have additional conclusion from (49) about influence of average productivity of human capital on the rates of production, manufactured and human capitals:

$$\frac{g_{Y^*} - g_{H^*}}{g_{Y^*} - g_{K^*}} = \frac{(p+1)\left[\left(\frac{Y^*}{H^*}\right)^2 + \rho\right]}{p\left[\left(\frac{Y^*}{H^*}\right)^2 + \rho\right]\dfrac{\beta\left(\frac{Y^*}{H^*}\right)^{p+1}}{\beta\left(\frac{Y^*}{H^*}\right)^p - 1} - \rho\left(\frac{Y^*}{H^*}\right)^2}. \tag{55}$$

4.3. Coexistence of the steady-state and optimal growth path

In this section we are going to investigate the properties of steady state, being simultaneously the optimal growth path. Combining condition (38) (constant combination of productivities of manufactured and human capitals) with the formula on optimal consumption growth rate (49):

$$g_{C^*} = -\left[1 + \frac{\rho}{\frac{Y^*}{H^*}}\right] - (\delta + \rho) < 0. \tag{56}$$

In view of equal growth rates given by $(34), (35)$ and (37), we immediately get, that the economy collapses. On the other side, by differentiation of (49) with respect to time, we conclude that $g_{Y^*} = 0$, and therefore $g_{C^*} = 0$, what gives contradiction. Therefore in our model it is impossible to have steady-state, which simultaneously realizes welfare maximum.

5. CONCLUSIONS

The main conclusion we can derive from the discussed model in the impossibility of the economic growth in the long run if the limited substitutability and constant natural capital are assumed. Technological progress either in the form of the increase of natural capital efficiency or in the form of increasing total factor productivity seems to be the only way to overcome this limit. If the stock of natural capital is abundant, we can assume steady state growth (again, we must note that steady-state is very unlikely due to necessity of $a = n$), although it is still impossible to have steady-state growth, which simultaneously realizes welfare maximum.

Other conclusions resulting from the model are:

1. Raising the average productivity of manufactured capital leads to greater consumption rate. Similar result for the growth rate of natural capital holds if growth rate of consumption is limited from above.
2. Raising the average productivity of human capital leads surprisingly to smaller consumption rate. Positive impact of this raise on the rate of growth for natural capital is possible, if the consumption growth rate is limited from below. It could be interpreted as follows: faster consumption growth inhibits economic growth by reducing investment opportunities, thus slowing down the growth of natural capital usage P, which is positively dependent on Y. As a result, a faster growth of natural capital is possible.
3. Greater depreciation rate δ results in smaller consumption rate.
4. Increase in p (so equivalently: decrease in substitutability) could lead to decrease in g_{C^*}. Increase in p could also have negative influence on g_{N^*}. In both cases decrease in substitutability means lower growth rates.

Obviously, more in-depth reflection on the results is needed as well as an attempt to modify the model to enable sustainable growth, i.e. constant positive growth rate of production and consumption with constant natural capital stock. The model presented in the paper is just one of the many possibilities for describing the behaviour of the economy, thus another specification of assumptions, in particular the assumption on substitutability of natural capital with other kinds of capital, would potentially allow the long-term growth with preserved natural capital.

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KAPITAŁ NATURALNY W MODELACH EKONOMICZNYCH

Streszczenie

Celem artykułu jest dokonanie krytycznej analizy wybranych modeli wzrostu proponowanych w ramach szkoły ekonomii ekologicznej oraz odwołujących się do kategorii kapitału naturalnego, jak również próba konstrukcji alternatywnego modelu. W szczególności traktujemy kapitał naturalny jako odnawialny zasób i używamy funkcji produkcji CES, tym samym ograniczając możliwości substytucji kapitału naturalnego innymi formami kapitału. Analizowane są optymalne (tj. maksymalizujące dobrobyt społeczny) ścieżki kapitału i konsumpcji. Artykuł kończy się wnioskami formułowanymi na podstawie modelu.

Słowa kluczowe: ekonomia ekologiczna, trwałość, wzrost gospodarczy, kapitał naturalny, funkcja produkcji CES

NATURAL CAPITAL IN ECONOMIC MODELS

Abstract

The goal of our paper is to make a critical analysis of selected growth models that use the notion of natural capital and to construct the alternative model. In particular we treat the natural capital as a renewable resource and we use CES production function, weakening the assumption of substitutability of natural capital with other forms of capital. We investigate the optimal paths for capital and consumption, giving their characterization in the dependence on the parameters of the model. The paper ends with conclusions derived from the model.

Keywords: ecological economics, sustainability, growth model, natural capital, CES production function