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ROBUST OPTIMISATION METAHEURISTICS FOR THE INVENTORY-ALLOCATION PROBLEM

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Abstract

As an example of a successful application of a relatively simple metaheuristics for a stochastic version of a multiple criteria optimisation problem, the inventory-allocation problem is discussed. Stochastic programming is introduced to deal with the demand of end consumers. It has been shown before that simple metaheuristics, i.e., local search may be a very competitive choice for solving computationally hard optimisation problems. In this paper, robust optimisation approach is applied to select more promising initial solutions which results in a significant improvement of time complexity of the optimisation algorithms. Furthermore, it allows more flexibility in choosing the final solution that need not always be minimising the sum of costs.

Keywords: robust optimisation, local search, stochastic programming, distribution.

1 Introduction

New metaheuristics paradigms are introduced and are getting popular in recent years and in recent decades because NP-hard problems provide challenging optimisation tasks (Talbi, 2009; Aarts and Lenstra, 1997; Sorensen et al., 2016). Despite of the dramatic increase of available computational power, the need for heuristic methods remains because also the size of practical problems to solve increases. While it is widely accepted that the most successful heuristics are those that use the very properties of a particular problem and the domain, it is

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not so commonly believed that simple metaheuristics are expected to be overperfoming the more complicated ones (for further discussion, see Žerovnik, 2015, 2003). In the talk given by one of the authors at the 8th International Workshop on Multiple Criteria Decision Making, arguments and examples were provided supporting the claim. A very general theoretical argument (Ferreira and Žerovnik, 1993) is that any local search asymptotically outperforms the often used heuristic simulated annealing (Kirkpatrick et al., 1983), on any problem (!). We mention here three examples. The first example of a simple local search type heuristic is the "remove and reinsert" heuristic that has been applied to the traveling salesman problem (Brest and Žerovnik, 1999), the probabilistic traveling salesman problem (Žerovnik, 1995), the resource-constrained project scheduling problem (Pesek et al., 2007) and the job shop scheduling problem (Zupan et al., 2016). The second example is the Petford-Welsh algorithm (Petford and Welsh, 1989), a heuristic for graph 3-colouring based on the antivoter model (Donelly and Welsh, 1983), that has later been applied to various generalised colouring problems including the k-colouring (Žerovnik, 1994), frequency assignment (Ubeda and Žerovnik, 1997), and very recently to the clustering problem (Ikica et al, 2019). For details of the close relation of the Petford-Welsh algorithm to the Boltzman machine and the simulated annealing algorithm, see Žerovnik (2000). The last example that will be elaborated in more detail in this paper is the application of local search heuristics to the inventory--allocation problem.

The rest of the paper is organised as follows. Inventory allocation in a supply chain is introduced in section 2. In section 3, the formal definition of the problem is given and the robust optimisation approach that extends our previous heuristics is described. The new approach allows a sizeable improvement in computation speed, as shown by the results of a computational experiment on a realistic example described in section 4. Conclusions are given in section 5.

2 Inventory allocation in a supply chain

A typical retail supply chain consists of one or several warehouses that distribute products to several stores, which have to deal with stochastic demand patterns. The idea is to align the decisions, reflecting the ordering policies that in retail companies are usually taken independently by several decision makers. On the one hand, we are dealing with warehouse managers, whose orders are naturally based on the price and availability of a product. On the other hand, we have store managers, whose orders are usually based on the actual requirements of the merchant. The ordered quantities from the external suppliers depend therefore on the stock market prices and not on the actual requirements. Of course there are many situations with higher or lower stock levels, causing overstocking effects or lost sales (state of stock-out with possible lower sale realisation).

In our previous research paper (Vizinger and Žerovnik, 2019) we have presented the idea of an on-going optimisation approach. In this approach we first find a tactical plan and then (re)define some strategic and operational decisions. Tactical planning for a chosen period (month, season, etc.) defines the appropriate inventory levels in warehouses and stores, and consequently the allocation of resources among retail facilities. The replenished quantities are defined on the operational level using the difference between the actual inventory and the pre-defined maximum level of a certain store inventory. Since the tactical plan already determines the necessary stock levels, the demand of the warehouses becomes more or less deterministic. With precise tactical planning, retailers may be able to contract constant supply quantities, which may result in a lower unit price and the corresponding higher profit. At the stores' side of the supply chain, precisely pre-defined inventory levels prevents stock accumulation, which may result in increased product quality and a related customer service level.

The model for product flow coordination in a retail supply chain that was introduced in Vizinger and Žerovnik (2019) considers optimisation of three criteria (distribution costs, overstocking effects and lost sales). While the costs are estimated on the basis of the expected demand, only the distribution cost can be calculated for the quantities predicted. On the other hand, both types of supply risk (overstocking effects and lost sales) are unknown a priori. Although the last two cannot appear at the same time (they are mutually exclusive), it is reasonable to deal with each of the three costs separately. On the one hand, we consider a single-item lot sizing problem and, on the other hand, a resource allocation problem. The stochastic model introduced in Vizinger and Žerovnik (2019) is the first that tackles the coordination problem at the tactical level of planning. The inventory-allocation problem may be seen as a generalized material flow problem, where the goal is to minimise the distribution costs of goods delivered from some supply points to a number of destination points (Anholcer, 2016). Below we refer to the combined inventory-routing problem, as there exist similarities to our inventory-allocation problem.

For the inventory-routing problem, the literature introduces mostly the use of mixed integer optimisation, multi-objective optimisation and stochastic programming (e.g., Liu and Papageorgiou, 2013; Grossman and Guillén-Gosálbez, 2010). The idea is to find an appropriate policy with minimal costs of distribution, minimal overstocking effects and maximal customer service level.

Liu and Papageorgiou (2013) interpret customer service level as the percentage of customer demand satisfied on time. Lower customer service level therefore causes lost sales or lost customers, and this results in profit loss of the supply chain.

Most of the applied stochastic models are two-stage programs, and are used to deal with demand uncertainties when assigning probability distributions (Franca et al., 2010). A stochastic transportation problem may be transformed into a deterministic one by removing the demand constraints, which are used to introduce a new cost function related to the expected extra cost (resulting in a difference between the delivered amount and the actual demand). Although risk is measured in our paper with the cost metric, the direct and indirect costs should not be summed up, because they have a totally different origin. As opposed to the well-known lot-sizing models or the Newsvendor approach (for coordination of the supply chain flow), we are not limited to use only a simple or a weighted sum of the criteria considered.

Beside stochastic programming and heuristics solution procedures, Grossman and Guillén-Gosálbez (2010) introduce robust optimisation and probabilistic programming. In many cases we are not able to identify the underlying probability distributions or such a stochastic description may simply not exist (Sarimveis et al., 2008). In such a situation it is reasonable to fit a suitable probability distribution for each parameter based on an expert's subjective knowledge derived from past experiences and feelings. Uncertain data are therefore unknown but bounded quantities, while constraints are satisfied for all realisations of the uncertain parameters. In robust programming, not every scenario represents a feasible solution. Once an uncertainty is realised, the solution obtained from robust optimisation ensures that constraints are satisfied with a certain probability.

The optimisation problem that arises from the model is a computationally hard problem. For time prohibitive stochastic programs, the use of heuristic approaches (which provides good feasible solutions) have become very popular. Several versions of the local search heuristics were adopted in Vizinger and Žerovnik (2019), including iterative improvement (a basic form of local search), tabu search and threshold accepting. The best performance was shown by the tabu search heuristic that proved to provide very good solutions on the instances tested. However, the computational time for a single product instances of moderate size is considerable. Even though these calculations are to be performed only occasionally, it is important to have a faster method if possible.

3 Formal definition of the problem

We represent the coordination problem for a retail supply chain product flow as a multi-objective discrete optimisation problem. A typical retail supply chain consists of one or several warehouses $i \in (1, \ldots, I)$ who deliver products to a number of stores $j \in (1, \ldots, J)$, where dc_{ij} is the distribution cost, and x_{ij} is the quantity of the product distributed. There are two types of vertices: a_i represents a given fixed supply available at each origin or warehouse, and C_j represents a fixed inventory holding capacities of stores. In addition, we are given the demand of the stores as random variables b_j. In other words, we model the customers' shopping habits with random variable b_j with some probability distribution that is not known a priori. Here we consider discrete distributions and assume that we are given hypothetical distributions based on past experience (managers' knowledge, information from the system) and/or intuition.

A feasible solution X is given by the matrix

$$X = [x_{ij}]_{i \in I, j \in J}, \tag{1}$$

where x_{ij} is the amount transferred from warehouse i to store j. A solution X is feasible if it satisfies the inventory holding capacity of stores C_j, and complies with the supply available at each origin or warehouse a_i.

A possible sale realisation is represented by the scenario, described by vector $L = [l_j]$, where l is the fixed scenario realised at store j.

3.1 Optimisation criteria

Given a scenario L, the cost of overstocking effects OS is calculated as:

$$OS(X, L) = \sum_j (\sum_i x_{ij} - l_j) \cdot c_{OS} \tag{2}$$

and the cost of lost sales LS is defined as:

$$LS(X, L) = \sum_j (l_j - \sum_i x_{ij}) \cdot c_{LS}. \tag{3}$$

In (2) and (3), c_{OS} is the cost of overstocking effects for a unit of product at store j, and c_{LS} is the cost of lost sales for a unit of product at store j. Here we can optimise only the expected values because the optimisation criteria depend on the a priori unknown values of the future sales. The expected cost of overstocking effects is represented as the weighted sum of the costs over all scenarios:

$$E(OS(X)) = \sum_L p(L) \cdot OS(X, L), \tag{4}$$

where $p(L)$ is the probability of scenario L. Similarly, for the expected cost of lost sales:

$$E(LS(X)) = \sum_{L} p(L) \cdot LS(X, L). \tag{5}$$

As indicated in Introduction, the relationship between the decision criteria is often represented by a (weighted or simple) sum of criteria. However, when defining the general mathematical model, we wish to consider the multi--objective optimisation problem in a more general and somewhat more natural way. The stochastic model and experimental study are described in detail in Vizinger and Žerovnik (2019). Note that the goal function to be minimised in the local search procedure was defined at first as a sum of the criteria. Simple local search heuristics have been shown to provide near optimal solutions of very good quality in a reasonable time (Vizinger and Žerovnik, 2019, 2018). Nevertheless, when larger instances and, in particular, when more products are considered, the computational time may be large, therefore we adopted the robust optimisation approach in order to restrict attention to a subset of promising feasible solutions. In short and roughly speaking robust optimisation here means that we attempt to speed up the optimisation procedure by focusing first on the two criteria modelling the risk and considering the third criterion only in the case when the first two are within reasonable bounds. In this way, costly optimisation of the distribution cost that involves linear programming is avoided. The optimisation criteria remain the same, but the set of feasible solutions and thus potential Pareto optimal solutions is reduced to those which have bounded risk costs. A preliminary report on this research, the robust optimisation approach for tactical planning of a retail supply chain product flow was announced in an extended abstract by Vizinger, Kokolj and Žerovnik (2017). Here we outline the entire solution procedure and test the model on a real-life instance.

3.2 Robust optimisation

A robust optimisation approach is performed in four consecutive steps (see Figure 1), where we first generate an initial solution (having at most some percent of lost sales or overstocking effects realisation). After examination of a limited number of testing scenarios, we iteratively seek solutions with minimal supply risks. For the set of best solutions (with minimal supply risk) we evaluate the distribution costs. This approach is closely related to robust programming, at least as regards the generation of scenarios. Moreover, the objective function to be minimised is no longer in the form of a sum of all three costs (as is the case in the exact solution approaches and previously used heuristics), but the criteria are rather placed in the hierarchy way. This allows us to exclude time prohibitive linear programming from the iterative improvement, which greatly speeds up the heuristic solution procedures.

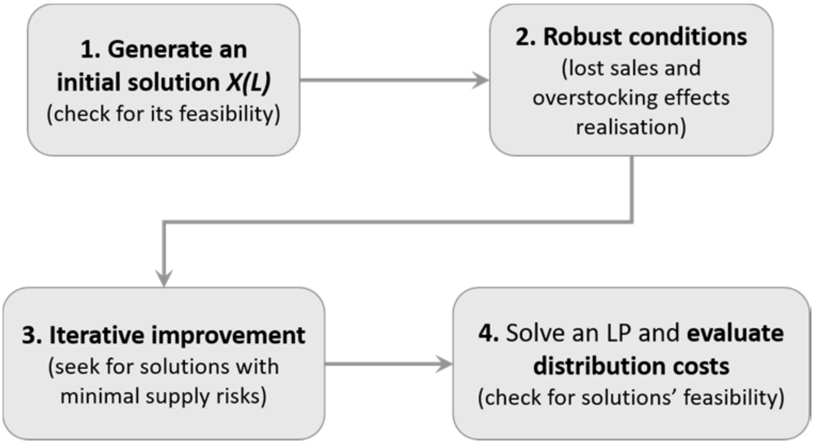

Figure 1: Four steps of a robust optimisation approach

3.2.1 Generating an initial solution

At first, an initial solution (or, it may be known from past experience) is generated at random. Let us assume that we know the future sales (in reality, actual demand is known a posteriori), given in a vector $l_j = [0, \dots, l_j(m)]$, where $l_j(m)$ represents the last possible (maximal) sold quantity at store j. Each l_j is assumed to take a limited number of values, and so may take the maximal possible value as does the maximal past sale (defined with the random variable b_j). Recall that the possible sale realisation is given by a scenario described in vector $L = [l_j]$, where l is the quantity needed at store j.

As indicated above, we first generate a scenario L and use $X(L)$ as the initial solution. Here $X(L)$ is the optimal solution of the linear problem solving the deterministic transportation problem corresponding to scenario L. A solution X represents the distribution plan given in a matrix, presented in expression (1).

3.2.2 Robust conditions

In contrast to the previous approach (see Vizinger and Žerovnik, 2019), the initial solution X is further checked for coincidence with the specified robust criteria (maximum allowable supply risks). In particular, here this means that the

initial solution must allow at most 25% of overstocking effects and/or with at most e.g. 30% of lost sales realisation. If the solution generated does not fit, a new solution is generated at random, until a feasible fitting solution is obtained.
Note that if we assume that the probability distributions of L are given by (independent) random realisations of the random variables l_j (defined by b_j), the probability of a scenario L is clearly:

$$p(L) = \prod_j p(j, l). \tag{6}$$

We calculate the expected overstocking effects for the solution generated initially as well as for the last possible solution (maximal distributed quantities). If the ratio of $E\big(OS(X)\big)$ for the solution tested to $E\big(OS(X)\big)$ for the last possible solution is less than some pre-defined percentage (for example 25%), we may, with reasonable confidence, accept the solution generated initially. Furthermore, we may search for feasible solutions with at most e.g. 30% of lost sales, where we consider the ratio of $E\big(LS(X)\big)$ for the tested solution to $E\big(LS(X)\big)$ for the first possible solution (having minimal distributed quantities). Note that solutions with higher probability for sales realisation are more likely to be generated.

3.2.3 Tabu search

The iterative improvement phase proceeds along the lines of our previous experiments, i.e. the tabu search heuristic is applied because this procedure had the best performance when several local search based heuristics were tested (see Vizinger and Žerovnik, 2019, 2018). Tabu search generates a random neighbour (random selection of a store and a random change of the amount to be delivered), and moves to the new solution based on the difference in the goal function. The goal function is improved if it is minimised compared to the objective function value of the previous solution. The objective function to be minimised is represented here as a sum of supply risks $E\big(OS(X)\big) + E\big(LS(X)\big)$. The best known solutions are reported as the final results.

3.2.4 Evaluation of the solutions

For the solutions with minimal supply risks we then solve a linear program and evaluate the distribution costs $DC(X)$. The preferred solution may be the one that appears most often in the set of solutions (with minimal supply risks):

$$\min_{X\in\chi}\left(E\big(OS(X)\big)+E\big(LS(X)\big)\right), \tag{7}$$

or the one with minimal total costs (trade-off between the direct and indirect costs):

$$\min_{X\in\chi}\left(E\big(DC(X)\big),E\big(OS(X)\big)+E\big(LS(X)\big)\right). \tag{8}$$

The choice of the best solution depends on the decision-maker's requests and preferences. If we are selling a product of higher value, the retailer would naturally like to minimise the supply risks costs and he might choose the solution obtained with equation 7. On the other hand, if we are selling a product of lower value, we would like for the solution to have minimal total costs (so we may sum up the costs in equation 8). If a highly demanded product with a rather low value is under investigation, we want the solution with minimal different types of costs (thus we observe separate direct distribution costs and indirect supply risks as presented in equation 8). We wish to stress that we have more options and the right one should be chosen on the basis of the decision maker's preferences.

Finally, in the robust optimisation approach we check the best solutions whether they satisfy the inventory holding capacities of stores C_j, and whether they do not exceed the supplies available at each origin or warehouse a_i. If none of the solution is feasible, we check the next set of the best solutions from the tabu search solution procedure.

4 Numerical example

The numerical example deals with the distribution of a non-substitutable perishable product from the fruit and vegetable program, i.e., bananas. Retailers usually sells products through stores of multiple formats; in our analysis we focus on the largest store format: megamarket. We assume that megamarkets have the most complete and well maintained databases regarding stocks, orders, etc.

The idea of this analysis is to set up a tactical plan for the selected sub-season of the chosen summer season. Actual sales data were statistically analysed and we found out that there are eight selling seasons (for the banana sales) and each of these we may further divide into at least three sub-seasons. In the selected summer season (July-August) we distinguish four sub-seasons (Monday--Wednesday, Thursday, Friday-Saturday, Sunday). In our example we set up a tactical plan for Fridays and Saturdays of the selected summer season.

The retailer distributes bananas between two warehouses and several hundreds of stores (we focus on 18 megamarkets and disregard other store formats). The chosen product (bananas) is packed into basic units (packages), each weighting 18 kg. We assume that transportation between warehouses and stores is provided once per day, and that the stores may order only a whole number of packages, as a package is a basic transportation unit. For a unit of product (package) we use cost estimates (in €) for daily distribution (transport, warehouse) and supply risks (overstocking effects, lost sales). Costs are estimated on the basis of the interviews with practitioners from the company: $c = €0.02/\text{package/km}$, $h_w = €0.25/\text{package/day}$, $h_s = €0.5/\text{package/day}$, $c_{OS} = €5/\text{package/day}$, and $c_{LS} = €6/\text{package/day}$. We also estimate entries d_{ij} of the distance matrix (in km) which are used to calculate the transportation costs $\left(c_{ij} = c \cdot d_{ij}\right)$. Note that the distribution cost (storing and transportation) from warehouse i to store j per unit is computed to be: $dc_{ij} = c_{ij} + h_w + h_s$.

Sales are recorded in kilograms of product sold. Because only whole-numbers of packages can be distributed, kilograms into packages have to be converted first. Since one package of bananas weights approximately 18 kg, we cannot fill the distribution classes with integers only, but need to divide them, e.g., into quarter, half, three-quarter and an entire package. For the case of megamarket 13 the sales distribution for Fridays and Saturdays of the summer season is shown in Figure 2. As we can see, megamarket 13 will sell up to ten packages of bananas in the chosen sub-season, and most probably it will sell between six and eight packages per day. Similar results hold true for other stores. When defining demand distributions we found out that all the stores considered have 20 to 75 sales possibilities (demand classes), and there are $8.6 \cdot 10^{27}$ possible scenarios or sales realisations in total.

For the stochastic model we have first tested the basic local search solution procedures (iterative improvement, tabu search, threshold accepting and a combination of all three) and showed that they are very efficient when addressing the inventory-allocation optimisation problem (Vizinger and Žerovnik, 2019, 2018). The convergence curve of the tabu search heuristic is shown in Figure 3 (note that here the objective function is represented by the sum of all three criteria). Since the tabu search turns out to be the most reliable among all the heuristics tested, we integrated this solution procedure into the robust optimisation approach. Instead of optimisation of the sum of criteria, we optimise the goal function that is defined as follows. First we optimise the cost of risk, which is the sum of two criteria: the expected lost sale cost and the expected overstock cost. Only feasible solutions with low cost of risk are then considered and their transportation costs are computed. Of course, we might optimise the criteria in some other way (hierarchy); for some systems it is perhaps important to minimise only the overstocks in the first stage of the optimisation procedure. Therefore, we may argue that the final decision about the importance of the criteria should be made by the decision makers and that their interactive involvement is definitely desirable.

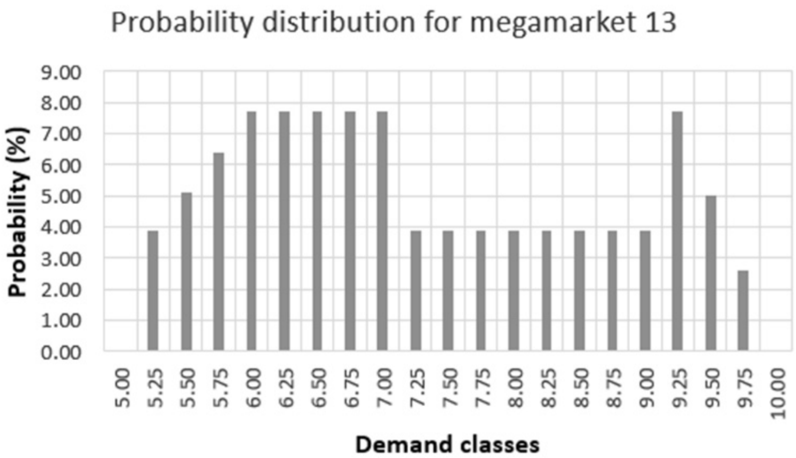

Figure 2: Example of a probability distribution for Fridays and Saturdays of the summer season

In the robust optimisation approach we first generate a scenario L, and use $X(L)$ as the initial solution, where $X(L)$ is the optimal solution of the linear program solving the deterministic transportation problem corresponding to scenario L. For the initial solution we first check if it coincides with the capacities and limitations of the warehouses and stores. For the initial solution, the first possible (minimal distributed quantities) and the last possible scenarios (maximal distributed quantities) we calculate the supply risks and check the robust conditions. In our case we request that the solution have at most 30% of the expected overstocking effects, as well as at most 30% of expected lost sales.

The robust optimisation approach was run 10 times for 1000 iterations. Figure 4 represents the iterative solution procedure (note that here the objective function is represented by the sum of expected overstocks and expected lost sales). As we can see, the expected supply risks amount up to €209 (€105 for overstocks and €104 for lost sales). The best solution from the tabu search procedure (without considering a robust 4-step approach) corresponds to the scenario with supply risk costs that amount up to €223.3 (€44.7 for overstocks and €178.6 for lost sales). Regarding the minimisation of supply risks, the criteria hierarchy definitely outperforms the simple tabu search procedure that uses only a simple or weighted sum of all criteria.

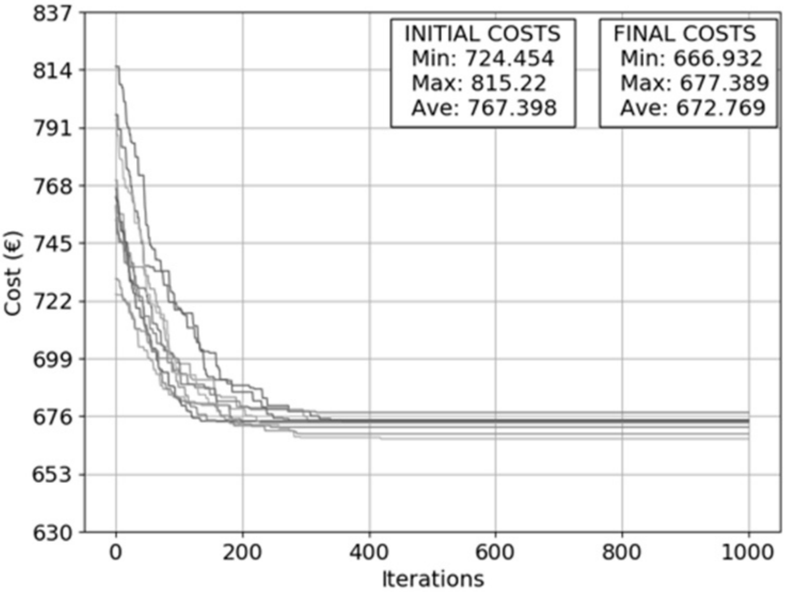

Figure 3: Convergence curve of the tabu search solution procedures (for a realistic case). The cost is the sum of expected risk costs and the distribution cost

For the best ten solutions (from the iterative improvement) we solve the linear program and evaluate the distribution costs. It turns out that the best solution with at most 30% of overstocking effects and at most 30% of lost sale realisations corresponds to the solution with total costs of €708 (distribution costs and supply risks). Nevertheless, although this solution has higher total costs by approximately €41 in comparison to the solution from the tabu search with the sum of the criteria (see also Figure 3), it has lower supply risk costs. Supply risks are also much more balanced (1:1 as compared to the previous result 1:4). The solution obtained by the robust optimisation approach is represented by matrix X, with 2 rows (2 warehouses) and 18 columns (18 megamarkets):

$$X = \begin{bmatrix} 12.5 & 10.5 & 17.75 & 9.75 & 22.75 & 14.0 & 0 & 8.25 & 0 & 8.5 & 0 & 0 & 0 & 0 & 10.0 & 11.0 & 20.25 & 9.75 \\ 0 & 0 & 0 & 0 & 0 & 0 & 12.0 & 0 & 6.75 & 0 & 25.25 & 11.0 & 7.5 & 7.75 & 0 & 0 & 0 & 0 \end{bmatrix}$$

From the distribution plan we see that the first warehouse should supply most of the megamarkets and the second warehouse, megamarkets 7, 9, 11, 12, 13 and 14. In the case of megamarket 13 we note that the appropriate stock level for Fridays and Saturdays of the summer season is seven and a half packages, while with the previous approach we have obtained six and a half packages. Note that we can distribute only a whole number of packages, so in this case we would distribute the difference between the needed level (the result from tactical planning) and the actual stock level, rounded up to the closest integer. We see that with the robust optimisation approach the distributed quantities are higher, as are also distribution costs and overstocking effects. As opposed to this, the costs of lost sales are lower and, most importantly, the costs of supply risk are balanced. The solution obtained can be also called a balanced or compromised solution.

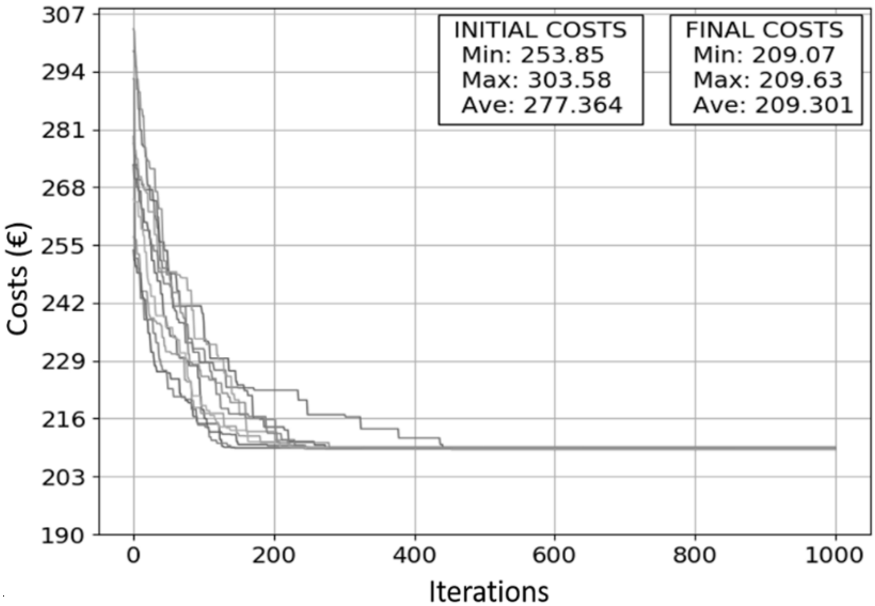

Figure 4: Convergence curve of a robust optimisation approach (for a realistic case). The cost is the expected risk costs

Algorithms have been implemented in a Python environment (the code was not optimised). Search runs were done on an Intel Xeon E3-1230 v3 (8M Cache, 3.3 GHz) processor. For the experimental case of 100 stores, 10 demand classes, 8 initially selected solutions and with 10 000 tabu search iterations performed for each solution, the basic tabu search computation (without considering a robust 4-step approach) took 40 minutes, and the robust computation took roughly 15 seconds. With the incorporation of robust criteria and criteria hierarchy, we have significantly increased the computation speed. Here we need to note that computational time increases when the number of stores changes, while the number of demand classes does not affect significantly the complexity of the problem.

5 Conclusion

This paper presents the robust optimisation approach for the inventory-allocation problem, which is appropriate for tactical planning of a retail supply chain product flow. We have considered a product whose sales figures are independent from those of other products. First, we randomly generated an initial solution (representing a distribution plan with defined inventory levels and allocation of resources), having at most some pre-defined percent of supply risk realisation (robust conditions). Then we used a tabu search algorithm to search for solutions with minimal supply risks (overstocking effects and lost sales). In a previous paper we have shown that local search, the most basic metaheuristics, is a very competitive choice. In fact, the tabu search was shown to be very efficient, therefore we have incorporated this solution procedure into the robust optimisation approach.

The initial solution was further evaluated by taking into the account also the distribution costs assessments. It was shown that exclusion of time prohibitive linear programming from the iterative improvement solution procedure greatly speeds up the computations. Therefore, the implementation that improves separately the distribution cost and the cost of risks is definitely reliable and also allows interactive decision making (e.g. defining robust criteria or choosing an appropriate cost function for optimisation).

From the general introductory discussion, we can conclude that the example discussed here is another argument supporting the claim that simple (meta)heuristics are usually a competitive choice when solving hard optimisation problems.

There are many interesting directions for future research. First of all, we will try to upgrade the model to deal with the distribution plans of the substitutable products. Of course, the optimisation problem will be harder, therefore we might also consider possible improvements of the heuristic solution procedure. For future research we also left out the natural extension that would include a dynamical self-adapting mechanism. Here the comparative model for the operative planning is one of the interesting research avenues, where the distribution quantities are going to be defined with a difference between an actual inventory and the pre-defined maximum level of a certain inventory (resulting from tactical planning).

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