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PULSE OF ECONOMY. IDENTIFICATION, DIAGNOSIS, DIRECTIONS OF DEVELOPMENT

Abstract

Samuelson and Nordhaus (2012: 436) claim that „There is no pattern in the type formulas, the trajectory of the planets or swing of the pendulum, which would be used to predict the duration and timing of business cycles. Rather, they resemble a change in the weather".

Nogalski and Klimek showed that in the years 1995-2010, the Polish GDP was forming in a way that made it possible to describe it using a mathematical formula (2015: 302-305). Having done further research, the authors are trying to establish whether Samuelson and Nordhaus are right. In their search for the answer, they conducted research on the „rhythm" of the world economy, European economy and that of several European states, including Poland, basing on the World Bank (WB) data for the years 1983-2013. The paper presents the results of these studies and the results of the analysis of the development of the Polish GDP based on the Central Statistical Office (CSO) data for the years 1995-2014.

JEL Classification Code: **E17**.

Keywords: GDP model, growth, cycles, forecast.

Research method

Mathematical models were sought from which conclusions could be drawn about the pace of economic growth and fluctuations in the economic situation of the examined entities. It was assumed that such conditions may fulfill the following functions: polynomial-trigonometric (1) and exponential-trigonometric[3] (2).

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[3] The exponential-trigonometric function has the feature that contains an element fulfilling the condition of constant growth rate s $P_{t+1} = (1+s) \cdot P_t$, where s = const.

$$GDP\,mod\_po = a_0 + a_1 \cdot N + a_2 \cdot N^2 + a_3 \cdot sin\left(2\pi \cdot \frac{N}{T-\alpha}\right) + \varepsilon \quad (1)$$

$$GDP\,mod\_ex = b_0 + b_1 \cdot (1+b_2)^{N-1} + b_3 \cdot sin\left(2\pi \cdot \frac{N}{T-\alpha}\right) + \varepsilon \quad (2)$$

where:
N – year number from 1 for 1983,
Searched parameters:
a_0, b_0 – fixed,
a_1, a_2, b_1, b_2 – growth rates,
a_3, b_3 – fluctuation amplitude,
T – length of the cycle,
α – phase period,
while maximizing the factor R^2 used as a measure of fitting the model to the actual data.
GDPmod_po – model (1) polynomial-trigonometric
GDPmod_ex – model (2) exponential-trigonometric

Parameters of the models developed on the basis of the World Bank data

The models were constructed on the basis of the data of the World Bank [WB] for the years 1983-2013[4] for the World (WLD)[5], the European Union (EUU), Germany (DEU), France (FRA), Great Britain (GBR), Greece (GRC) and Poland[6] (POL).

Table 1 shows the calculated parameters of functions (1) and (2) and the level of adjustment of the models to the data for the mentioned entities.

Table 1. Summary of the model parameters (1) and (2) for the examined entities

	T=15, α=π										
	Model (1)						Model (2)				
	a_0	a_1	a_2	a_3	R^2		b_0	b_1	b_2	b_3	**R^2**
WLD	16.72089	-0.08227	0.06327	3.47116	98.88%		6.76827	8.662	7.25%	3.26079	**99.30%**
EUU	4.19383	0.23139	0.00779	1.54775	97.33%		-6.01252	10.2446	3.00%	1.52071	97.27%
DEU	0.99628	0.07474	0.000334	0.378875	96.93%		-7.24098	8.31827	0.90%	0.37938	96.94%
FRA	0.771798	0.030799	0.001223	0.261578	97.36%		-0.39911	1.181632	3.50%	0.258304	97.37%
GBR	0.406961	0.065198	0.000439	0.105443	94.08%		-2.40911	2.93081	2.00%	0.1106	93.89%
GRC	4.19383	0.23139	0.00779	1.54775	91.40%		-6.01252	10.2446	3.00%	1.52071	91.15%
POL	-0.03082	0.005185	0.000434	0.043256	97.20%		-0.31632	0.267125	4.00%	0.044071	97.18%

Source: Own elaboration based on the World Bank data [WB].

[4] The World Bank provides data about the GDP for 249 entities (states and groups of states).

[5] The symbols come from the table of the World Bank [WB].

[6] The data for Poland cover the period 1990-2013.

It turned out that for all the analyzed entities the best adjustment is reached for the 15-year- cycle, wherein all test agents are in the same phase of the cycle.

The size of GDP can be represented as the sum of three components:

$$GDP = G\_GDP + CGDP + R\_GDP$$

where:
G_GDP – describes the growth rate,
C_GDP – describes the amplitude and phase of the cycle,
R_GDP – describes the short-term fluctuations[7].

The World GDP

Models (1) and (2)for the global GDP are shaped as follows:

$$GDP\ mod\_ex = 6.76827 + 8.662 \cdot (1+0{,}0725)^{N-1} + 3.26079 \cdot sin\left(2\pi \cdot \frac{N}{15-\pi}\right)$$

$$GDP\ mod\_po = 16.72089 - 0.08227 \cdot N + 0.06327 \cdot N^2 - 3.47116 \cdot sin\left(2\pi \cdot \frac{N}{15}\right)$$

The real data (provided by the World Bank[8]) and the data calculated on the basis of these models are given in the table and the graph (Figure 1). The high level of adjustment (99%) allows one to state that the world economy during the period 1983-2013 „passed" through two 15-year cycles.

The analysis of the differences between the real GDP and the data obtained from the models indicates the existence of fairly regular shorter cycles overlapping 15-year-cycles. These differences are shown in Figure 2 in which two nine-year cycles (1985-1993, 1994-2002) and two four-year cycles (2003-2006 , 2007-2010) can be observed.

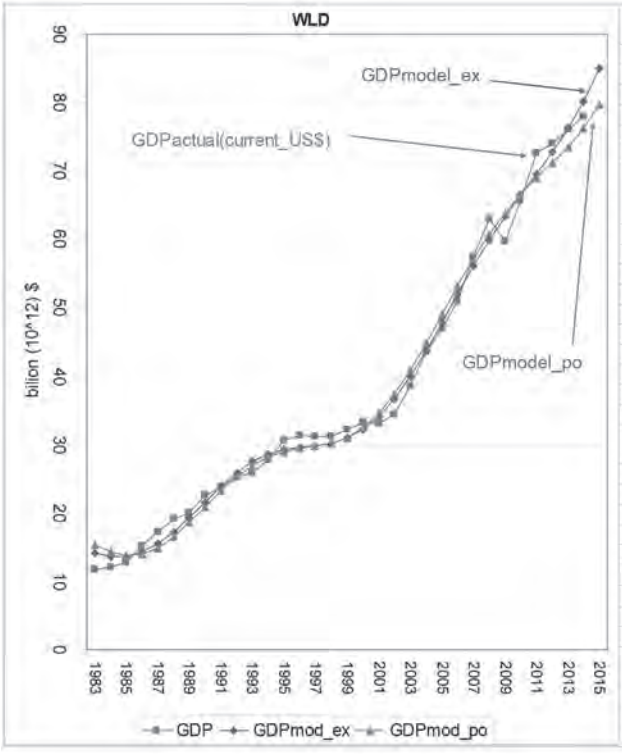
For the global GDP (WLD) model (2) thus it can be written

$$GDP = G\_GDP\ mod\_ex + CGDP\ mod\_ex + R\_GDP\ mod\_ex$$

$$GDP\ mod\_ex = \left[6.76827 + 8.662 \cdot (1+0{,}0725)^{N-1}\right] + \left[3.26079 \cdot sin\left(2\pi \cdot \frac{N}{15-\pi}\right)\right] + (-3.758864 + 3.030148) \quad (3)$$

[7] See further: the World GDP and the GDP of the Union and selected European states

[8] The models were developed on the basis of the World Bank data for the period before January 1, 2015. On July 1 the data were updated. GDP was introduced for 2014 and the „older" figures were updated. Changes to the „older" data were a fraction of a percent (with the exception of GBR for the period 1987 to 1993, during which the average changes amounted to 2.2%). The adjustment of the models developed on the basis of the data for the period 1983-2013 to the data for the period 1983-2014, with the exception of Germany, has slightly increased.



Year	GDP	GDPmod_ex	GDPmod_po
1983	11,680	14,104	15,290
1984	12,126	13,635	14,230
1985	12,749	13,631	13,742
1986	15,084	14,211	13,952
1987	17,180	15,405	14,885
1988	19,080	17,143	16,465
1989	20,046	19,273	18,524
1990	22,547	21,585	20,634
1991	23,670	23,848	23,146
1992	25,341	25,855	25,231
1993	25,775	27,453	26,924
1994	27,679	28,576	28,148
1995	30,631	29,254	28,924
1996	31,289	29,612	29,382
1997	31,198	29,845	29,723
1998	31,080	30,192	30,190
1999	32,233	30,890	31,028
2000	33,276	32,136	32,438
2001	33,079	34,058	34,546
2002	34,359	36,691	37,377
2003	38,586	39,972	40,655
2004	43,465	43,757	44,612
2005	47,034	47,844	49,020
2006	50,971	52,012	53,230
2007	57,452	56,060	57,214
2008	62,983	59,848	60,805
2009	59,705	63,320	63,925
2010	65,489	66,517	66,601
2011	72,572	69,576	68,957
2012	74,042	72,707	71,196
2013	76,124	76,161	73,561
2014	77,869	80,191	76,297
R-squared		99,34%	98,95%

Figure 1. The real GDP and models (1) and (2) [current US $ (billion=1012)] for the world (WLD)

Source: own elaboration based on the World Bank data [WB].

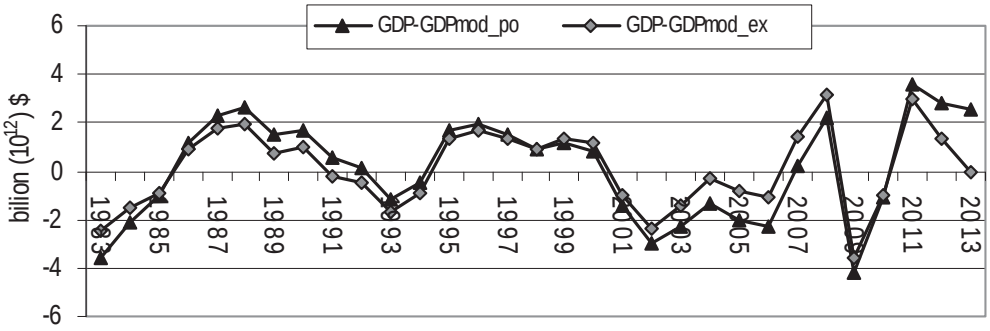


Figure 2. A list of the differences between the real GDP and the models (= R_GDP)

Source: own elaboration.

The GDP of the European Union and selected European states

A similar study was carried out for the European Union, Germany, France, Great Britain, Greece and Poland. Considering the compatibility of the cycle and the phases of these entities with the cycle and phase for the world GDP (see Table 1), an analysis of the differences between the real GDP and the results of model (2) was made:

$$R\_GDP = GDP - GDP\,mod\_ex$$

As a measure of accuracy of synchronizing the individual states' economies, a correlation R_GDP of the examined entities was adopted. The correlation coefficients are included in Table 2.

Table 2. Matrix of correlation differences (R_GDP)

	WLD	EUU	DEU	FRA	GBR	GRC	POL*
WLD	1.00	0.60	0.76	0.66	0.32	0.09	0.75
EUU	0.60	1.00	0.60	0.96	0.79	0.73	0.60
DEU	0.76	0.60	1.00	0.75	0.18	0.04	0.47
FRA	0.66	0.96	0.75	1.00	0.63	0.61	0.59
GBR	0.32	0.79	0.18	0.63	1.00	0.67	0.31
GRC	0.09	0.73	0.04	0.61	0.67	1.00	0.37
POL*	0.75	0.60	0.47	0.59	0.31	0.37	1.00

* – the WB provides the data for Poland for the period 1990-2013.
Source: own elaboration.

Almost all correlation coefficients indicate the existence of fairly significant co-dependences. It is confirmed by the value of the determinant of this matrix which is close to zero.

It is also worth looking at the summary of charts R_GDP (Figure 3). It can be observed that R_GDP are similar for all the examined entities.

Because the presented differences are applicable to all models of the same cycle of years (T = 15 years) and the same phase ($\alpha = \pi$), it seems reasonable to state that in the period from 1983 to 2013, two 15-year cycles are a fact. Synchronised shorter cycles (sub-cycles), to a large extent, overlapped these cycles. Thus, the economy operated in the „rhythm" which is defined with a relatively high accuracy.

It can also be noted that the economies of the European Union, Germany and France are working in the „rhythm of the World". The economies of Great Britain, Greece and Poland reveal less similarity.

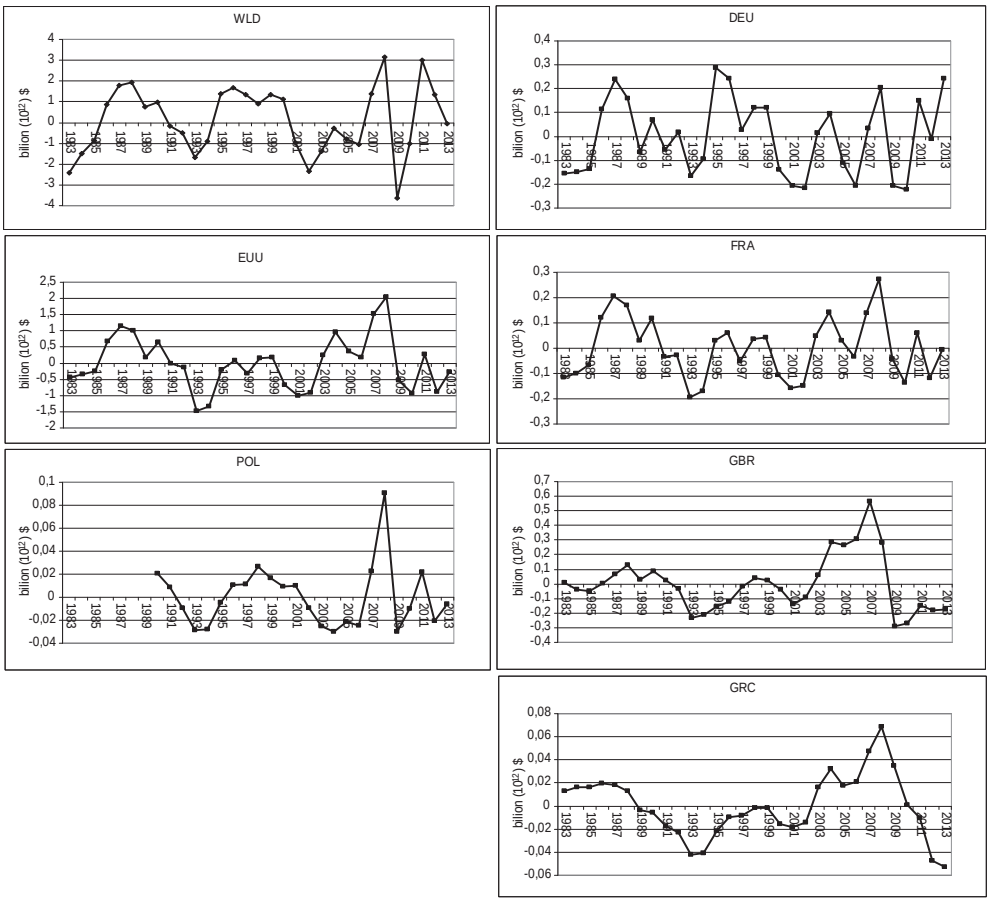


Figure 3. Deviations from the model for the examined entities R_GDP = GDP - GDPmod_ex
Source: own elaboration.

Observations of the Polish GDP 1995-2014

Continuing research on the Polish GDP as presented in Nogalski and Klimek (2015), the CSO data were used, calculated in the system of national accounts ESA 2010[9] [GDP GUS] and the period up to and including 2014 was taken into consideration. In addition, the CSO data were analyzed for the formation of the investment [INV GUS]. The CSO data are presented in the table in Figure 4.

[9] In the cited study, the data of the Central Statistical Office were used, collected in the system of the National Accounts ESA 1995 (the analysis results for both sets of data differ only slightly).

GDP at current prices

On the basis of the available research studies, a mathematical model was searched to describe the forming of the GDP in the period 1995-2010 in the form of a polynomial-trigonometric function (1). Table 3 shows the parameters found for this period at a very high coefficient of adjustment of 99.94% (left part of the table).

The data for the period 2011 - 2014 deviated significantly from the developed model. It was then decided to find a way to revise the model in order to maintain the level of adjustment throughout the period 1995-2014 which is not lower than for the models developed on the basis of the data for the years 1995-2010.

In Table 3 (right part) revised parameters are presented. The changes made to the model are indicated in bold – they consisted in adding a fixed amount to the coefficient a_0 at the amount of 84 billion PLN. The revision introduced in 2011 made it possible to reach the adjustment at the level of 99.97%[10].

Table 3. Parameters of the model (1) for the „nominal GDP" before the change and after the change

	1995-2010				1995-2010		2011-				1995-2014
Model (1)	T=10			α=5/10·π			T=10			α=5/10·π	
	a_0	a_1	a_2	a_3	R^2		$a_{0+84000}$	a_1	a_2	a_3	R^2
	327 636	5 8179,9	505,7	49 869,9	99,94%		411 636	58 179,9	505,7	49 869,9	99,97%

Source: own elaboration.

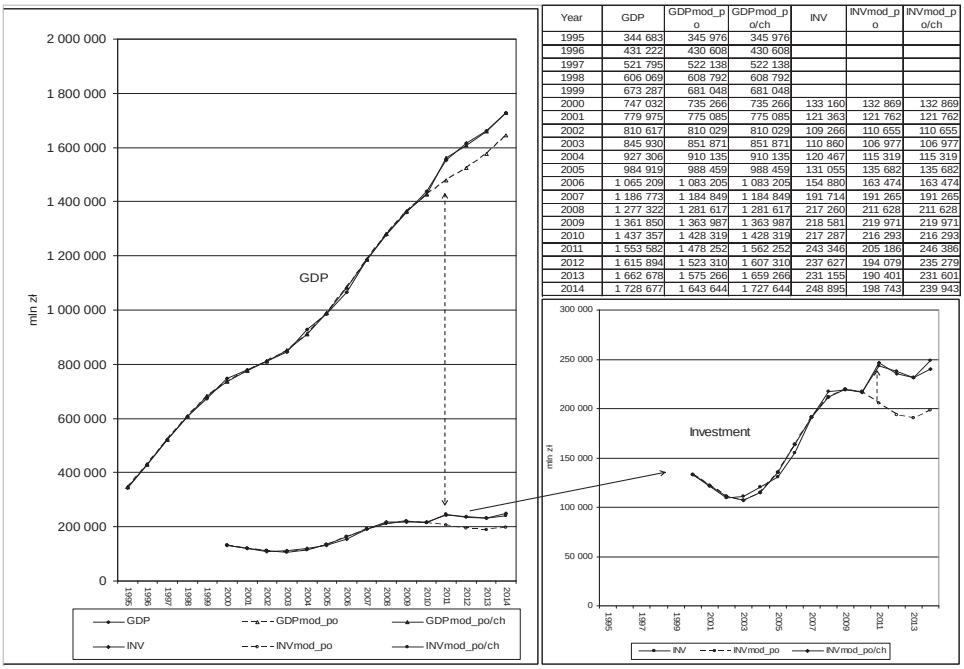
Seeking an explanation for the introduced revision, it was decided to analyze the amount of investment in the individual years. The analysis was carried out for the CSO data for the period 2000-2014. The analysis was conducted in the same way as for the GDP – the model parameters were found for the data up to 2010 (with the adjustment of 99.15%), and having concluded that the further data significantly differed from the developed model, the model was revised, reaching the adjustment level of 99.37% (Table 4). The revision consisted in adding a fixed amount equaling 41.2 billion PLN to the coefficient a_0 in 2011.

[10] The business cycle for the CSO data is different from for the WB data.

Table 4. The model parameters (1) for the „Investments nominal" before the change and after the change

	2000-2010				2000-2010		2011-				2000-2014
Model (1)	T=10			α=4/10·π			T=10			α=4/10·π	
	a_0	a_1	a_2	a_3	R^2		$a_{0+41200}$	a_1	a_2	a_3	R^2
	63364,82	8342,4	0	33089,37	99,15%		104564,8	8342,4	0	33089,37	99,37%

Figure 4 shows graphs reflecting the actual data and the data from the models.



Year	GDP	GDPmod_po	GDPmod_po/ch	INV	INVmod_po	INVmod_po/ch
1995	344 683	345 976	345 976			
1996	431 222	430 608	430 608			
1997	521 795	522 138	522 138			
1998	606 069	608 792	608 792			
1999	673 287	681 048	681 048			
2000	747 032	735 266	735 266	133 160	132 869	132 869
2001	779 975	775 085	775 085	121 363	121 762	121 762
2002	810 617	810 029	810 029	109 266	110 655	110 655
2003	845 930	851 871	851 871	110 860	106 977	106 977
2004	927 306	910 135	910 135	120 467	115 319	115 319
2005	984 919	988 459	988 459	131 055	135 682	135 682
2006	1 065 209	1 083 205	1 083 205	154 880	163 474	163 474
2007	1 186 773	1 184 849	1 184 849	191 714	191 265	191 265
2008	1 277 322	1 281 617	1 281 617	217 260	211 628	211 628
2009	1 361 850	1 363 987	1 363 987	218 581	219 971	219 971
2010	1 437 357	1 428 319	1 428 319	217 287	216 293	216 293
2011	1 553 582	1 478 252	1 562 252	243 346	205 186	246 386
2012	1 615 894	1 523 310	1 607 310	237 627	194 079	235 279
2013	1 662 678	1 575 266	1 659 266	231 155	190 401	231 601
2014	1 728 677	1 643 644	1 727 644	248 895	198 743	239 943

Figure 4. GDP and INV (current prices in million PLN) and the models

Source: own calculations based on the CSO data [GDP GUS, INV GUS].

The graphs in Figure 4 reflecting the data obtained from the uncorrected models are marked using a dotted line.

Having eliminated the C_GDP factor reflecting the volatility of the economic conditions and the residue of R_GDP from the GDP model and the INV model, it was decided to look for the relations between the increases for these models. In 2011, the relation was clearly disturbed (Figure 5).

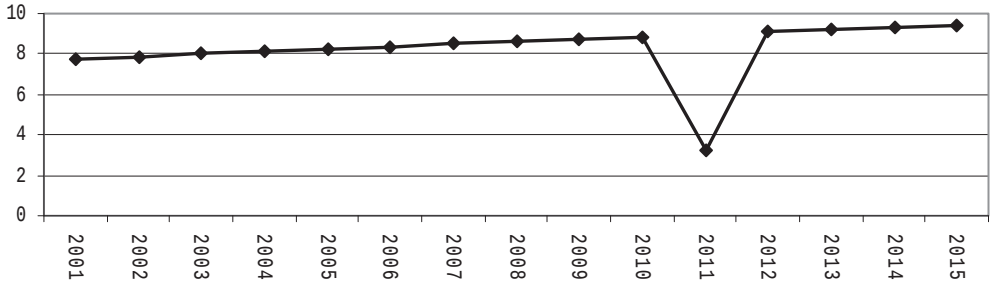


Figure 5. ΔG_GDP / ΔG_INV

Source: own elaboration.

Real GDP

The research was also continued for the real GDP. The prices of 1995 were adopted as basic (after the denomination). The real GDP was calculated by dividing the nominal GDP by the cumulative consumer price index [WCEN GUS]. The real GDP was estimated for the model parameters (1) and (2).

As was the case with the nominal GDP, models were developed on the basis of the data for the period 1995-2010. Table 5 shows the parameters found for this period, reaching a very high coefficient of 99.78% of the adjustment level for model (1) and 99.56% for model (2) (left part of the table).

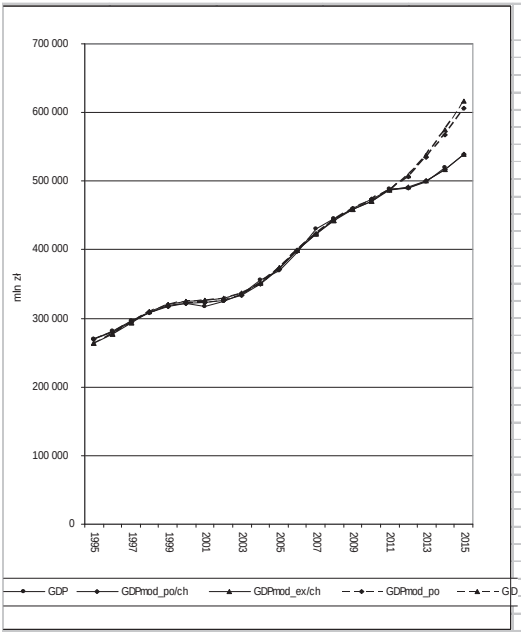
Unlike for the nominal GDP, the data deviated significantly from the developed models only since 2012. In the attempt to find a way to revise them in order to maintain the level of adjustment, it was concluded that the revision of the models must be deeper than for the nominal GDP.

Table 5. Model parameters (1) and (2) „real GDP" before the change and after the change

	1995-2011				1995-2011		2012-				1995-2014
Model (1)	T=9		N^2	$\alpha=5/9\cdot\pi$			T=9		$N^{2\text{-}0,185}$	$\alpha=5/9\cdot\pi+2/9\cdot\pi$	
	a_0	a_1	a_2	a_3	R^2		$a_{0+65000}$	a_1	a_2	a_3	R^2
	278945	1882,5	640,2	13708,5	99,82%		343945	1882,5	640,2	13708,5	**99,88%**
Model (2)	T=9		N^2	$\alpha=5/9\cdot\pi$			T=9		N^2	$\alpha=5/9\cdot\pi+2/9\cdot\pi$	
	b_0	b_1	b_2	b_3	R^2		$b_{0+85000}$	b_1	$b_{2\text{-}0,0245}$	b_3	R^2
	192885,9	81915,2	8,5%	13450	99,64%		277885,9	81915,2	6,05%	13450	**99,77%**

Source: own elaboration.

Figure 6 shows the graphs which reflect the real data and the data from the models.



Year	GDP	GDPmod_ po/ch	GDPmod_ ex/ch	GDPmod_ po	GDPmod_ ex
1995	269 705	269 596	263 153	269 596	263 153
1996	281 417	280 582	277 164	280 582	277 164
1997	296 367	295 043	293 919	295 043	293 919
1998	307 900	308 590	309 163	308 590	309 163
1999	318 778	317 863	319 654	317 863	319 654
2000	321 248	322 099	324 704	322 099	324 704
2001	317 928	323 492	326 528	323 492	326 528
2002	324 258	326 166	329 242	326 166	329 242
2003	335 698	334 243	336 967	334 243	336 967
2004	355 547	349 918	351 937	349 918	351 937
2005	369 869	372 428	373 495	372 428	373 495
2006	396 060	398 412	398 438	398 412	398 438
2007	430 497	423 483	422 567	423 483	422 567
2008	444 667	444 279	442 697	444 279	442 697
2009	458 061	460 039	458 205	460 039	458 205
2010	471 207	472 956	471 377	472 956	471 377
2011	488 312	487 154	486 403	487 154	486 403
2012	489 775	490 535	491 595	506 755	507 486
2013	499 460	500 258	500 447	533 953	536 951
2014	519 286	516 848	516 311	567 986	574 234
2015		539 536	538 488	605 494	616 240

Figure 6. The real GDP and the models (million PLN)

Source: Own calculations based on the CSO data [GDP GUS, WCEN GUS]

The graphs in Figure 6 reflecting the data derived from the unrevised models are marked using a dotted line.

In model (1) an amount of 65 billion PLN was added to factor a_0, and also the growth rate was reduced by changing the exponent of 2 to 1,815 and the phase of the cycle was revised by 2/9 π (extending the cycle), reaching the level of adjustment 99.88%. In model (2), the amount of 85 billion PLN was added to the coefficient b_0 and also the growth rate was reduced by reducing the coefficient b_2 by 2.45%. The cycle phase was revised in the same way as for model (1).

In Table 5 (the right part) the revised parameters are presented. The obtained adjustment was 99.77%.

The prospect of 2050 - the world and Poland

In order to evaluate the development of the world economy in 2050, the following assumptions were made (see (3)):

1. G_GDP - growth parameters (A0, B0, a1, b1, a2, b2) do not change - the growth rate for model (1) decreases and model (2) tends to b2.
2. C_GDP – economy 'fluctuates" in 15-year cycles (a3, b3, T, α do not change - the amplitude and phase of the cycle remain unchanged).
3. R_GDP - sub-cycles occurring in the last 15 years to be repeated[11] in the next 15-year - periods.

Indicators GDP_{2050} / GDP_{2014} and average annual growth rates $((GDP_{2050} / GDP_{2014})^{(1/36)} - 1)$ were calculated on the basis of the models.

The results obtained on the basis of the developed models were compared with the results published by PricewaterhouseCoopers [PWC][12] (Table 6).

Table 6. GDP growth forecast for 2050 in relation to 2014

	GDP (current US$) 2050/2014			Average annual grow th rates		
	Model (1)	Model (2)	Pw C	Model (1)	Model (2)	Pw C
WLD	4.0	11.8	3.0	3.9%	7.1%	3.1%
EUU	3.1	3.7	nd	3.1%	3.7%	nd
DEU	2.2	2.3	1.7	2.3%	2.4%	1.5%
FRA	3.0	4.0	2.0	3.1%	4.0%	1.9%
GBR	2.4	3.0	2.3	2.5%	3.1%	2.4%
GRC	3.2	3.2	nd	3.2%	3.3%	nd
POL*	4.3	6.1	2.5	4.1%	5.2%	2.6%

* – an indicator for the Polish Central Statistical Office stated in Table 7.
Source: own elaboration.

The course of the annual forecast growth rates for WLD is shown in Figure 7.

The bold line indicates the growth rate calculated on the basis of the real data in the period 1983-2013, and in the subsequent years it shows the growth rate for model (2) (top) and (1) (bottom) resulting from the forecast, assuming the following are fulfilled: 1-3 (G_GDP + C_GDP + R_GDP). Solid lines (regular) reflect rates of growth respectively for models (2) and (1) assuming that the fol-

[11] The assumption was adopted to show a sample diversity of a future growth rate.

[12] The forecasted average growth rates of real increase in GDP (in % on an annualized basis, in the years 2015-2050). PwC analysis was conducted on the basis of the UN projections on population size.

lowing are fulfilled 1-2 (G_GDP + C_GDP), and the dotted lines assuming that the following is fulfilled 1 (G_GDP).

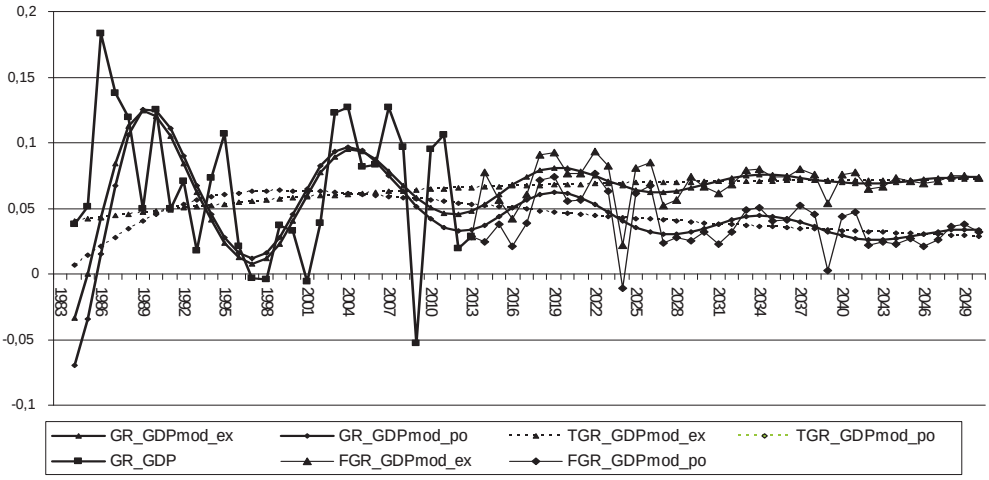


Figure 7. Forecasted growth rate for the WLD

Source: own elaboration.

The upper graph in Figure 8 shows the course of cycles: resulting from the World Bank and resulting from the data from the Polish Central Statistical Office for the Polish GDP (it seems from the graph that, according to the CSO data , Poland is now entering a phase of recovery, which will last until 2018/2019 and then it will enter a recession phase; the global economy should be in the recovery phase at the moment). In the lower diagram, Poland's cycles according to the CSO were referred to the sub-cycles resulting from the World Bank for the world.

The forecast for Poland for 2050 was also prepared on the basis of the CSO data. The result is shown in Table 7. The table also gives the forecast of PricewaterhouseCoopers [PwC].

Table 7. Polish GDP growth forecast for the period 2014-2050 (based on the CSO data)

	PwC	GDPC mod_po	GDPR mod_po	GDPR mod_ex
GDPx 2050/2014	2,5	3,1	2,7	4,5
Average annual growth rates	2,6%	3,2%	2,8%	4,3%

Source: own elaboration.

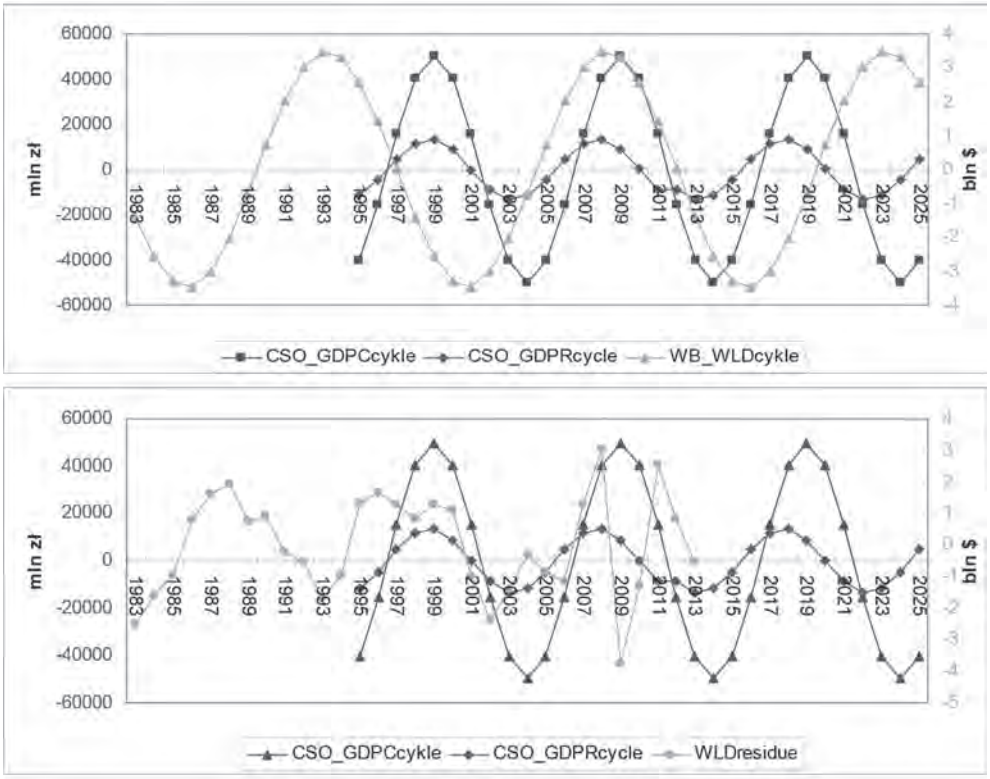


Figure 8. Cycles according to the data from the WB and CSO (for Poland)
Source: own elaboration.

On the basis of the developed models, attempts were made to compare the forecasts outlined in the Medium-Term National Development Strategy 2020 [MNDS] with the forecast resulting from the revised models[13] (Table 8).

Table 8. Polish GDP growth forecast for the period 2010-2020

	MNDS	GDPC mod_po	GDPR mod_po	GDPR mod_ex
GDPx 2020/2010	140%-144%	161,5%	133,2%	134,4%

Source: own elaboration.

[13] While maintaining the „old" parameters, indicators calculated as based on the models would be the following: GDPCmod_po - 155,6%, GDPRmod_po - 159,0%, GDPRmod_ex - 172,7%.

The weakened growth rate of the GDP „captured" by the models looks consolidated. Its acceleration requires implementing systemic and institutional changes.

The authors consider public administration, whose activities largely contribute to the prolongation of the investment process, as one of the brakes of economic growth. They believe that one of the key factors leading to the way in which administration functions is, among others, lack of an institution responsible for maintaining the legal order of the land[14].

Conclusions

Answering the question whether Samuelson and Nordhaus were right in saying that „There is no pattern in the type formulas, the trajectory of the planets or swing of the pendulum, which would be used to predict the duration and timing of business cycles. They resemble rather the change of weather", the authors conclude that precise description of the future size of the GDP is impossible, however, it can be possible to mathematically describe domestic product not only of states but also of groups of states, including the duration and time of cycles when using the same functions which are needed to calculate a deviation of a pendulum, with accuracy that is significant at a macroeconomic level.

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