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SOME EFFECTIVE ESTIMATION PROCEDURES UNDER NON-RESPONSE IN TWO-PHASE SUCCESSIVE SAMPLING

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ABSTRACT

This work is designed to assess the effect of non-response in estimation of the current population mean in two-phase successive sampling on two occasions. Sub-sampling technique of non-respondents has been used and exponential methods of estimation under two-phase successive sampling arrangement have been proposed. Properties of the proposed estimation procedures have been examined. Empirical studies are carried out to justify the suggested estimation procedures and suitable recommendations have been made to the survey practitioners.

Key words: non-response, successive sampling, two-phase sampling, mean square error, optimum replacement strategy.

1. Introduction

In collecting information through sample surveys, there may arise numerous problems; one of them is non-response. It frequently occurs in mail surveys, where some of the selected units may refuse to return back the filled in questionnaires. An estimate obtained from such an incomplete survey may be misleading, especially when the respondents differ significantly from the non-respondents, because the estimate may be a biased one. Hansen and Hurwitz (1946) suggested a technique of sub-sampling of non-respondents to handle the problem of non-response. Cochran (1977) and Fabian and Hyunshik (2000) extended the Hansen and Hurwitz (1946) technique for the situation when besides the information on the character under study, information on auxiliary character is also available. Recently, Choudhary *et al*. (2004) Singh and Priyanka (2007), Singh and Kumar

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(2009, 2010), Singh *et al.* (2011) and Garcia Luengo (2013) used the Hansen and Hurwitz (1946) technique for the estimation of population mean on the current occasion in two-occasion successive sampling.

If the study character of a finite population is subject to change over time, a single occasion survey is insufficient. For such a situation successive sampling provides a strong tool for generating reliable estimates over different occasions. Sampling on successive occasions was first considered by Jessen (1942) in the analysis of farm data. The theory of successive (rotation) sampling was further extended by Patterson (1950), Eckler (1955), Rao and Graham (1964), Sen (1971, 1972, 1973), Gupta (1979), Das (1982) and Singh and Singh (2001) among others.

In sample surveys, the use of auxiliary information has shown its significance in improving the precision of estimates of unknown population parameters. When the population parameters of auxiliary variable are unknown before start of the survey we go for two-phase (double) sampling structure to provide the reliable estimates of the unknown population parameters. Singh and Singh (1965) used two-phase (double) sampling for stratification on successive occasions. Recently, Singh and Prasad (2011) and Singh and Homa (2014) applied two-phase sampling scheme with success in the estimation of the current population mean in two-occasion successive sampling.

The aim of the present work is to study the effect of non-response when it occurs on various occasions in two-occasion successive (rotation) sampling. Recently, Bahl and Tuteja (1991), Singh and Vishwakarma (2007) and Singh and Homa (2013) suggested exponential type estimators of population mean under different realistic situations. Motivated with the dominating nature of these estimators and utilizing the information on a stable auxiliary variable with unknown population mean over both occasions, some new exponential methods of estimation have been proposed to estimate the current population mean in two-phase successive (rotation) sampling arrangement.. The Hansen and Hurwitz (1946) technique of sub-sampling of non-respondents has been used to reduce the negative effects of non-response. Properties of the proposed estimators are examined and their empirical comparisons are made with the similar estimator and with the natural successive sampling estimator when complete response is observed on both occasions. Results are interpreted and followed by suitable recommendations.

2. Sample structures and symbols

Let $U = (U_1, U_2, -, -, -, U_N)$ be the finite population of N units, which has been sampled over two occasions. The character under study is denoted by x(y) on the first (second) occasion respectively. It is assumed that the non-response occurs only in study variable x(y) and information on an auxiliary variable z (stable over occasion), whose population mean is unknown on both occasions, is available and positively correlated with study variable. Since we have assumed that non-response occurs on both occasions, the population can be divided into two classes – those who will respond at the first attempt and those who will not on both occasions. Let the sizes of these two classes be N_1^* and N_2^* respectively on the first occasion and the corresponding sizes on the current (second) occasion be N_1 and N_2, respectively. To furnish a good estimate of the population mean of the auxiliary variable z on the first occasion, a preliminary sample of size n' is drawn from the population by the simple random sampling without replacement (SRSWOR) method, and information on z is collected. Further, a second-phase sample of size n ($n' > n$) is drawn from the first-phase (preliminary) sample by the SRSWOR method and henceforth the information on the study character x is gathered. We assume that out of selected n units, n_1 units respond and n_2 unit do not respond. Let n_{2h} denote the size of sub-sample drawn from the non-responding units in the sample on first occasion. A random sub-sample s_m of $m = n\lambda$ units is retained (matched) from the responding units on the first occasion for its use on the second occasion under the assumption that these units will give complete response on the second occasion as well. Once again, to furnish a fresh estimate of the population mean of the auxiliary variable z on the second occasion, a preliminary (first-phase) sample of size u' is drawn from the non-sampled units of the population by the SRSWOR method and information on z is collected. A second-phase sample of size $u = (n-m) = n\mu$ ($u' > u$) is drawn from the first-phase (preliminary) sample by the SRSWOR method and the information on study variable y is gathered. It is obvious that the sample size on the second occasion is also n. Here λ and μ ($\lambda+\mu=1$) are the fractions of the matched and fresh samples, respectively, on the second (current) occasion. We assume that in the unmatched portion of the sample on the current (second) occasion u_1 units respond and u_2 units do not respond. Let u_{2h} denote the size of the sub-sample drawn from the non-responding units in the fresh sample (s_u) on the current (second) occasion. Hence, onwards, we use the following notations:

$\bar{X}, \bar{Y}, \bar{Z}$: The population means of the variables x, y and z respectively.

$\bar{y}_m, \bar{y}_u, \bar{y}_{u_1}, \bar{y}_{u_{2h}}, \bar{x}_n, \bar{x}_{n_1}, \bar{x}_{n_{2h}}, \bar{x}_m, \bar{z}_m, \bar{z}_u$: The sample means of the respective variables based on the sample sizes shown in suffices.

$\bar{z}_n^{'}, \bar{z}_u^{'}$: The sample means of the auxiliary variable z and based on the first-phase samples of sizes u' and n' respectively.

$\rho_{yx}, \rho_{xz}, \rho_{yz}$: The population correlation coefficients between the variables shown in suffices.

S_x^2, S_y^2, S_z^2 : The population variances of the variables x, y and z respectively.

S_{2x}^2, S_{2y}^2 : The population variances of the variables x and y respectively in the non-responding units of the population.

C_x, C_y, C_z : The coefficients of variation of the variables x, y and z respectively.

C_{2x}, C_{2y} : The coefficients of variation of the variables x and y in the non-responding units of the population.

$W^* = \frac{N_2^*}{N}$: The proportion of non-responding units in the population at first occasion. $W = \frac{N_2}{N}$: The proportion of non-responding units in the population on the current (second) occasion.

$$f_1 = \frac{n_2}{n_{2h}} \text{ and } f_2 = \frac{u_2}{u_{2h}}.$$

3. Formulation of estimation strategy

To estimate the population mean $\bar{Y}$ on the current (second) occasion, two different estimators are considered – one estimator T_u based on sample s_u of size u drawn afresh on the second occasion and the second estimator T_m based on the sample s_m of size m, which is common to both occasions. Since the non-response occurs in the samples s_n and s_u, we have used the Hansen and Hurwitz (1946) technique to propose the estimators T_u and T_m. Hence, the estimators T_u and T_m for estimating the current population mean $\bar{Y}$ are formulated as

$$T_u = \bar{y}_u^* \exp\left(\frac{\bar{z}_u' - \bar{z}_u}{\bar{z}_u' + \bar{z}_u}\right) \text{ and } T_m = \bar{y}_m \exp\left(\frac{\bar{x}_n^* - \bar{x}_m}{\bar{x}_n^* + \bar{x}_m}\right)\left(\frac{\bar{z}_n'}{\bar{z}_m}\right)$$

where

$$\bar{x}_n^* = \frac{n_1 \bar{x}_{n_1} + n_2 \bar{x}_{n_{2h}}}{n} \text{ and } \bar{y}_u^* = \frac{u_1 \bar{y}_{u_1} + u_2 \bar{y}_{u_{2h}}}{u}.$$

Combining the estimators T_u and T_m, finally we have the following estimator of population mean $\bar{Y}$ on the current (second) occasion

$$T = \varphi\, T_u + (1-\varphi) T_m \tag{3.1}$$

where $\varphi\ (0 \le \varphi \le 1)$ is the unknown constant (scalar) to be determined under certain criterions.

4. Properties of the estimator T

Since the estimators T_u and T_m are exponential type estimators, the population mean $\bar{Y}$ are biased, therefore the resulting estimator T defined in equation (3.1) is also a biased estimator of $\bar{Y}$. The bias B (.) and the mean square error M (.) of the estimator T are derived up to the first order of approximations using the following transformations:

$\bar{y}_m = (1+e_1)\bar{Y}$, $\bar{y}_u = (1+e_2)\bar{Y}$, $\bar{y}_u^* = (1+e_3)\bar{Y}$, $\bar{x}_m = (1+e_4)\bar{X}$, $\bar{x}_n = (1+e_5)\bar{X}$, $\bar{x}_n^{'} = (1+e_6)\bar{X}$, $\bar{x}_n^* = (1+e_7)\bar{X}$, $\bar{z}_m = (1+e_8)\bar{Z}$, $\bar{z}_u = (1+e_9)\bar{Z}$, $\bar{z}_u^{'} = (1+e_{10})\bar{Z}$, $\bar{z}_n^{'} = (1+e_{11})\bar{Z}$, such that $E(e_i) = 0$, $|e_i| < 1 \ \forall \ i = 1, 2, 3, ---, 11$. Under the above transformations, the estimators T_u and T_m take the following forms:

$$T_u = \bar{Y}(1+e_3)\exp\left[\frac{1}{2}(e_{10}-e_9)\left(1+\frac{1}{2}(e_{10}+e_9)\right)^{-1}\right] \tag{4.1}$$

and

$$T_m = \bar{Y}(1+e_1)(1+e_{11})(1+e_8)^{-1}\exp\left[\frac{1}{2}(e_7-e_4)\left(1+\frac{1}{2}(e_7+e_4)\right)^{-1}\right] \tag{4.2}$$

Thus, we have the following theorems:

Theorem 4.1.

Bias of the estimator T to the first order of approximations is obtained as

$$B(T) = \varphi B(T_u)+(1-\varphi)B(T_m) \tag{4.3}$$

where

$$B(T_u) = \bar{Y}\left(\frac{1}{u}-\frac{1}{u'}\right)\left(\frac{3}{8}C_z^2-\frac{1}{2}\rho_{yz}C_yC_z\right)$$

and

$$B(T_m) = \bar{Y}\begin{Bmatrix}\left(\frac{1}{m}-\frac{1}{n}\right)\left(\frac{3}{8}C_x^2+\frac{1}{2}\rho_{xz}C_xC_z-\frac{1}{2}\rho_{xy}C_yC_x\right)\\ -\frac{1}{8}\frac{(f_1-1)}{n}W^*C_{2x}^2+\left(\frac{1}{m}-\frac{1}{n'}\right)\left(C_z^2-\rho_{yz}C_yC_z\right)\end{Bmatrix}$$

Proof

The bias of the estimator T is given by

$$B(T) = E\left[T-\bar{Y}\right] = \varphi E(T_u-\bar{Y}) + (1-\varphi)\,E(T_m-\bar{Y})$$
$$= \varphi B(T_u)+\left(1-\varphi\right)B(T_m) \tag{4.4}$$

where

$$B(T_u) = E\left[T_u-\bar{Y}\right] \text{ and } B(T_m) = E\left[T_m-\bar{Y}\right].$$

Substituting the expressions of T_u, and T_m from equations (4.1) and (4.2) in equation (4.4), expanding the terms binomially and exponentially, taking expectations and retaining the terms up to the first order of sample sizes, we have the expressions for the bias of the estimator T as described in equation (4.3).

Theorem 4.2.

The mean square error of the estimator T to the first order of approximations is obtained as

$$M(T) = \varphi^2 M\left(T_u\right)+\left(1-\varphi\right)^2 M\left(T_m\right)+2\varphi\left(1-\varphi\right)C \tag{4.5}$$

where

$$M(T_u) = E\left(T_u-\bar{Y}\right)^2 = \left[\left(\frac{1}{u}-\frac{1}{u'}\right)\left(\frac{1}{4}-\rho_{yz}\right)+\left(\frac{1}{u}-\frac{1}{N}\right)+\frac{W(f_2-1)}{u}\right]S_y^2 \tag{4.6}$$

$$M(T_m) = E\left(T_m-\bar{Y}\right)^2 = \begin{bmatrix} \left\{\left(\frac{1}{m}-\frac{1}{n}\right)\left(\frac{1}{4}+\rho_{xz}-\rho_{yx}\right)\right\}+\left(\frac{1}{m}-\frac{1}{N}\right) \\ +\left\{\left(\frac{1}{m}-\frac{1}{n'}\right)\left(1-2\rho_{yz}\right)\right\}+\frac{1}{4}\frac{(f_1-1)}{n}W^* \end{bmatrix} S_y^2 \tag{4.7}$$

and

$$C = E\left[\left(T_u-\bar{Y}\right)\left(T_m-\bar{Y}\right)\right] = -\frac{S_y^2}{N}. \tag{4.8}$$

Proof

It is obvious that the mean square error of the estimator T is given by

$$M(T) = E\left[T-\bar{Y}\right]^2 = E\left[\varphi\left(T_u-\bar{Y}\right)+\left(1-\varphi\right)\left(T_m-\bar{Y}\right)\right]^2$$

$$= \varphi^2 E\left(T_u-\bar{Y}\right)^2+\left(1-\varphi\right)^2 E\left(T_m-\bar{Y}\right)^2+2\varphi\left(1-\varphi\right)E\left[\left(T_u-\bar{Y}\right)\left(T_m-\bar{Y}\right)\right]$$

$$= \varphi^2 M(T_u)+\left(1-\varphi\right)^2 M\left(T_m\right)+2\varphi\left(1-\varphi\right)C \tag{4.9}$$

Substituting the expressions of T_u, and T_m from equations (4.1)-(4.2) in equation (4.9), expanding the terms binomially and exponentially, taking expectations and retaining the terms up to the first order of sample sizes, we have the expression of the mean square error of the estimator T as it is given in equation (4.5).

Remark 4.1.

The expression of the mean square error in the equation (4.5) is derived under the assumptions (i) that the coefficients of variation of non-response class are similar to that of the population, i.e. $C_{2x} = C_x$ and $C_{2y} = C_y$, and (ii) since x and y are the same study variable over two occasions and z is the auxiliary variable correlated to x and y, looking at the stability nature of the coefficients of variation, viz. Reddy (1978), the coefficients of variation of the variables x, y and z in the population are considered equal, i.e. $C_x = C_y = C_z$.

5. Minimum mean square error of the estimator T

Since the mean square error of the estimator T in equation (4.5) is the function of unknown constant φ, it is minimized with respect to φ, and subsequently the optimum value of φ is obtained as

$$\varphi_{opt} = \frac{M(T_m)-C}{M(T_u)+M(T_m)-2C} . \qquad (5.1)$$

Now, substituting the value of φ_{opt} in equation (4.5), we get the optimum mean square error of T as

$$M(T)_{opt} = \frac{M(T_u).M(T_m)-C^2}{M(T_u)+M(T_m)-2C} . \qquad (5.2)$$

Further, substituting the values from equations (4.6)-(4.8) in equation (5.2), we get the simplified value of $M\,(T)_{opt}$ which is given below:

$$M(T)_{opt} = \frac{a_3+\mu a_2+\mu^2 a_1}{a_6+\mu a_5+\mu^2 a_4} \frac{S_y^2}{n} . \qquad (5.3)$$

where

$a_1 = ac+k^2 f^2$, $a_2 = ad+cb-k^2 f^2$, $a_3 = bd$, $a_4 = c-a+2kf$, $a_5 = a-b+d-2kf$, $a_6 = b$,

$a = -(f+t_1 a_0)$, $b = a_0+1+(f_2-1)W$, $c = t_2 d_1+c_1+f-\frac{1}{4}(f_1-1)W^*$,

$d = 1-f+(1-t_2)d_1+\frac{1}{4}(f_1-1)W^*$, $k = -1$, $a_0 = \frac{1}{4}-\rho_{yz}$, $c_1 = \frac{1}{4}+\rho_{xz}-\rho_{xy}$, $d_1 = 1-2\rho_{yz}$,

$f = \frac{n}{N}$, $f_1 = \frac{n_2}{n_{2h}}$, $f_2 = \frac{u_2}{u_{2h}}$, $t_1 = \frac{n}{u'}$ and $t_2 = \frac{n}{n'}$.

6. Optimum replacement strategy

Since the mean square error of the estimator T given in equation (5.3) is the function of μ (fractions of the sample to be drawn afresh at the second occasion), the optimum value of μ is determined to estimate the population mean $\bar{Y}$ with maximum precision and lowest cost. To determine the optimum value of μ, we minimized the mean square error of the estimator T given in equation (5.3) with respect to μ, which results in quadratic equation in μ and the respective solutions of μ, say $\hat{\mu}$, are given below:

$$p_1\mu^2+2p_2\mu+p_3=0 \tag{6.1}$$

$$\hat{\mu}=\frac{-p_2\pm\sqrt{p_2^2-p_1p_3}}{p_1} \tag{6.2}$$

where

$$p_1=a_1a_5-a_2a_4,\ p_2=a_1a_6-a_3a_4 \text{ and } p_3=a_2a_6-a_3a_5.$$

From equation (6.2) it is obvious that real values of $\hat{\mu}$ exist iff the quantities under square root are greater than or equal to zero. For any combinations of correlations ρ_{yx}, ρ_{xz} and ρ_{yz}, which satisfy the conditions of real solutions, two real values of $\hat{\mu}$ are possible. Hence, while choosing the values of $\hat{\mu}$, it should be remembered that $0\le\hat{\mu}\le 1$. If both the values of $\hat{\mu}$ satisfy the stated condition, we chose the smaller value of $\hat{\mu}$ as it will help in reducing the cost of the survey. All other values of μ are inadmissible. Substituting the admissible value of $\hat{\mu}$, say $\mu^{(0)}$, from equation (6.2) into equation (5.3), we have the optimum value of the mean square error of T, which is shown below:

$$M(T^0)_{opt}=\frac{a_3+\mu^{(0)}a_2+\mu^{(0)2}a_1}{a_6+\mu^{(0)}a_5+\mu^{(0)2}a_4}\frac{S_y^2}{n}. \tag{6.3}$$

7. Some special cases

Case 1: When non-response occurs only at first occasion

When non-response occurs only at first occasion, the estimator for the mean $\bar{Y}$ on the current occasion may be obtained as

$$T^*=\varphi^*\xi_{1u}+\left(1-\varphi^*\right)T_m \tag{7.1}$$

where

$\xi_{1u} = \bar{y}_u \exp\left(\frac{\bar{z}_u^{'} - \bar{z}_u}{\bar{z}_u^{'} + \bar{z}_u} \right)$ and T_m is defined in section 3, where $\varphi^* \ (0 \le \varphi^* \le 1)$ is the unknown constant (scalar) to be determined under certain criterions.

7.1. properties of the estimator T^*

Since the estimator T^* is exponential type estimator, it is biased for the population mean $\bar{Y}$. The bias B (.) and the mean square error M (.) of the estimator T^* are derived up to the first order of approximations similar to that of the estimator T.

Theorem 7.1.

The bias of the estimator T^* to the first order of approximations is obtained as

$$B(T^*) = \varphi^* B(\xi_{1u}) + (1-\varphi^*) B(T_m) \tag{7.2}$$

where

$$B(\xi_{1u}) = \bar{Y}\left(\frac{1}{u} - \frac{1}{u^{'}}\right)\left(\frac{3}{8}C_z^2 - \frac{1}{2}\rho_{yz}C_y C_z\right)$$

and $B(T_m)$ is defined in section 4.

Theorem 7.2.

The mean square error of the estimator T^* to the first order of approximations is obtained as

$$M(T^*) = \varphi^{*2} M\left(\xi_{1u}\right) + \left(1-\varphi^*\right)^2 M\left(T_m\right) + 2\varphi^*\left(1-\varphi^*\right)C^* \tag{7.3}$$

where

$$M(\xi_{1u}) = E\left(\xi_{1u} - \bar{Y}\right)^2 = \left[\left(\frac{1}{u} - \frac{1}{u^{'}}\right)\left(\frac{1}{4} - \rho_{yz}\right) + \left(\frac{1}{u} - \frac{1}{N}\right)\right] S_y^2 \tag{7.4}$$

$$C^* = E\left[\left(\xi_{1u} - \bar{Y}\right)\left(T_m - \bar{Y}\right)\right] = -\frac{S_y^2}{N} \tag{7.5}$$

and $M(T_m)$ is defined in section 4.

Since the mean square error of the estimator T^* in equation (7.3) is the function of unknown constant φ^*, it is minimized with respect to φ^*, and subsequently the optimum value of φ^* is obtained as

$$\varphi^*_{opt} = \frac{M(T_m) - C^*}{M(\xi_{1u}) + M(T_m) - 2C^*}. \tag{7.6}$$

Now substituting the value of φ^{*}_{opt} in equation (7.6), we get the optimum mean square error of the estimator T^{*} as

$$M(T^{*})_{opt}=\frac{M(\xi_{1u}).M(T_m)-C^{*2}}{M(\xi_{1u})+M(T_m)-2C^{*}}, \tag{7.7}$$

Further, substituting the values from equations (4.7), (7.4) and (7.5) in equation (7.7), we get the simplified value of M $(T^{*})_{opt}$, which is given below:

$$M(T^{*})_{opt} = \frac{a_3^{*}+\mu^{*}a_2^{*}+\mu^{*2}a_1}{a_6^{*}+\mu^{*}a_5^{*}+\mu^{*2}a_4}\frac{S_y^2}{n} \tag{7.8}$$

where

$a_2^{*}=ad+cb^{*}-k^2f^2$, $a_3^{*}=b^{*}d$, $a_5^{*}=a-b^{*}+d-2kf$, $a_6=b^{*}$, $b^{*}=a_0+1$, a_1 and a_4 are defined in section 5.

To determine the optimum values of μ^{*}, we minimized the mean square error of the estimator T^{*} given in equation (7.8) with respect to μ^{*} , which results in quadratic equation in μ^{*} and the respective solutions of μ^{*}, say $\hat{\mu}^{*}$, are given below:

$$p_1^{*}\mu^{*2}+2p_2^{*}\mu^{*}+p_3^{*}=0 \tag{7.9}$$

$$\hat{\mu}^{*} = \frac{-p_2^{*}\pm\sqrt{p_2^{*2}-p_1^{*}p_3^{*}}}{p_1^{*}} \tag{7.10}$$

where

$$p_1^{*}=a_1a_5^{*}-a_2^{*}a_4,\ p_2^{*}=a_1a_6^{*}-a_3^{*}a_4 \text{ and } p_3^{*}=a_2^{*}a_6^{*}-a_3^{*}a_5^{*}.$$

Substituting the admissible value of $\hat{\mu}^{*}$, say $\mu^{*(0)}$, from equation (7.10) into equation (7.8), we have the optimum value of the mean square error of the estimator T^{*}, which is shown below:

$$M(T^{*0})_{opt} = \frac{a_3^{*}+\mu^{*(0)}a_2^{*}+\mu^{*(0)2}a_1}{a_6^{*}+\mu^{*(0)}a_5^{*}+\mu^{*(0)2}a_4}\frac{S_y^2}{n}\ . \tag{7.11}$$

Case 2: When non-response occurs only at second occasion

When non-response occurs only at current (second) occasion, the estimator for the mean $\bar{Y}$ at current occasion may be obtained as

$$T^{**} = \varphi^{**}T_u+\left(1-\varphi^{**}\right)\xi_{1m} \tag{7.12}$$

where

$\xi_{1m}=\bar{y}_m\exp\left(\frac{\bar{x}_n-\bar{x}_m}{\bar{x}_n+\bar{x}_m}\right)\left(\frac{\bar{z}_n'}{\bar{z}_m}\right)$ and T_u is defined in section 3, where φ^{**} $(0 \le \varphi^{**} \le 1)$ is the unknown constant (scalar) to be determined under certain criterions.

7.2. Properties of the estimator T**

Since the estimator T^{**} is exponential type estimator, it is biased for the population mean $\bar{Y}$. The bias B (.) and the mean square error M (.) of the estimator T^{**} are derived up to the first order of approximations similar to that of the estimator T.

Theorem 7.3.

The bias of the estimator T^{**} to the first order of approximations is obtained as

$$B(T^{**}) = \varphi^{**}B(T_u)+(1-\varphi^{**})B(\xi_{1m}) \tag{7.13}$$

where

$$B(\xi_{1m}) = \bar{Y}\left\{\left(\frac{1}{m}-\frac{1}{n}\right)\left(\frac{3}{8}C_x^2+\frac{1}{2}\rho_{xz}C_xC_z-\frac{1}{2}\rho_{xy}C_yC_x\right)+\left(\frac{1}{m}-\frac{1}{n'}\right)\left(C_z^2-\rho_{yz}C_yC_z\right)\right\}$$

and $B(T_u)$ is defined in section 4.

Theorem 7.4.

The mean square error of the estimator T^{**} to the first order of approximations is obtained as

$$M(T^{**}) = \varphi^{**2}M(T_u)+\left(1-\varphi^{**}\right)^2 M(\xi_{1m})+2\varphi^{**}\left(1-\varphi^{**}\right)C^{**} \tag{7.14}$$

where

$$M(\xi_{1m}) = E\left(\xi_{1m}-\bar{Y}\right)^2 = \left[\left\{\left(\frac{1}{m}-\frac{1}{n}\right)\left(\frac{1}{4}+\rho_{xz}-\rho_{yx}\right)\right\}+\left(\frac{1}{m}-\frac{1}{N}\right)+\left(\frac{1}{m}-\frac{1}{n'}\right)\left(1-2\rho_{yz}\right)\right]S_y^2 \tag{7.15}$$

$$C^{**} = E\left[\left(T_u-\bar{Y}\right)\left(\xi_{1m}-\bar{Y}\right)\right] = -\frac{S_y^2}{N} \tag{7.16}$$

and $M(T_u)$ is defined in section 4.

Since the mean square error of the estimator T^{**} in equation (7.14) is the function of unknown constant φ^{**}, it is minimized with respect to φ^{**} and subsequently the optimum value of φ^{**} is obtained as

$$\varphi^{**}_{opt} = \frac{M(\xi_{1m})-C^{**}}{M(T_u)+M(\xi_{1m})-2C^{**}}. \tag{7.17}$$

Now substituting the value of φ_{opt}^{**} in equation (7.17), we get the optimum mean square error of T^{**} as

$$M(T^{**})_{opt} = \frac{M(T_u).M(\xi_{1m})-C^{**2}}{M(T_u)+M(\xi_{1m})-2C^{**}} \tag{7.18}$$

Further, substituting the values from equations (4.6), (7.15) and (7.16) in equation (7.18), we get the simplified value of $M\,(T^{**})_{opt}$ which is given below:

$$M(T^{**})_{opt} = \frac{a_3^{**}+\mu^{**}a_2^{**}+\mu^{**2}a_1^{**}}{a_6+\mu^{**}a_5^{**}+\mu^{**2}a_4^{**}}\frac{S_y^2}{n} \tag{7.19}$$

where

$a_1^{**} = ac^*+k^2f^2$, $a_2^{**} = ad^*+c^*b-k^2f^2$, $a_3^{**} = bd^*$, $a_4^{**} = c^*-a+2kf$, $a_5^{**} = a-b+d^*-2kf$, $a_6 = b$, $c^* = f+c_1+t_2d_1$, $d^* = (1-f)+d_1(1-t_2)$.

To determine the optimum values of μ^{**}, we minimized the mean square error of the estimator T^* given in equation (7.19) with respect to μ^{**}, which results in quadratic equation in μ^{**}, and the respective solutions of μ^{**}, say $\hat{\mu}^{**}$, are given below:

$$p_1^{**}\mu^{**2}+2p_2^{**}\mu^{**}+p_3^{**}=0 \tag{7.20}$$

$$\hat{\mu}^{**} = \frac{-p_2^{**}\pm\sqrt{p_2^{**2}-p_1^{**}p_3^{**}}}{p_1^{**}} \tag{7.21}$$

where

$$p_1^{**} = a_1^{**}a_5^{**}-a_2^{**}a_4^{**},\ p_2^{**} = a_1^{**}a_6-a_3^{**}a_4^{**} \text{ and } p_3^{**} = a_2^{**}a_6-a_3^{**}a_5^{**}.$$

Substituting the admissible values of $\hat{\mu}^{**}$, say $\mu^{**(0)}$, from equation (7.21) into equation (7.19), we have the optimum value of the mean square error of T^{**}, which is shown below:

$$M(T^{**0})_{opt} = \frac{a_3^{**}+\mu^{**(0)}a_2^{**}+\mu^{**(0)2}a_1^{**}}{a_6+\mu^{**(0)}a_5^{**}+\mu^{**(0)2}a_4^{**}}\frac{S_y^2}{n}. \tag{7.22}$$

8. Comparison of efficiencies

The percentage relative loss in efficiencies of the estimator T, T^* and T^{**} is obtained with respect to the similar estimator and natural successive sampling estimator when the non-response is not observed on any occasion. The estimator ξ_1 is defined under similar circumstances as the estimator T but under complete response, whereas the estimator ξ_2 is the natural successive sampling estimator, and they are given as

$$\xi_j = \psi_j\xi_{ju}+\left(1-\psi_j\right)\xi_{jm} \quad ;\ (j= 1, 2) \tag{8.1}$$

where

$$\xi_{1u} = \bar{y}_u \exp\left(\frac{\bar{z}_u^{'} - \bar{z}_u}{\bar{z}_u^{'} + \bar{z}_u}\right), \xi_{2u} = \bar{y}_u, \xi_{1m} = \bar{y}_m \exp\left(\frac{\bar{x}_n - \bar{x}_m}{\bar{x}_n + \bar{x}_m}\right)\left(\frac{\bar{z}_n^{'}}{\bar{z}_m}\right), \xi_{2m} = \bar{y}_m + \beta_{yx}(\bar{x}_n - \bar{x}_m)$$

Proceeding on a similar line as discussed for the estimator T the optimum mean square errors of the estimators ξ_j (j=1,2) are derived as

$$M(\xi_1^0)_{opt} = \left[\frac{b_3 + \mu^{'} b_2 + \mu^{'2} b_1}{b_6 + \mu^{'} b_5 + \mu^{'2} b_4}\right]\frac{S_y^2}{n} \tag{8.2}$$

and

$$M(\xi_2^0)_{opt} = \left[\frac{1}{2}\left\{1 + \sqrt{(1-\rho_{xy}^2)}\right\} - f\right]\frac{S_y^2}{n}. \tag{8.3}$$

where

$$\mu^{'} = \frac{-q_2 \pm \sqrt{q_2^2 - q_1 q_3}}{q_1} \text{ (fraction of the fresh sample for the estimator } \xi_1\text{)},$$

$b_1 = ac^* + k^2 f^2$, $b_2 = ad^* + c^* b^* - k^2 f^2$, $b_3 = b^* d^*$, $b_4 = c^* - a + 2kf$, $b_5 = a - b^* + d^* - 2kf$, $b_6 = b^*$, $q_1 = b_1 b_5 - b_2 b_4$, $q_2 = b_1 b_6 - b_3 b_4$ and $q_3 = b_2 b_6 - b_3 b_5$.

Remark 8.1.

To compare the performances of the estimators T, T^* and T^{**} with respect to the estimators ξ_j (j=1, 2), we introduce the following assumptions:

(i) $\rho_{xz} = \rho_{yz}$, which is an intuitive assumption, also considered by Cochran (1977) and Feng and Zou (1997), (ii) $W=W^*$ (iii) $f_1=f_2$.

The percentage relative losses in the precision of the estimators T, T^* and T^{**} with respect to ξ_j (j=1, 2) under their respective optimality conditions are given by

$$L_j = \frac{M\left(T^{(0)}\right)_{opt} - M\left(\xi_j\right)_{opt}}{M\left(T^{(0)}\right)_{opt}} \times 100, \; L_j^* = \frac{M\left(T^{*(0)}\right)_{opt} - M\left(\xi_j\right)_{opt}}{M\left(T^{*(0)}\right)_{opt}} \times 100$$

$$\text{and } L_j^{**} = \frac{M\left(T^{**(0)}\right)_{opt} - M\left(\xi_j\right)_{opt}}{M\left(T^{**(0)}\right)_{opt}} \times 100; \; (j=1, 2)$$

For N = 5000, n' = 1000, u' = 1000, n = 500, t_1=0.50 ,t_2=0.50, f=0.1 and different choices of f_1, ρ_{yx} and ρ_{yz}, Tables 1-6 give the optimum values of

$\mu^{(0)}, \mu^{*(0)}, \mu^{**(0)}$ and percentage relative losses L_j, L_j^* and L_j^{**} (j=1, 2) in the precision of the estimators T ,T^* and T^{**} with respect to estimators ξ_j (j=1, 2).

Table 1. Percentage relative loss L_1 in the precision of the estimator T with respect to ξ_1

W			0.05		0.10		0.15		0.20	
ρ_{yx}	f_2	ρ_{yz}	$\mu^{(0)}$	L_1	$\mu^{(0)}$	L_1	$\mu^{(0)}$	L_1	$\mu^{(0)}$	L_1
0.6	1.5	0.6	0.8640	3.2181	0.7182	5.9332	0.5829	8.2004	0.4580	10.0748
		0.7	0.4617	2.5435	0.3950	4.7480	0.3298	6.6502	0.2666	8.2850
		0.8	0.3161	2.5098	0.2740	4.7029	0.2320	6.6173	0.1903	8.2870
		0.9	0.2404	2.7911	0.2106	5.2223	0.1803	7.3438	0.1495	9.1988
	2.0	0.6	0.7182	5.9332	0.4580	10.0748	0.2385	12.8584	0.0558	14.6753
		0.7	0.3950	4.7480	0.2666	8.2850	0.1472	10.8815	0.0385	12.7754
		0.8	0.2740	4.7029	0.1903	8.2870	0.1087	11.0137	0.0312	13.0986
		0.9	0.2106	5.2223	0.1495	9.1988	0.0880	12.2538	0.0277	14.6331
0.8	1.5	0.6	0.5890	2.4511	0.5527	4.6664	0.5179	6.6733	0.4844	8.4959
		0.7	0.4775	2.4663	0.4511	4.7018	0.4254	6.7339	0.4004	8.5863
		0.8	0.3985	2.6439	0.3787	5.0344	0.3591	7.2037	0.3398	9.1796
		0.9	0.3381	3.0190	0.3234	5.7243	0.3085	8.1616	0.2936	10.3684
	2.0	0.6	0.5527	4.6664	0.4844	8.4959	0.4214	11.6712	0.3636	14.3348
		0.7	0.4511	4.7018	0.4004	8.5863	0.3525	11.8338	0.3077	14.5833
		0.8	0.3787	5.0344	0.3398	9.1796	0.3024	12.6435	0.2667	15.5811
		0.9	0.3234	5.7243	0.2936	10.3684	0.2640	14.2120	0.2353	17.4529

Table 2. Percentage relative loss L_2 in the precision of the estimator T with respect to ξ_2

W			0.05		0.10		0.15		0.20	
ρ_{yx}	f_2	ρ_{yz}	$\mu^{(0)}$	L_2	$\mu^{(0)}$	L_2	$\mu^{(0)}$	L_2	$\mu^{(0)}$	L_2
0.6	1.5	0.6	0.8640	-6.7964	0.7182	-3.8003	0.5829	-1.2986	0.4580	0.7698
		0.7	0.4617	-19.2197	0.3950	-16.5229	0.3298	-14.1959	0.2666	-12.1961
		0.8	0.3161	-38.1243	0.2740	-35.0170	0.2320	-32.3048	0.1903	-29.9392
		0.9	0.2404	-65.7067	0.2106	-61.5624	0.1803	-57.9460	0.1495	-54.7839
	2.0	0.6	0.7182	-3.8003	0.4580	0.7698	0.2385	3.8414	0.0558	5.8463
		0.7	0.3950	-16.5229	0.2666	-12.1961	0.1472	-9.0197	0.0385	-6.7029
		0.8	0.2740	-35.0170	0.1903	-29.9392	0.1087	-26.0759	0.0312	-23.1220
		0.9	0.2106	-61.5624	0.1495	-54.7839	0.0880	-49.5762	0.0277	-45.5204
0.8	1.5	0.6	0.5890	2.0316	0.5527	4.2565	0.5179	6.2720	0.4844	8.1025
		0.7	0.4775	-11.1328	0.4511	-8.5855	0.4254	-6.2701	0.4004	-4.1593
		0.8	0.3985	-29.5093	0.3787	-26.3293	0.3591	-23.4436	0.3398	-20.8151
		0.9	0.3381	-56.2099	0.3234	-51.8524	0.3085	-47.9266	0.2936	-44.3720
	2.0	0.6	0.5527	4.2565	0.4844	8.1025	0.4214	11.2914	0.3636	13.9665
		0.7	0.4511	-8.5855	0.4004	-4.1593	0.3525	-0.4591	0.3077	2.6738
		0.8	0.3787	-26.3293	0.3398	-20.8151	0.3024	-16.2073	0.2667	-12.2995
		0.9	0.3234	-51.8524	0.2936	-44.3720	0.2640	-38.1811	0.2353	-32.9609

Table 3. Percentage relative loss L_1^* in the precision of the estimator T^* with respect to ξ_1

W			0.05		0.10		0.15		0.20	
ρ_{yx}	f_2	ρ_{yz}	$\mu^{*(0)}$	L_1^*	$\mu^{*(0)}$	L_1^*	$\mu^{*(0)}$	L_1^*	$\mu^{*(0)}$	L_1^*
0.6	1.5	0.6	*	-	*	-	*	-	*	-
		0.7	0.5489	0.1481	0.5667	0.2827	0.5832	0.4057	0.5984	0.5185
		0.8	0.3748	0.3426	0.3911	0.6617	0.4065	0.9596	0.4212	1.2385
		0.9	0.2828	0.5457	0.2961	1.0599	0.3089	1.5452	0.3212	2.0041
	2.0	0.6	*	-	*	-	*	-	*	-
		0.7	0.5667	0.2827	0.5984	0.5185	0.6258	0.7180	0.6497	0.8890
		0.8	0.3911	0.6617	0.4212	1.2385	0.4484	1.7458	0.4732	2.1954
		0.9	0.2961	1.0599	0.3212	2.0041	0.3445	2.8508	0.3663	3.6142
0.8	1.5	0.6	0.6356	0.1025	0.6443	0.1997	0.6525	0.2920	0.6604	0.3798
		0.7	0.5141	0.2075	0.5234	0.4057	0.5324	0.5951	0.5411	0.7764
		0.8	0.4277	0.3345	0.4368	0.6552	0.4456	0.9628	0.4542	1.2582
		0.9	0.3613	0.4977	0.3698	0.9754	0.3780	1.4345	0.3861	1.8759
	2.0	0.6	0.6443	0.1997	0.6604	0.3798	0.6752	0.5430	0.6887	0.6916
		0.7	0.5234	0.4057	0.5411	0.7764	0.5574	1.1163	0.5727	1.4292
		0.8	0.4368	0.6552	0.4542	1.2582	0.4705	1.8150	0.4859	2.3308
		0.9	0.3698	0.9754	0.3861	1.8759	0.4016	2.7098	0.4163	3.4842

Note: ‘*’ indicates $\mu^{*(0)}$ does not exist.

Table 4. Percentage relative loss L_2^* in the precision of the estimator T^* with respect to ξ_2

W			0.05		0.10		0.15		0.20	
ρ_{yx}	f_2	ρ_{yz}	$\mu^{*(0)}$	L_2^*	$\mu^{*(0)}$	L_2^*	$\mu^{*(0)}$	L_2^*	$\mu^{*(0)}$	L_2^*
0.6	1.5	0.6	*	-	*	-	*	-	*	-
		0.7	0.5489	-22.1500	0.5667	-21.9853	0.5832	-21.8349	0.5984	-21.6969
		0.8	0.3748	-41.1948	0.3911	-40.7427	0.4065	-40.3205	0.4212	-39.9255
		0.9	0.2828	-69.5345	0.2961	-68.6579	0.3089	-67.8305	0.3212	-67.0482
	2.0	0.6	*	-	*	-	*	-	*	-
		0.7	0.5667	-21.9853	0.5984	-21.6969	0.6258	-21.4529	0.6497	-21.2436
		0.8	0.3911	-40.7427	0.4212	-39.9255	0.4484	-39.2068	0.4732	-38.5697
		0.9	0.2961	-68.6579	0.3212	-67.0482	0.3445	-65.6050	0.3663	-64.3036
0.8	1.5	0.6	0.6356	-0.3271	0.6443	-0.2294	0.6525	-0.1367	0.6604	-0.0486
		0.7	0.5141	-13.7064	0.5234	-13.4806	0.5324	-13.2648	0.5411	-13.0583
		0.8	0.4277	-32.5814	0.4368	-32.1549	0.4456	-31.7456	0.4542	-31.3527
		0.9	0.3613	-60.2711	0.3698	-59.5015	0.3780	-58.7621	0.3861	-58.0511
	2.0	0.6	0.6443	-0.2294	0.6604	-0.0486	0.6752	0.1153	0.6887	0.2646
		0.7	0.5234	-13.4806	0.5411	-13.0583	0.5574	-12.6709	0.5727	-12.3144
		0.8	0.4368	-32.1549	0.4542	-31.3527	0.4705	-30.6119	0.4859	-29.9258
		0.9	0.3698	-59.5015	0.3861	-58.0511	0.4016	-56.7079	0.4163	-55.4606

Note: ‘*’ indicates $\mu^{*(0)}$ does not exist.

Table 5. Percentage relative loss L_1^{**} in the precision of the estimator T^{**} with respect to ξ_1

W			0.05		0.10		0.15		0.20	
ρ_{yx}	f_2	ρ_{yz}	$\mu^{**(0)}$	L_1^{**}	$\mu^{**(0)}$	L_1^{**}	$\mu^{**(0)}$	L_1^{**}	$\mu^{**(0)}$	L_1^{**}
0.6	1.5	0.6	0.8515	3.2070	0.6634	5.8247	0.4538	7.7922	0.2206	9.0181
		0.7	0.4376	2.3271	0.3382	4.1649	0.2312	5.5017	0.1163	6.3162
		0.8	0.2964	2.0922	0.2305	3.7242	0.1600	4.9027	0.0847	5.6273
		0.9	0.2250	2.1709	0.1772	3.8495	0.1257	5.0602	0.0707	5.8180
	2.0	0.6	0.6634	5.8247	0.2206	9.0181	*	-	*	-
		0.7	0.3382	4.1649	0.1163	6.3162	*	-	*	-
		0.8	0.2305	3.7242	0.0847	5.6273	*	-	*	-
		0.9	0.1772	3.8495	0.0707	5.8180	*	-	*	-
0.8	1.5	0.6	0.5785	2.3221	0.5297	4.3575	0.4801	6.1286	0.4298	7.6552
		0.7	0.4668	2.2289	0.4284	4.1749	0.3891	5.8623	0.3490	7.3121
		0.8	0.3883	2.2786	0.3573	4.2545	0.3254	5.9574	0.2925	7.4125
		0.9	0.3288	2.4916	0.3038	4.6280	0.2776	6.4507	0.2504	7.9947
	2.0	0.6	0.5297	4.3575	0.4298	7.6552	0.3272	10.0446	0.2222	11.6475
		0.7	0.4284	4.1749	0.3490	7.3121	0.2666	9.5722	0.1818	11.0819
		0.8	0.3573	4.2545	0.2925	7.4125	0.2245	9.6656	0.1538	11.1591
		0.9	0.3038	4.6280	0.2504	7.9947	0.1934	10.3593	0.1333	11.9082

Note: '*' indicates $\mu^{**(0)}$ does not exist.

Table 6. Percentage relative loss L_2^{**} in the precision of the estimator T^{**} with respect to ξ_2

W			0.05		0.10		0.15		0.20	
ρ_{yx}	f_2	ρ_{yz}	$\mu^{**(0)}$	L_2^{**}	$\mu^{**(0)}$	L_2^{**}	$\mu^{**(0)}$	L_2^{**}	$\mu^{**(0)}$	L_2^{**}
0.6	1.5	0.6	0.8515	-6.8086	0.6634	-3.9201	0.4538	-1.7490	0.2206	-0.3963
		0.7	0.4376	-19.4844	0.3382	-17.2361	0.2312	-15.6008	0.1163	-14.6045
		0.8	0.2964	-38.7159	0.2305	-36.4038	0.1600	-34.7341	0.0847	-33.7074
		0.9	0.2250	-66.7641	0.1772	-63.9026	0.1257	-61.8388	0.0707	-60.5469
	2.0	0.6	0.6634	-3.9201	0.2206	-0.3963	*	-	*	-
		0.7	0.3382	-17.2361	0.1163	-14.6045	*	-	*	-
		0.8	0.2305	-36.4038	0.0847	-33.7074	*	-	*	-
		0.9	0.1772	-63.9026	0.0707	-60.5469	*	-	*	-
0.8	1.5	0.6	0.5785	1.9021	0.5297	3.9462	0.4801	5.7249	0.4298	7.2581
		0.7	0.4668	-11.4032	0.4284	-9.1858	0.3891	-7.2633	0.3490	-5.6113
		0.8	0.3883	-29.9953	0.3573	-27.3668	0.3254	-25.1016	0.2925	-23.1658
		0.9	0.3288	-57.0593	0.3038	-53.6183	0.2776	-50.6823	0.2504	-48.1954
	2.0	0.6	0.5297	3.9462	0.4298	7.2581	0.3272	9.6578	0.2222	11.2676
		0.7	0.4284	-9.1858	0.3490	-5.6113	0.2666	-3.0360	0.1818	-1.3158
		0.8	0.3573	-27.3668	0.2925	-23.1658	0.2245	-20.1687	0.1538	-18.1818
		0.9	0.3038	-53.6183	0.2504	-48.1954	0.1934	-44.3867	0.1333	-41.8919

Note: '*' indicates $\mu^{**(0)}$ does not exist.

9. Interpretations of results

The following conclusions may be drawn from Tables 1-6:

(1) From Tables 1 and 5 it is clear that

(a) For the fixed values of W, ρ_{yx} and f_2, the values of $\mu^{(0)}, \mu^{**(0)}$ decrease with the increasing values of ρ_{yz}. This implies that the higher the value of ρ_{yz}, the lower the fraction of a fresh sample required on the current occasion.

(b) For the fixed values of W, ρ_{yx} and ρ_{yz}, the values of $\mu^{(0)}, \mu^{**(0)}$ decrease and L_1, L_1^{**} increase with the increasing values of f_2.

(c) For the fixed values of W, ρ_{yz} and f_2, no pattern is observed with the increasing values of ρ_{yx}.

(d) For the fixed values of f_2, ρ_{yz} and ρ_{yx}, the values of $\mu^{(0)}, \mu^{**(0)}$ decrease and L_1, L_1^{**} increase with the increasing values of W. This behaviour shows that with the higher non-response rate one may require to draw the smaller sample on the current occasion, which reduces the cost of a survey.

(2) From Tables 2 and 6 it may be seen that

(a) For the fixed values of W, ρ_{yx} and f_2, the values of $\mu^{(0)}, \mu^{**(0)}$ and L_2, L_2^{**} decrease with the increasing values of ρ_{yz}.. This implies that if one uses the information on highly correlated auxiliary variable, there is a significant gain in the precision of estimates.

(b) For the fixed values of W, ρ_{yx} and ρ_{yz}, the values of $\mu^{(0)}, \mu^{**(0)}$ decrease and L_2, L_2^{**} increase with the increasing values of f_2.

(c) For the fixed values of W, ρ_{yz} and f_2, the values of L_2, L_2^{**} increase with the increasing values of ρ_{yx}.

(d) For the fixed values of f_2, ρ_{yz} and ρ_{yx}, the values of $\mu^{(0)}, \mu^{**(0)}$ decrease and L_2, L_2^{**} increase with the increasing values of W. This pattern shows that the higher the non-response rate, the greater the loss. This behaviour is practically justified.

(3) From Table 3 it is clear that

(a) For the fixed values of W, ρ_{yx} and f_2, the values of $\mu^{*(0)}$ decrease and L_1^{*} increase with the increasing values of ρ_{yz}. This behaviour indicates that if the information on highly correlated auxiliary variable is available, it plays an important role in improving the precision of estimates.

(b) For the fixed values of W, ρ_{yx} and ρ_{yz}, the values of $\mu^{*(0)}$ and L_1^{*} increase with the increasing values of f_2.

(c) For the fixed values of W, ρ_{yz} and f_2, no pattern is visible with the increasing values of ρ_{yx}.

(d) For the fixed values of f_2, ρ_{yz} and ρ_{yx}, the values of $\mu^{*(0)}$ and L_1^* increase with the increasing values of W.

(4) From Table 4 it may be seen that

(a) For the fixed values of W, ρ_{yx} and f_2, the values of $\mu^{*(0)}$ and L_2^* decrease with the increasing values of $\rho_{yz.}$ This implies that negative loss is observed due to the presence of high correlation between the auxiliary variables. This behaviour is highly desirable.

(b) For the fixed values of W, ρ_{yx} and ρ_{yz}, the values of $\mu^{*(0)}$ and L_2^* increase with the increasing values of $f_{2.}$ This indicates that if a smaller size of sub-sample is chosen, the loss in precision increases, as it was expected.

(c) For the fixed values of W, ρ_{yz} and f_2 no pattern is seen with the increasing values of ρ_{yx}.

(d) For the fixed values of f_2, ρ_{yz} and ρ_{yx}, the values of $\mu^{*(0)}$ and L_2^* increase with the increasing values of W.

10. Conclusions and recommendations

It may be seen from the above tables that for all cases the percentage relative loss in precisions is observed wherever the optimum value of μ exists, when non-response occurs on both occasions. From Tables 1, 3 and 5, it is seen that the loss is present due to the presence of non-response on each occasion, but the negative impact of non-response is very low, which justifies the use of Hansen and Hurwitz (1946) technique in the proposed estimation procedures. From Tables 2, 4 and 6, when the proposed estimators are compared with the natural successive sampling estimator, substantial profit is visible, which justifies the intelligible use of auxiliary information in the form of exponential methods of estimation. Finally, looking at good behaviours of the proposed estimators one may recommend them to survey statisticians and practitioners for their practical applications.

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