CONSTRUCTION OF MULTI-STATE LIFE TABLES FOR CRITICAL ILLNESS INSURANCE – INFLUENCE OF AGE AND SEX ON THE INCIDENCE OF HEALTH INEQUALITIES

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Joanna Dębicka, Beata Zmyślona
Wrocław University of Economics

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Summary: The aim of the paper is twofold. The first one is to present the multi-state life tables associated with the insurance against the risk for lung cancer. Probabilistic structure of the model regarding incidence and mortality rates of lung cancer takes into account many factors such as a patient's health condition (mild and critical), the probability of remaining in mild state of health and the probability of state of health deterioration. The lifetime in a critical state is analyzed in detail. The probability of death of a patient is also analysed according to the health condition. The analysis of the influence of inequalities in health caused by gender, biological sex and age on the probabilistic structure pose the second aim of paper.

Keywords: multi-state life tables, health insurance, inequalities in health, mortality rate, fatality rate, incidence rate, lung cancer, unhealthy behaviours.

1. Introduction

The phenomenon of an aging population, unhealthy eating and lifestyle habits, and pollution of the environment have led to the development of civilization diseases which in many cases have a very difficult course. Serious diseases belong to the so-called more expensive causes of death [Gutman et al. 2013]. Although health expenditure usually increases for most people with time to death, it should be noted that the growth of end-of-life expenditures in case of serious diseases is particularly high. The cost of serious illnesses treatment in their final stage represents a significant proportion of total expenditure on health [Hogan et al. 2001; Felder et al. 2000; Lillard et al. 1997]. Public health care systems in many countries are not able to bear the burden of end-of-life medical services financing. The aging population exacerbates the effect of the collapse of public budgets. Therefore, the concept of using private health insurance as an additional source of funding end-of-life spending appears to be an essential element of modern health care systems.

A probabilistic structure of the model used to estimate an insurance premium should take into account a severity of a disease and distinguish the end-of-life, terminal period of the patient's treatment process. In particular it should provide the possibility to obtain additional services by the insured during the terminal period. An example of such a model is characterized in detail in two manuscripts [Dębicka, Zmyślona 2016; Dębicka, Zmyślona Manuscript I]. In [Dębicka, Zmyślona 2016] the procedure of modelling the probabilistic structure of the model is described, while [Dębicka, Zmyślona Manuscript] focuses on costing such insurances. The model is illustrated by an example of health insurance against the risk of lung cancer. The obtained results allow for stating clearly that significant differences between insurance premiums designated for men and women are observable. These differences are the result of the occurrence of inequalities in health caused by biological sex, gender and age of patients.

The aim of this article is twofold. The analysis of the influence of inequalities in health caused by gender, biological sex and age on the probabilistic structure poses the first aim. The second one is to present the derived multi-state life tables in connection with insurance against the risk of serious diseases on the example of lung cancer.

The paper is organized as follows. Section 2 elucidates the phenomenon of health inequalities arising from the existence of biological sex and gender. In Section 3, the differences in the incidence, morbidity, mortality and fatality rates between the sexes are discussed on the example of lung cancer at the residents of Lower Silesia. Section 4 describes the probabilistic structure of the dread disease insurance model. Multi-state life tables are provided in Appendix.

2. Inequalities in health

Recent studies in genetics and medicine seem to confirm the fact that the pattern of illness and health is not determined by the expression of a particular gene but the entire gene set. This expression depends on one's lifestyle and environmental factors and can be changed by them. This relationship causes that the diversity of health status, morbidity and mortality rates are the result of an influence of various factors on an occurrence of so-called inequalities in health. Place of residence, education level, lifestyle, nutrition manner and socioeconomic status determined by income belong to the most frequent factors indicated those influencing the level of health. However, numerous studies confirm that biological sex and gender are also one of the most important determinants of inequalities in health status. Two phenomena should be taken into account in the analysis of the impact of gender on health. The first is associated with biological determinants and the second with socio-cultural factors. Biological factors make incidence or mortality rates from certain diseases higher for one sex whereas socio-cultural factors are the consequence of different roles ascribed to the sexes in society. The roles often have a determining influence on lifestyle, attitude towards health and disease and the relationship to the so-called unhealthy behaviour [Laskowska 2012; Królikowska 2011; WHO 2012; Kosecka, Jankowska 2007].

Many lifestyle diseases due to numerous complications and high mortality become serious diseases. One of them is cancer. Malicious tumours, after heart diseases, pose the most common illnesses and are the second cause of death in developed countries. Malignant tumours are the most common cause of death in middle age (from 45 to 65 years of age). In Poland, 34% of deaths in middle age are caused by cancer, with division according to sex it is 48% of cases for women and 28% for men. However, due to significant excess of mortality of men in this age group, the level of threat to men life due to cancer is higher by 60% in comparison to women [Laskowska 2012].

In this paper lung cancer is considered as an example of a critical illness due to its increased morbidity and the highest mortality rates among cancers. Additionally, lung cancer belongs to the group of tumours with unfavourable prognosis. Epidemiological data confirms that smoking cigarettes is the main risk factor for cancer of the respiratory system, especially lung cancer. The risk of lung cancer among smokers depends on the number of cigarettes smoked and first of all on the duration of addiction. It is estimated that in European countries every fifth case of cancer is caused by smoking, which is determined as the most common anti-health social behaviour around the world, including Poland. Entourage of smokers is also more exposed to greater risk of increased morbidity due to the adverse effect because of passive smoking. According to the data of the Central Statistical Office in Poland, the consumption of cigarettes was growing from the twenties to the eighties of the 20th century. Since this time the general downward trend is noticed [WHO 2009]. This general trend is different in the populations of men and women. On the basis of data contained in four reports [Czapliński 2011; CBOS 2012; WHO 2012; WHO 2009] the percentage of smokers in male and female populations in Poland are shown in Figure 1.

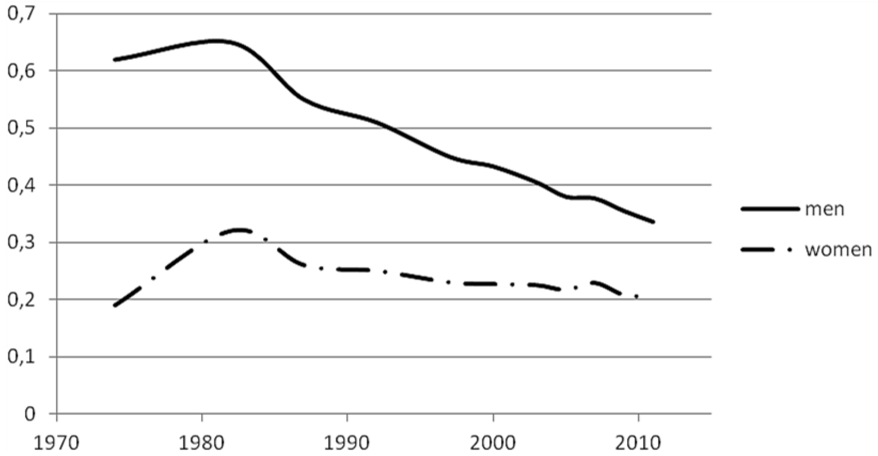

Figure 1. The percentage of smokers in men and female population in Poland

Source: own presentation on the basis of [Czapliński 2011; CBOS 2012; WHO 2009, 2012].

Analyzing the trends shown in Figure 1, the following conclusions can be drawn. A significantly larger proportion of smokers occurs in male population. In female population the percentage of smokers increased in the eighties of the 20th century, then decreased slightly in the nineties, and this trend persists to this day. The disease process associated with lung cancer develops with a considerable lag after a dozen or so and even several dozen of years after the beginning of addiction. Therefore, the current situation concerning the incidence rate depends largely on the prevailing attitudes towards smoking in the past. On the other hand, the analysis of current statistics on smoking allows for estimating what morbidity rates will be like in the future. This movement of morbidity in time and the fact that older people are more prone to cancer make morbidity and mortality rates dependent on the age of a patient.

The reluctance to visit a doctor is another anti-health behaviour. Numerous sociological studies confirm that an average man is far less likely to visit doctors, compared to an average woman. Reluctance to visit doctors and start treatment results in later detection of chronic and serious diseases. In case of many diseases including lung cancer, this factor determines the state of health at diagnosis. Of course, the subsequent prognosis is much worse in people delaying a doctor's appointment. Men also have a different attitude to the recommendations received from the doctors regarding the treatment process, follow-up examination and advice related to the change in diet and lifestyle [Laskowska 2012; Królikowska 2011; WHO 2012].

3. The influence of age and sex on epidemiological rates

The previously described inequalities in health due to biological sex and gender make epidemiological characteristics different in male and female populations. Differences in morbidity and mortality are observed depending on place of residence. In Poland, epidemiological indicators take different values depending on the voivodeship.

We consider populations of male and female living in Lower Silesia (one of the voivodeships in Poland). The analysis is made on the basis of three datasets. First of them consists of life expectancy tables. Secondly, the most precise information about the incidence and mortality rates in populations of Lower Silesia residents is obtained from the National Cancer Registry for the Lower Silesia Region. This register is created on the basis of declarations and death certificates from hospitals and out-patients clinics. The third data base is created on the basis of the data set of individual hospitalizations and visits to doctors from the Lower Silesia Department of the National Health Fund. The number of patients with lung cancer was identified using the disease code (C33 and C34 according to the international system of codes from ICD-10). Patients were identified using the coded numbers of the Universal Electronic System for Registration of the Population (Social Security).

Available data cover the period between 2006 and 2011. On the basis of these data the histories of disease course were analyzed. We distinguish between two disease states, namely mild (without distant metastases) and critical (diagnosed with distant metastases). Additionally, on the basis of data on the treatment history of patients, fatality rates for patients in critical state were estimated.

The year 2008, as one of the middle periods, has been established as the reference year. The choice of the middle period allows for considering the histories of hospitalization of these patients in the time frame from 2006 to 2011.

3.1. The incidence and mortality rates

Age, gender, race and geographic region belong to the group of factors listed in the world literature as modifying the incidence and mortality of lung cancer. Other causes of lung cancer are smoking and environmental pollution. Geographical region determines the level of environmental pollution. Gender influences the spread of smoking habit, which is much higher in men. There are no studies explicitly stating the impact of diet on the pathogenesis of lung cancer.

Crude incidence rates calculated per 100 000 inhabitants of Lower Silesia and of Poland observed in years 1999 to 2013 are shown in Figure 2. The incidence rate is several times higher in men population. This is a result of the occurrence of inequalities in health caused by the greater prevalence of cigarette smoking among men. Due to the fact that the percentage of smokers among males has stabilized and even slightly decreased since the eighties of the 20th century, incidence rate of lung cancer also shows the tendency to stabilize and to decrease in the future. A different tendency occurs in case of women. The number of cancer cases continues to grow, which is undoubtedly affected by cultural changes. These changes are caused by an increase in the number of smokers among women in the post-war generation. Due to the growing number of smokers among women, we should expect a further increase in the number of cancer cases in the population of women. In addition, some researchers suggest that smoking exposes more threat to healthy women than men. Hypotheses are put forward that a female body is more sensitive to the toxic effects of cigarette smoke. The mechanisms and causes of this phenomenon are acknowledged [Wojciechowska, Didkowska 2016]. Biological sex has thus some impact on the incidence rate. The mentioned factors mean that a further increase in incidence rates in the female population will be expected.

The mortality rate depends directly on the incidence rate, therefore similar trends are observed for incidence and mortality rates. In Figure 3,

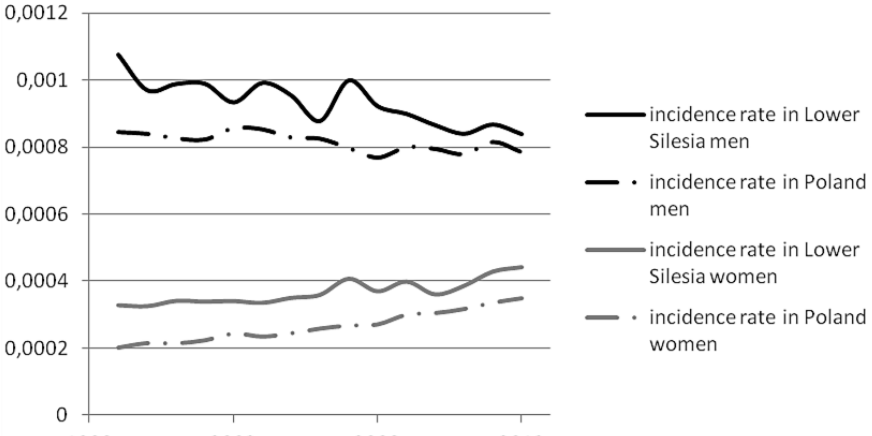

Figure 2. Crude incidence rates in male and female populations in Lower Silesia and Poland

Source: own calculations on the basis of [Wojciechowska, Didkowska 2014].

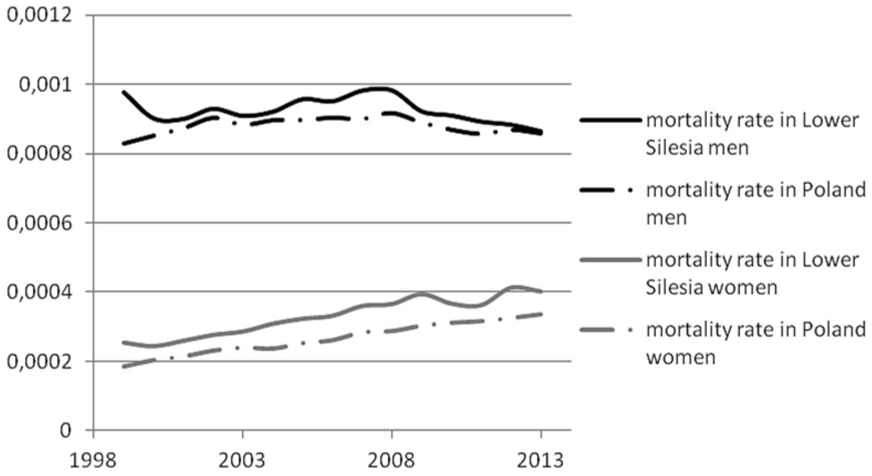

Figure 3. Crude mortality rates in male and female populations in Lower Silesia and Poland

Source: own calculations on the basis of [Wojciechowska, Didkowska 2014].

crude mortality rates calculated per 100 000 inhabitants of Lower Silesia and of Poland observed from 1999 to 2013 are presented.

Construction of multi-state life tables requires consideration of the effect of age on epidemiological indicators. Both morbidity and mortality due to lung cancer rates depend on age. Let ω_x and ζ_x denote respectively mortality and incidence rate for a person of age x in the

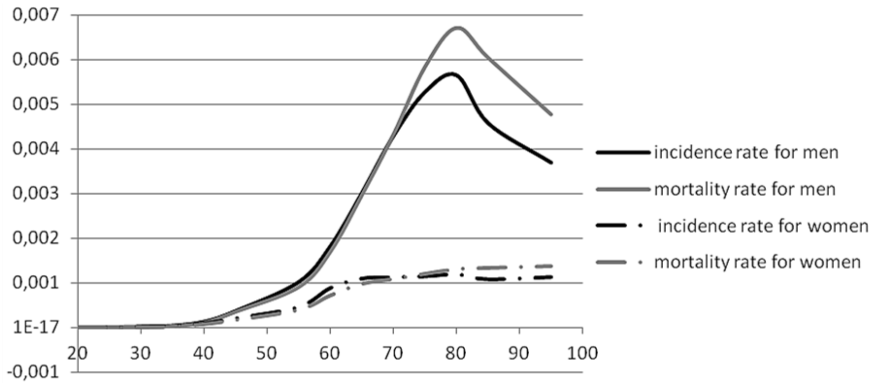

Figure 4. The average incidence and mortality rates
(on the basis of data from the period 2006–2010)

Source: own calculations on the basis of data from the National Cancer Registry [Wojciechowska, Didkowska 2014].

population of inhabitants of Lower Silesia. Due to the fact that data concerning the analysis of diseases histories is considered in time frame of 2006 to 2011, incidence and mortality rates are calculated as the average of that period. The values of the indicators depending on the age are shown in Figure 4.

The incidence of lung cancer patients is incidental to 40 years of age, both in men and women population. Then, this ratio is increasing and reaches maximum values in the sixth and seventh decade of life, then falls, but in case of women this decline is less conspicuos. The same dependencies are observed in case of mortality.

3.2. The degree of disease severity

Another aspect connected with the disease course is its state. In the event of malignant tumors metastases to distant organs considetably determines the state of disease. Two cases are considered, the first one without metastases, which means that the state of patient is mild, and the second with diagnosed metastases, which is connected with the deterioration of health state. Metastases can be diagnosed during the first visit or can develop or may be discovered later. From the point of view of health insurance aimed to provide financial resources in a serious condition, the moment of diagnosis of metastases is important.

In order to estimate the percentage of people who fell ill in the reference year (2008) and were diagnosed with metastatic disease during the first diagnosis, two cohorts of patients with lung cancer who, during 2008, fell ill with lung cancer have been separated. The first cohort consisted of 1353 men, the second of 605 women, who were diagnosed with lung cancer in the region of Lower Silesia. In male population, the youngest patient was 29 years old, the oldest one 89. The first quartile of age amounted to 58 years, median – 68, while the third quartile 74 years. In female population, the characteristics of age were the following: minimum age was equal to 42, maximum 93, the first quartile 56, median 62 and the third quartile 72. Patients received one of the two diagnoses. The first option was the recognition of metastases to lymph nodes in the chest and so called distant metastases. The analysis included an additional period of four weeks after making the first diagnosis. This period, treated as the additional time which is required to obtain the results of diagnostic tests, is taken into account in the model. If, during this period, the existence of metastases was confirmed, the patient was classified to the same group as patients who received a diagnosis of metastatic disease during the first visit.

Let β_x denote the percentage of patients who are diagnosed with metastases during the first diagnosis in age x. At the first stage, a logistic regression model was constructed, expressing the dependence of diagnosis of metastases of patient's age and sex. Let T denote the random variable used to model the risk of detection of metastases during the first diagnosis, where

$$\beta_x = P(T=1) =$$
$$= \begin{cases} \dfrac{1}{1+\exp-\left(1.400879-0.02439x+0.177567\right)} & \text{for men above 40 years of age,} \\ \dfrac{1}{1+\exp-\left(1.400879-0.02439x\right)} & \text{for women above 40 years of age.} \end{cases}$$

The values of probability are estimated only for patients above 40 years of age due to the low representativeness of the sample for those under this age. The values of probability are presented in Figure 5 by graph depicted by continuous lines. These relationships turned out to be significant and indicated that the probability of detection of metastases during the first visit decreases with age. The analysis also showed that the probability is lower for women than for men.

Lower average values of probability for women may indicate that women are more liable to medical appointments and examinations than men, which is known as lack of prevention habits. However, this difference is not very significant, which may stem from the fact that lung cancer in Poland is diagnosed at a late stage, not giving any characteristic symptoms before. This means that patients seek medical care at an advanced stage of the disease. Lower values of the probability for the elderly are caused by the decrease of aggressiveness of tumour cells with age. Despite a fairly good fitting of the model to the data, a negative result of Hosmer-Lemeshow test was obtained (the value of test statistic 28.3916 and $p = 0.000186708$). It points out uneven fitting of the model to the data. Certain age groups diverge significantly from the average trend expressed by the logit model (for men there are patients 40–45 years of age and for women there are patients in age groups 40–50 and 70–75 years). Therefore, the probability of detecting metastases during the first diagnosis was estimated separately in five-year age groups as a percentage of patients with detected metastases. The choice of five-year age groups results from the estimation of morbidity and mortality exactly in these groups. The patients in the age group of 20–40 years pose an exception. In this age group lung cancer occurs extremely rarely, therefore a twenty-year age limit was used for the estimation. The values of parameter β_x are presented in Figure 5 by graph depicted by dash lines.

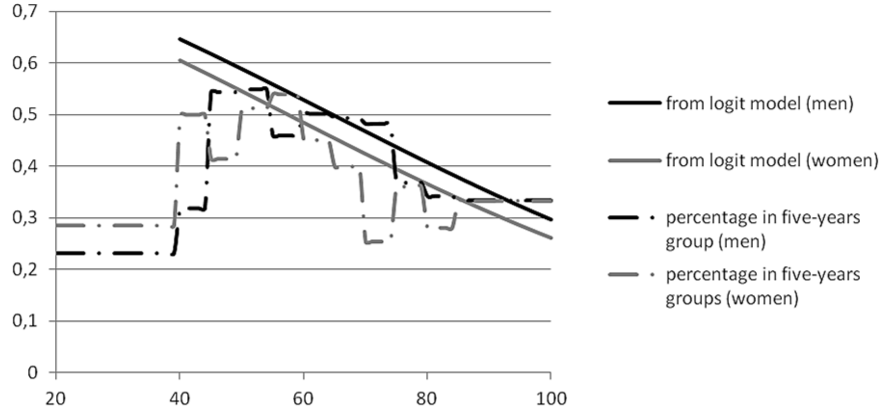

Figure 5. The values of probability of metastases detection during first diagnosis obtained using logit model and percentage of patients in five-year groups of patients

Source: own calculations on the basis of analysis of individual data from the National Health Fund.

The health state of patient can be changed after metastases are diagnosed. The examination of treatment histories is made not from the perspective of a patient but from that of an insurance company, this change of perspective is necessary in construction of an insurance product and it enables an insured person to remain without metastases apparently higher than it results from the epidemiological data. The studied cohort at this stage of the analysis consists of patients who, during 2008, suffered from lung cancer and in the initial diagnosis in 2008 they did not have metastases. The diseases histories of 1098 men and 533 women were singled out to the study. In the studied groups, the following characteristics of age were observed: for men (min 30, max 88, first quartile 58, median 66, third quartile 73) and for women (min 31, max 93, the first quartile 56, median 63, the third quartile 72).

The dependence of the risk of metastases diagnosis upon patient's age was modelled using a logit regression. A calendar year was considered as time horizon from the perspective of an insurance company. The fact that the study is carried out from the perspective of a calendar year and not from the point of view of the one-year history of each patient makes the probability of metastases detection within one year lower than the results obtained from epidemiological data. The thorough description of the obtained results and the evaluation of goodness of fit of the model to the data are presented in [Dębicka, Zmyślona 2016]. In order to achieve better fit of the model to data, patients were divided into several groups. In both populations, patients below 45 years of age occur very rarely and we assume that for these groups of patients the probability is constant and equals the probability of diagnosis for a 45 years-old person. Additionally, we divided the men population into two groups, the first form 45 to 59 years of age, and the second above 59 years of age. The values of probability of metastases diagnosis depending on age are presented in Figure 6.

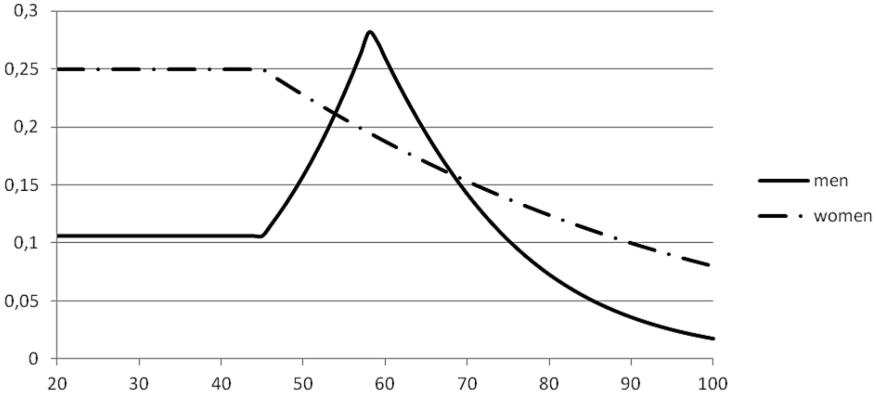

Figure 6. The values of probability of metastases detection during a year using logit model

Source: own calculations on the basis of analysis of individual data from the National Health Fund.

The course of the curves representing the probability of metastastes diagnosis during a calendar year is different for men and women in dependence on age. The probability is lower in the elderly (more than 60 years old) populations.This phenomenon is related to the hypothesis that many tumors are less aggressive in the elderly. The proliferation of all cells, including cancer, in organs of an elderly is weaker and thus a spread of cancer is less dynamic. However, in male population the decrease of probability values with age is significantly greater than for women.

The differences between genders are also observed for middle age patients (between 45 and 60 years). In the age group from 45 to 55 years, the risk of metastases detection is significantly higher in women population. However, the comparison of the probability shown in

Figure 6 and the probability presented in Figure 5 (detection of metastases during the first diagnosis) has revealed that in this age group, men are more likely to have detectable metastases at an initial diagnosis. The earlier described reluctance to medical appointments is one of the reasons for this. Additionally, in men population in this age group, the risk of detection of metastases grows rapidly. The risk of metastases detection, in the group of patients from 55 to 70 years of age, in which the highest incidence rates are observed, is significantly higher in male population. The probability of metastases detection during the first year after the diagnosis for a patient in age x is denoted as ρ_x.

In conclusion, we can claim that the risk of deterioration is moderately higher in women population than in men. The exception poses the group of patients aged 55 to 70 years. Considering the fact that they create the largest group of patients suffering from lung cancer, it turns out that the deterioration of health during a year occurs more frequently in men population.

3.3. The fatality rate for patients in critical state

Separation of the terminal state is important from the point of view of end-on-life cost estimation. Various options of financial and insurance products, in which the payment of benefits occurs precisely in a period when the patient is in the terminal state, may be some alternative or supplementing to financing public health care. Examples of such products are described in papers [Dębicka, Zmyślona Manuscript] and [Dębicka i in. 2015]. Despite many kinds of cancers, prognosis in a terminal state is similar for older and younger people, however, higher fatality rates are observed for the elderly. This is caused by the fact that the organism of the elderly regenerates much longer. The elderly are often burdened with other diseases, therefore the risk of complications after the treatment is much greater. In addition, due to the increased risk of complications, a radical treatment is avoided for these patients, which shortens life expectancy. All of these factors make mortality rates dependent of a patient's age.

Modelling of the fatality rates is identical with the estimation of survival time of patients in the critical state. These patients who fell ill and were diagnosed with metastases in 2008 pose a studied cohort, including 845 males and 324 females. In the male group the minimum value of age equals 40, the maximum 89. 25% of patients were not more than 58 years of age, 50% not more than 65 and 75% not more than 72. In the female group, the youngest patient was 40 years old, the oldest 88. The quartiles of age are equal to 55, 60 and 69, respectively.

Variable Y is introduced, which is defined as the number of years that the patient from the analyzed cohort survived after the metastases diagnosis. On the basis of empirical data we observed that patients had survived maximum four years. For that reason we assume that the variable takes values from 0 to 3. If $Y = 0$, which means that a patient died during the first year (calculating from the first hospitalization with diagnosed metastases during 2008). If $Y = 1$, it means that a patient died during the second year, etc. Due to the fact that lung cancer occurs rarely in population to 40 years of age, the model was estimated only for patients of above 40 years of age.

The empirical distribution of the number of patients who died in particular years and the probabilities of death in particular years in male and female populations are presented in Table 1. The probabilities of death defined as $P(T = 0)$ and $P\left(T = i \middle| T > i-1\right)$ for $i = 0, 1, \ldots, 3$ are used to estimate the fatality rates for patients in the terminal state.

Table 1. The empirical distribution of number of survived years and the fatality rates

Number of survived years	Number of patients who died in particular year (men)	Empirical distribution (men)	Fatality rates (men)	Number of patients who died in particular year (women)	Empirical distribution (women)	Fatality rates (women)
0	732	0.8662722	0.8662722	268	0.8271605	0.8271605
1	84	0.0994083	0.7433630	43	0.1327160	0.7678569
2	27	0.0319527	0.9310363	12	0.0370370	0.9230750
3	2	0.0023668	1	1	0.0030865	1

Source: own calculations.

The empirical distributions and fatality rates seem to be quite similar, but if we take into account the influence of age we can observe some differences. Modelling of the dependence age and fatality rates was made using the logistic regression for ordered categorical variable in men population and the Poisson regression with identity link function in women population. For patients between 20 and 39 years of age, we assume that the fatality rate is equal to the probability of death for a 40-year-old person.

The details connected with estimation of parameters and assessment of fitting of model to data are described in [Dębicka, Zmyślona 2016]. The values of fatality rates are presented in Figure 7.

The analysis of the curves shown in Figure 7 reveals some regularities. Firstly, the fatality rate in the subsequent years of duration

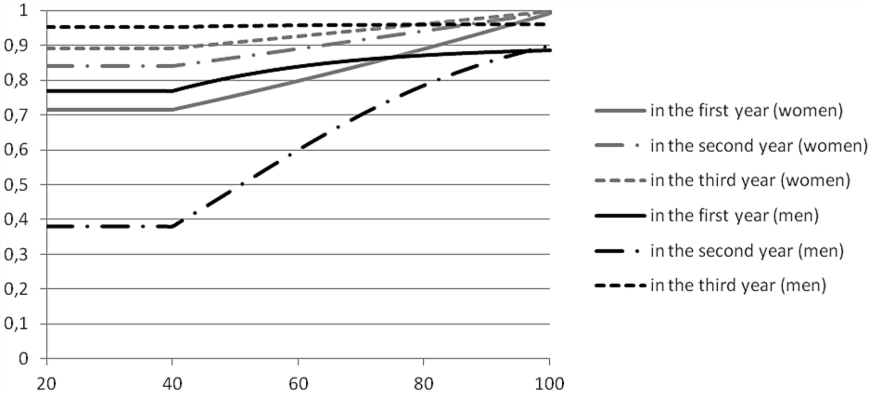

Figure 7. The fatality rates in particular years in male and female population

Source: own calculations on the basis of analysis of individual data from National Health Fund, presented in [Dębicka, Zmyślona 2016].

in the critical condition increases with age, regardless of gender, but the growth rate is lower in women population than in men. Secondly, in the first and third year after diagnosis with metastases the fatality rate is higher for men up to 75 years of age (in case of the first year) and up to 79 years of age (in case of the third year). In older people, aged above 75 and 79 years, respectively, this trend is reversed and the risk of mortality is higher in women population.

The death probability for a critically ill patient in age x in consecutive years assumes quite a different shape in case of men and women population. In female population, the fatality rate in consecutive years increases, but the differences between these rates in particular years decrease with patient's age. In male population the fatality rate in the first year of duration in critical state is very high and slightly increases with age. In the second year, it decreases significantly. The difference between the values of fatality rates in the first and second year is greater for a younger patient, it gradually decreases with age and approaches zero for patients aged 97. The probability of death in the third year drastically increases and remains at a similar level, regardless of the age of the patient (for a 20-year person it is 0.9532 and 0.9592 for a centenarian). Generally, in male population the values of fatality rate in consecutive years for a patient in age x are more varied.

The probabilities of death for a patient in age x in the first, the second, the third year after the diagnosis with metastases are denoted as $d_x^{(3,4)}$, $d_x^{(4,5)}$, $d_x^{(5,6)}$, respectively.

We conclude that lung cancer with metastases belongs to cancers with poor prognosis. The patients with advanced disease have a poor chance of survival for more than four years. Fatality rate in the first year after diagnosis with metastases is very high in men and women populations. The age of patient slightly increases the risk of death.

4. Multistate life tables

Mortality due to lung cancer or other causes, incidence of malignant cancer and diagnosed metastases are three random events, which should be taken into account in the analysis connected with financing of public health care and health insurance concerning cancer. We introduce the *multiple state model* (S, T), where S is a state space and T denotes a set of direct transitions between states of the state space [Dębicka 2013]. The state space consists of eight states, which are described as $S = \{1, 2, \ldots, 8\}$. The meaning of particular states is as follows [Dębicka, Zmyślona 2016] and particular states mean that an insured:

1 – is alive and healthy (without lung cancer),
2 – is ill with lung cancer without metastases,
3 – is suffering from lung cancer for a year after diagnosis of metastases,
4 – is suffering from lung cancer for the second year after diagnosis of metastases,
5 – is suffering from lung cancer for the third year after diagnosis of metastases,
6 – is suffering from lung cancer for the fourth year after diagnosis of metastases,
7 – died for reasons other than cancer or being sick with cancer without diagnosed metastases,
8 – died being sick with lung cancer with diagnosed metastases.

The graphic representation of the multiple state model (S, T) is shown in Figure 8, where circles represent the states and arcs correspond to direct transitions between the states. In the model, we distinguish two potential conditions of patient's health. The state 2 on our model is connected with the mild health condition (without diagnosis of metastases) and states between 3 to 6 model the survival time in the critical health condition (diagnosis with distant metastases). The distinction between the two conditions is important in the analysis connected with health care cost and health insurance. In case of critical state costs generated in the treatment process are often higher and some additional sources are needed to finance medical services. In [Dębicka, Zmyślona 2016] and [Dębicka i in. 2015] examples of financial-insurance products in case of terminal stage of health are presented, which can be used to finance additional expenditures resulting from palliative care, treatment of complications and other.

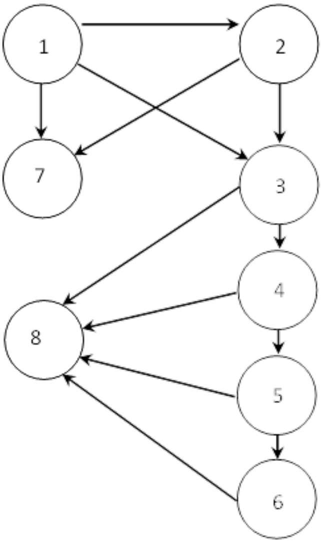

Figure 8. A multiple state model for critical illness insurances

Source: [Dębicka, Zmyślona 2016].

Let x is the *age of a person*. The preparation of multiple increment-decrement tables for each age x is cumbersome and not always needed. Then we assume that the *hypothesis of aggregation* is satisfied. We focus on discrete-time model, where $X(x)$ denotes the state of an individual at the age x ($x = 20, 21, 22, \ldots$). Hence the evolution of the insured risk is described by a discrete-time stochastic process $\{X(t);\ t \in \{20, 21, 22, \ldots\}\}$. Under the assumption that $\{X(t);\ t \in \{20, 21, 22, \ldots\}\}$ is a nonhomogeneous Markov chain we have described the probabilistic structure of the model based on chain of the transition matrices $\left\{\mathbf{Q}(x)\right\}_{k=0}^{n-1}$, where $\mathbf{Q}(x) = \left(q_{ij}(x)\right)_{i,j=1}^{N}$ and $q_{ij}(x) = P(X(x+1) = j \mid X(x) = i)$. The transition matrix for the considered model is as follows

$$\mathbf{Q}(k) = \begin{pmatrix} q_{11}(k) & q_{12}(k) & q_{13}(k) & 0 & 0 & 0 & q_{17}(k) & 0 \\ 0 & q_{22}(k) & q_{23}(k) & 0 & 0 & 0 & q_{27}(k) & 0 \\ 0 & 0 & q_{33}(k) & q_{34}(k) & 0 & 0 & 0 & q_{38}(k) \\ 0 & 0 & 0 & 0 & q_{45}(k) & 0 & 0 & q_{48}(k) \\ 0 & 0 & 0 & 0 & 0 & q_{56}(k) & 0 & q_{58}(k) \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

The following rates and estimators of probabilities are used in estimation of the transition probabilities of matrix $\mathbf{Q}(k)$:

ω_{x+k} – the mortality rate for a person aged $x + k$,

ζ_{x+k} – the incidence rate for a person aged $x + k$,

β_{x+k} – the percentage of patients who are diagnosed with metastases during the first diagnosis aged $x + k$,

ρ_{x+k} – the probability of metastases detection during the first year after diagnosis for patient aged $x + k$,

q_{x+k} – the probability of death for person aged $x + k$ from (Life Tabels for Poland),

$d_{x+k}^{(3,8)}$ – the fatality rate in the first year after diagnosis with metastases for patient aged $x + k$,

$d_{x+k}^{(4,8)}$ – the fatality rate in the second year after diagnosis with metastases for patient aged $x + k$,

$d_{x+k}^{(5,8)}$ – the fatality rate in the third year after diagnosis with metastases for patient aged $x + k$.

The estimates are obtained by adopting the methology used in the construction of life tables and are presented in Table 1 (the details concerning of estimation are described in [Dębicka, Zmyślona 2016]).

The multiple increment-decrement tables based on mulistate model presented in in Figure 8 may be presented in following form

$$\left\{q_{11}(x), q_{12}(x), q_{13}(x), q_{22}(x), q_{23}(x), q_{34}(x), q_{45}(x), q_{56}(x)\right\}_{x=20}^{100}$$

The number of state	Estimates of transition probability
1	$q_{11}(k)=1-(q_{x+k}-\omega_{x+k})-\zeta_{x+k}$ $q_{12}(k)=\zeta_{x+k}(1-\beta_{x+k})$ $q_{13}(k)=\zeta_{x+k}\cdot\beta_{x+k}$ $q_{17}(k)=q_{x+k}-\omega_{x+k}$
2	$q_{21}(k)=0$ $q_{22}(k)=1-q_{x+k}-\rho_{x+k}$ $q_{23}=\rho_{x+k}$ $q_{27}(k)=q_{x+k}$
3	$q_{34}(k)=1-d_{x+k}^{(3,8)}$ $q_{38}(k)=d_{x+k}^{(3,8)}$
4	$q_{45}(k)=1-d_{x+k}^{(4,8)}$ $q_{48}(k)=d_{x+k}^{(4,8)}$
5	$q_{56}(k)=1-d_{x+k}^{(5,6)}$ $q_{58}(k)=d_{x+k}^{(5,8)}$
6	$q_{68}(k)=1$

The estimators of transition probabilities depending on the age of a person (x) are presented in multistate life tables calculated separately for male and female populations in Appendix.

5. Conclusions

Existing inequalities in health due to biological sex and gender strongly differentiate incidence, mortality and fatality rates for many kinds of chronic and critical illnesses which depend on lifestyle habits. Therefore, the construction of financial and insurance products and cost analyses concerning these illnesses should be carried out separately for the populations of men and women. In the paper we describe an example of the multiple state model connected with insurance against lung cancer, which takes into account these inequalities. The multi-state life tables presented in the paper can be used in the analysis of many phenomena connected with health economics.

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APPENDIX

Multistate Life Tables for men

Age (x)	$q_{11}(x)$	$q_{12}(x)$	$q_{13}(x)$	$q_{22}(x)$	$q_{23}(x)$	$q_{34}(x)$	$q_{45}(x)$	$q_{56}(x)$
20	0.998878	0.000003	0.000001	0.893019	0.105861	0.231514	–	–
21	0.998878	0.000003	0.000001	0.893019	0.105861	0.231514	0.619088	–
22	0.998908	0.000003	0.000001	0.893049	0.105861	0.231514	0.619088	0.046846
23	0.998928	0.000003	0.000001	0.893069	0.105861	0.231514	0.619088	0.046846
24	0.998918	0.000003	0.000001	0.893059	0.105861	0.231514	0.619088	0.046846
25	0.998886	0.000005	0.000001	0.893029	0.105861	0.231514	0.619088	0.046846
26	0.998836	0.000005	0.000001	0.892979	0.105861	0.231514	0.619088	0.046846
27	0.998786	0.000005	0.000001	0.892929	0.105861	0.231514	0.619088	0.046846
28	0.998726	0.000005	0.000001	0.892869	0.105861	0.231514	0.619088	0.046846
29	0.998656	0.000005	0.000001	0.892799	0.105861	0.231514	0.619088	0.046846
30	0.998561	0.000012	0.000004	0.892709	0.105861	0.231514	0.619088	0.046846
31	0.998461	0.000012	0.000004	0.892609	0.105861	0.231514	0.619088	0.046846
32	0.998341	0.000012	0.000004	0.892489	0.105861	0.231514	0.619088	0.046846
33	0.998211	0.000012	0.000004	0.892359	0.105861	0.231514	0.619088	0.046846
34	0.998061	0.000012	0.000004	0.892209	0.105861	0.231514	0.619088	0.046846
35	0.997876	0.000031	0.000009	0.892029	0.105861	0.231514	0.619088	0.046846
36	0.997686	0.000031	0.000009	0.891839	0.105861	0.231514	0.619088	0.046846
37	0.997456	0.000031	0.000009	0.891609	0.105861	0.231514	0.619088	0.046846
38	0.997206	0.000031	0.000009	0.891359	0.105861	0.231514	0.619088	0.046846
39	0.996878	0.000061	0.000009	0.891069	0.105861	0.231514	0.619088	0.046846
40	0.996579	0.000061	0.000041	0.890739	0.105861	0.231514	0.619088	0.046846
41	0.996199	0.000061	0.000041	0.890359	0.105861	0.243359	0.676901	0.046846
42	0.995769	0.000061	0.000041	0.889929	0.105861	0.238507	0.668097	0.062688
43	0.995299	0.000061	0.000041	0.889459	0.105861	0.233786	0.659173	0.062301
44	0.994769	0.000061	0.000041	0.888929	0.105861	0.229195	0.650135	0.061929
45	0.994202	0.00017	0.000202	0.888359	0.105861	0.224731	0.640988	0.061571
46	0.993572	0.00017	0.000202	0.878686	0.114904	0.220394	0.631737	0.061226
47	0.992902	0.00017	0.000202	0.868309	0.124611	0.216180	0.622388	0.060895
48	0.992192	0.00017	0.000202	0.857197	0.135013	0.212089	0.612948	0.060577
49	0.991452	0.00017	0.000202	0.845331	0.146139	0.208117	0.603422	0.060271
50	0.990565	0.000461	0.000562	0.832666	0.158014	0.204263	0.593816	0.059977
51	0.989745	0.000461	0.000562	0.819199	0.170661	0.200525	0.584139	0.059694
52	0.988885	0.000461	0.000562	0.804901	0.184099	0.196890	0.574396	0.059422
53	0.987995	0.000461	0.000562	0.789768	0.198342	0.193386	0.564594	0.059161
54	0.987065	0.000461	0.000562	0.773781	0.213399	0.189980	0.554743	0.058909
55	0.986077	0.000986	0.000839	0.756929	0.229271	0.186681	0.544848	0.058668
56	0.985037	0.000986	0.000839	0.739204	0.245956	0.183486	0.534917	0.058435
57	0.983957	0.000986	0.000839	0.720641	0.263439	0.180392	0.524959	0.058212
58	0.982807	0.000986	0.000839	0.701229	0.281701	0.177397	0.514980	0.057998
59	0.981587	0.000986	0.000839	0.707843	0.273867	0.174499	0.504990	0.057791
60	0.980375	0.001505	0.001515	0.721203	0.259217	0.171695	0.494996	0.057593

61	0.979015	0.001505	0.001515	0.733974	0.245086	0.168983	0.485006	0.057402
62	0.977565	0.001505	0.001515	0.746125	0.231485	0.166360	0.475028	0.057219
63	0.976025	0.001505	0.001515	0.75765	0.21842	0.163825	0.465069	0.057043
64	0.974385	0.001505	0.001515	0.768535	0.205895	0.161375	0.455139	0.056874
65	0.972709	0.002178	0.002123	0.77877	0.19391	0.159007	0.445244	0.056711
66	0.970829	0.002178	0.002123	0.788338	0.182462	0.156719	0.435392	0.056555
67	0.968809	0.002178	0.002123	0.797233	0.171547	0.154509	0.425591	0.056405
68	0.966619	0.002178	0.002123	0.805435	0.161155	0.152374	0.415848	0.056260
69	0.964239	0.002178	0.002123	0.812932	0.151278	0.150313	0.406170	0.056121
70	0.962163	0.002733	0.002548	0.819695	0.141905	0.148324	0.396565	0.055988
71	0.959293	0.002733	0.002548	0.82571	0.13302	0.146404	0.387039	0.055860
72	0.956123	0.002733	0.002548	0.830948	0.124612	0.144550	0.377599	0.055737
73	0.952593	0.002733	0.002548	0.835367	0.116663	0.142762	0.368250	0.055618
74	0.948673	0.002733	0.002548	0.838952	0.109158	0.141037	0.358999	0.055504
75	0.944822	0.003574	0.002079	0.84168	0.10208	0.139373	0.349852	0.055395
76	0.939982	0.003574	0.002079	0.843508	0.095412	0.137767	0.340815	0.055290
77	0.934652	0.003574	0.002079	0.844454	0.089136	0.13622	0.331891	0.055189
78	0.928792	0.003574	0.002079	0.844495	0.083235	0.134727	0.323087	0.055092
79	0.922392	0.003574	0.002079	0.843638	0.077692	0.133289	0.314406	0.054999
80	0.915867	0.003016	0.001566	0.841912	0.072488	0.131902	0.305853	0.054909
81	0.908387	0.003016	0.001566	0.839312	0.067608	0.130565	0.297432	0.054823
82	0.900357	0.003016	0.001566	0.835857	0.063033	0.129277	0.289146	0.054740
83	0.891777	0.003016	0.001566	0.831561	0.058749	0.128035	0.280999	0.054660
84	0.88263	0.003016	0.001566	0.826421	0.0547388	0.126839	0.272993	0.054584
85	0.872499	0.002465	0.001232	0.820432	0.050988	0.125687	0.265131	0.054510
86	0.862139	0.002465	0.001232	0.81358	0.04748	0.124577	0.257415	0.054440
87	0.851139	0.002465	0.001232	0.805857	0.044203	0.123508	0.249848	0.054372
88	0.839439	0.002465	0.001232	0.797217	0.04114	0.122478	0.24243	0.054307
89	0.826989	0.002465	0.001232	0.787625	0.038285	0.121486	0.235163	0.054244
90	0.813839	0.002465	0.001232	0.777141	0.035619	0.120532	0.228049	0.054184
91	0.799759	0.002465	0.001232	0.765548	0.033132	0.119612	0.221088	0.054126
92	0.784929	0.002465	0.001232	0.753037	0.030813	0.118727	0.214280	0.054070
93	0.769349	0.002465	0.001232	0.739618	0.028652	0.117875	0.207626	0.054017
94	0.753019	0.002465	0.001232	0.725302	0.026638	0.117055	0.201126	0.053966
95	0.735969	0.002465	0.001232	0.710128	0.024762	0.116265	0.194779	0.053916
96	0.718189	0.002465	0.001232	0.694095	0.023015	0.115505	0.188585	0.053869
97	0.699729	0.002465	0.001232	0.677261	0.021389	0.114773	0.182544	0.053823
98	0.680589	0.002465	0.001232	0.659635	0.019875	0.11407	0.176654	0.053780
99	0.660829	0.002465	0.001232	0.641284	0.018466	0.113392	0.170914	0.053738
100	0.640459	0.002465	0.001232	0.622224	0.017156	0.112740	0.165324	0.053697

Multistate Life Tables for women

Age (x)	$q_{11}(x)$	$q_{12}(x)$	$q_{13}(x)$	$q_{22}(x)$	$q_{23}(x)$	$q_{34}(x)$	$q_{45}(x)$	$q_{56}(x)$
20	0.999726	0.000004	0.000001	0.750187	0.249543	0.284497	–	–
21	0.999746	0.000004	0.000001	0.750207	0.249543	0.284497	0.158063	–
22	0.999746	0.000004	0.000001	0.750207	0.249543	0.284497	0.158063	0.108409
23	0.999746	0.000004	0.000001	0.750207	0.249543	0.284497	0.158063	0.108409
24	0.999736	0.000004	0.000001	0.750197	0.249543	0.284497	0.158063	0.108409
25	0.999729	0.000002	0.000001	0.750187	0.249543	0.284497	0.158063	0.108409
26	0.999709	0.000002	0.000001	0.750167	0.249543	0.284497	0.158063	0.108409
27	0.999689	0.000002	0.000001	0.750147	0.249543	0.284497	0.158063	0.108409
28	0.999669	0.000002	0.000001	0.750127	0.249543	0.284497	0.158063	0.108409
29	0.999649	0.000002	0.000001	0.750107	0.249543	0.284497	0.158063	0.108409
30	0.999611	0.000012	0.000005	0.750077	0.249543	0.284497	0.158063	0.108409
31	0.999581	0.000012	0.000005	0.750047	0.249543	0.284497	0.158063	0.108409
32	0.999551	0.000012	0.000005	0.750017	0.249543	0.284497	0.158063	0.108409
33	0.999511	0.000012	0.000005	0.749977	0.249543	0.284497	0.158063	0.108409
34	0.999461	0.000012	0.000005	0.749927	0.249543	0.284497	0.158063	0.108409
35	0.999399	0.00002	0.000008	0.749867	0.249543	0.284497	0.158063	0.108409
36	0.999329	0.00002	0.000008	0.749797	0.249543	0.284497	0.158063	0.108409
37	0.999249	0.00002	0.000008	0.749717	0.249543	0.284497	0.158063	0.108409
38	0.999159	0.00002	0.000008	0.749627	0.249543	0.284497	0.158063	0.108409
39	0.999049	0.00002	0.000008	0.749517	0.249543	0.284497	0.158063	0.108409
40	0.9989246	0.000044	0.000044	0.749397	0.249543	0.284497	0.158063	0.108409
41	0.9987846	0.000044	0.000044	0.749257	0.249543	0.280597	0.158063	0.108409
42	0.9986246	0.000044	0.000044	0.749097	0.249543	0.276677	0.155645	0.108409
43	0.9984446	0.000044	0.000044	0.748917	0.249543	0.272735	0.153222	0.106701
44	0.9982546	0.000044	0.000044	0.748727	0.249543	0.268771	0.150795	0.104991
45	0.9980107	0.00012	0.000085	0.748507	0.249543	0.264786	0.148362	0.103279
46	0.9977807	0.00012	0.000085	0.752831	0.244989	0.260779	0.145925	0.101566
47	0.9975307	0.00012	0.000085	0.757078	0.240492	0.25675	0.143482	0.099851
48	0.9972607	0.00012	0.000085	0.761249	0.236051	0.252699	0.141035	0.098134
49	0.9969707	0.00012	0.000085	0.765343	0.231667	0.248626	0.138583	0.096415
50	0.9966352	0.000223	0.000235	0.769359	0.227341	0.244531	0.136126	0.094694
51	0.9963052	0.000223	0.000235	0.773299	0.223071	0.240414	0.133664	0.092972
52	0.9959452	0.000223	0.000235	0.77715	0.21886	0.236274	0.131198	0.091248
53	0.995565	0.000223	0.000235	0.780925	0.214705	0.232112	0.128726	0.089523
54	0.995165	0.000223	0.000235	0.784621	0.210609	0.227927	0.126249	0.087795
55	0.99462	0.000406	0.000477	0.78822	0.20657	0.223719	0.123768	0.086066
56	0.99416	0.000406	0.000477	0.791742	0.202588	0.219488	0.121282	0.084335
57	0.99367	0.000406	0.000477	0.795176	0.198664	0.215234	0.118791	0.082603
58	0.99315	0.000406	0.000477	0.798522	0.194798	0.210957	0.116295	0.080869
59	0.99261	0.000406	0.000477	0.801791	0.190989	0.206657	0.113794	0.079133
60	0.992109	0.0006	0.000494	0.804993	0.187237	0.202333	0.111288	0.077395
61	0.991529	0.0006	0.000494	0.808108	0.183542	0.197986	0.108777	0.075655
62	0.990929	0.0006	0.000494	0.811147	0.179903	0.193615	0.106262	0.073914
63	0.990309	0.0006	0.000494	0.814108	0.176322	0.18922	0.103741	0.072171
64	0.989639	0.0006	0.000494	0.816964	0.172796	0.184801	0.101216	0.070427
65	0.988985	0.000673	0.000449	0.819703	0.169327	0.180359	0.098685	0.068681
66	0.988155	0.000673	0.000449	0.822287	0.165913	0.175892	0.09615	0.0669327
67	0.987205	0.000673	0.000449	0.824695	0.162555	0.1714	0.09361	0.065183
68	0.986105	0.000673	0.000449	0.826899	0.159251	0.166884	0.091065	0.063432
69	0.984795	0.000673	0.000449	0.828837	0.156003	0.162344	0.088515	0.061679
70	0.983361	0.000853	0.000292	0.830502	0.152808	0.157778	0.08596	0.059924
71	0.981541	0.000853	0.000292	0.831823	0.149667	0.153188	0.0834	0.058167

72	0.979391	0.000853	0.000292	0.83276	0.14658	0.148573	0.080836	0.056409
73	0.976881	0.000853	0.000292	0.833284	0.143546	0.143933	0.078266	0.054649
74	0.973941	0.000853	0.000292	0.833326	0.140564	0.139267	0.075691	0.052888
75	0.970602	0.000758	0.000428	0.832856	0.137634	0.134576	0.073112	0.051124
76	0.966672	0.000758	0.000428	0.831805	0.134755	0.12986	0.070528	0.049359
77	0.962182	0.000758	0.000428	0.830142	0.131928	0.125117	0.067938	0.047593
78	0.957102	0.000758	0.000428	0.827839	0.129151	0.120349	0.065344	0.045824
79	0.951392	0.000758	0.000428	0.824856	0.126424	0.115555	0.062745	0.044054
80	0.945168	0.000775	0.000303	0.821174	0.123746	0.110735	0.060141	0.042283
81	0.938148	0.000775	0.000303	0.816783	0.121117	0.105888	0.057532	0.040509
82	0.930458	0.000775	0.000303	0.811673	0.118537	0.101015	0.054919	0.038734
83	0.922088	0.000775	0.000303	0.805836	0.116004	0.096116	0.0523	0.036958
84	0.913028	0.000775	0.000303	0.799261	0.113519	0.09119	0.049676	0.035179
85	0.903248	0.000752	0.000376	0.791931	0.111079	0.086236	0.047048	0.033399
86	0.892708	0.000752	0.000376	0.783784	0.108686	0.081256	0.044414	0.031617
87	0.881398	0.000752	0.000376	0.774821	0.106339	0.076249	0.04178	0.029834
88	0.869238	0.000752	0.000376	0.764964	0.104036	0.071215	0.039132	0.0280488
89	0.856158	0.000752	0.000376	0.754143	0.101777	0.066153	0.036484	0.026262
90	0.842208	0.000752	0.000376	0.742408	0.099562	0.061063	0.033831	0.024474
91	0.827068	0.000752	0.000376	0.72944	0.09739	0.055946	0.031173	0.022683
92	0.810988	0.000752	0.000376	0.71549	0.09526	0.0508	0.02851	0.020892
93	0.793968	0.000752	0.000376	0.700558	0.093172	0.045628	0.025842	0.019098
94	0.776008	0.000752	0.000376	0.684645	0.091125	0.040426	0.023169	0.017303
95	0.757128	0.000752	0.000376	0.667771	0.089119	0.035197	0.020491	0.015506
96	0.737348	0.000752	0.000376	0.649957	0.087153	0.029939	0.017809	0.013708
97	0.716698	0.000752	0.000376	0.631234	0.085226	0.024652	0.015121	0.011908
98	0.695228	0.000752	0.000376	0.611653	0.083337	0.019336	0.012428	0.010106
99	0.672978	0.000752	0.000376	0.591253	0.081487	0.013991	0.009731	0.008303
100	0.649998	0.000752	0.000376	0.570086	0.079674	0.008618	0.007029	0.006498