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Evaluation and choice of an anti-aircraft missile system under uncertain conditions based on fuzzy-integral calculus and hierarchical cluster analysis

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Abstract

Evaluation and choice of anti-aircraft missile weapon systems (AAMWSs) are a relevant problem in many countries. Estimates are often reduced to one top-level criterion. This is questionable since subordinated criteria are usually difficult to compare and decision-makers excluded from the final decision. The solution to the problem should also take into account non-statistical uncertainty. We propose to use a combination of two algorithms. The first algorithm calculates the estimates of AAMWSs in the criteria hierarchy using fuzzy-integral calculus. The hierarchy relates the characteristics of the AAMWSs to the three top-level criteria. In the space of these criteria, the second algorithm based on hierarchical clustering forms a class of the best AAMWSs equivalent to each other. The decision-maker makes the final choice from the obtained class, taking into account non-formalized factors. These algorithms are tested using the example of medium-range AAMWSs.

Keywords: *evaluation, uncertainty, fuzzy-integral calculus, hierarchical clustering, anti-aircraft missile system*

1. Introduction

Analyzing the most famous arms exhibitions [27], we can see that the number of countries that can produce the entire range of weapons is very small. Production of weapons requires high technologies and significant investments, which can only be provided by leading countries with a developed scientific base and industry. Member countries of the North Atlantic Alliance dominate the list of the leading arms manufacturers [6]. Countries with less developed economies meet the needs of the army and navy through external purchases. The solution to the problem of choosing any weapon systems (WSs) is the consensus of many criteria. This statement is valid for the anti-aircraft missile weapon systems (AAMWSs).

Only a few countries such as the USA, Germany, France, Italy, Norway, Russia, China, and India produce AAMWSs. Most of them are members of the North Atlantic Alliance. AAMWSs are a rather

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broad class of WSs, some of which are functionally very distant from each other and the comparison between them does not always make sense. Therefore, we introduced the first limitation of our research – it is aimed at considering land-based AAMWSs produced by member countries of the North Atlantic Alliance with a target damage range from 40 to 100 km.

The problem of evaluating AAMWSs has many aspects, the nature of which is quite different. The main one is the effectiveness of AAMWSs during the main stages of exploitation. It can be described by three relevant criteria: combat use, logistics, maintenance and repair. These criteria exhaustively describe why a country buys AAMWS. The emphasis on exploitation efficiency implies taking into account the conditions of modern conflicts, the latest of which show significant changes. If earlier the main efforts of the developers of AAMWSs were aimed at combating ballistic missiles, today the enemy is massively using low-flying cruise missiles and drones. In addition, the possibility of gaps in logistics supply has increased significantly. Today, the delivery of missiles, ammunition, fuel must be provided in off-road conditions. Active opposition from the enemy determines the need to ensure stealth, mobility, and rapid change of positions. Thus, the set of priority requirements for AAMWSs has changed.

Here we do not consider economic parameters, such as the price of AAMWS, the price of ammunition (missiles), and the financial risks of the transaction. These parameters are rarely advertised and are often formed in a non-market way, for example, using political preferences. Therefore, in this research, we consider the evaluation problem solely from the viewpoint of the expected effectiveness of AAMWSs at the mentioned stages of exploitation. This is the second limitation of our research.

The evaluation of AAMWSs has several questions related to uncertainty. Firstly, the relationships between the evaluation criteria are not obvious and give rise to fuzziness. Secondly, the evaluation is prognostic, since we are estimating the expected effectiveness of the AAMWSs in the future. Orientation on the future also breeds uncertainty. And thirdly, data on the AAMWSs' parameters is often incomplete, classified, or improved to increase market attractiveness. Sometimes tactical and technical characteristics (TTCs) are measured in ideal conditions, and in real conditions, they are not confirmed.

The problem of choice is associated with the question of the advisability of using a single-criteria choice and the degree of participation of the decision-maker in the choice procedure. Researchers often reduce multi-criteria choice to choice based on a single generalized criterion. As shown by Nogin [49], this is permissible, assuming that the person will think rationally. But this de facto excludes him from the decision-making process since the choice is reduced to the automatic choice of the alternative with the highest estimation. In addition, as mentioned above, the exploitation effectiveness of AAMWSs must be evaluated in the context of three generalized criteria: combat use, logistics, maintenance, and repair. To ensure a single-criteria choice, it is necessary to assign the importance of these criteria. However, they are poorly comparable with each other, since they characterize different stages of the AAMWSs exploitation. They are equally important since any AAMWS must hit targets with a high probability, and be easy in logistics, maintenance, and repair. Therefore, it can be expected that when assigning criteria weights, the expert will experience difficulties and will subjectify the choice, unconsciously based on individual adherence. Since AAMWSs are very complex systems and formal methods cannot always take into account all the nuances and circumstances of their exploitation, it is advisable to leave the final choice to a person, simultaneously offering a way to reduce the number of alternatives.

As a general approach, we proposed not to reduce the multi-criteria evaluation to an evaluation in one criterion, but to calculate the AAMWS estimates in three generalized top-level criteria and, on this basis, to form a set of solutions as a class of the best AAMWSs which are equivalent to each other. The decision-maker makes the final choice, taking into account additional non-formalized factors.

To calculate the AAMWS estimates, we proposed using a criteria hierarchy that relates the AAMWSs TTCs to the three top-level criteria. The calculation algorithm is based on fuzzy-integral calculus methods to take into account non-statistical uncertainty. In particular, we proposed using:

- Sugeno fuzzy measures for describing the importance of criteria in the hierarchy,
- fuzzy membership functions for describing AAMWS estimates in the criteria hierarchy,
- Sugeno fuzzy integral for generalizing AAMWS estimates along the criteria hierarchy.

To form the class of the best equivalent AAMWSs, we proposed to use the hierarchical clustering algorithm in the space of three top-level criteria. This algorithm forms a similarity relation from previously calculated estimates based on the Hamming distance, and an equivalence relation using transitive closure. To reveal the proposed approach, we will consider the following questions:

- a review of known research regarding the evaluation and choice of WSs, including AAMWSs,
- clarification of the research problem, taking into account the results of the review,
- hierarchy of AAMWS evaluation criteria,
- AAMWSs evaluating algorithm,
- algorithm for choice of the best AAMWSs,
- discussion of the proposed approach to demonstrate the working capacity of the proposed algorithms and the quality of results.

2. Literature review

The results of well-known studies from the viewpoint of the following aspects have been considered:

- conceptual questions of evaluation and choice of WSs, including AAMWSs,
- generalized approaches (ideas) in methods for solving problems of evaluation and choice of WSs,
- mathematical constructs that are used to evaluate and choice of WSs.

2.1. Conceptual questions of evaluating and choice of WSs including AAMWSs

The nature of applied problems. The characteristics of the purchase process of WSs for the armed forces are disclosed by Schwartz [57]. Kangaspunta et al. [36] and Yajie et al. [63] consider the problem of evaluating and choosing a portfolio of WSs, taking into account WSs interactions with each other and using the criterion of economic efficiency. The WSs portfolio must not be viewed as a set of individual systems but as a holistic collection of systems that must be properly combined to ensure effective sharing. The WSs portfolio may include old and new systems. In particular, the study of Albert [2] is devoted to evaluating the economic efficiency of air strike weapons systems, taking into account the portfolio of old and new types of weapons and the limitations of appropriations for large purchases. Kim et al. [39] affect an important question about the readiness of law enforcement agencies and the state to acquire modern WSs, since modern WSs are technically complex and require special knowledge for efficient use, maintenance, and logistics.

Part of the research focuses on methods for calculating some important generalized criteria for evaluating WSs. For example, the research of Monn et al. [48] aims to evaluate the readiness of WSs, which is one of the most important partial criteria of effectiveness. Another research, in particular Robbe et al. [53], can also be considered as an evaluation of one of the partial criteria for the effectiveness of non-lethal kinetic weapons. Magbagbeola [46] considers an additional aspect of evaluating lethal autonomous WSs, in particular, possible violations of the laws of war in the case of a technical failure. Most of the well-known approaches use a single rating scale, in which 0 means bad and 1 means good. However, Pei et al. [51] formulate an inverse problem, for example, evaluating the presence and criticality of gaps in WSs. Research of Cernis and Hasall [8] is devoted to the development of a universal environment for modeling the characteristics of WSs. Since there are many WSs today, a universal approach to describing and combining WSs provides a significant simplification of modeling defense planning. The book of Eriskin and Gunal [26] discloses post-purchase testing methods of WSs to evaluate their compliance with desired combat capabilities.

Criteria for evaluating WSs. Note that well-known studies consider the evaluation of WSs as a multi-criteria decision-making problem, but reduce it to a single-criteria problem. Many studies take into account conditions of uncertainty. At the top level of the criteria hierarchy, most studies consider the effectiveness of WSs regardless of war theater conditions. As an exception, we can consider the study of Zhang et al. [66], in which the authors proposed to evaluate WSs by analyzing their abilities in various conditions of combat use. This study uses the method of correlating a sample of weapons with a reference sample (TOPSIS - The Technique for Order of Preference by Similarity to Ideal Solution) for evaluation. The relationship between the evaluation of the WSs effectiveness and the character of combat missions is taken into account in the research of Zhao et al. [67]. This research was devoted to determining the best composition of various WSs for mission operation, presented as a hierarchy of subtasks. Some research considers ability as the main criterion for evaluating WSs. For example, in the article of Jiang et al. [35], the authors propose to evaluate WSs using a hierarchy of abilities, at the lower level of which are TTCs, and at the upper level – a generalized evaluation. The intermediate criteria of this hierarchy describe abilities at the corresponding levels of generalization. The approach ensures the use of both qualitative and quantitative data. The aggregation of estimates in the hierarchy of criteria is carried out using the basic probability mass. Kim et al. [38] evaluated the partial criterion: the convenience of a graphical user interface in human-machine WSs. The research method is based on the analytical hierarchical process (AHP). Galal [28] considers how the criteria for evaluating new WSs can change when they acquire qualitatively new tactical and technical characteristics. The study of Han et al. [32] is devoted to the formation of a system of indices (criteria) for evaluating the effectiveness of intelligent WSs. This system was proposed based on the analysis of combat situations in which WSs will be used.

2.2. Generalized approaches (ideas) in methods for solving problems of evaluation and choice of WSs

Today, only two generalized approaches to evaluating and choosing alternatives are known, including under conditions of uncertainty. Let's call them conditionally relative and absolute.

The relative approach involves comparing alternatives with each other. The best-known representative of this approach is the AHP method proposed by Saaty [54]. The main disadvantage of the relative approach is the inadequacy of the solution if there is not a single acceptable alternative among the considered alternatives. The computational complexity (number of pairwise comparisons) increases significantly with the growth of the number of alternatives. This approach has been used in many researches related to the evaluation and choice of WSs. In particular, Dos Santos et al. [24] propose using the AHP method to select a warship for the Brazilian Navy. Greiner et al. [31] solve the problem of evaluating and choosing WSs projects taking into account the optimal set of WSs projects in the portfolio based on AHP and integer optimization. Wu et al. [61], Cheng and Mon [17, 18], and Mon et al. [47] use various combinations of AHP methods and triangular fuzzy numbers to evaluate the effectiveness of WSs.

Othman et al. [50] solve the problem of ranking WSs based on fuzzy rules extracted from the input data. Kozakiewicz and Wróblewski [40] use the AHP method to choose the best 4th generation aircraft. The work of Cheng and Lin [15] is devoted to the choice of the main battle tank based on expert estimates. These estimates are described using linguistic terms, which are approximated by trapezoidal fuzzy numbers. To determine the group estimates, the authors use the fuzzy Delphi method with averaging the fuzzy rating of experts and the values of the criteria weights. The authors of the work also proposed a procedure for ranking alternatives. The results of similar research are described by Zhang et al. [65].

The absolute approach (TOPSIS technique) involves comparing the considered alternatives with a certain standard – the best alternative. This approach has no disadvantages to the relative approach. Note that the approach can use the AHP method, not to compare alternatives, but to determine the importance of partial criteria. An example is the research of Bai and Wang [5] devoted to the determination of optimal WSs. Another example is the paper of Sánchez-Lozano and Rodríguez [56], which is aimed at the choice of a training military aircraft for the Spanish Air Force. These researchers use the AHP method to obtain weights of the criteria and use fuzzy numbers to describe the estimates of the alternatives in the criteria. Sánchez-Lozano et al. [55], Dağdeviren et al. [22], Cheng [13], Chu and Shih [20] carried out research on a similar basis. The work of Chen [12] can also be attributed to the absolute approach. The author uses fuzzy numbers to describe the importance of criteria and estimates of alternatives, as well as simplified arithmetic operations to aggregate estimates. This simplifies and speeds up calculations. Chen [11] uses a similar method for evaluating WSs.

Li and Liu [44] modernize the AHP method by proposing to determine the relative weights using the Kullback-Leibler divergence, which is a measure of the difference in the information entropy of two distributions. To rank the alternatives, the authors proposed to use the method of ordering by similarity with a standard. Research by Erdal et al. [25] is directed towards the choice of anti-tank guided missiles. The authors use Shannon's entropy based on alpha-level sets to obtain criteria weights and a combination of the fuzzy combined compromise solution method with the Bonferroni method to rank alternatives. Maêda et al. [45] have also used a combination of AHP and TOPSIS methods to select a Brazilian Navy helicopter. Akgün and Erdal [1] used a combined AHP-TOPSIS method and geographic information system to design a multi-level ammunition distribution network. The geographic information system is used to determine the optimal location of warehouses, which minimizes transportation costs and risks. The TOPSIS method is used to calculate warehouse location risk estimates, and the AHP method is used to determine risk attribute weights.

Cho and Kim [19] developed a standard for subsequent estimation of correlation WSs with it. The standard was created based on the AHP method, in which the dependencies between the evaluation criteria are identified using the DEMATEL (decision-making trial and evaluation laboratory) method is an approach to structural modeling, that identifies the cause and effect relationships between elements of a system). Karadayi et al. [37] propose to evaluate WSs by comparison with an ideal solution, the structure of which is multi-criteria and takes into account data fuzziness. Ashari and Parsaei [4] proposed to modify the ELECTRE (in French – elimination et choix traduisant la realite) method — a family of methods for helping decision-making, the peculiarity of which is the study of the relationship between alternatives based on agreement and disagreement). The modified method was used to choose an infantry rifle.

2.3. Mathematical constructs used to evaluate and choice of WSs

In most cases, the choice of mathematical constructs is determined by the presence of uncertainty, the sources of which are most often input data and dependencies between criteria. A somewhat outdated but useful review of mathematical and simulation models for evaluating WSs is given by Cline [21]. To evaluate WSs, Cheng [14] uses triangular fuzzy numbers that describe the importance of the criteria. The choice of the best WS is based on the results of ranking fuzzy numbers. Cheng [13] uses the AHP method to determine the criteria weights and the membership function to describe the input data. Lide et al. [42] propose a methodology for evaluating the effectiveness of a short-range anti-aircraft missile system based on simulation modeling using experimental data. Wu and Mendel [62] propose to use linguistic variables represented by fuzzy numbers to describe the weights of criteria and data that describe the values of alternative characteristics. The fuzzy centroid (weighted average of the mass centers of the criterion and estimate) is used for the aggregation of estimates. The proposed approach allows you to operate with both numbers and words-estimates. The proposed criteria structure indirectly takes into account the combat missions that the WS must solve.

A feature of the solution proposed by Cheng et al. [16] is the absence of the requirement to determine the criteria weights. The approach uses a combination of triangular fuzzy numbers and catastrophe theory. The proposed algorithm was used to evaluate the missile destroyer. To generalize estimates according to a hierarchical system of criteria, Gao et al. [29] proposed an inference mechanism based on the use of interval fuzzy numbers. Research of Bi et al. [7] also uses interval fuzzy numbers to represent estimates. Jabbarova [34] proposed a hierarchical decision-making algorithm based on data representation in the form of Z-numbers, which Zadeh used to describe inaccurate and incompletely reliable data. Liao et al. [41] proposed an estimation method based on Hwang's relative distance between two triangular fuzzy numbers that describe the evaluations of experts in the AHP scale. The choice of the best alternative is based on the ranking of fuzzy numbers. The approach proposed by Chen and Shyu [10] is based on the use of target programming to find a solution in a multi-criteria environment in which the criteria take values of zero or one. The study of Ziyuan et al. [68] presents one of the new methods of multicriteria evaluation for solving problems in which both fuzziness and randomness must be taken into account. The proposed method is based on the use of the center of gravity of a p-dimensional synthetic cloud, which describes a multidimensional space of index weights.

Summing up, modern research pays much attention to the problems of evaluating and choosing WS, which indicates their relevance in practice. Researchers formulate the evaluation problem as a hierarchical multi-criteria decision-making problem, but most often they consider one criterion at the top level of generalization. On the other hand, we did not find research that formulated the evaluation problem as an estimate of the distance (including weighted) between points in the TTCs space. At the same time, only a few researchers consider the TTCs system in the context of combat missions or abilities. We have not found approaches that consider the problem in the context of evaluating the effectiveness of exploitation at all stages of the life cycle.

Many authors group TTCs based on generic affiliation, for example, tactical characteristics, noise immunity, and others. It can sometimes be assumed that TTCs grouping is done to reduce the criteria number in a pairwise comparison since this procedure allows comparison of up to 7-9 criteria to ensure their distinguishability. That is, known approaches can use artificial techniques that can introduce distortions into the conceptual model of solving the problem.

Accounting for uncertainty is carried out using fuzzy numbers and membership functions. Most often, the importance of criteria is obtained using the AHP method and described by additive functions, which are probability distributions. To aggregate the estimates of alternatives in a hierarchy of criteria, researchers most often use different types of weighted averages. We did not find works using fuzzy measures and integrals, which have some advantages.

3. Generalized scheme for solving the research problem

We propose an approach based on a combination of the TOPSIS technique and the hierarchical clustering method. A generalized scheme of the proposed approach is shown in Figure 1. The proposed approach involves the implementation of two steps: first, we calculate the estimates of all considered AAMWSs, and then we form a set of best solutions for further choice.

In the first step, we form a hierarchical system of evaluation criteria with three criteria at the top level of the hierarchy. They describe the exploitation efficiency of any AAMWSs: combat use, logistics, maintenance and repair. We do not use a generalized criterion, since it is difficult to determine the importance of indicated criteria. The bottom level of the hierarchy consists of criteria that describe AAMWSs TTCs. Intermediate criteria connect three efficiency criteria with AAMWSs TTCs. We proposed to describe the importance of the criteria using Sugeno fuzzy measures since the solution of our problem requires taking into account non-statistical uncertainty. The hierarchical system of criteria plays the role of an implicit standard that describes the ideal AAMWS. The ideal AAMWS best suits the current (forecast) conditions of exploitation. In this case, we will consider the calculated estimates of the considered AAMWSs as the degree of similarity with the implicit standard.

We proposed to describe the estimates of AAMWSs TTCs using fuzzy membership functions which experts form on the base of Harrington's "desirability" function.

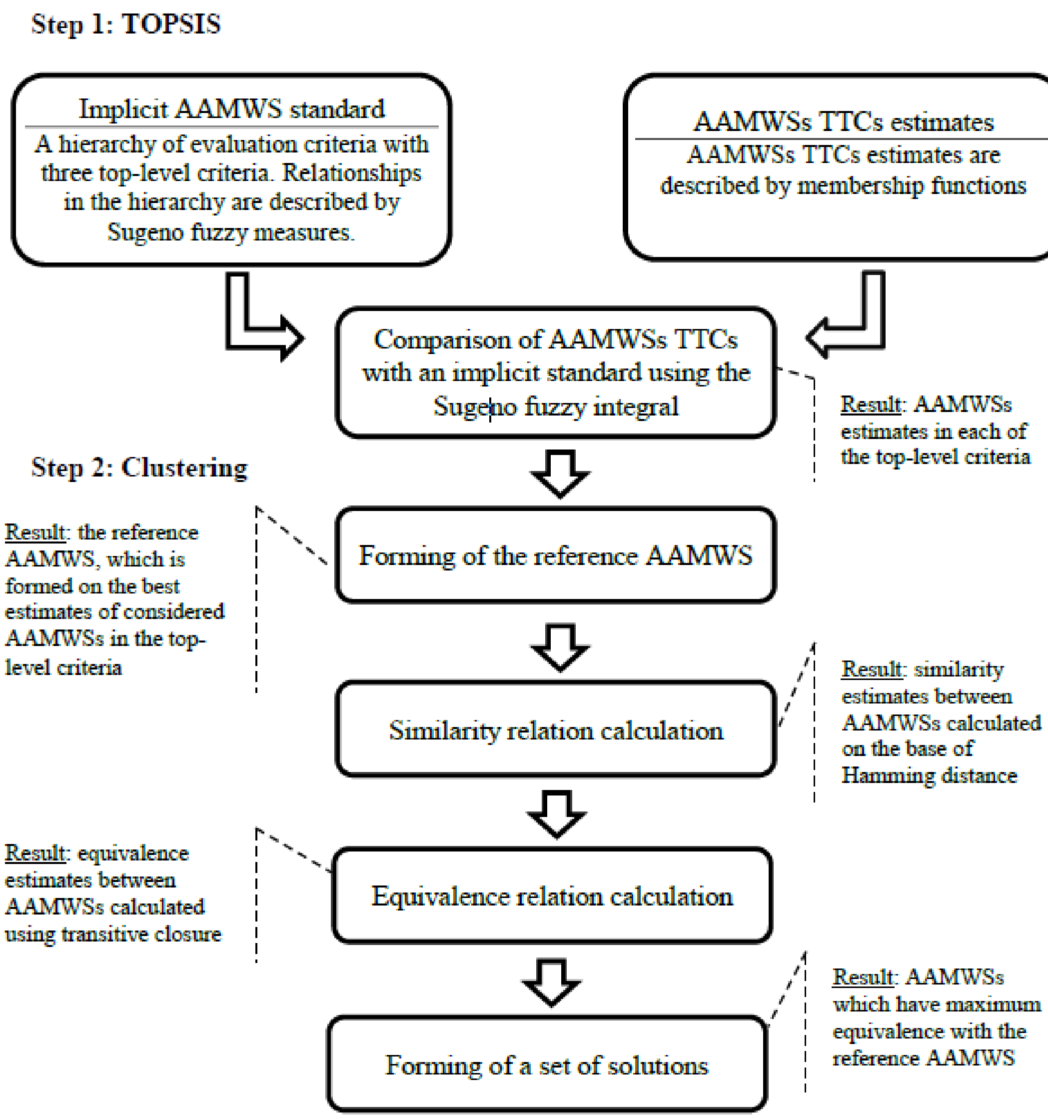

Figure 1. Generalized outline of the proposed approach to solving the research problem.

To calculate the estimates of the considered AAMWSs, we proposed to use the Sugeno fuzzy integral of the fuzzy membership function (which is composed of the estimates in the criteria) over the fuzzy measure of criteria importance. As a result of sequential integration over the entire hierarchy of criteria, the algorithm calculates the estimates of the considered AAMWSs in each of the three top-level criteria. These estimates are input data for the calculations in the second step.

Compared to widely used additive approaches to the generalization of estimates (in particular, AHP), fuzzy-integral calculus does not have the drawback related to mutual compensating of partial estimates. Its use allows you to customize the properties of the generalization procedure. Concerning the basis of logical operations, the fuzzy integral can model the logic of generalization from AND until OR. The use of fuzzy measures makes it possible to model interactions between criteria, that is, it allows the presence of dependent criteria in the hierarchy, while the AHP method requires independent criteria. Otherwise, it is necessary to use a very complex method of taking into account intercriteria interactions. In general, the use of fuzzy-integral calculus increases the adequacy of the presentation of expert estimates and provides more opportunities for their processing.

The second step aims to form a set of solutions by reducing the set of considered AAMWSs. As the review of the literature has shown, the well-known methods form an order relation on a set of alternatives. However, this approach de facto excludes a person from the decision-making process and can encourage errors, especially in the case of a large number of heterogeneous criteria and high uncertainty.

Therefore, we proposed to provide the decision-maker with a class of best AAMWSs that are equivalent to each other. The equivalence relation can be obtained using clustering methods. The study by Madhulatha [43] provides an overview of the most common types of clustering and clustering methods. We used hierarchical clustering because it is important for us that the number of classes is not given a priori. Specifying the number of classes as a precondition for clustering limits the choice of the decision-maker.

Hierarchical clustering allows you to form classes of alternatives equivalent to each other to some degree. However, in our problem, we must form a set of AAMWSs not only equivalent to each other but also best. We proposed to add the reference AAMWS to the set of AAMWSs under consideration. The estimates of the reference AAMWS were established as the best estimates from the estimates of the considered AAMWSs. In this case, the solutions set will be the class with the maximum equivalence level including the reference AAMWS.

Taking into account the above scheme, the problem of our research is to develop an approach to evaluating AAMWSs under uncertainty based on fuzzy-integral calculus, as well as an approach to the formation of a set of solutions based on hierarchical clustering. To solve the problem, it is necessary to solve the following sub-problems.

1. Build a hierarchy of AAMWSs evaluation criteria that relate TTCs of AAMWSs to three generalized criteria: combat use, logistics, maintenance and repair.
2. Develop an algorithm for estimating AAMWSs in each of these criteria based on fuzzy-integral calculus.
3. Develop an algorithm for choosing the best AAMWSs based on hierarchical clustering.
4. Demonstrate the working capacity and discuss the use of the developed algorithms on the example of evaluating modern medium-range AAMWSs manufactured by NATO countries.

4. Hierarchy of AAMWSs evaluation criteria

The AAMWSs evaluation criteria hierarchy is a set of criteria that are related to each other by subordination. Each criterion is assigned a value from the interval $[0, 1]$, which means the importance of this criterion.

Hierarchy of criteria. The hierarchy of AAMWSs evaluation criteria must satisfy the following requirements, which follow from the theory of systems and the above features of our problem.

1. The main focus of the evaluation is the exploitation of the AAMWSs. We evaluate AAMWSs from the viewpoint of exploitation efficiency within a three-stage cycle: combat use, logistics, maintenance and repair. These stages correspond to top-level criteria. Note that the stages of exploitation do not intersect with each other, sequentially alternating in time.
2. The priority of criteria in the hierarchy should be determined in the context of typical combat tasks for the solution of which we choose AAMWS. We assume that long-term strategic planning of

military operations should precede the choice of AAMWS. Planning should determine the theater of war, potential adversaries, scenarios of military operations, and possible groupings of troops. Based on this data, we will be able to make assumptions about the possible composition of air targets, their priority for destruction, as well as about typical AAMWS combat missions.

3. The criteria hierarchy should reflect the relationship between the AAMWSs TTCs and the top-level criteria. Criteria should not be artificially grouped. Criteria with approximately the same level of generalization should be located at the same level of the hierarchy.
4. To reduce the influence of uncertainty, the criteria should be as independent as possible, that is, the intersection of their domains of definition (subsets of TTCs) should be minimal.

Following the above requirements, we proposed a hierarchy of AAMWSs evaluation criteria, which is shown in Table 1.

Table 1. Hierarchy of AAMWSs evaluation criteria and criteria designations

(z_1) Combat use			
(y_1) Target type	$((y_2)$ Shooting conditions	(y_3) Readiness to shoot	$((y_4)$ Maneuverability
(x_1) non-maneuvering ballistic missiles	(x_{10}) all-weather	(x_{13}) deployment time	(x_{17}) multi-fuelness
(x_2) maneuvering ballistic missiles	(x_{11}) noise immunity	(x_{14}) technical readiness	(x_{18}) cruising range
(x_3) hypersonic ballistic missiles	(x_{12}) blind area	(x_{15}) size of ammunition load	(x_{19}) movement on off-road
(x_4) cruise missiles		(x_{16}) vulnerability to damages	(x_{20}) March-speed
(x_5) 4th generation aircraft			(x_21) transfer-time to the stowed position
(x_6) 5th generation aircraft			
(x_7) Helicopters			
(x_8) Tactical drones			
(x_9) operational and strategic drones			

(z_2) Logistics		
(y_5) Missiles and ammunition	(y_6) Fuel	(y_7) Repair kit
(x_{22}) ammunition load on launchers	(x_{26}) multi-fuelness	(x_{29}) compliance with the reliability level
(x_{23}) ammunition load on transport vehicle	(x_{27}) refuelers	(x_{30}) national production
(x_{24}) delivery on off-road	(x_{28}) delivery on off-road	
(x_{25}) national production		

(z_3) Maintenance and repair	
(y_8) Missiles and ammunition	(y_9) Fuel
(x_{31}) need for evacuation	(x_{36}) need for evacuation
(x_{32}) maintenance unit	(x_{37}) repair unit
(x_{33}) frequency	(x_{38}) manufacturability
(x_{34}) manufacturability	(x_{39}) mechanisms and materials
(x_{35}) mechanisms and materials	

The criteria hierarchy has three independent branches which correspond to the three top-level criteria. These criteria are denoted by symbols z_1, z_2, z_3. Each of these criteria is defined through subsets of the second-level criteria $\{y_1, \ldots, y_9\}$. In turn, the second-level criteria are defined through subsets of the third-level criteria $\{x_1, \ldots, x_{39}\}$.

Let us briefly consider the AAMWSs evaluation criteria. The combat use criterion characterizes the fulfillment of the main goal function of the AAMWS as a purposeful system — the destruction of air targets. Execution of this function depends on target type, shooting conditions, readiness to shoot, and maneuverability of the AAMWS.

The target type criterion characterizes the ability of AAMWS to destroy modern flying machines with a given probability at a given range and height. The nomenclature of air targets includes flying machines which can be used by a potential adversary. We emphasize that for different theaters of war and scenarios of military operations, the nomenclature may be the same, but the priority of targets should be determined based on typical combat missions.

The shoot conditions criterion characterizes the ability of AAMWS to maintain effectiveness in difficult combat conditions. In particular, AAMWS must be able to be used in all weather, day and night, and conditions of electromagnetic noise. The size of the blind area reflects the restrictions on the minimum shooting range and the maximum elevation angle.

The readiness to shoot criterion characterizes the ability of AAMWS to perform its main function at any time. This ability depends on the deployment time after arriving at the position, on the level of technical readiness, on the size of the ammunition load, and on the ability to shoot in conditions of partial damage from enemy fire.

The maneuverability criterion characterizes the ability of AAMWS to arrive at a combat position to complete a combat mission, as well as to depart from a combat position to avoid an enemy strike. Maneuverability depends on the possibility of using different types of fuel in the engines of combat vehicles (multi-fuelness), on the cruising range, the ability to move off-road, the march speed, and the transfer time to the stowed position.

The logistics criterion characterizes the ability of the logistics subsystem to provide the AAMWS with everything necessary for shooting, in particular missiles, ammunition, fuel, and a repair kit.

The missiles and ammunition criterion characterizes the ability of the logistics subsystem to provide AAMWS with missiles and ammunition. The following criteria characterize this ability. The larger the size of the ammunition load, which is located on the launchers, the less logistical procedures will be required to replenish the ammunition load. A similar logic is used to the ammunition load that transport and transport-loading vehicles can carry with them. The ability to move transport and transport-loading vehicles off-road improves the evaluation of logistics. The ability to somehow organize the production of missiles and ammunition on the national territory also improves the evaluation of logistics.

The fuel criterion characterizes the ability to provide AAMWS with fuel. The use of different types of fuel in different vehicles complicates the logistics. The presence of refuelers makes it possible to replenish fuel reserves on its own, without involving the transport of logistics units. This increases the independence of shooting from combat conditions. The ability to move refuelers off-road also improves logistics evaluation.

The criterion repair kit characterizes the ability to provide materials for the AAMWS repair. The composition and size of the repair kit should correspond to the level of reliability of AAMWS, the character of equipment failures, and the character of possible damage from enemy fire. The greater the amount of repair provided by the repair kit, the less time required for logistical procedures. The ability to somehow organize the production of the repair kit in the national territory also improves the evaluation of logistics.

The maintenance and repair criterion characterizes the ability of the maintenance and repair subsystem to maintain and restore the combat capability of the AAMWS.

The maintenance criterion characterizes the simplicity of maintenance of combat vehicles from the AAMWS. The need to evacuate combat vehicles from combat positions complicates maintenance. Requiring maintenance by special units worsens the evaluation of the criterion and vice versa, the evaluation of the criterion will be improved if the maintenance can be performed by the crew. The high frequency of maintenance of the key elements of AAMWS and the low manufacturability of the procedures worsen the evaluation of the criterion. The need to use complex, expensive, rare mechanisms and/or materials in general contributes to the complexity of maintenance.

The repair criterion characterizes the ability of the repair subsystem to provide minor, medium, and major repairs of AAMWS combat vehicles. The need to evacuate combat vehicles from combat positions in the case of minor and medium repairs increases the downtime of AAMWS. In the case of major repair, it is necessary to take into account the need for evacuation abroad. The requirement to perform repairs by special repair units worsens the criteria estimates. Low manufacturability (the prerequisites of which are low standardization, low unification, and non-modular construction) also worsens the criterion estimation. The need to use complex, expensive, rare mechanisms and/or materials, in general, complicates repairs and contributes to an increase in repair time and, accordingly, AAMWS downtime.

Importance of criteria. Criteria importance values depend on the chosen mathematical tool used to evaluate the alternatives. We propose to use fuzzy measures introduced by Sugeno [58]. The Sugeno fuzzy measure $g(\cdot)$ is a non-additive function of the set $g(\cdot)$: $2^X \to [0,\ 1]$, where 2^X is the set of all subsets of the universal set X. The Sugeno fuzzy measure satisfies the following rule. Let $A,\ B \subseteq X,\ A \cap B = \emptyset$. Then $g(A \cup B) = g(A) + g(B) + \lambda \cdot g(A) \cdot g(B)$, where λ is the normalization parameter of the Sugeno fuzzy measure $g(\cdot)$.

The advantages of the Sugeno fuzzy measure are considered in detail by Sveshnikov et al. [59]. The use of fuzzy-integral calculus makes it possible to use the whole range of different modalities to describe the importance of criteria. Since the fuzzy measure is a non-additive set function, it allows taking into account the interactions between the criteria, which is important for ensuring the adequacy of evaluation. However, the disadvantage of a fuzzy measure is the complexity of its determination. In our case, this shortcoming is not critical, since the importance of the criteria must be formed once before evaluation.

Fuzzy measures of the importance of criteria from Table 1, as well as the sets of their definition, are presented in Table 2. This table consists of 12 groups of rows (3 top-level criteria and 9 second-level criteria). Each row group contains three rows: a criterion designation, criteria from its definition set, and the corresponding importance values, which are represented as fuzzy measure densities.

For the convenience of the subsequent writing of proposed algorithms, the designations of the criteria in this table are defined in a special way. Three top-level criteria z_k are combined into set Z. Nine intermediate criteria y_j are combined into set Y, which in turn is divided into three subsets Y_k according to the number of top-level criteria. The lower level criteria x_i are combined into a set X, which is also divided into nine subsets X_j according to the number of intermediate criteria.

Table 2. Fuzzy measures of the importance of criteria and their domains of definition

Criterion	z_1								
Subordinate criteria Y_1	y_1	y_2	y_3	y_4					
Fuzzy measure $g_{Y_1}(\cdot)$	0.156	0.099	0.095	0.095					
Criterion	y_1								
Subordinate criteria X_1	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
Fuzzy measure $g_{X_1}(\cdot)$	0.057	0.519	0.39	0.462	0.238	0.338	0.057	0.214	0.281
Criterion	y_2								
Subordinate criteria X_2	x_{10}	x_{11}	x_{12}						
Fuzzy measure $g_{X_2}(\cdot)$	0.209	0.237	0.1						
Criterion	y_3								
Subordinate criteria X_3	x_{13}	x_{14}	x_{15}	x_{16}					
Fuzzy measure $g_{X_{13}}(\cdot)$	0.307	0.255	0.08	0.175					
Criterion	y_4								
Subordinate criteria X_4	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}				
Fuzzy measure $g_{X_4}(\cdot)$	0.131	0.183	0.286	0.08	0.141				
Criterion		z_2							
Subordinate criteria Y_2	y_5	y_6	y_7						
Fuzzy measure $g_{Y_2}(\cdot)$	0.235	0.099	0.192						
Criterion	y_5								
Subordinate criteria X_5	x_{22}	x_{23}	x_{24}	x_{25}					
Fuzzy measure $g_{X_5}(\cdot)$	0.269	0.147	0.203	0.123					
Criterion	y_6								
Subordinate criteria X_6	x_{26}	x_{27}	x_{28}						
Fuzzy measure $g_{X_6}(\cdot)$	0.161	0.232	0.284						
Criterion	y_7								
Subordinate criteria X_7	x_{29}	x_{30}							
Fuzzy measure $g_{X_7}(\cdot)$	0.293	0.397							
Criterion	z_3								
Subordinate criteria Y_3	y_8	y_9							
Fuzzy measure $g_{Y_3}(\cdot)$	0.405	0.405							
Criterion	y_8								
Subordinate criteria X_8	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}				
Fuzzy measure $g_{X_8}(\cdot)$	0.187	0.154	0.089	0.042	0.117				
Criterion	y_9								
Subordinate criteria X_9	x_{36}	x_{37}	x_{38}	x_{39}					
Fuzzy measure $g_{X_9}(\cdot)$	0.269	0.155	0.052	0.188					

$$Z = \{z_k, k = \overline{1,\ 3}\}$$

$$Y = \{y_j, j = \overline{1, 9}\} = \bigcup_{k=\overline{1,3}} Y_k$$

$$X = \{x_j, j = \overline{1, 39}\} = \bigcup_{j=\overline{1,9}} X_j$$

where k is an index for listing top-level criteria, j is an index for listing intermediate criteria, i is an index for listing lower level criteria:

$Y_1 = \{y_1, y_2, y_3, y_4\}, Y_2 = \{y_5, y_6, y_7\}, Y_3 = \{y_8, y_9\}$,

$X_1 = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}, X_2 = \{x_{10}, x_{11}, x_{12}\}, X_3 = \{x_{13}, x_{14}, x_{15}, x_{16}\}$,

$X_4 = \{x_{17}, x_{18}, x_{19}, x_{20}, x_{21}\}, X_5 = \{x_{22}, x_{23}, x_{24}, x_{25}\}, X_6 = \{x_{26}, x_{27}, x_{28}\},$
$X_7 = \{x_{29}, x_{30}\}, X_8 = \{x_{31}, x_{32}, x_{33}, x_{34}, x_{35}\};$
$X_9 = \{x_{36}, x_{37}, x_{38}, x_{39}\}.$

The importance of the criteria was obtained by the method of successive approximation described by Pospelov [52]. The method refers to psychological measurements [3]. It involves the formation of initial importance using paired comparisons and subsequent iterative approximations to a given value of λ parameter. The priority of the criteria takes into account the experience of modern military conflicts in Eastern Europe, in which tactical and operational drones, cruise missiles, helicopters began to play a special role, as well as the ability to operate in off-road conditions and with poor logistics. In general, the criteria hierarchy can be viewed as an implicit standard of the best AAMWS.

5. Algorithm for estimating AAMWSs

The AAMWSs estimating algorithm must provide a generalizing of the TTCs estimates according to the criteria hierarchy described above. The estimating scheme consists of calculating a generalized estimation in a criterion based on the estimates in the subordinate criteria, taking into account their importance. The algorithm starts by estimating the AAMWSs TTCs and stops when the estimates in each of the three top-level criteria have been calculated. Since an evaluation of AAMWSs is one of the most important procedures in defense planning, the analysis and estimation of TTCs must be carried out by several experts. Therefore, the algorithm must provide the processing of group estimates.

Based on our proposal regarding the use of fuzzy-integral calculus, the estimates of AAMWSs in the hierarchy criteria are presented as membership functions. To generalize the estimates, we used the Sugeno fuzzy integral (s) of the fuzzy membership function $h(b_i) : B \to [0,\, 1]$ defined on the discrete set $B = \{b_i, i = \overline{1, N_B}\}$, $N_B = card\,(B)$, along the fuzzy measure $g_B(\cdot)$ of criteria importance:

$$(s)\int\limits_R h(b) \circ g_B(\cdot) = \max_{i=\overline{1,N_B}}(\min(h(b_i), g_B(H_m))),$$

where $H_m = \{b_n | h(b_n) \geq h(b_m),\ j = \overline{1, N_B}\}$.

Here, first of all, raises the question of determining the initial estimates of AAMWSs TTCs in the lower-level criteria, which can be obtained by different methods. A review of these methods is presented in the known literature, for example, in works of Chameau and Santamarina [9], Dombi [23], and others. Sveshnikov et al. [59] use a method based on the desirability function of Harrington [33], which showed good results. In particular, the use of the Harrington function makes it possible to increase the accuracy of reflecting the opinions of experts in numerical estimates and, due to this, also increase the accuracy of measuring the input estimates. Therefore, here we will omit from consideration the procedure for determining initial TTCs estimates. Let us describe further the algorithm for their processing, which consists of performing the following steps.

Step 1. Aggregation of expert estimates in the lower level criteria. Denote by $F = \{f_s, s = \overline{1, N_F}\}$ the set of experts who evaluate TTCs, where N_F is the number of experts. We also denote the AAMWSs set as $A = \{a_l, l = \overline{1, N_A}\}$, where N_A is the number of AAMWSs. Then the expert estimates in each lower level criterion can be represented using the membership function of the fuzzy relation

$\mu(x_i, a_l, f_s) X \times A \times F \rightarrow [0, 1]$. The fuzzy measure $g_F(\cdot) : 2^F \rightarrow [0, 1]$ describes the competence of experts. The Sugeno fuzzy integral of the membership function $\mu(x_i, a_l, f_s)$ along the fuzzy measure $g_F(\cdot)$ calculates the AAMWSs group estimates in the lower level criteria $x_i \in X$ for all objects $a_l \in A$:

$$\varepsilon(x_i, a_l) = (s) \int_F \varepsilon(x_i, a_l, f_s) \circ g_F(\cdot) \forall x_i \in X, \ \forall a_l \in A$$

Step 2. Aggregation of AAMWS estimates in intermediate criteria. The membership function $\varepsilon(x_i, a_l) : X \rightarrow [0, 1]$ describes the AAMWS estimates in the lower level criteria. The estimation of each AAMWS in the intermediate criterion $y_j \in Y$ is based on the estimates from the set of lower-level criteria $X_j \in X$. Therefore, to calculate the estimate $a_l \in A$, we have determined fuzzy measures $g_X^j(\cdot) : 2^X \rightarrow [0, 1]$, which are concentrated on subsets X_j (Table 2). Then the Sugeno fuzzy integral of the membership function $\varepsilon(x_i, a_l)$ along the fuzzy measure $g_X^j(\cdot)$ calculates the AAMWS estimates in the intermediate criteria:

$$v(y_j, a_l) = (s) \int_{X_j \in X} \varepsilon(x_i, a_l) \circ g_X^j(\cdot) \forall y_j \in Y, \ \forall a_l \in A$$

Step 3. Aggregation of the AAMWS estimates in the top-level criteria. The membership function $v(y_j, a_l)$ describes the AAMWS estimates in the intermediate criteria. The estimation of each AAMWS in the top-level criterion $z_k \in Z$ is based on estimates from the set of intermediate criteria $Y_k \in Y$. Therefore, to calculate the estimate $a_l \in A$, we have determined fuzzy measures $g_Y^k(\cdot) : 2^Y \rightarrow [0, 1]$, which are concentrated on subsets of Y_k (Table 2). Then the Sugeno fuzzy integral of the membership function $v(y_j, a_l)$ along the fuzzy measure $g_Y^k(\cdot)$ calculates the AAMWS estimates in the top-level criteria:

$$\sigma(z_k, a_l) = (s) \int_{Y_k \in Y} v(y_j, a_l) \circ g_Y^k(\cdot) \forall z_k \in Z, \ \forall a_l \in A.$$

The result is the estimates of all AAMWSs in the top-level criteria. The fuzzy relation $\sigma(z_k, a_l) : Z \times A \rightarrow [0, 1]$ describes these estimates, which are input data for the best AAMWSs choice algorithm.

6. Algorithm for choosing the best AAMWSs

Recall that the choosing algorithm should ensure the grouping of AAMWSs based on estimates in three criteria: combat use, logistics, maintenance and repair. We added a reference AAMWS (with index $l = 1$) to the set of AAMWSs to be able to observe which AAMWSs form a common cluster with it. Since we proposed to use hierarchical clustering, the number of classes cannot be known in advance. The general clustering scheme consists of obtaining a similarity relation between alternatives based on the Hamming distance and transforming the similarity relation into an equivalence relation using transitive closure (see, e.g., [30]). The need for transitive closure is because the similarity relation does not have transitivity. We add that the equivalence relation can also be constructed in other ways (see, e.g. [60]).

An equivalence relation defines a nested system of partitions on a set of objects. The number of AAMWSs in each class will depend on the chosen equivalence level. The result of the algorithm is the AAMWSs class, which has the maximum equivalence level and which contains at least one AAMWS along with the reference AAMWS.

We propose to form the estimates of the reference AAMWS in the top-level criteria as a convex hull of the set of estimates of the considered AAMWSs:

$$\sigma(z_1, a_1) = \max_{l=\overline{2,N_A}} \sigma(z_1, a_l), \quad \sigma(z_2, a_1) = \max_{l=\overline{2,N_A}} \sigma(z_2, a_l), \quad \sigma(z_3, a_1) = \max_{l=\overline{2,N_A}} \sigma(z_3, a_l)$$

Note that the power of the AAMWSs set, taking into account the reference AAMWS, will be increased by one: $N_A^* = N_A + 1$. The choosing algorithm consists of the following steps.

Step 1. Calculation of the fuzzy similarity relation. For each AAMWS, the previous algorithm calculated estimates $\sigma(z_k, a_l), z = \overline{1, 3}, l = \overline{2, N_A^*}$. These estimates reflect the level of similarity of AAMWSs with an implicit standard, which is described using a hierarchy of criteria. The level of similarity between AAMWSs a_m and a_n can be calculated as follows:

$$s(a_m, a_n) = 1 - \vartheta(a_m, a_n), \ \forall m, n \in \overline{1, N_A^*},$$

where $\vartheta(a_m, a_n) = \dfrac{1}{3} \sum_{z_i \in Z} |\sigma(z_k, a_m) - \sigma(z_k, a_n)|$ is the normalized Hamming distance.

All similarity estimates between AAMWSs form a similarity relation $S(a_m, a_n)$. Note that the Hamming distance is widely used to obtain the similarity relation (see, e.g., [64].

Step 2. Calculation of the fuzzy equivalence relation. As stated above, the transitive closure of the similarity relation gives us an equivalence relation:

$$R(a_m, a_n) = \bigcup_{r=1}^{r_{max}} S^r(a_m, a_n)$$

where $S^r(a_m, a_n) = S^{r-1}(a_m, a_n) \circ S(a_m, a_n)$ is the maximin composition [52]), r is the index of iterations, r_{max} is the iteration index value at which $S^r(a_m, a_n) = S^{r-1}(a_m, a_n)$.

Step 3. Partitioning the equivalence relation into classes and forming a set of solutions. To form classes, it is sufficient to consider the α levels of the membership function of the fuzzy relation $R(a_m, a_n)$. The α level representation of a fuzzy equivalence relation can be written as follows:

$$R(a_m, a_n) = \left\{ \bigcup_{\alpha \in [0,\,1]} \alpha \cdot R_\alpha(a_m, a_n) | R_\alpha(a_m, a_n) : A \times A \to [0,\,1] \right\}$$

The α level of the fuzzy relation R is the usual relation R_α which is defined as follows

$$R_\alpha = \{(a_m, a_n) \in A \times A\} | R(a_m, a_n) \geq \alpha$$

If the usual relation R_α is identified with its characteristic function, then we can write

$$R_\alpha(a_m, a_n) = \begin{cases} 1, R(a_m, a_n) > 0, \\ 0, R(a_m, a_n) \leq 0. \end{cases}$$

If α = const, the set A can be partitioned into N_c subsets (classes) that do not intersect

$$R_\alpha(a_m, a_n) = \left\{ \bigcup_{c=\overline{1,N_c}} R_\alpha^c(a_m, a_n) | \forall R_\alpha^c \cap R_\alpha^r = \emptyset, r \neq c \right\}$$

So, after choosing α-level, we obtained a set of partitions on the set A. The sense of the α level in our problem is as follows. The higher the α level, the stricter the requirements for the equivalence of objects in a common class. If $\alpha = 1$, then $N_c = N_A^*$ and each class includes only one alternative or several alternatives that are completely equivalent to each other. If $\alpha = 0$, then $N_c = 1$, and we have a single class that includes all alternatives. Both of these cases are trivial solutions.

The result of the algorithm for choosing the best AAMWSs will be a class $A^d \in A$, which will contain the reference AAMWS a_1. Since the trivial solution is $A^d = A$, it is necessary to limit the size of A^d. There are two potential ways:

1. By determining the level $\alpha_{est} \neq 0$ for which $a_1 \in A^d : A^d = a_1 \cup A', A' = \{a_m | R_{\alpha_{est}}(a_1, a_m) = 1\} \neq \emptyset$;
2. By determining the number of elements N_{A^d} of the subset $A^d : A^d = a_1 \cup A', A' = \{a_m | R_\alpha(a_1, a_m) = 1\}, card(A') = N_{A^d}$.

In other words, for choosing a subset from the entire set of AAMWSs for decision-making, we must establish either an equivalence level or a maximum class size of equivalent AAMWSs.

7. Discussion

Since the proposed algorithms are intended to form recommendations regarding the decision about the choice of the best AAMWS, the question of the quality of these recommendations requires discussion. Here we will not take into account the reliability of the initial data that are used in the algorithms, because ensuring their reliability is not the goal of our research. Then, in general, it can be assumed that the quality of recommendations depends on two questions. Firstly, it is the adequacy of the mathematical constructions used for data transformations. And secondly, this is the composition of the criteria used and the dependencies between them. The adequacy of the used mathematical constructions (fuzzy-integral calculus) was analyzed in the other research (e.g. [59]). Therefore, here we consider the resistance of recommendations to the composition of criteria. In particular, we want to discuss the following questions:

1. Calculation of estimates of modern AAMWSs with a range of 40-100 km and grouping them into classes in the space of the three top-level criteria.
2. Study of the stability and sensitivity of estimates and clustering results from the composition of evaluation criteria (see Section 4).
3. Study of the dependence of clustering results on the composition of the criteria that used as the basis of clustering.

Question 1. Calculation of estimates of modern AAMWSs with a range of 40-100 km and grouping them into classes in the space of the three top-level criteria. For evaluation, we considered the NATO AAMWSs which are present in the arms market. Due to imprecision, ambiguity, and lack of data, we determined the initial estimates of AAMWSs TTCs using experts. A description of the main AAMWSs TTCs is shown in Table 3.

It is not possible to disclose AAMWS estimates in all criteria due to their large number. Therefore, in Table 4, we have presented AAMWS estimates (including estimates of the reference AAMWS) only in the top-level level criteria.

Table 3. Description of the main AAMWSs TTCs

AAMWSs	Description
Patriot PAC-2 (USA)	AAMWS is designed to destroy aerodynamic and ballistic targets (tactical missiles). Aerodynamic target: probability of defeat is 0.8-0.9; maximum shooting range is 100 km; the maximum target height is 25 km. Ballistic target: probability of defeat – 0.3-0.4; maximum shooting range – 25 km; the maximum target height is 11 km. The number of simultaneously shooting targets is 8.
Patriot PAC-3 (USA)	AAMWS is designed to destroy aerodynamic and ballistic targets (tactical missiles). Aerodynamic target: probability of defeat is 0.8-0.9; maximum shooting range is 100 km; the maximum target height is 25 km. Ballistic target: probability of defeat is 0.6-0.8; maximum shooting range is 40 km; the maximum target height is 20 km. The number of simultaneously shooting targets is 8.
FSAF SAMP/T (Italy, France)	AAMWS is designed for air defense of troops on the march, as well as for covering stationary objects from a massive air attack (tactical missiles, all types of aircraft, drones) in all weather conditions and conditions of high-intensity interference. Aerodynamic target: probability of defeat is 0.8-0.9; maximum shooting range is 100 km; the maximum target height is 25 km. Ballistic target: probability of defeat is 0.5-0.6; maximum shooting range is 35 km; the maximum target height is 25 km. The number of simultaneously shooting targets is 10. The missile is capable of providing a direct hit on a ballistic target.
Arrow-2 (Israel, USA)	AAMWS is designed to intercept ballistic missiles with a range of up to 3000-5000 km in interference conditions. Aerodynamic target: no data. Ballistic target: probability of defeat - no data; maximum shooting range is 100 km; the maximum target height is 50 km. The number of simultaneously shooting targets is 2. The rocket engine can control the thrust vector.
Arrow-3 (Israel, USA)	AAMWS differs from Arrow-2 in hitting a target with a kinetic strike.
Iron Dome (Israel, USA)	AAMWS is designed to destroy unguided missiles, artillery shells, mortar mines, helicopters and drones. Aerodynamic target: probability of defeat - no data; maximum shooting range is 70 km; the maximum target height is 10 km. Ballistic target: probability of defeat is 0.85; maximum shooting range is 17 km; the maximum target height is 10 km. The number of simultaneously shooting targets - no data.
MEADS (USA, Germany, Italy)	The promising AAMWS is designed to destroy operational-tactical ballistic missiles with a range of up to 1000 km, cruise missiles, aircraft and drones. Aerodynamic target: probability of defeat - no data; maximum shooting range is 100 km; the maximum target height is 25 km. Ballistic target: probability of defeat - no data; maximum shooting range is 35 km; the maximum target height is 25 km. The number of simultaneously shooting targets is 10.
IRIS-T SLM (Germany, Sweden, Norway, Italy, Spain, Greece)	AAMWS is designed to destroy aerodynamic targets and air-to-ground missiles. The maximum range of destruction is 40 km. The maximum target height is 20 km. The number of simultaneously shooting targets is 24. The rocket engine can control the thrust vector.
NASAMS-3 (Norway, USA)	AAMWS is designed to destroy maneuvering aerodynamic targets, as well as drones and ballistic missiles at low and medium altitudes. The probability of defeat is 0.85; the shooting range is 20-180 km (depending on the rocket type); the maximum target height is 21 km. AAMWS is compatible with "Patriot" and can use 7 types of missiles.
i-Hawk (USA)	AAMWS is designed to destroy aerodynamic targets, as well as drones and tactical ballistic missiles at low and medium altitudes. Aerodynamic target: probability of defeat is 0.85; maximum shooting range is 40 km; maximum target height is 17.7 km. Ballistic target: no data. The number of simultaneously shooting targets is 1.

Recall that we interpret the estimations of an alternative in the top-level criterion as the degree of compliance of this alternative with an implicit standard, which is described by a hierarchy of subordinate criteria. In addition, we emphasize that the estimates should be considered only as an approximation to reality since the data about TTCs should be refined when analyzing commercial offers.

Table 4. AAMWS estimates in the top-level criteria

AAMWSs	Top-level criteria		
	Combat use	Logistics	Maintenance and repair
Reference	0.602	0.388	0.531
Patriot PAC-2	0.392	0.293	0.405
Patriot PAC-3	0.392	0.293	0.405
FSAF SAMP/T	0.602	0.293	0.434
Arrow-2	0.385	0.249	0.424
Arrow-3	0.385	0.249	0.424
Iron Dome	0.35	0.305	0.418
MEADS	0.602	0.235	0.474
IRIS-T SLM	0.602	0.302	0.531
NASAMS-3	0.602	0.293	0.405
i-Hawk	0.392	0.388	0.431

The overall level of AAMWS estimates in the combat use criterion is not high — the maximum estimate does not exceed 0.602. Reference AAMWS is estimated similarly. This can be explained by the fact that we have established the importance of the criteria in the hierarchy following the requirements of the recent conflicts in Eastern Europe. However, the considered AAMWSs do not take these requirements into account well, as the nature of conflicts has changed. Estimates in the criteria "Maintenance and repair" and "Logistics" vary in a small range and are generally low. This is related to the fact that we avoided giving high estimates for TTCs because the data was incomplete. We also had no reason to make large differences in estimates.

Table 5. Criteria subordinated to the criterion Combat uses

AAMWSs	Level of subordination	
	First	Second
Patriot PAC-2	shooting conditions	noise immunity
	maneuverability	movement on off-road, transfer-time to the stowed position, multi-fuelness
Patriot PAC-3	shooting conditions	noise immunity
	maneuverability	movement on off-road, transfer-time to the stowed position, multi-fuelness
FSAF SAMP/T	maneuverability	movement on off-road, transfer-time to the stowed position, multi-fuelness
Arrow-2	maneuverability	movement on off-road, transfer-time to the stowed position, Multi-fuelness
Arrow-3	maneuverability	movement on off-road, transfer-time to the stowed position, multi-fuelness
Iron Dome	readiness to shoot	deployment time, vulnerability to damages
	maneuverability	transfer-time to the stowed position, multi-fuelness
MEADS	maneuverability	cruising range, multi-fuelness
IRIS-T SLM	maneuverability	cruising range, multi-fuelness
NASAMS-3	maneuverability	movement on off-road, multi-fuelness
i-Hawk	shooting conditions	noise immunity
	maneuverability	movement on off-road, multi-fuelness, march-speed, transfer-time to the stowed position

Table 5 can be used to improve the explanation of the estimates. Given the importance of the combat use criterion, we have shown subsets of the subordinate criteria in which AAMWSs are estimated as non-preferred. That is, this table shows what requirements of combat use the considered AAMWSs meet the worst. A significant part of the low estimates was received in the maneuverability criterion. Here the most common reason is multi-fuel. Unfortunately, modern AAMWSs are not equipped with engines that can use several types of fuel, for example, gasoline, kerosene, and alcohol. This does not allow the functioning of AAMWSs in the conditions of a break in logistics supplies. The second most common reason can be considered the use of a wheeled platform, which does not allow off-road marching in Eastern Europe during the rainy season. In addition, several AAMWSs received low estimates due to being semi-stationary. As the experience of recent conflicts shows, the listed characteristics are very important in conditions when the battlefield is saturated with technical means of intelligence and countering.

The result of the clustering algorithm is partitioning AAMWSs into equivalence classes, shown in Figure 2 in the form of a dendrogram.

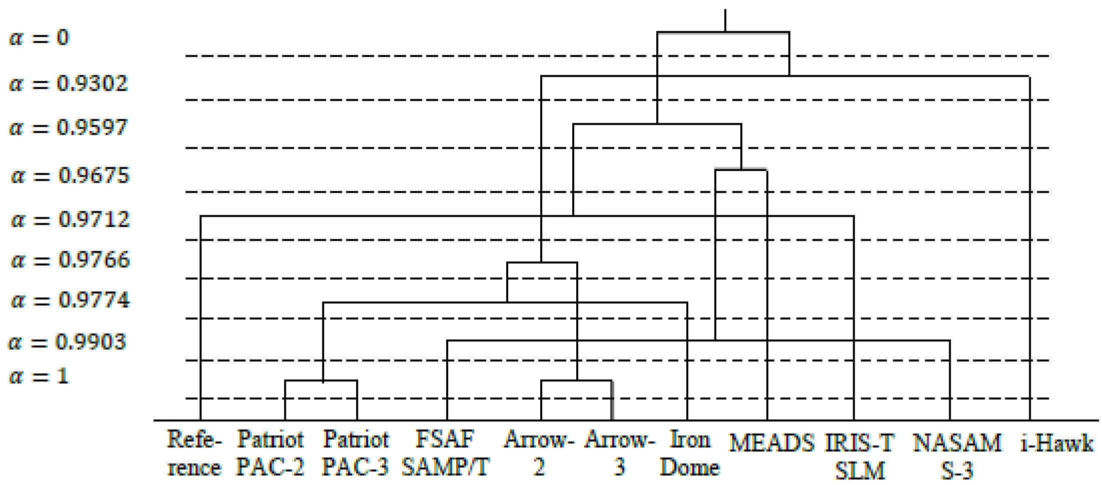

Figure 2. Partitioning of AAMWSs into equivalence classes

The dendrogram shows which alternatives are equivalent at an established level if the levels are arranged in descending order. For example, AAMWS IRIS-T SLM becomes equivalent with the reference AAMWS at the maximum level $\alpha = 0.9712$. Recall that the level of equivalence can be identified with the requirements for distinguishability of alternatives.

The analysis provides three confirmations regarding the consistency of the clustering result with common sense. Firstly, the difference $\alpha_{\max} - \alpha_{\min} = 0.9903 - 0.9302 = 0.0601$ is insignificant, which may indicate that all AAMWSs correspond to approximately the same class, namely, medium-range AAMWSs. Secondly, let's analyze the sequence of class forming, moving from $\alpha_{\max} = 0.9903$ to $\alpha_{\min} = 0.9302$. We can see that the merging occurs in two directions: earlier US-Israeli-made AAMWSs and later European-made AAMWSs. Indeed, the technical requirements for US-Israeli systems differ from those embodied in more modern European AAMWSs. Thirdly, the clustering algorithm combined the AAMWSs Patriot PAC-2 and Patriot PAC-3 and Arrow-2 and Arrow-3 into the corresponding classes at the equivalence level $\alpha = 1$, by accepting them as indistinguishable. This decision is natural since these AAMWSs are modifications that differ only in improvements to defeat ballistic missiles, which is not paramount in modern conflicts.

As a solution, the algorithm determined the set of the best AAMWSs $A^d = \{IRIS - TSLM\}$. If we relax the requirements for equivalence, the following set can be used as a solution: $A^d = IRIS\text{-}T\ SLM; MEADS; FSAFSAMP/T; NASAMS\text{-}3\}$.

Question 2. Study of the stability and sensitivity of estimates and clustering results from the composition of evaluation criteria (see Section 2). Above we noted that in our problem there is a significant uncertainty in the data on some TTCs, since we used only open Internet sources. Therefore it is necessary to study how the estimates of AAMWSs might change if we reduce the importance of the criteria associated with "uncertain" TTCs. Reducing the importance of criteria is equivalent to reducing (compressing) the criteria hierarchy. Firstly, this will make it possible to evaluate how the received recommendations are resistant to uncertainty, and secondly, how the calculated estimates are sensitive to changes in the composition of the criteria.

We reduced the importance of some third-level criteria while maintaining the value of the normalization parameter of fuzzy measures with the help of proportionally changing the importance of other criteria. This will preserve the modality of the estimates and ensure the comparability of the estimates since obtained for different hierarchies: full and reduced. In particular, we reduced the importance of the following criteria to zero:

(x_{11}) noise immunity
(x_{14}) technical readiness
(x_{16}) vulnerability to damages
(x_{12}) cruising range
(x_{19}) movement on off-road
(x_{24}) delivery of missiles and ammunition on off-road
(x_{28}) delivery of fuel on off-road
(x_{7}) repair kit
(x_{33}) frequency of maintenance
(x_{39}) mechanisms and materials for repair
(x_{35}) mechanisms and materials for maintenance

The composition of these criteria was determined based on an analysis of the uncertainty of data about TTCs in open Internet sources. For ease of comparison, new AAMWS estimates are shown in Table 6 together with the estimates obtained earlier (see Table 4).

Table 6. AAMWS estimates in the top-level criteria

AAMWSs	Top-level criteria[1]		
	Combat use	Logistics	Maintenance and repair
Reference	0.602/0.602	0.388/0.402	0.531/0.574
Patriot PAC-2	0.392/0.337	0.293/0.402	0.405/0.405
Patriot PAC-3	0.392/0.337	0.293/0.402	0.405/0.405
FSAF SAMP/T	0.602/0.56	0.293/0.402	0.434/o.434
Arrow-2	0.385/0.337	0.249/0.365	0.424/0.574
Arrow-3	0.385/0.337	0.249/0.365	0.424/0.574
Iron Dome	0.35/0.337	0.305/0.402	0.418/0.405
MEADS	0.602/0.602	0.235/0.365	0.474/0.564
IRIS-T SLM	0.602/0.602	0.302/0.365	0.531/0.531
NASAMS-3	0.602/0.461	0.293/0.339	0.405/0.405
i-Hawk	0.392/0.337	0.388/0.365	0.431/0.408

[1] Considering/without considering the specified criteria.

An analysis of the estimates shows that decreasing the importance of the above criteria can affect the resulting AAMWS estimates in different ways. This influence depends on the place of the criterion in the hierarchy and the importance of other criteria located at the same level. As a result, one can see the preservation of order among the estimations in the most important criterion of combat use. In particular, the estimates for the best five AAMWSs did not change much. The composition of this subset remained unchanged, only NASAMS-3 and FSAF SAMPT/T changed places. However, we see significant changes in the criteria logistics as well as maintenance and repair.

In general, the results testify to the stability of the estimates of changes in the composition of the third-level criteria in the most important top-level criterion combat use. In addition, the stability of the estimates shows that the impact of uncertainty can be reduced by excluding from consideration the criteria that are related to the most uncertain TTCs. On the other hand, the results of the study demonstrate the sensitivity of the estimates in the criteria logistics and maintenance and repair.

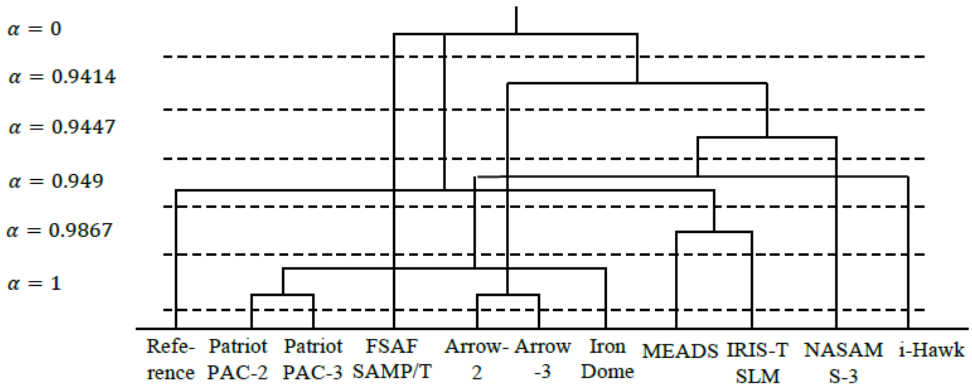

Figure 3. Partitioning of AAMWSs into equivalence classes for the case of reduced criteria importance

Figure 3 shows the partitioning of AAMWSs into equivalence classes for the case of reduced criteria importance. Analysis of the dendrogram confirms the conclusions made earlier. In particular, the AAMWSs IRIS-T SLM and MEADS become equivalent with the reference AAMWS at a maximum level of $\alpha = 0.949$. As a solution, the algorithm gives one set of the best AAMWSs, which intersects with the solution considered in Question 1: $A^d = \{IRIS - TSLM; MEADS\}$. This confirms the stability of solutions to changes in the composition of criteria related to the most uncertain TTCs. This also confirms the stability of the solutions obtained using the proposed combination of algorithms.

Question 3. Study of the dependence of clustering results on the composition of the criteria that used as the basis of clustering.

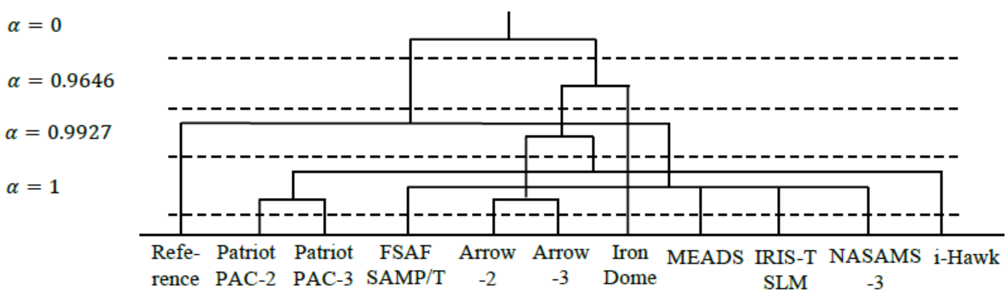

Figure 4. Partitioning of AAMWSs into equivalence classes, if take into account only the combat use criterion

If we consider AAMWSs a purposeful system, then the criterion of combat use is the main characteristic of purposefulness. Therefore, let us consider further how the clustering results change if only this criterion is used in the hierarchical clustering algorithm. To study this question, we excluded the criteria logistics, maintenance and repair from the hierarchical clustering algorithm and performed the calculations. Figure 4 shows the partitioning of AAMWSs into equivalence classes, considering only the combat use criterion. Taking into account only the combat use criterion in the clustering problem somewhat destroys the overall picture of the results, although the set of solutions remains identical: $A^d = \{IRIS\text{-}T\,SLM; MEADS; FSAFSAMP/T; NASAMS\text{-}3\}$.

The main conclusion is the reduction in the distinguishability of the AAMWSs classes, which can be explained by the narrowing of the criteria space and the decrease in the range of estimates. It follows from this that when choosing AAMWS, it is necessary to consider other criteria in addition to the combat use criterion, that is, to evaluate alternatives more fully. The decrease in distinguishability indicates a diminishing of the sensitivity of the proposed hierarchy of criteria and algorithms.

8. Conclusions

To choose the best AAMWS, we proposed to abandon the reduction of a multi-criteria problem to a single-criteria evaluation and proposed the use of three top-level criteria and a combination of two algorithms. The first algorithm was used to estimate AAMWSs under non-statistical uncertainty. The second algorithm is used to form a class of the best equivalent AAMWSs for the final choice by the decision-maker.

We have developed a hierarchy for evaluation according to three top-level criteria of generalization, which reflect the predictive effectiveness of the exploitation of AAMWSs in the conditions of recent modern conflicts, in particular, when the enemy massively uses low-flying unmanned aerial vehicles and cruise missiles when the timely delivery of missiles, ammunition, and fuel off-road is an important condition for the effectiveness of combat use when the enemy actively uses the means of counter-fighting. The proposed hierarchy can be seen as an implicit AAMWS standard that maximizes efficiency for established exploitation conditions.

We also developed an algorithm for estimating AAMWSs under conditions of uncertainty: fuzzy criteria and incompleteness of initial data. In this algorithm, we proposed to use: the Sugeno fuzzy measures to describe the importance of criteria, and fuzzy membership functions to describe AAMWSs estimations in criteria, and the Sugeno fuzzy integral to generalize partial estimations taking into account the importance of the criteria. The algorithm provides the assignment of initial estimates of TTCs AAMWSs by several experts, the calculation of group estimations taking into account the competence of experts, and the generalization of partial estimations of AAMWSs along the hierarchy of criteria.

Oppositely to widely used additive approaches to the generalization of estimates (in particular, AHP), fuzzy-integral calculus does not have the drawback related to compensating of partial estimates. Its use allows you to customize the properties of the generalization procedure. The fuzzy integral can model the entire range of logical operations from AND to OR. The use of fuzzy measures makes it possible to model interactions between criteria, that is, it enables the presence of dependent criteria in the hierarchy. Fuzzy-integral calculus in this case does not require labor-intensive procedures to consider interactions between criteria. The use of fuzzy-integral calculus increases the adequacy of the presentation of expert estimates and provides more opportunities for their processing.

As a result, the algorithm calculates AAMWS estimates in three top-level criteria: combat use, logistics, and maintenance and repair. These estimates reflect the degree of similarity to the AAMWS standard described by the hierarchy of criteria.

To form a class of the best equivalent AAMWSs, we proposed to use the AAMWSs hierarchical clustering algorithm in the space of the three top-level criteria with the addition of a reference AAMWS whose estimates in the top-level criteria are defined as the best estimates of the considered AAMWSs. The algorithm forms a similarity relation between AAMWSs based on the Hamming distance and transforms it into an equivalence relation using transitive closure. The solutions class definition is based on the principle of maximum equivalence with the reference AAMWS. From this class, the decision-maker chooses the best AAMWS considering his experience and factors difficult to formalize. Thus, the proposed algorithms form a set of solutions without specifying the priority of alternatives. These algorithms do not exclude a person from the decision-making process, as happens when many criteria are reduced to one in known methods.

We have demonstrated the performance of the proposed algorithms using the example of evaluating and choosing the highest priority medium-range AAMWSs that are present on Western arms markets. We have confirmed that the obtained solutions do not contradict common sense. In addition, the proposed algorithms ensure the stability of solutions to changes in the composition of criteria associated with the most uncertain TTCs. Also, our study showed that the impact of uncertainty can be reduced by excluding from consideration the criteria that are associated with the most "uncertain" TTCs. On the other hand, the results demonstrated the sensitivity of the estimates and the completeness of the set of top-level criteria.

In general, our research demonstrates an alternative to data preparation methods for decision-making, which are based on the reduction of many criteria to one generalized criterion. Our results may be useful for future research on a choosing problem which have poorly comparable criteria or a high degree of uncertainty in general or in some part of the criteria.

The main question that requires attention in practice is to ensure: the most accurate reflection of the conditions of the war theater in the hierarchy of criteria; and the accurate describing criteria importance with the help of fuzzy measures. Another important question is to ensure complete data about AAMWSs TTCs. Promising directions for further research are the improvement of the hierarchy of evaluation criteria and the procedure for setting fuzzy measures.

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