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How can one improve SAW and max-min multi-criteria rankings based on uncertain decision rules?

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Abstract

The paper aims to improve a simple additive weighting method (SAW) and the max-min rule designed for M-DMC (multi-criteria decision making under certainty) based on already developed extensions for the Laplace and Wald rules (applied to one-criterion decision making under uncertainty, i.e., 1-DMU). Some evident analogies between scenario-based 1-DMU and M-DMC have been recently revealed in the literature, which gives the possibility to implement necessary amendments in M-DMC procedures, particularly in the multiple solutions case. The suggested modifications consist of applying additional decision tools (for SAW) and using the lexicographic approach (for the max-min rule). Thanks to them, options, treated as equivalent according to original M-DMC procedures, may obtain different ranks in the ranking. Such an improvement facilitates the decision making process. Both modified methods are illustrated by employing an example concerning the ranking creation for UE countries.

Keywords: *rankings, multiple solutions case, one-criterion decision making under uncertainty, multi-criteria decision making under certainty, SAW and max-min rule, Laplace and Wald rules*

1. Introduction

Rankings are generated very often for different purposes (Best Countries 2020, Global MBA ranking 2020, Gross Domestic Product 2019), [33, 49]. Some of them are based on one criterion (e.g., Gross Domestic 2019), others take into account more than one feature (e.g., Global MBA ranking 2020). They enable indicating the best person, institution, region, investment project, marketing strategy, and so on. They show the relation (superiority, inferiority, equivalence) between analyzed objects. They also allow us to make our final decisions more consciously.

The ranking creation is one of the issues investigated within multi-criteria optimization or multi-criteria decision making (M-DM). M-DM involves two groups of areas: multiple attribute decision problems (MADP) and multiple objective decision problems (MODP). In MADP, the number of possible

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options (decision variants, courses of action) is precisely defined at the beginning of the decision making process and the levels of considered attributes are assigned to each alternative [46]. For MODP the cardinality of the set of potential decision variants is not exactly known. The decision maker only knows the mathematical optimization model, i.e., the set of objective functions and constraints that create the set of possible solutions [7, 22]. The ranking creation is a domain belonging to MADP where the considered objects are treated as potential options.

Numerous methods have been already developed to establish diverse rankings, e.g., the goal programming [4, 5], TOPSIS [22, 23, 53], AHP [42]. In this paper, we examine in detail two selected existing procedures, i.e., the SAW (simple additive weighting) method and the max-min method. Both approaches are well-known techniques applied to diverse problems [1, 24, 34, 45, 52]. However, it is worth stressing that each of them has some drawbacks.

We intend to modify the original versions of the aforementioned methods. To improve them, we:

- refer to some analogies which occur between two different issues (multi-criteria decision making under certainty and scenario-based one-criterion decision making under uncertainty); these similarities have been recently revealed [16],
- present similarities between procedures already developed for both problems,
- indicate some modifications suggested for classical uncertain techniques,
- attempt to apply the same modifications to SAW and max-min approaches.

The rest of the paper is organized as follows. Section 2 presents in detail the analogies between scenario-based one-criterion decision making under uncertainty (1-DMU) and multi-criteria decision making under certainty (M-DMC). Section 3 reminds the idea of the SAW method (developed for M-DMC) and the Laplace rule (developed for 1-DMU). This part discusses their similarities and defects. It also refers to an improvement proposed for the Laplace rule and recommends applying the same modification to the SAW procedure. Section 4 describes, compares and critically analyzes the max-min method devoted to M-DMC and the Wald rule designed for 1-DMU. It also suggests improving the existing max-min approach by introducing a modification already proposed for the Wald rule. Section 5 uses an example to show how both amended methods may be applied to generate rankings. We illustrate the procedures using the example of EU countries. The features of the suggested approaches are discussed in Section 6. Conclusions are gathered in the last section.

2. Analogies between multi-criteria optimization under certainty and scenario-based one-criterion optimization under uncertainty

The structure of M-DM under certainty (M-DMC) is extremely similar to the structure of scenario-based 1-DMU, i.e., one-criterion decision making under uncertainty based on scenario planning (SP). The first area is related to cases where the decision maker assesses particular alternatives in terms of many criteria (at least two). „Under certainty" signifies that the parameters of the problem are supposed to be known. The second area is connected with situations in which the DM (decision maker) evaluates a given decision variant in terms of one objective function, but, due to numerous unknown future factors, the parameters of the problem are not deterministic. A set of potential scenarios is available [3]. These scenarios may be defined by experts, decision makers or by a person who is simultaneously an expert and a DM. “Scenario” means a possible way in which the future might unfold.

The scenario-based 1-DMU is investigated by many researchers and practitioners since real economic decision problems (e.g., choice of investment projects, selection of marketing strategies, choice of technology, human resource management) are usually uncertain [8–11, 13, 14, 17, 19, 20, 25, 28, 30, 39, 47].

It is worth underlining that there are diverse uncertainty levels [6, 15, 51]:

- Uncertainty with known probabilities (the DM knows the options, scenarios, scenario probabilities and particular payoffs).
- Uncertainty with partially known probabilities (the DM knows the options, scenarios, partial scenario probabilities and particular payoffs – probabilities may be given as interval values, sometimes scenarios are ordered according to their approximate chance of occurrence).
- Uncertainty with unknown probabilities (the DM knows the options, scenarios and particular payoffs – scenario probabilities are not known).
- Uncertainty with unknown scenarios (the DM knows the options only).

In this article, the third level is investigated since in connection with the fact that the set of scenarios in SP does not need to be exhaustive, the use of probabilities seems to be unjustifiable [31, 48]. Furthermore, von Mises [32] adds that the probability of a single event should not be expressed numerically because probabilities only concern repetitive situations which are not frequent in real economic problems (innovative or innovation projects, turbulent times, etc.).

In the list of possible uncertainty levels the notion of “payoff” appears many times. The words “payoff”, “result” or “outcome” signify the effect gained by the DM if he or she selects a given alternative and a given scenario occurs. Table 1 shows the payoff matrix related to M-DMC while Table 2 represents the payoff matrix connected with 1-DMU.

Table 1. Payoff matrix for M-DMC (after [18])

Criterion	Alternative				
	A_1	$\cdots$	A_j	$\cdots$	A_n
C_1	$b_{1,1}$	$\cdots$	$b_{1,j}$	$\cdots$	$b_{1,n}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
C_k	$b_{k,1}$	$\vdots$	$b_{k,\,j}$	$\vdots$	$b_{k,n}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
C_p	$b_{p,1}$	$\cdots$	$b_{p,j}$	$\cdots$	$b_{p,n}$

n – number of alternatives, p – number of criteria, $b_{k,\,j}$ – performance of criterion C_k if option A_j is selected.

Based on Tables 1 and 2, one can formulate the following conclusions. First, there is a significant difference between M-DMC and 1-DMU consisting in the fact that within 1-DMU, if A_j is chosen, the outcome ($a_{i,\,j}$) is single and depends on the real scenario which will occur, meanwhile, within M-DMC, if A_j is selected, there are p outcomes, i.e., $b_{1,j}, \ldots,\ b_{k,\,j}, \ldots,\ b_{p,j}$, because particular decision variants are evaluated in terms of p essential objectives. The next difference results from the fact that in the case of M-DMC initial values usually have to be normalized since they represent the performance of different criteria. Hence, they are expressed through different scales and units.

Table 2. Payoff matrix for 1-DMU with unknown probabilities (after [18])

Scenario	Alternative				
	A_1	$\cdots$	A_j	$\cdots$	A_n
S_1	$a_{1,1}$	$\cdots$	$a_{1,j}$	$\cdots$	$a_{1,n}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
S_i	$a_{i,1}$	$\vdots$	$a_{i,j}$	$\vdots$	$a_{i,n}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
S_m	$a_{m,1}$	$\cdots$	$a_{m,j}$	$\cdots$	$a_{m,n}$

n – number of alternatives, m – number of scenarios, $a_{i,j}$ – payoff obtained if option A_j is selected and scenario S_i occurs.

Equation (1) is designed for criteria which are maximized. Equation (2) can be applied to minimized criteria.

$$b(n)_{k,j} = \frac{b_{k,j} - \min_j b_{k,j}}{\max_j b_{k,j} - \min_j b_{k,j}} \tag{1}$$

$$b(n)_{k,j} = \frac{\max_j b_{k,j} - b_{k,j}}{\max_j b_{k,j} - \min_j b_{k,j}} \tag{2}$$

In the case of 1-DMU, the problem is related to one criterion. Thus, the normalization is useless. However, some similarities between both issues (M-DMC and 1-DMU) are also very visible [15, 16, 18]. The construction of both payoff matrices is extremely similar. In both cases, there is a set of potential options. The set of significant objectives in M-DMC can correspond to the set of possible scenarios in 1-DMU. Another analogy is related to the final step of the decision-making process. The decision maker, in both decision problems, can select and execute only one option (so-called pure strategy) or a combination of several options (mixed strategy). Mixed strategies are especially common in portfolio construction and cultivation of different plants [10, 29, 37]. Of course, in some decision situations, only pure strategies can be considered (e.g., choice of a place to organize the wedding – it would be rather inconvenient to have a wedding in 5 countries simultaneously). Due to the investigated problem (ranking creation), we focus on pure strategies.

3. From the modified Laplace rule to a modification of the SAW method

The Laplace rule (Bayes rule) is one of the classical decision rules developed for 1-DMU. It is also called the principle of insufficient reason, the principle of indifference or the Bernoulli rule. It consists in computing the arithmetical average of all the payoffs. Thus, within this approach, the same probability is assigned to each scenario, because it is assumed that the DM does not have sufficient knowledge to estimate the probability of occurrence of particular scenarios [40].

Note that a very similar method has been created for M-DMC. It is the SAW method. This procedure also takes into account each payoff connected with a given option and, just like the Laplace rule, it consists in calculating the sum of products of all the outcomes multiplied by some weights. The only difference is that in the case of the Laplace rule the weights are equal, while in the case of the SAW method, the weights may be diverse for particular criteria because they represent the subjective importance of subsequent attributes and they are declared by the decision maker. The SAW is a very popular method and it has been extended by numerous researchers. Here are some examples. Salehi and Izadikhah [43] developed the SAW technique for the case with input data stated in intervals. The original version of SAW has been modified by Niroomand et al. [35] to cope with interval values. Gündoğdu and Yörükoğlu [21] applied spherical fuzzy sets to create the fuzzy version of SAW. Roszkowska and Kacprzak [41] used ordered fuzzy numbers for the SAW fuzzy version. Piasecki et al. [38] equipped the SAW method with a fuzzy ranking of evaluated alternatives. Irvanizam et al. [26] used triangular fuzzy numbers to facilitate fuzzy multiple-attribute decision making. All these examples are not directly relevant to our research (here we concentrate on multi-criteria decision making under certainty). Nevertheless, it is worth emphasizing that the SAW procedure is consistently adjusted to new needs or circumstances.

The Laplace rule is a very simple procedure but one of its main drawbacks results from the fact that it only focuses on the average of payoffs. Other disadvantages are discussed in [14], but they are not significant for the SAW method.

Table 3. Payoff matrix: Laplace (Bayes) rule 1-DMU (Example 1)

Scenario	Alternative				
	A_1	A_2	A_3	A_4	A_5
S_1	–9	13	53	-1000	10
S_2	12	14	0	-5000	60
S_3	100	12	0	4000	60
S_4	–50	14	0	2053	–77
L_j	13.25	13.25	13.25	13.25	13.25

Table 3 presents fictitious data for 4 potential scenarios and 5 considered alternatives (investment projects). The last row shows the Laplace (Bayes) index computed for each decision variant. Hence, the values of the applied measure are equal for all the options, which means that when creating a ranking, each option would obtain the same position - all the courses of action would be treated as equivalent. This phenomenon is undoubtedly alarming, because if the decision maker does not analyze the payoff matrix very carefully, he or she may really think that the projects are equally attractive. The DM will not notice that the dispersion of outcomes for particular variants is totally different (for A1 from –50 to 100, for A2 from 12 to 14, for A3 from 0 to 53, for A4 from –5000 to 4000, and for A5 from –77 to 60).

We are certainly aware of the fact that cases, where different sets of data are characterized by equal index levels, occur in other domains as well. For example [2] has found Anscombe's quartet, i.e., four data sets for which numerical calculations (means of the independent variable data, means of the dependent variable data, variances, correlation between variables, parameters of the linear regression line, coefficient of determination) were exact, but the graphical representation was essentially distinct!

In Table 4, we also can compare 5 alternatives based on the Laplace index. This time, each option receives a different rank in the ranking, but can we state that A4 is better than A2? The Laplace index is lower for A2 but this alternative, regardless of the real scenario, does never lead to losses which could be very severe.

Table 4. Payoff matrix: Laplace (Bayes) rule
1-DMU (Example 2)

Scenario	Alternative				
	A_1	A_2	A_3	A_4	A_5
S_2	12	14	0	-5000	60
S_3	103	12	0	4000	62
S_4	-50	14	0	2060	-77
L_j	14.00	13.25	13.50	15.00	13.75

The above analysis demonstrates that the use of one measure to create the ranking is unsatisfactory. That is why, [25] suggests using the standard deviation as an auxiliary decision making tool. This tool allows controlling the payoff dispersion. Additionally, [12] recommends applying the difference between extreme values and even their levels (i.e., the maximal payoff and the minimal one). Thanks to the aforementioned supplementary measures the DM has got the chance to make the final decision more reasonably.

Now, let us look at the SAW method. This time particular outcomes can be multiplied by criteria weights since the decision maker has the opportunity to declare the importance of each criterion. According to the fictitious data given in Table 5 (we assume that all the criteria are expressed in different units and scales, so the normalization is necessary), the SAW procedure may treat many options as equivalent even if the dispersion of the normalized outcomes connected with subsequent alternatives is different. If we examine each payoff more carefully, we will observe for instance that option A3 is theoretically as attractive as A5 but the first one has two values equal to 0 (which means that its performance degrees are the worst for two criteria) while the second one is never (i.e., for any criterion) the weakest. Thus, if the decision maker tends to select a variant that performs each objective at a decent level, he or she ought to consider a supplementary measure to assess the various options properly.

Table 5. Payoff matrix: SAW method
M-DMC (Example 3)

Criterion	Alternative				
	A_1	A_2	A_3	A_4	A_5
C_1 (0.2)	0.45	1.00	0.00	0.30	0.80
C_2 (0.3)	1.00	0.00	0.30	0.80	0.30
C_3 (0.1)	1.00	1.00	0.00	0.30	1.00
C_4 (0.4)	0.00	0.475	1.00	0.40	0.35
SAW_j	0.49	0.49	0.49	0.49	0.49

We notice that the DM using the original version of the SAW approach in M-DMC is exposed to a similar danger as the DM using the original version of the Laplace rule in 1-DMU. Therefore, the improvement of the SAW method seems to be necessary and the use of the standard deviation sounds reasonable. When a variant has a relatively small standard deviation, it means that it contains few extreme performance degrees, i.e., both very high and very low degrees. We have two possibilities. The first one assumes the use of the same measure, i.e., the standard deviation. According to the second concept, we should not directly apply the modification suggested for the Laplace rule, since in the case of the SAW procedure particular payoffs have different weights. In connection with that difference, the use of the weighted standard deviation might be more appropriate:

$$s_j^w = \sqrt{\sum_{k=1}^{p} w_k \left(b_{k,j} - SAW_j\right)^2}, \quad j = 1, \ldots, n \tag{3}$$

The suggested modified SAW technique based on the weighted standard deviation contains the following steps (see also Figure 1):

1. Define the set of options, the set of criteria, their weights (they should add up to one) and the payoff matrix.
2. Normalize initial payoffs based on Equations (1) and (2).
3. For each option, calculate the SAW_j indices.
4. Divide the options into separate sets containing decision variants with the same (or almost the same) SAW values. The equivalence of two or more courses of action depends on the DM's preferences.
5. Compute the weighted standard deviation (Equation 3) for each option belonging to a set with more than one element.
6. If each option from a given set has a weighted standard deviation different enough, generate the ranking (list options in descending order of SAW_j values and then, within each set defined in step 4, rank options in ascending order of s_j^w values). Otherwise, go to step 7.
7. If there are variants with very similar SAW_j and s_j^w values within a given set (the similarity of two courses of action depends on the DM's preferences), the DM may analyze additional factors (e.g., the maximal value, the minimal value or the difference between extreme values) in order to diversify the position of particular options in the ranking.

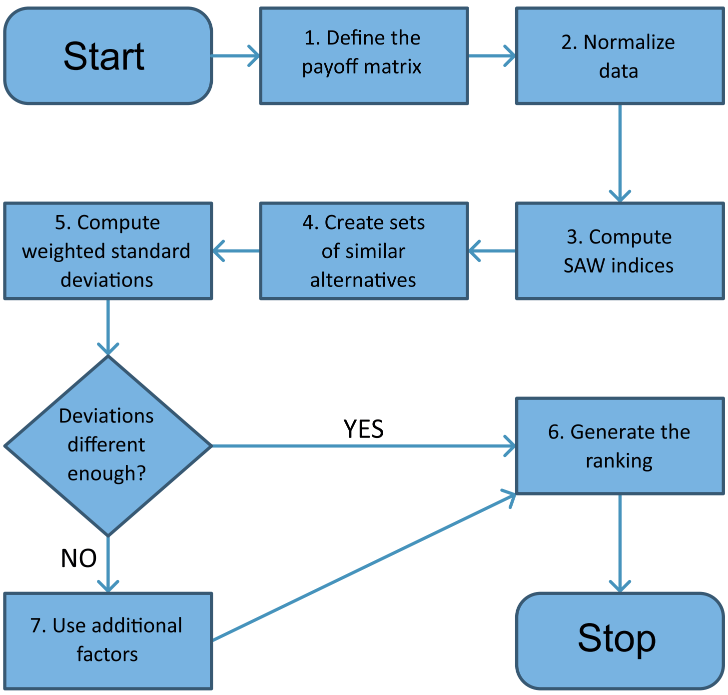

Figure 1. Modified SAW technique based on the weighted standard deviation

In example 3, all the variants certainly belong to the same set (step 4). The weighted standard deviations are as follows: $s_1^w = 0.44766$, $s_2^w = 0.38749$, $s_3^w = 0.43232$, $s_4^w = 0.20712$, $s_5^w = 0.25278$ (step 5).

We see that for alternatives A1 and A3 the weighted standard deviation is almost the same (step 7). That is why, before generating the final ranking, the DM may want to compare the aforementioned options based on a third decision tool, e.g., the minimal performance degree, the maximal performance degree, the difference between them or the arithmetical average of payoffs. For the first three measures alternatives A1 and A3 obtain the same values, but the arithmetical average of payoffs is lower in the case of A3 (0.3250<0.6125). Therefore, A1 can be treated as slightly better than A3. The final ranking is as follows:

1. A4 (the lowest standard deviation)
2. A5
3. A2
4. A1
5. A3

In our example, the alternatives achieving the highest places (i.e., A4 and A5) perform each criterion at a level equal to at least 30%.

The use of the weighted standard deviation (instead of the standard deviation) in the modified SAW technique requires an additional justification. Such a measure is commonly applied to descriptive statistics when the probabilities (frequencies) of particular results are varied. In such a case each value is compared with the weighted average (not the arithmetical one). Nevertheless, in deterministic multi-criteria issues (which are investigated in this paper) the application of the weighted standard deviation has a different background because scenarios are replaced here with criteria. These circumstances significantly change the interpretation of the weighted standard deviation. This time, each performance degree is compared with the SAW index which is a weighted average of partial utilities. Hence SAW_j represents a synthetic measure for each option where payoffs connected with the most important criteria have the greatest impact on the value of this index. By applying different weights for each performance degree in equation (3) we favour options which are characterized by small deviations between payoffs related to the most vital factors and the synthetic measure. As we can see sjw has a more informative value than the standard deviation. If we applied the first concept (standard deviation), alternatives with critical performance degrees being very far from the synthetic index could be considered promising, but this would be against the assumptions made in this research. By referring to the weighted standard deviation the measure still represents the outcome dispersion (the more differentiated payoffs are, the higher the deviation is), but not strictly in a statistical sense. However, the use of a special variety of standard deviations in the SAW method allows taking into account the criteria importance.

4. From the modified Wald rule to a modification of the max-min rule

The Wald rule [50] is also one of the classical decision rules developed for 1-DMU. It is also named the max-min rule and it consists of defining the security level (i.e., the worst possible outcome, w_j) for each alternative and choosing the option with the maximal security level. This rule represents extreme prudence.

It is worth stressing (because this observation has not yet been revealed in the literature) that a max-min rule for M-DMC also exists! It is described for instance in [45]. Its goal is to indicate the worst performance degree for each option and show which criterion is performed the worst by particular variants. Then, the ranking of options is generated according to the decreasing level of the minimal performance degree. Within the max-min rule for M-DMC it is assumed that each criterion has the same weight.

The Wald rule has numerous drawbacks [14] but the disadvantages which are essential from the point of view of the max-min rule for M-DMC are as follows. First, the Wald rule does not distinguish between alternatives with the same security level. Second, the Wald rule discriminates options with even very high outcomes if at least one of their payoffs is lower than the lowest values connected with the remaining variants. Table 6 presents fictitious data for 4 potential scenarios and 5 considered projects. The last row shows the Wald index computed for each decision variant.

Table 6. Payoff matrix: Wald rule 1-DMU (Example 4)

Scenario	Project				
	P_1	P_2	P_3	P_4	P_5
S_1	5	7	5	4	25,000
S_2	2000	8	5	10,000	4
S_3	3000	8	5	10,000	30,000
S_4	4000	5	10	20,000	45,000
W_j	5	5	5	4	4

When assessing the alternatives based on the Wald rule, we obtain a ranking: I. P1, P2, P3, II. P4, P5. The first three projects are treated as equivalent due to the same security level (5) but even if the decision maker is an extreme pessimist, he or she would rather prefer project P1 to P2 or P3, since the remaining outcomes related to the first one are much more attractive than the remaining outcomes connected with P2 or P3. Data given in Table 6 indicate that the Wald rule does not distinguish between decision variants with the same minimal payoff (first drawback) but they also demonstrate that this procedure has discriminated projects P4 and P5 (second drawback). Although their outcomes are remarkable in comparison with payoffs related to projects P1, P2, and P3, they obtain the second rank because their security level (4) is lower.

To overcome the first deficiency, the literature offers the lexicographic max-min (lex-min) rule which compares the 2nd-worst outcomes (3rd-worst, 4th-worst, ect., if necessary) of the options with the same level w_j and recommends the one with the highest value [44]. However, thanks to the lex-min rule the second deficiency can also be, at least partially, avoided since if security levels are not the same, but relatively close to each other, they may be treated as equivalent and then the 2nd-worst outcome comparison will be also recommended. If the DM treats payoffs 5 and 4 as almost the same, the use of the lex-min rule is going to lead us to the conclusion that project P5 is the most attractive (ranking: I. P5, II. P4, III. P1, IV. P2, V. P3).

Now, let us investigate the max-min rule for M-DMC. Due to a very similar construction, this procedure is criticised for almost the same analogical reasons as the Wald rule is. Thus, the max-min rule for M-DMC does not distinguish between alternatives with the same minimal performance degree. Second, it discriminates options with even very high-performance degrees if at least one of their results is lower than the lowest values connected with the remaining variants.

The suggested modified max-min rule for M-DMC contains the following steps (see also Figure 2):

1. Define the set of options, the set of criteria and the payoff matrix.
2. Normalize initial payoffs based on equations (1) and (2).
3. For each option, find the minimal performance degree.
4. Generate the ranking according to the 1st-worst performance degree. If for some options the minimal performance degrees are equal or almost the same (the equivalence of two or more courses of action depends on the DM's preferences), go to step 5. Otherwise, stop the procedure.
5. Find the 2nd-worst performance degree for the aforementioned subset of variants. Diversify the ranks of the options belonging to the subset based on the 2nd-worst performance degree. If for some alternatives from the subset the 2nd-worst performance degrees are equal or almost the same, continue the procedure by finding the 3rd-worst results for those options and modifying the ranking. Repeat the step until the kth-worst performance degrees are different (k denotes the level of the worst results). If two alternatives have the same sets of values, they should obtain the same rank.

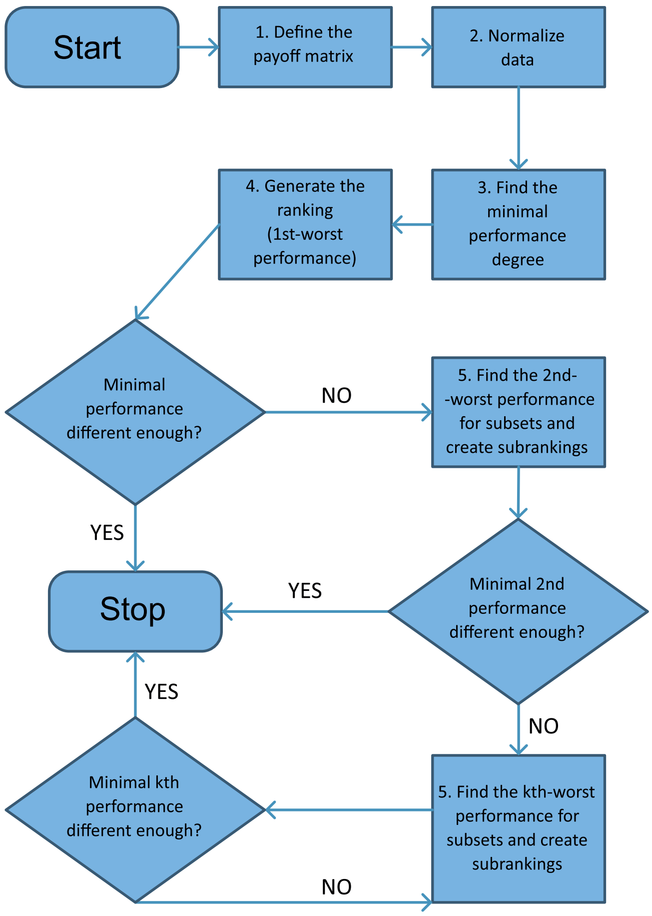

Figure 2. Modified max-min rule for M-DMC

Let us analyze the suggested technique by referring to data given in table 5. If we apply this method, we will see that alternatives A4 and A5 have the first rank and the remaining options occupy the second place. However, it is desired to look at the 2nd-worst performance degree, because A3 is the worst option for two criteria while A1 and A2 – for one criterion only. After applying step 5, we obtain the following sequence:

1. A5
2. A4
3. A2
4. A1
5. A3

On the one hand, the modified max-min rule for M-DMC may extend the computation time, which is certainly unfavourable for the decision maker. On the other hand, the novel procedure is not computationally complex and enables better control of a given decision-making situation.

5. Example. Ranking for UE countries

In this section, we analyze a larger problem. We assume that the goal is to assess the quality of life in the European Union countries for an average inhabitant. Ten factors have been selected for the research: F1 – GDP per capita (USD, maximized), F2 – remaining life expectancy (years, maximized), F3 – quantity of fuel that can be bought with an average payment (litres, maximized), F4 – number of months the inhabitant is supposed to work in order to acquire Renault Clio (minimized), F5 – ratio of the average net salary to the gross salary (%, maximized), F6 – average number of years spent in retirement (maximized), F7 – average Internet speed (Mbps, maximized), F8 – average flat area (square metres, maximized), F9 – number of non-working days within a year (maximized), F10 – ratio of the number of dead people due to SARS coronavirus to the number of infected people (%, minimized).

For factor F3, the fuel price (PB95) comes from 07.11.2019. The Renault Clio purchase prices (F4) relate to 2019. Factor F6 is computed as the difference between the life expectancy and the pensionable age. The data related to F7, F8 and F10 come from the end of 2019, the end of 2018 and 3.05.2020, respectively. The data for the rest of factors (F1, F2, F3 – salary, F4 – salary, F5, F6) refer to the first quarter of 2020.

The data have been collected from the following websites:

- International Monetary Fund
- populationof.net
- www.e-petrol.pl
- Renault websites
- List of European countries by average wage (wikipedia)
- https://biqdata.wyborcza.pl
- https://www.cable.co.uk/broadband/speed/worldwide-speed-league
- https://www.locja.pl
- dniwolne.eu
- https://www.worldometers.info/coronavirus

The choice of the criteria aforementioned was not random. The aim of the research was to include factors covering diverse areas (e.g., healh, earnings, technology) and to take into consideration at least several indicators which are not usually applied when generating rankings (e.g., number of non-working days).

Table 7 presents the data related to subsequent UE countries. To apply the modified version of SAW, criteria weights are needed. Therefore, a small survey was arranged amongst 100 respondents from Poland in March 2020 (22 respondents from age brackets $[15, 25]$, 28 from $[26, 35]$, 19 from $[36, 45]$, 17 from $[46, 55]$ and 14 from $[56, \infty)$). The respondents were supposed to declare criteria weights in the following way. They had to assign a value from the set $\{1, 2, 3, 4, 5\}$ to each factor where 1 meant that a given criterion was irrelevant and 5 that this criterion was extremely significant. The average criteria weights were as follows: $w_1 = 3.25$, $w_2 = 3.21$, $w_3 = 3.36$, $w_4 = 3.43$, $w_5 = 3.46$, $w_6 = 3.24$, $w_7 = 3.30$, $w_8 = 3.14$, $w_9 = 3.19$, $w_{10} = 3.19$. We can observe that they are very similar, which means that the set of potential criteria has been aptly selected. In the research, we apply transformed weights, i.e., weights that do total 1 (100%).

Table 7. Data concerning UE countries

Country	Criterion									
	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}
Austria	53764	38.90	38929	2.82	67.7	19.7	19.33	99.7	13	3.83
Belgium	51237	40.10	31982	3.83	63.8	14.7	35.69	124.3	12	15.72
Bulgaria	9810	31.40	7789	6.33	77.5	10.2	16.95	73.0	13	4.47
Croatia	15533	34.80	10651	9.98	74.4	14.7	17.22	81.6	13	3.69
Cyprus	30520	42.50	23350	6.01	83.2	16.1	11.00	141.4	13	1.74
Czech Republic	26113	36.80	19215	5.47	75.1	15.2	23.27	78.0	12	3.16
Denmark	66195	39.50	36311	3.99	63.0	15.3	49.19	118.1	10	5.05
Estonia	25560	35.30	17660	6.04	85.4	14.9	31.55	66.7	11	3.24
Finland	54869	39.30	35728	3.94	74.2	19.0	29.34	88.6	12	4.38
France	46793	41.40	28235	3.98	75.2	21.1	30.44	93.7	12	14.70
Germany	53275	36.70	33254	3.22	60.8	16.6	24.64	94.3	14	4.13
Greece	22077	37.70	12184	8.96	84.0	14.9	13.41	88.6	9	5.46
Hungary	17717	33.50	14066	7.54	68.4	12.3	31.10	75.6	12	11.34
Ireland	84826	43.90	52949	2.64	77.7	16.1	23.87	80.8	9	6.07
Italy	37231	37.90	21463	5.11	72.4	16.6	17.30	93.6	12	13.72
Latvia	19923	31.60	14088	7.28	74.1	11.9	32.74	62.5	11	1.82
Lithuania	21242	32.40	16432	6.83	63.1	12.5	30.66	63.2	12	3.26
Luxembourg	125364	42.70	94555	1.51	67.9	17.4	41.69	131.1	10	2.41
Malta	33952	39.10	21794	5.17	74.0	18.4	18.16	150.0	14	0.84
Netherlands	57902	40.20	29776	3.94	75.4	16.4	40.21	106.7	11	12.39
Poland	17130	36.20	13389	8.53	72.1	15.6	24.38	75.2	12	4.95
Portugal	25439	37.10	14573	8.15	81.3	15.9	22.75	106.4	13	4.06
Romania	13664	33.60	10661	10.05	61.1	12.9	21.80	43.9	10	5.93
Slovakia	22031	36.30	14993	5.90	75.2	15.2	29.45	87.4	15	1.70
Slovenia	29303	38.00	20883	4.95	65.4	16.5	27.83	80.3	14	6.67
Spain	34281	39.70	23685	4.17	80.5	18.6	36.06	99.1	14	10.22
Sweden	60436	41.70	37466	3.50	77.0	18.1	55.18	103.3	10	12.09

The use of SAW requires criteria normalization – performance degrees are gathered in Table 8 (the column with SAW_j indices includes values expressed to four decimal places to facilitate the comparison of the results). This table shows the initial ranking. Let us assume that if the difference between two SAW indices is less than 0,005 (0,5%) the evaluated countries are regarded as equivalent.

Thus, we can distinguish sets Cyprus, Sweden, Ireland, Finland, Denmark, France, Slovenia, Belgium, Greece, Croatia and compute the weighted standard deviation for at least those states. We notice that for some couples of countries, the suggested modified SAW technique requires a priority change. In the final ranking: Sweden will overtake Cyprus; Finland will overtake Ireland; and Croatia will overtake Greece (see arrows in the first column, of the table). According to the applied procedure, Luxembourg is the best country in which to live and Romania is the worst one.

Table 8. Initial ranking (SAW) and revisions (modified SAW)

Country	Criterion											
	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	SAW_j	s_w^j
Luxembourg	1.000	0.904	1.000	1.000	0.289	0.661	0.695	0.822	0.167	0.894	0.7427	0.284
Malta	0.209	0.616	0.161	0.753	0.537	0.752	0.162	1.000	0.833	1.000	0.5981	0.309
Cyprus ↓	0.179	0.888	0.179	0.696	0.911	0.541	0.000	0.919	0.667	0.940	0.5901	0.336
Sweden ↑	0.438	0.824	0.342	0.866	0.659	0.725	1.000	0.560	0.167	0.244	0.5854	0.265
Spain	0.212	0.664	0.183	0.821	0.801	0.771	0.567	0.520	0.833	0.370	0.5755	0.237
Ireland ↓	0.649	1.000	0.520	0.924	0.687	0.541	0.291	0.348	0.000	0.649	0.5642	0.280
Finland ↑	0.390	0.632	0.322	0.836	0.545	0.807	0.415	0.421	0.500	0.762	0.5635	0.177
Austria	0.380	0.600	0.359	0.912	0.280	0.872	0.189	0.526	0.667	0.799	0.5566	0.243
Denmark	0.488	0.648	0.329	0.833	0.089	0.468	0.864	0.699	0.167	0.717	0.5285	0.258
France	0.320	0.800	0.236	0.833	0.585	1.000	0.440	0.469	0.500	0.069	0.5268	0.272
The Netherlands	0.416	0.704	0.253	0.836	0.593	0.569	0.661	0.592	0.333	0.224	0.5202	0.193
Slovakia	0.106	0.392	0.083	0.704	0.585	0.459	0.418	0.410	1.000	0.942	0.5086	0.291
Germany	0.376	0.424	0.293	0.885	0.000	0.587	0.309	0.475	0.833	0.779	0.4931	0.267
Portugal	0.135	0.456	0.078	0.552	0.833	0.523	0.266	0.589	0.667	0.784	0.4880	0.246
Estonia	0.136	0.312	0.114	0.694	1.000	0.431	0.465	0.215	0.333	0.839	0.4582	0.288
Slovenia	0.169	0.528	0.151	0.768	0.187	0.578	0.381	0.343	0.833	0.608	0.4527	0.235
Belgium	0.359	0.696	0.279	0.843	0.122	0.413	0.559	0,758	0.500	0.000	0.4519	0.260
Czech Republic	0.141	0.432	0.132	0.733	0.581	0.459	0.278	0.321	0.500	0.844	0.4428	0.224
Italy	0.237	0.520	0.158	0.757	0.472	0.587	0.143	0.468	0.500	0.134	0.3987	0.206
Poland	0.063	0.384	0.065	0.526	0.459	0.495	0.303	0.295	0.500	0.724	0.3810	0.196
Greece ↓	0.106	0.504	0.051	0.497	0.943	0.431	0.055	0.421	0.000	0.690	0.3723	0.299
Croatia ↑	0.050	0.272	0.033	0.428	0.553	0.413	0.141	0.355	0.667	0.808	0.3708	0.244
Latvia	0.088	0.016	0.073	0.611	0.541	0.156	0.492	0.175	0.333	0.934	0.3438	0.280
Lithuania	0.099	0.080	0.100	0.641	0.093	0.211	0.445	0.182	0.500	0.837	0.3181	0.256
Hungary	0.068	0.168	0.072	0.593	0.309	0.193	0.455	0.299	0.500	0.294	0.2962	0.170
Bulgaria	0.000	0.000	0.000	0.000	0.679	0.000	0.135	0.274	0.667	0.756	0.2500	0.307
Romania	0.033	0.176	0.033	0.424	0.012	0.248	0.244	0.000	0.167	0.658	0.1990	0.199
Weight	0.099	0.098	0.103	0.105	0.106	0.099	0.101	0.096	0.097	0.097	–	–

Now, let us try to generate the ranking for the case where people treat all the criteria as equivalent and tend to maximize the minimal performance degree. The modified max-min rule for M-DMC consists in finding the worst result for each country (Table 9) and ranking states according to the decreasing worst payoff. However, if the difference between two minimal performance degrees is less than for instance 0,005 (0,5%), the evaluated countries may be regarded as equivalent. Therefore, we must distinguish sets Luxembourg, Sweden, Slovakia, Lithuania, Portugal, France, Hungary, Cyprus, Ireland, Germany, Belgium, Greece, Bulgaria, Romania and find the 2nd-worst result for at least those states.

For some countries the suggested modified max-min technique requires a priority change. In the final ranking: Portugal will overtake Slovakia and Lithuania (see arrows in the first column, Table 9). Additionally, in the last 7-element set the difference between the 2nd-worst performance degrees for Ireland and Germany is less than for instance 0.005 (0.5%). That is why the 3rd-worst results have to be found for those states. They are equal to 0.348 and 0.309, respectively, which means that Germany will not overtake Ireland. The subranking in the set aforementioned is as follows: Ireland, Germany, Cyprus, Belgium, Greece, Romania, and Bulgaria. According to the applied procedure, Finland is the best country to live in and Bulgaria is the worst one.

Table 9. Initial ranking (max-min rule) and revisions (modified max-min rule)

Country	Criterion											
	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	min(1)	min(2)
Finland	0.390	0.632	0.322	0.836	0.545	0.807	0.415	0.421	0.500	0.762	0.322	0.390
The Netherlands	0.416	0.704	0.253	0.836	0.593	0.569	0.661	0.592	0.333	0.224	0.224	0.253
Austria	0.380	0.600	0.359	0.912	0.280	0.872	0.189	0.526	0.667	0.799	0.189	0.280
Spain	0.212	0.664	0.183	0.821	0.801	0.771	0.567	0.520	0.833	0.370	0.183	0.212
Luxembourg	1.000	0.904	1.000	1.000	0.289	0.661	0.695	0.822	0.167	0.894	0.167	0.289
Sweden	0.438	0.824	0.342	0.866	0.659	0.725	1.000	0.560	0.167	0.244	0.167	0.244
Malta	0.209	0.616	0.161	0.753	0.537	0.752	0.162	1.000	0.833	1.000	0.161	0.162
Slovenia	0.169	0.528	0.151	0.768	0.187	0.578	0.381	0.343	0.833	0.608	0.151	0.169
Italy	0.237	0.520	0.158	0.757	0.472	0.587	0.143	0.468	0.500	0.134	0.134	0.143
Czech Republic	0.141	0.432	0.132	0.733	0.581	0.459	0.278	0.321	0.500	0.844	0.132	0.141
Estonia	0.136	0.312	0.114	0.694	1.000	0.431	0.465	0.215	0.333	0.839	0.114	0.136
Denmark	0.488	0.648	0.329	0.833	0.089	0.468	0.864	0.699	0.167	0.717	0.089	0.167
Slovakia ↓	0.106	0.392	0.083	0.704	0.585	0.459	0.418	0.410	1.000	0.942	0.083	0.106
Lithuania ↓	0.099	0.080	0.100	0.641	0. 093	0.211	0.445	0.182	0.500	0.837	0.080	0.093
Portugal. ↑↑	0.135	0.456	0.078	0.552	0.833	0.523	0.266	0.589	0.667	0.784	0.078	0.135
France	0.320	0.800	0.236	0.833	0.585	1.000	0.440	0.469	0.500	0.069	0.069	0.236
Hungary	0.068	0.168	0.072	0.593	0.309	0.193	0.455	0.299	0.500	0.294	0.068	0.072
Poland	0.063	0.384	0.065	0.526	0.459	0.495	0.303	0.295	0.500	0.724	0.063	0.065
Croatia	0.050	0.272	0.033	0.428	0.553	0.413	0.141	0.355	0.667	0.808	0.033	0.050
Latvia	0.088	0.016	0.073	0.611	0.541	0.156	0.492	0.175	0.333	0.934	0.016	0.073
Cyprus ↓↓	0.179	0.888	0.179	0.696	0.911	0.541	0.000	0.919	0.667	0.940	0.000	0.179
Ireland ↑	0.649	1.000	0.520	0.924	0.687	0.541	0.291	0.348	0.000	0.649	0.000	0.291
Germany ↑	0.376	0.424	0.293	0.885	0.000	0.587	0.309	0.475	0.833	0.779	0.000	0.293
Belgium	0.359	0.696	0.279	0.843	0.122	0.413	0.559	0.758	0.500	0.000	0.000	0.122
Greece	0.106	0.504	0.051	0.497	0.943	0.431	0.055	0.421	0.000	0.690	0.000	0.051
Bulgaria ↓	0.000	0.000	0.000	0.000	0.679	0.000	0.135	0.274	0.667	0.756	0.000	0.000
Romania ↑	0.033	0.176	0.033	0.424	0.012	0.248	0.244	0.000	0.167	0.658	0.000	0.012

6. Discussion and conclusions

In this section, some issues will be discussed in detail.

- We have shown that the construction of the simple additive weighting method and the max-min rule for multi-criteria optimization under certainty ought to be improved due to their drawbacks. We have also underlined that both methods may be modified by using the same amendments that were applied to the Laplace rule and the Wald rule. Although these procedures are designed for a different domain, i.e., one-criterion optimization under uncertainty, both issues (M-DMC and 1-DMU) have a lot in common. The paper investigates only the discrete version of M-DMC since the research focuses on the possibility of creating rankings. Nevertheless, SAW and the max-min rule may also be used in the continuous version of multi-criteria decision making to find the optimal mixed strategy. That is why, in the future, it would be desirable to explore that area and check whether analogical modifications are necessary and possible to implement in the methods aforementioned.
- The SAW technique and the max-min rule are theoretically designed for maximized and minimized criteria, but the performance degree for neutral criteria (in such a case the DM tends to reach a specific value, not the highest/lowest one) can also be computed, which means that these methods have a wider range of applications.

- In the previous section, the use of the modified procedures has been illustrated using an example concerning UE countries. We are aware of the fact that the generated rankings are very sensitive to numerous factors. As for data related to the pandemic situation (F10), we must recognize that they are partially affected by error since not everyone has been subjected to tests. Furthermore, in the case of SAW the sequence of countries depends on criteria weights. In our research, the weights were very similar, which may be indicative of a proper initial selection of criteria. However, if the questionnaire had been addressed to more people, the weights could have significantly changed, which would have influenced the ranking. It is also worth emphasizing that the structure of the ranking depends on the period during which the study is carried out because the majority of data is systematically changing. The set of criteria applied to the procedures has also an essential impact on the final results. Notwithstanding the foregoing (i.e., the huge ranking sensitivity), the main goal of section 5 was to explain the suggested methods in detail.
- A lot of factors can affect the ranking. One of them is the choice of the method applied to estimate criteria weights. In our research, weights were set based on questionnaire results. Nevertheless, it is not the only possible way. SAW allows the decision maker to use other procedures, e.g., entropy or variance method. Within entropy, the weights of different indicators are determined according to the degree of dispersion [54]. The statistical variance procedure is described for instance in [36]. Both approaches are objective weighting methods. However, we focus on tools enabling us to deal with the multiple solutions case. Hence, the choice of the weighting method is of secondary importance in this investigation.
- It is worth stressing that UE countries' rankings generated based on the modified SAW technique and the modified max-min rule are quite different, which might appear as somewhat surprising. Cyprus and Ireland are in the top 10 countries in the modified SAW ranking while in the modified max-min ranking they are ranked at the bottom of the list. Nevertheless, such a situation is possible, since the investigated methods have different assumptions. Within the modified SAW technique the ranking depends on each performance degree, while within the modified max-min rule the sequence is dependent on the worst and quasi-worst results since the second approach is designed for DMs who do not intend to select an option with at least one relatively low minimal value.
- In the modified SAW method, the use of the weighted standard deviation has been suggested although the analogical amended Laplace rule is based on the standard deviation. Such a novelty has been introduced to take the criteria' importance into account. But the standard deviation without weights would also be justified.
- The suggested amendments are particularly crucial in the multiple solutions case, i.e., when some options in the ranking have identical or similar indices. The original procedures do not take into consideration such situations and treat variants as equivalent, but the novel approaches allow the decision makers to explore the problem more deeply and diversify the ranks. It is a significant advantage.
- SAW and the max-min rule are rather designed for quantitative criteria, but there is a possibility to express numerous qualitative factors numerically, so the methods aforementioned are quite universal.
- As mentioned in the paper, there are numerous methods allowing generating the ranking. They can be divided into the following groups: additive methods (e.g., SAW, SMART), analytical hierar-

chy methods (e.g., AHP), verbal methods (e.g., ZAPROS), outranking methods (e.g., ELECTRE), methods based on reference points (e.g., VIKOR) and interactive methods (e.g., INSDECM). It is quite difficult to compare the modified SAW procedure and the modified max-min approach with techniques representing other groups, because in each category decision maker's preferences are declared in a different way and particular rules are designed for different purposes. Nevertheless, the existence of so many varieties gives the possibility to adjust the choice of method to our needs. Another possible way to improve the original versions of SAW and max-min in the multiple solutions case could consist in combining them with ELECTRE. Then, if more than one option obtains the same synthetic value (according to SAW or max-min), the aforementioned procedure could be applied to decide which one is better.

- The modified SAW and max-min techniques can only be applied under certainty, which may be regarded as their limitation. However, there are no obstacles to developing a hybrid enabling to handle both uncertainty and multiple criteria. Such procedures already exist (e.g., [27]), but they do not take the multiple solutions case and scenario planning into account.
- In connection with the fact that some analogies between multi-criteria optimization under certainty and one-criterion optimization under uncertainty have been revealed in this article, possible future research directions could be connected with other potential adjustments of existing procedures formulated for one area to the second, analogical one.

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