ZESZYT NAUKOWY
Wyższa Szkoła Zarządzania i Bankowości w Krakowie

Anna Grobel-Kijanka
Faculty of Management, Finance and Computer Science
The School of Banking and Management in Krakow

APPLICATION OF DYNAMICAL SYSTEMS METHOD IN MODELING AND COMPUTER SIMULATION OF ECONOMIC GROWTH

I have found out what economics is; it is the science of confusing stocks with flows.
M. Kalecki

Key words

system dynamics, simulation modeling, economic growth ,

Introduction

The modeling of the heterogeneous structure of economic systems involves several problems due to their complexity and dynamics. A model of economic growth, despite several simplifying assumptions that have to be made when constructing it as regards the identification of the most significant driving forces of the process under investigation and the relations between them, must reflect the multidimensional structure and dynamics of the economic system. An effective modeling of complex systems can be supported by an approach referred to as the system dynamics (SD). Simulation models help understand more completely the development processes thanks to the presentation of complex systems as coherent entities with the consideration of their dynamics. The tools applied in the modeling process enable intuitive mapping of relationships, better understanding of the functioning and an effective analysis of the behavior of the system components over time and at various scenarios.

The aim of the article is to present the benefits of the application of SD when modeling economic growth. The opportunities and benefits of such research approach are illustrated by single-sector growth models of R.Harrod and R.Solow and the Kalecki's business cycle model. In the dynamic modeling the Vensim[1] software was applied.

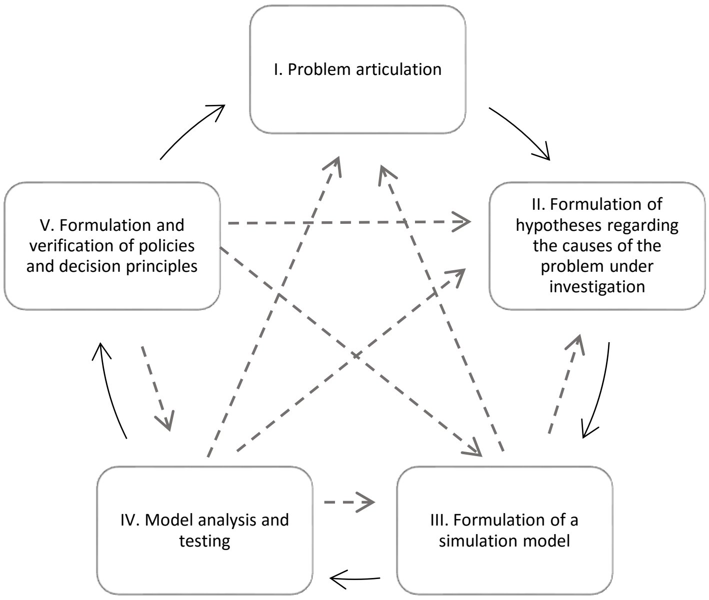
[1] http://vensim.com. A free PLE software version is available on the website of Ventana Systems, Inc. for research and educational purposes.

1. The concept of the SD modeling approach

SD is a computer simulation method that was developed in the late 1950s by J.W. Forrester and his team in the Massachusetts Institute of Technology and it provides a set of conceptual tools and techniques of computer modeling. The principles of the SD modeling approach was presented in numerous publications[2].

According to J.D. Sterman, the development of a model should consider five crucial steps (Fig.1) that are based on a correct definition of the research problem.

Figure 1. Modeling steps in the SD approach



Source: Author's research based on Sterman J.D., *Business Dynamics, Systems Thinking and Modeling for a Complex World*, Irwin McGraw-Hill 2000, p. 87.

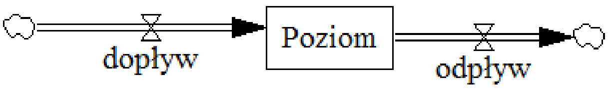
In the course of the conceptualization step an attempt should be made to explain the problem and to describe the structures of the system that is subject to modeling with the application of the existing scientific theories regarding the problem in question, the expertise and the researcher's intuition. When formulating the hypotheses that explain the behavior of the system, one should define the key factors that affect the investigated problem and determine the main feedback loops that describe the cause-and-effect dependencies between the elements of the system structure. According to the founder of the SD approach, feedback

[2] J.D. Sterman, *Business Dynamics, Systems Thinking and Modeling for a Complex World*, Wyd. Irwin McGraw-Hill 2000; R. Hoffmann, T. Protasowicki, *Metoda dynamiki systemowej w modelowaniu złożonych systemów i procesów* [in:] Biuletyn Instytutu Systemów Informatycznych 12 19-28 (2013); A. Wierciszewska, M. Lasek, *Dynamika systemów jako narzędzie modelowania procesów rozwoju gospodarczego*, Optimum. Studia Ekonomiczne No. 1 (49) 2011.

loops constitute a basic element of every complex system[3] and are decisive as regards its particular dynamic behavior. The existence of positive feedback loops that tend to strengthen the changes in the system is considered to be the cause of the exponential growth of the values of particular variables. Negative feedback loops, which counterbalance the positive ones, result in the decrease of the values of variables and their approximation to a particular boundary. A complex dynamic behavior of the system is the result of the interactions of numerous feedback loops. The search for and mapping the cause-and-effect relations in the investigated phenomenon are the indispensible activities in the SD modeling method.

Further steps involve the differentiation of the variables (accumulation and flow ones) and then stock and flow diagrams are introduced to the cause-and –effect feedback loop scheme. The idea of diagrams originated by J.W. Forrester refers to the concept of a hydraulic system of tanks, pipes, valves, etc. (fig.2)

Figure 2. Stock and flow diagram



Source: J.D. Sterman, *Business Dynamics, Systems Thinking and Modeling for a Complex World*, McGraw-Hill 2000, p. 193.

As the process under investigation is treated as a network of stocks with system boundaries (sources and sinks), inflows, outflows and decision variables that control the size of the flows, the mathematical grounds of the system dynamics can be easily explained: the stocks accumulate (integrate) the flows. The amount of stock in a tank at t is given by equation[4]:

$$\text{Stock}(t) = \int_{t_0}^{t} [\text{inflow}(s) - \text{outflow}(s)]ds + \text{Stock}(t_0) \qquad (1)$$

Stock(t_0) means the initial value of the integrative variable[5]. Thus, the differential equation presenting the change dynamics of the accumulation variable is:

[3] Jay. W. Forrester, Urban Dynamics (1969), p. 107.

[4] J.D. Sterman, *Business Dynamics…,* op. cit., p. 194.

[5]The equation that defines the amount of stock in several existing software programmes of dynamic modeling (e.g. *Vensim*, *Stella*, *IThink*) has the folowing form: $\text{State} = INTEGER(\text{inflow} - \text{outflow}, \text{State}_{t_0})$

$$\frac{d(\text{State})}{dt} = \text{inflow}(t) - \text{outflow}(t) \tag{2}$$

The flow and decision variables are given by algebraic equations. Accumulation variables (stocks) are crucial as regards the generation of the system's dynamic behavior. Integrating causes to a time-character of behaviors in the system and is the source of delays between the inflows[6].

The mathematical model concretizes the relations between the values of the system in a schematic and quantitative way. Linking the variables with quantitative relations, i.e. introducing mathematical formalization of the model is necessary for further computer simulations and verifications of the functioning of the designed model.

The results of the simulation experiments are confronted with the data that come from the system observations that were carried out in the past. Modeling is an iterative process and the discovery of irregularities at any step of the model development results in operations that aim at its improvement and, finally, at the modification of the operations that were conducted in the preceding phases. The initial structure is improved in the course of subsequent computer simulations until the model reflects the real behavior of the system in a satisfactory way.

In the final step of modeling, the results of the model quantitative analysis are applied to design and assess the strategies and decision rules. Thanks to the simulations performed, the effect is investigated and the results of introducing the changes to the system are analyzed, which constitutes the basis for planning and implementing adequate policies in real time.

2. Reasons for applying the SD method in modeling the economic growth

Economic growth consists in the increase of the whole economy as a result of the changes in its components[7]. Due to the complexity and variety of social and economic mechanisms, feedback loops and delays between the processes, the modeling of economic growth is extremely difficult. Despite the fact that a model is a schematic simplification of the process under investigation, it is necessary that, apart from distinguishing crucial causative agents of the investigated process, the relationships between them should be reflected.

Thanks to a clear presentation of the system structure, the systems dynamics facilitates a better understanding of the development processes. The modeling tools that are offered

[6] R. Hoffmann, T. Protasowicki, *Metoda dynamiki systemowej …,* op. cit., p. 20.
[7] M.G. Woźniak, *Wzrost gospodarczy. Podstawy teoretyczne*, Wydawnictwo Uniwersytetu Ekonomicznego w Krakowie, Kraków 2008, p. 10.

make it possible to map intuitively the relationships and help grasp the essence of the dynamic cause-and-effect relations, which enables a deeper understanding of the functioning of the system . Mathematical relations result clearly from the system structure. When defining the relations between the variables, the modeling person is required to differentiate and consider adequate variables in the categories of streams and stocks. With reference to M.Kalecki's words *that economy is a science where economist constantly mix the concepts of stocks and flows and, as a result, make mistakes*[8] , it can be stated that the SD method, thanks to the rigors imposed in the course of the mathematical formalization of the model, helps to solve this issue.

Due to the accessibility of the software based on graphic modeling (e.g. Vensin), the modeling has many supporters. One should not underestimate the possibility to automate the process, conduct numerous computer experiments and simulations, which makes it possible to determine parameters that are optimal for the accepted strategy[9]. SD enables easy adaptations of models to the existing needs. The SD-based models result in better understanding of structures that may generate undesired behavior of the economic system. Through the presentation of the system performance at different scenarios, they help make adequate decisions as regards problem solving and make it possible to investigate the consequences of the solutions implemented.

SD has been used many times to model economic growth in various countries. It is mainly the modeling of the global economy development that resulted in a widespread use of this approach. In 1972, on the commission the Club of Rome a report *The Limits to Growth* was prepared for which – with the use of the SD method – a model of the global economy development was developed in order to show mutual relationships and influences between demographic processes, the development of industry, the depletion of natural resources and the food production. The simulation demonstrated the risk of reaching the limits of growth and the collapse of production in the mid 21st century unless adequate measures are taken to counteract the increasing trends in population, industrialization and the depletion of the resources[10]. The authors of the report pointed also to the conditions that have to be met to make sustainable growth possible.

[8] O. Lange, *Teoria reprodukcji i akumulacji*, 2nd edition, PWN, Warszawa1965, p. 21.

[9] A. Wierciszewska, M. Lasek, *Dynamika systemów jako narzędzie ...,* op. cit., p. 78.

[10] D.H. Meadows, D.L. Meadows., J. Randers, W.W. Behrens., *The Limits to Growth, A Report for The Club of Rome's Project on the Predicament of Mankind*, Universe Books, New York 1972, p. 124.

3. Introduction to the theory of economic growth in the SD approach

The structure of the models of economic growth is closely connected with the existing theories of economy. The majority of current theories of growth are limited to the determination and justification of the path of sustainable economic growth with a complete application of production factors that most commonly include the resources of physical capital and labour. Thus, the models function as patterns that should be followed if the economy is to develop[11]. The models of Harrod and Solov may serve as examples. The theory of economic fluctuations of a Polish outstanding economist M.Kalecki aims at the explanation of cyclical development of national income. However, while investigating the mechanism of investment fluctuations, it does not take into consideration the growth trend.

The understanding of the rules how to construct models that are based on the fundamental theories of economic growth should be a solid ground for conducting further research.

A. Harrod's model of economic growth – the SD approach

The Harrod's Keynesian model of economic growth is the starting point for numerous considerations regarding economic growth. The objective of constructing the model was to explain the process of sustainable growth and particularly to find what national income growth rate warranties sustainability. Harrod's model is based on the following assumptions[12]:

a) the output level Y (national product) determines savings S (savings are a proportional function of national product, with a constant average and marginal saving rate s),
b) savings S equal investment I,
c) capital K does not decrease and its increase is determined by the value of investment I,
d) labour resources L increase with a constant (natural) growth rate n, proportionally to the increase of the whole population,

[11] M.G. Woźniak, *Wzrost …*, op. cit., p. 145.
[12] D. Pieńkowski, *Rola producentów w wybranych koncepcjach wzrostu gospodarczego* [in:] Optimum. Studia Ekonomiczne nr 2 (62) 2013, p. 44.

e) there is a zero substitution of capital and labour investment; both capital and labour ratio are required to be constant, the volume of national product is given by formula:

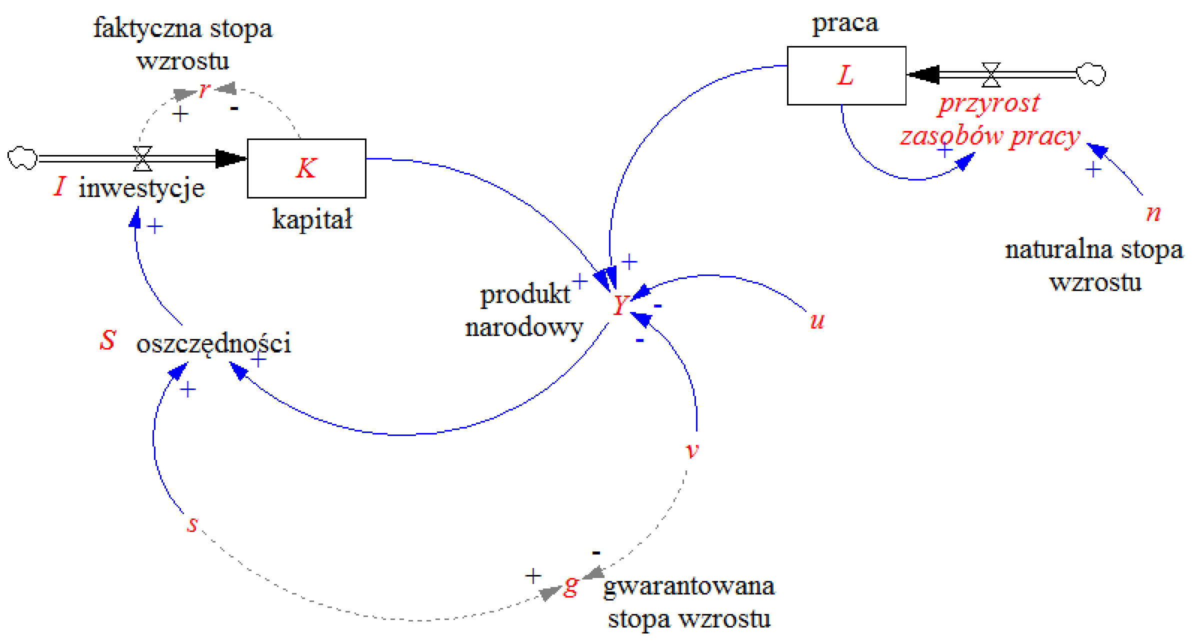
$$Y = min\left[\frac{K}{v}, \frac{L}{u}\right],$$

where v – capital intensity ratio,

u – constant labour share required for achieving complete output.

The hypotheses identify the key growth determinants and the existing cause-and effect relations between them. The dependent value is the value of output (usually identified with the GDP), while the capital and labour resources are the explanatory values, i.e. the determinants of the economic growth. There are two positive feedback loops in the model: one is responsible for the increase of capital resources, and the other controls the increase of labour resources. This information helps develop a logical diagram of stocks and flows (figure 3).

Figure 3. Stock and flow diagram in the Harrod's model



Source: Author' research

The mathematical formalization of the model is the result of the assumptions presented above. Equations that combine particular variables are given by:

a) $S = s \cdot Y$

b) $S = I$

c) L = INTEGER[13] (the growth in labour resources, $L0$),

where *the growth in labour resources* = $n \cdot L$

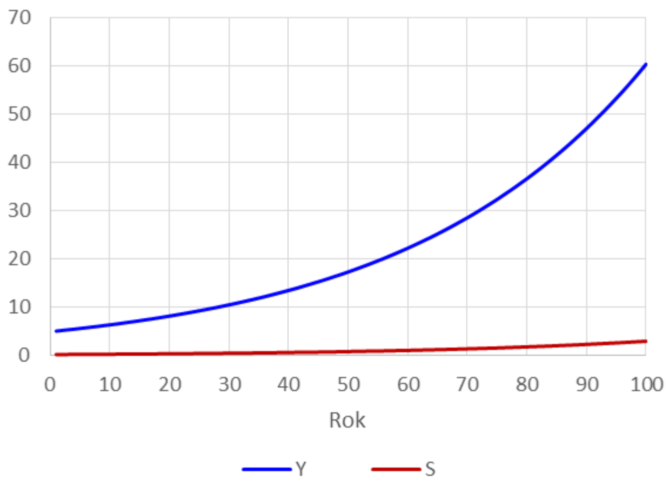
d) K = INTEGER (I , $K0$)

e) Y = MIN(K/v, L/u)

A computer simulation of the model will be possible after constant values of *s*, *n* and *v* are specified. It is also necessary to determine the system initial parameters.

The presentation of economic processes in the form of a clear diagram of feedback loops facilitates the understanding of relations between them and makes it possible to indicate in a simple way the determinants of economic growth. It shows that savings (and particularly the propensity to save *s*), which are the only sources of investment necessary to generate capital are significant. Moreover, improvements resulting in the increase in labour effectiveness or capital productivity (which consequently leads to the decrease of capital intensity ration *v*) are a potential growth factor. National product grows also due to the increase in labour force, which is determined by the rate of population growth *n*. Thanks to the simulations, quantitative relations between the variables can be investigated (fig.4) and optimal growth parameters can be determined.

Figure 4. Exponential growth of national product *Y* and savings *S* in the equilibrium state (v = 2, s = 0,06, n = 0,03, $K0$ = 10)



Source: Author’s research

[13] The equivalent of the integer in equation (1) in Vensim notation .

In line with formulas $g = s / v$ and $r = I / K$, a value of rate that guarantees a balanced economic growth g and actual rate of economic growth r were introduced to the stock and flow diagram. This helped to define and compare their values within the model. By means of a simple value change of variables in the model, it is possible to investigate the behavior of the system at different scenarios, and particularly to present the instability of the equilibrium of the Harrod's model – the effect of a narrow path of balanced growth.

The basic Solow model of economic growth in the SD approach

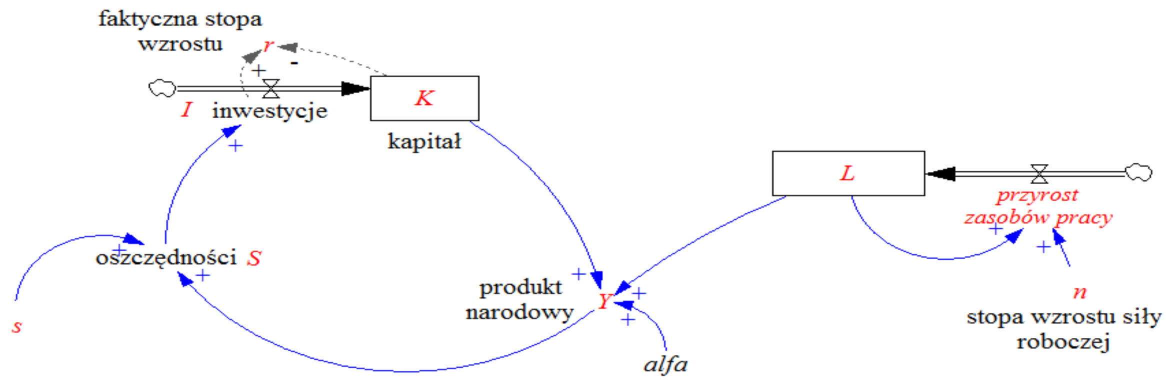
R.M. Solow modified the model presented by Harrod. While looking for the conditions of the equilibrium growth, he rejected the assumption on the lack of flexibility between production factors. Solow introduced a two-argument neoclassical production function and, particularly, he used Cobb-Douglas function as the example[14]:

$$Y = K^{\alpha} \cdot L^{1-\alpha}$$

where α should be interpreted as the flexibility of production against the changes in capital stock.

A graphic presentation of relations in the SD approach is significantly helpful when comparing models under investigation. It is easily noticeable that a qualitative mapping of the assumptions of the Solow basic growth model differs insignificantly from the Harrod stock and flow diagram (fig.5)

Figure 5. Stock and flow diagram in the Solow model



Źródło: opracowanie własne.

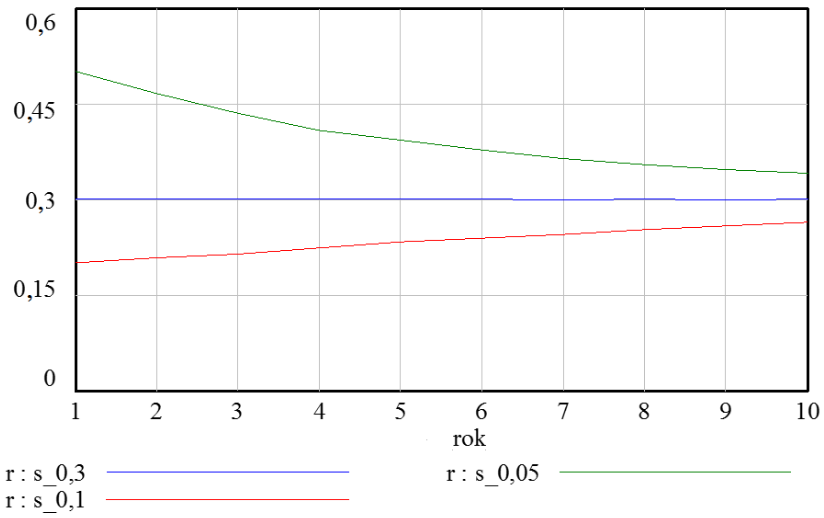
[14] R.M. Solow, *A Contribution to the Theory of Economic Growth* [in:] *The Quarterly Journal of Economics*, Vol. 70, No. 1 (1956), p. 76.

Source: Author's research

There is a basic difference in the mathematical model where the formula defining the value of national product is given by: $Y = K$^*alfa* · L^(1-*alfa*).

As in the previous case, a simulation investigation of the model dynamics can be started after initial parameters have been specified. Experiments with various values *s* lead clearly to the crucial conclusion of the Solow theory that a higher rate of savings results in the increase of the national product but does not increase permanently the actual rate of economic growth. Irrespectively of the initial point, the economy is aiming at the equilibrium state (figure 6)

Figure 6. Changes in the rates of economic growth *r* for *s*=0,1, *s*=0,3 and *s*=0,05.



Source: Author's research

The model that was constructed in line with the SD approach explains in a simple way the mechanism of growth, the impact of particular factors on changes in dynamics; it facilitates the analysis and is helpful in reaching conclusion. It can be easily developed by adding new variables, considering – for example – the decrease in fixed capital. To do this, it is sufficient to add new "outflow" with a depreciation rate that regulates the value of flow and to modify the accumulation formula (in line with equation 2) of variable *K*. SD in this approach is a perfect tools for learning and teaching.

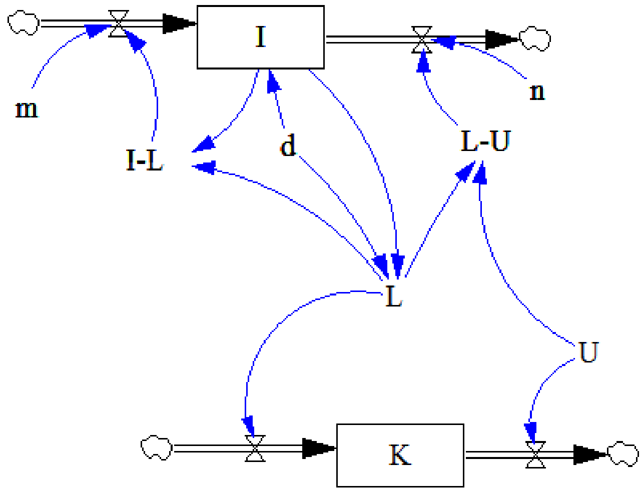
B. SD in the Kalecki's model of cyclical growth

In the early 1930s M.Kalecki pointed to the basic reason of cyclical investment fluctuations and, consequently, explained the mechanism of cyclical fluctuation of national product. The starting point of the Kalecki's concept of business cycles is the investment process which includes three subsequent steps: investment orders I, production of goods A and their supply L . Further considerations lead to the fundamental equation of dynamics[15]:

$$I'(t) = \frac{m}{d}[I(t) - I(t-d)] - n[I(t-d) - U]$$

where d indicates delay period, constant m (m>0), U is the depreciation rate of capital stock and $I(t-d) = L(t)$. On the basis of the above investment formula of business cycle, an SD-based model can be developed (fig.7).

Fig. 7. Stock and flow diagram in Kalecki's model



Source: Author's research based on D. Soto Torres, *A System Dynamics Approach to Kalecki's Model,* [in:] System Dynamics'93, *p*. 509.

where:

I=INTEGER(*m*·(*I-L*)/*d* - *n*·(*L-U*), *I*0),

K=INTEGER(*L-U*, *K*0)

L=DELAY FIXED(*I* , d)

M.Kalecki proved mathematically that the delay between investment decisions and the supply of investment goods results in cycle fluctuations. Computer simulations facilitate an easy presentation of the dynamic behavior of economic systems and confirm the impact of the investment delay on the development of cyclical fluctuations.

[15] M. Kalecki, *A Macrodynamic Theory of Business Cycles* [in:] Econometrica, vol.3, No. 3 (1935), p. 328.

Conclusion

Due to a combination of the formal methods of analysis and the intuitive instruments of modeling the structure and dynamics of systems, the SD method constitutes a perfect tool for modeling such a complex process as economic growth since it enables a simple presentation and comprehension of extremely complicated nonlinear relations. The understanding of process dynamics in fundamental theories of economic growth constitutes a solid basis for further research. The presented examples show the usefulness and advantages of such approach in effective learning and teaching.

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Abstract

The main aim of the article is to present the issues of modeling complex economic processes with the application of the Systems Dynamics (SD) method. Cognitive and application opportunities of this research approach are presented through the examples based on the economic growth modeling concepts of R.Harrod , R.Solow and Kalecki's theory of cyclical development.

The article presents the steps of developing the model; simulations of selected variables of the system were conducted and the results were analyzed. It points to the usefulness of modeling in economic decision making processes and the advantages that result from the application of SD approach in the areas of learning and teaching.