OPERATIONS RESEARCH AND DECISIONS

No. 1 2022
DOI: 10.37190/ord220101

LOGARITHMIC SIMILARITY MEASURES ON PYTHAGOREAN FUZZY SETS IN THE ADMISSION PROCESS

HARI DARSHAN ARORA, ANJALI NAITHANI*

Department of Mathematics, Amity University Uttar Pradesh, Noida-201313 (U.P.), India

The intuitionistic fuzzy sets (IFSs) have a more significant contribution to describing and dealing with uncertainty. The intuitionistic fuzzy measure is a significant consideration in the field of IFSs theory. However, Pythagorean fuzzy sets (PFSs) are an extension of the IFSs. PFSs are more capable of modelling uncertainties than IFSs in real-world decision-making scenarios. The majority of PFSs research has concentrated on establishing decision-making frameworks. A similarity measure is a key concept which measures the closeness of PFSs. IFSs-based similarity measures have been proposed in the literature. This type of similarity measure, however, has a drawback since it cannot satisfy the axiomatic definition of similarity by offering counter-intuitive examples. For this study, a similarity-based on logarithmic function for Pythagorean fuzzy sets (PFSs) is proposed as a solution to the problem. A decision-making approach is presented to ascertain the suitability of careers for aspirants. Additionally, numerical illustration is applied to determine the strength and validity of the proposed similarity measures. The application of the proposed similarity measures is also presented in this article. A comparison of the suggested measures with the existing ones is also demonstrated to ensure the reliability of the measures. The results show that the proposed similarity measures are efficient and reasonable from both numerical and realistic assessments.

Keywords: *intuitionistic fuzzy sets, Pythagorean fuzzy sets, similarity measures, weighted similarity measures, logarithmic function, decision-making*

1. Introduction

Decision-making is undoubtedly one of the most fundamental activities of human beings. The process of decision-making involves deciding (choosing or selecting) one or more alternatives (or action or solution) from the available (finite) set of alternatives, which satisfy a finite set of constraints (or criteria). Many real-life decision-making problems, such as pattern recognition, medical diagnosis, image processing, clustering,

*Corresponding author, email address: anathani@amity.edu
Received 10 September 2021, accepted 16 February 2022

etc., require some degree of analogy between the objects. This encourages the introduction of similarity measures and their practical applications. Because all the issues stated above deal with uncertainty, the concept of similarity measure is investigated in several generalised set-theoretical concepts that can deal with ambiguity.

The concept of fuzzy set theory, proposed by Zadeh [63], is extensively applied to prototype ambiguity which appears in real-life problems. In this concept, every aspect is measured with a membership grade between 0 and 1 that correspond to the limited communication, although the concept of fuzzy sets theory appears to be uncertain because of the exclusion of non-membership function and the disregard for the possibility of hesitation margin. Intuitionistic fuzzy sets (IFSs) proposed by Atanassov [1, 2] are believed to be an extension of a fuzzy set (FS). It considers non-membership grade (ζ) of a component besides membership grade (δ), with hesitation margin η to fulfil an inequality $\delta + \zeta \leq 1$ and $\delta + \zeta + \eta = 1$. There are instances in IFSs when $\delta + \zeta \geq 1$. Pythagorean fuzzy sets (PFSs) suggested by Yager [58, 59] play a vital role in extending this constraint by fulfilling the condition of $\delta + \zeta \leq 1$ or $\delta + \zeta \geq 1$ such that $\delta^2 + \zeta^2 + \eta^2 = 1$, where η is the degree of indeterminacy. PFSs have a larger accessible region than IFSs, allowing them to handle uncertainty more effectively and accurately.

As per our findings, most PFSs similarity functions are introduced as a generalisation of IFSs equivalents. On the other hand, the complex mathematical nature of some similarity measures restricts their usefulness and effectiveness as mathematical tools. Furthermore, none of the available measures could distinguish between uncertain PFSs with limited membership and non-membership grades. Such PFSs occur when there is simply insufficient knowledge or information about a system. As a result, dealing with such PFSs necessitates the use of an appropriate similarity measure. To fulfil the gaps, the goal of this paper is to suggest an innovative logarithmic similarity measure for PFSs in a simple mathematical form that can successfully handle extremely uncertain PFSs, while also being similarly efficient in other scenarios.

2. Literature review

For measuring the degree of association between two items, similarity measures are significant and valuable tools. IFSs give a comprehensive framework for elaborating uncertainty and ambiguity through similarity measures. IFSs appears to be useful in simulating a variety of real-world circumstances, including pattern recognition [22, 26, 66], medical diagnosis [20, 32, 46, 47], career determination [9], selection process [6], clustering [23, 50], and multi-criteria decision-making [3, 17, 55]. However, similarity measures for trigonometric function for FSs and IFSs are also presented [44, 48, 49].

PFSs initiated by Yager [58] and Yager and Abbasov [61] are regarded as useful tools for modelling circumstances when intuitionistic fuzzy sets are unable to consider all available data while making decisions. A Pythagorean fuzzy set is characterised by membership degree (δ), non-membership degree (ζ), and hesitancy degree (η) in such a way that the sum of the squares of each of the parameters is one. The description of similarity/distance measure between two objects is one of the most fascinating issues in PFSs theory. Similarity measures between two PFSs are significant because they have a range of applications in domains including multi-criteria decision-making [6, 7, 34, 66], pattern recognition [10, 33, 35, 62], and career placements [9]. Zeng et al. [64] propose several PFSs distance and similarity measures, which are then used in the MADM problem. Ullah et al. [52] create a set of T-spherical fuzzy set similarity measures and apply them to building material recognition challenges. Mahmood et al. [28, 29] and Ullah et al. [53] study MADM problems in bipolar-valued hesitant fuzzy settings, whereas Jan et al. [24] and Mahmood et al. [30] discuss medical diagnosis and MADM problems using T-spherical fuzzy and linguistic cubic fuzzy information. Peng and Selvachandran [36] establish a state-of-the-art study of PFSs, and Peng et al. [39] develop a study of PFSs information measures. Peng and Yang [37] present Choquet integrals for PFSs, whereas Peng and Yang [38] various aggregation operations for PFSs. Many authors, including Hussain and Yang [21], Garg [15], Ren et al. [45], Gou et al. [14], and Xiao and Ding [55] discuss distance, similarity measures, and divergence of PFSs.

To demonstrate that the proposed method has superior similarity identification and practicability, Xu et al. [57] establish a variation coefficient similarity measure based on the extension of the Dice and a cosine similarity measure. Zhang [65] originally introduced a new distance measure for PFSs and described its benefits and presented a simple and successful Pythagorean fuzzy group decision-making method.

Some formulae of Pythagorean fuzzy information measures on similarity measures and corresponding transformation relationships are also developed [39]. They study the association between the measure of distance, the measure of similarity, and the measure of entropy, and propose a transformation of these along with novel findings. Wei and Wei [54] and Mohd and Abdullah [31] establish similarity measures between PFSs based on the cosine function and apply this similarity and weighted similarity measures to PFSs for pattern recognition and medical diagnosis.

The idea of PFSs can be utilised to describe imprecise data more adequately and precisely than IFSs. Garg [16] introduces an improved ranking order interval-valued PFSs using the TOPSIS technique. Indeed, the hypothesis of PFSs has been widely considered, as demonstrated by various researchers [17, 27, 38]. In connection with the uses of PFSs, Rahman et al. [41] work on some aggregation operators on interval-valued PFSs and utilise it in the decision-making process. Rahman et al. [43] propose a few ways to deal with multi-attribute group decision-making. Overall, the possibility of PFSs has pulled incredible considerations from numerous researchers, and the idea has been applied to a few application regions, viz., aggregation operators, multi-criteria decision-making, information measures, and many more [25, 42, 60].

From a statistical standpoint, a new method for determining the correlation coefficient between PFSs is developed [11, 12]. In contrast to other existing correlation coefficient approaches in the Pythagorean fuzzy context, which only assesses the strength of relationship, the correlation coefficient obtained via this technique shows the strength of correlation between the Pythagorean fuzzy sets under consideration and indicates whether the Pythagorean fuzzy sets under consideration are related negatively or positively. By merging the principles of Pythagorean fuzzy deviation, variance, and covariance, Ejegwa et al. [13] offer several unique techniques of computing correlation between PFSs via the three characteristic parameters of PFSs. The criteria for selecting an air traffic control radar station that properly fulfils the job of the radar in air traffic management are developed [40]. Gergin et al. [19] propose a five-stage intuitionistic fuzzy multi-criteria decision-making (IFMCDM) model. To help the Libyan Iron and Steel Company, Badi and Pamucar [3] design a hybrid Grey theory-MARCOS technique for decision-making about supplier selection.

Because of the broader scope of PFSs applicability to solve real-life problems involving imprecision, we are driven to examine the efficiency of PFSs in the admission process in this paper. The purpose of this research is to investigate the concept of logarithmic similarity measures and to demonstrate how it may be applied to career placements based on academic performance using PFSs. The paper is organised as follows: a literature review is included in section 2, while some basic notions of FSs, IFSs, and PFSs are discussed in Section 3. Section 4 introduces logarithmic similarity measures and weighted similarity measures of the PFSs and their numerical computations to validate our measures. An application of proposed similarity measures is given in a hypothetical case study in Section 5. Section 6 compares the new logarithmic similarity measures with the existing similarity measure by an example. Finally, Section 7 summarises the article and provides recommendations for future research.

3. Preliminaries

In this section, we bring in some basic theories related to fuzzy sets, intuitionistic fuzzy sets, and Pythagorean fuzzy sets used in the outcome.

Definition 3.1 (Zadeh [63]). Let E be a nonempty set. A fuzzy set M in $E = \{x_1, x_2, ..., x_n)$ is characterised by a membership function

$$M = \{x, \delta_M(x) | x \in E\} \tag{1}$$

$\delta_M: E \to [0,1]$ is a measure of belongingness or degree of membership of an element $x \in E$ in *M*.

Definition 3.2 (Atanassov [1]). An IFS *M* in *E* is given by

$$M = \{x, \delta_M(x), \zeta_M(x) | x \in E\} \tag{2}$$

$\delta_{\mathrm{M}}, \zeta_{\mathrm{M}}: E \to [0,1]$, where $0 \le \delta_M(x) + \zeta_M(x) \le 1$, $\forall\ x \in E$. The number $\delta_M(x)$ and $\zeta_M(x)$ represent, respectively, the membership degree and non-membership degree of the element *x* to the set *M*. For each IFS *M* in *E*, if

$$\eta_M(x) = 1 - \delta_M(x) - \zeta_M(x), \quad \forall\ x \in E \tag{3}$$

Then, $\eta_M(x)$ is called the degree of indeterminacy of *x* to *N*. For convenience, Xu [56] denotes this pair as $M = (\delta_M, \zeta_M)$.

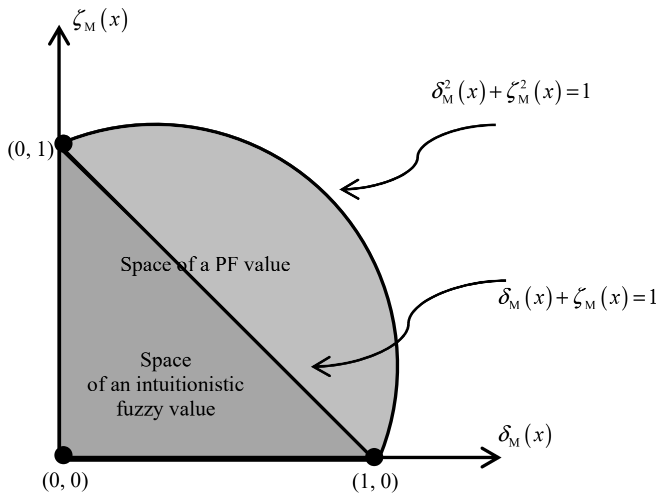

Fig. 1. Comparison of space of IFSs and PFSs [38]

Definition 3.3 (Yager [58]). An IFS $\tilde{A}$ in *X* is given by

$$M = \{x, \delta_M(x), \zeta_M(x) | x \in E\}, \quad \delta_M, \zeta_M: E \to [0,1]$$

with the condition that

$$0 \le \delta_M^2(x) + \zeta_M^2(x) \le 1 \tag{4}$$

and the degree of indeterminacy (η) for any PFSs M and $x \in E$ is given by

$$\eta_M(x) = \sqrt{1 - \delta_M^2(x) - \zeta_M^2(x)} \tag{5}$$

This distinction of imperative conditions gives a more extensive inclusion of evidence which can be shown in Fig. 1.

4. Similarity measures

4.1. Proposition 1 [6]

Let X be a nonempty set and $P, Q, R \in$ PFS (E). The similarity measure Sim between P and Q is a function $\mathrm{Sim}: PFS \times PFS \to [0, 1]$ that satisfies

P1. Boundedness: $0 \le \mathrm{Sim}(P, Q) \le 1$.

P2. Separability: $\mathrm{Sim}(P, Q) = 1 \Leftrightarrow P = Q$.

P3. Symmetric: $\mathrm{Sim}(P, Q) = \mathrm{Sim}(Q, P)$.

P4. Inequality: If R is a PFS in E and $P \subseteq Q \subseteq R$, then $\mathrm{Sim}(P, R) \le \mathrm{Sim}(P, Q)$ and $\mathrm{Sim}(P, R) \le \mathrm{Sim}(Q, R)$.

In several circumstances, the weight of the elements $x_i \in X$ must be considered. For instance, in decision-making, the attributes usually have distinct significance, and thus ought to be designated unique weights.

Keeping the concept of weights in mind, we also propose the following logarithmic similarity measures between P and Q, as follows:

Let $P, Q \in$ PFS (E) such that $E = \{x_1, x_2 ..., x_n\}$, then

$$S_{\mathrm{PFS}L1}(P, Q) = 1 - \frac{1}{2n \ln 2} \sum_{i=1}^{n} \ln\left(\left(1 + \left|\delta_P^2(x_i) - \delta_Q^2(x_i)\right|\right)\left(1 + \left|\zeta_P^2(x_i) - \zeta_Q^2(x_i)\right|\right)\right) \tag{6}$$

$$S_{\mathrm{PFS}L2}(P, Q) = 1 - \frac{1}{3n \ln 2} \sum_{i=1}^{n} \ln\left(\left(1 + \left|\delta_P^2(x_i) - \delta_Q^2(x_i)\right|\right)\left\{1 + \left|\zeta_P^2(x_i) - \zeta_Q^2(x_i)\right|\right\} \times \left(1 + \left|\eta_P^2(x_i) - \eta_Q^2(x_i)\right|\right)\right) \tag{7}$$

$$S_{\mathrm{WPFS}L1}(P, Q) = 1 - \frac{1}{2n \ln 2} \sum_{i=1}^{n} \omega_i \ln\left(\left(1 + \left|\delta_P^2(x_i) - \delta_Q^2(x_i)\right|\right)\left(1 + \left|\zeta_P^2(x_i) - \zeta_Q^2(x_i)\right|\right)\right) \quad (8)$$

$$\begin{aligned} S_{\mathrm{WPFS}L2}(P, Q) = 1 - \frac{1}{3n \ln 2} \sum_{i=1}^{n} \omega_i \ln\Big(&\left(1 + \left|\delta_P^2(x_i) - \delta_Q^2(x_i)\right|\right)\left(1 + \left|\zeta_P^2(x_i) - \zeta_Q^2(x_i)\right|\right) \\ &\times\left(1 + \left|\delta_P^2(x_i) - \delta_Q^2(x_i)\right|\right)\left(1 + \left|\zeta_P^2(x_i) - \zeta_Q^2(x_i)\right|\right)\Big) \end{aligned} \quad (9)$$

where

$$\eta_P(x_i) = \sqrt{1 - \delta_P^2(x_i) - \zeta_P^2(x_i)}, \ \eta_Q(x_i) = \sqrt{1 - \delta_Q^2(x_i) - \zeta_Q^2(x_i)}, \ \omega = (\omega_1, \omega_2, ..., \omega_n)^T$$

is the weight vector of $x_i\,(i = 1, 2, ..., n)$, with $\omega_k \in [0, 1]$, $k = 1, 2, ..., n, \sum_{k=1}^{n} \omega_k = 1$. If $\omega = \left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right)^T$, then the weighted logarithmic similarity measure reduces to proposed logarithmic similarity measures, i.e., if we take $\omega_k = 1$, $k = 1, 2, ..., n$, then $S_{\mathrm{WPFS}L1}(P, Q) = S_{\mathrm{PFS}L1}(P, Q)$. Similarly, it can be verified that $S_{\mathrm{WPFS}L2}(P, Q) = S_{PFSL2}(P,Q)$.

Theorem 4.1. The Pythagorean fuzzy logarithmic similarity measures $S_{\mathrm{PFS}L}(P, Q)$ and $S_{WPFSL}(P,Q)$ defined in equations (6) - (9) are valid measures of Pythagorean fuzzy similarity.

Proof. All the necessary four conditions to be a similarity measure are satisfied by the new similarity measures as follows:

P1. Boundedness: $0 \le S_{\mathrm{PFS}L1}(P, Q), S_{\mathrm{PFS}L2}(P, Q) \le 1$

Proof. For $S_{\mathrm{PFS}L1}(P, Q)$: As

$$0 \le \left|\delta_P^2(x_i) - \delta_Q^2(x_i)\right| \le 1 \text{ or } 1 \le 1 + \left|\delta_P^2(x_i) - \delta_Q^2(x_i)\right| \le 2$$

and also,

$$1 \le 1 + \left|\zeta_P^2(x_i) - \zeta_Q^2(x_i)\right| \le 2$$

therefore,

$$1 \le \left(1+\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|\right)\left(1+\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|\right) \le 4$$
$$\Rightarrow 0 \le \ln\left(\left(1+\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|\right)\left(1+\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|\right)\right) \le 2\ln 2$$
$$\Rightarrow 0 \le \frac{1}{2n\ln 2}\sum_{i=1}^{n}\ln\left(\left(1+\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|\right)\left(1+\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|\right)\right) \le 1$$
$$\Rightarrow 0 \le 1-\frac{1}{2n\ln 2}\sum_{i=1}^{n}\ln\left(\left(1+\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|\right)\left(1+\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|\right)\right) \le 1$$

Thus, $0 \le S_{\mathrm{PFS}L1}(P, Q) \le 1.$

Measure $S_{\mathrm{PFS}L2}(P,Q)$ can be proved similarly.

P2. Separability: $S_{PFSL1}(P,Q), S_{PFSL2}(P,Q)=1 \Leftrightarrow P=Q$.

Proof. For $S_{\mathrm{PFST1}}(P, Q)$: For two PFSs P and Q in $X=\{x_1, x_2, ..., x_n\}$, if $P=Q$, then $\delta_P^2(x_i)=\delta_Q^2(x_i)$ and $\zeta_P^2(x_i)=\zeta_Q^2(x_i)$.

Thus,

$$\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|=0 \text{ and } \left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|=0$$
$$\Rightarrow\left(1+\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|\right)\left(1+\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|\right)=1 \Rightarrow In1=0$$
$$\Rightarrow\frac{1}{2n\ln 2}\sum_{i=1}^{n}\ln 1=0\Rightarrow 1-\frac{1}{2n\ln 2}\sum_{i=1}^{n}\ln 1=1$$

Therefore,

$$S_{PFSL1}(P,Q)=1. \text{ If } S_{PFST1}(P,Q)=1,$$

then this implies

$$\frac{1}{2n\ln 2}\sum_{i=1}^{n}\ln\left(\left(1+\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|\right)\left(1+\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|\right)\right)=0$$
$$\Rightarrow\left(1+\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|\right)\left(1+\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|\right)=1$$
$$\Rightarrow\left(1+\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|\right)=1, \left(1+\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|\right)=1$$

Either $\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|=0$ or $\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|=0$. Therefore, $\delta_P^2(x_i)=\delta_Q^2(x_i)$ and $\zeta_P^2(x_i)=\zeta_Q^2(x_i)$. Hence $P = Q$.

Measure $S_{\mathrm{PFS}L2}(P, Q)$ can be proved similarly.

P3. Symmetric: $S_{\mathrm{PFS}L1}(P, Q)=S_{\mathrm{PFS}L1}(Q, P)$ and $S_{\mathrm{PFS}L2}(P, Q)=S_{\mathrm{PFS}L2}(Q, P)$. Proofs are self-explanatory and straightforward.

P4. Inequality: If R is a PFS in X and $P\subseteq Q\subseteq R$, then

$$S_{\mathrm{PFS}T1}(P, R)\le S_{\mathrm{PFS}T1}(P, Q),\ S_{\mathrm{PFS}T2}(P, R)\le S_{\mathrm{PFS}T2}(Q, R)$$

and

$$S_{\mathrm{PFS}T2}(P, R)\le S_{\mathrm{PFS}T2}(P, Q),\ S_{\mathrm{PFS}T2}(P, R)\le S_{\mathrm{PFS}T2}(Q, R)$$

Proof. For $S_{\mathrm{PFS}T1}(P, Q)$: If $P\subseteq Q\subseteq R$, then for $x_i \in$ X, we have

$$0\le\delta_P(x_i)\le\delta_Q(x_i)\le\delta_R(x_i)\le 1 \text{ and } 1\ge\zeta_P(x_i)\ge\zeta_Q(x_i)\ge\zeta_R(x_i)\ge 0$$

This implies that

$$0\le\delta_P^2(x_i)\le\delta_Q^2(x_i)\le\delta_R^2(x_i)\le 1 \text{ and } 1\ge\zeta_P^2(x_i)\ge\zeta_Q^2(x_i)\ge\zeta_R^2(x_i)\ge 0$$

Thus, we have

$$\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|\le\left|\delta_P^2(x_i)-\delta_R^2(x_i)\right|,\ \left|\delta_Q^2(x_i)-\delta_R^2(x_i)\right|\le\left|\delta_P^2(x_i)-\delta_R^2(x_i)\right|$$

and

$$\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|\le\left|\zeta_P^2(x_i)-\zeta_R^2(x_i)\right|,\ \left|\zeta_Q^2(x_i)-\zeta_R^2(x_i)\right|\le\left|\zeta_P^2(x_i)-\zeta_R^2(x_i)\right|\ ;$$

From the above we can write

$$1+\left(\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|\right)\left(\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|\right)\le 1+\left(\left|\delta_P^2(x_i)-\delta_R^2(x_i)\right|\right)\left(\left|\zeta_P^2(x_i)-\zeta_R^2(x_i)\right|\right)$$

$$\Rightarrow \frac{1}{2n\ln 2}\sum_{i=1}^{n}\ln\left(\left(1+\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|\right)\left(1+\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|\right)\right)$$
$$\leq \frac{1}{2n\ln 2}\sum_{i=1}^{n}\ln\left(\left(1+\left|\delta_P^2(x_i)-\delta_R^2(x_i)\right|\right)\left(1+\left|\zeta_P^2(x_i)-\zeta_R^2(x_i)\right|\right)\right)$$
$$\Rightarrow 1-\frac{1}{2n\ln 2}\sum_{i=1}^{n}\ln\left(\left(1+\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|\right)\left(1+\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|\right)\right)$$
$$\leq 1-\frac{1}{2n\ln 2}\sum_{i=1}^{n}\ln\left(\left(1+\left|\delta_P^2(x_i)-\delta_R^2(x_i)\right|\right)\left(1+\left|\zeta_P^2(x_i)-\zeta_R^2(x_i)\right|\right)\right)$$
$$\Rightarrow S_{\mathrm{PFS}L1}(P, R)\leq S_{\mathrm{PFS}L1}(P, Q) \text{ similarly } S_{\mathrm{PFS}L1}(P, R)\leq S_{\mathrm{PFS}L1}(Q, R)$$

Similar proofs can be made for $S_{\mathrm{PFS}L2}(P, R)\leq S_{\mathrm{PFS}L2}(P, Q)$ and $S_{\mathrm{PFS}L2}(P, R) \leq S_{\mathrm{PFS}L2}(Q, R)$. Analogous to the proofs done above, we can also validate properties depicted in Preposition 1 for weighted similarity measures $S_{\mathrm{WPFS}L1}(P, Q)$ and $S_{\mathrm{WPFS}L2}(P, Q)$ accordingly.

4.2. Numerical verification of the similarity measures

Based on the parameters suggested by Wei and Wei [54], we verify whether the proposed similarity measures satisfy the above four properties:

Example 1. Let $P, Q, R\in \mathrm{PFS}(X)$ for $X=\{x_1, x_2, x_3\}$. Suppose
$P=\{x_1, 0.6, 0.2, x_2, 0.4, 0.6, x_3, 0.5, 0.3\}$,
$Q=\{x_1, 0.8, 0.2, x_2, 0.7, 0.3, x_3, 0.6, 0.3\}$,
$R=\{x_1, 0.9, 0.1, x_2, 0.8, 0.2, x_3, 0.7, 0.1\}$.

The values for the proposed similarity measure are as follows:

$$\begin{aligned} S_{\mathrm{PFS}L1}(P, Q)=1-\frac{1}{6\ln 2}\Big(&\ln\left(\left(1+\left|0.6^2-0.8^2\right|\right)\left(1+\left|0.2^2-0.2^2\right|\right)\right)\\ &+\ln\left\{\left(1+\left|0.4^2-0.7^2\right|\right)\left(1+\left|0.6^2-0.3^2\right|\right)\right\}\\ &+\ln\left(\left(1+\left|0.5^2-0.6^2\right|\right)\left(1+\left|0.3^2-0.3^2\right|\right)\right)\Big)\\ &=1-\frac{1}{6\ln 2}(0.24686+0.5241958+0.10436)=1-\frac{0.8754158}{4.158883}=0.7895069\end{aligned}$$

$$S_{\mathrm{PFS}L1}(P, R) = 1 - \frac{1}{6\ln 2}\Big(\ln\big\{\big(1+\left|0.6^2 - 0.9^2\right|\big)\big(1+\left|0.2^2 - 0.1^2\right|\big)\big\}$$
$$+\ln\Big(\big(1+\left|0.4^2 - 0.8^2\right|\big)\big(1+\left|0.6^2 - 0.2^2\right|\big)\Big)$$
$$+\ln\Big(\big(1+\left|0.5^2 - 0.7^2\right|\big)\big(1+\left|0.3^2 - 0.1^2\right|\big)\Big)\Big)$$
$$= 1 - \frac{1}{6\ln 2}(0.40112 + 0.66967 + 0.29207) = 1 - \frac{1.3628685}{4.158883} = 0.6722993$$

$$S_{\mathrm{PFS}L1}(Q, R) = 1 - \frac{1}{6\ln 2}\Big(\ln\Big(\big(1+\left|0.8^2 - 0.9^2\right|\big)\big(1+\left|0.2^2 - 0.1^2\right|\big)\Big)$$
$$+\ln\Big(\big(1+\left|0.7^2 - 0.8^2\right|\big)\big(1+\left|0.3^2 - 0.2^2\right|\big)\Big)$$
$$+\ln\Big(\big(1+\left|0.6^2 - 0.7^2\right|\big)\big(1+\left|0.3^2 - 0.1^2\right|\big)\Big)\Big)$$
$$= 1 - \frac{1}{6\ln 2}(0.18656 + 0.18855 + 0.19917) = 1 - \frac{0.5742927}{4.158883} = 0.86191165$$

The detailed computation for the proposed measures is summarised in Table 1.

Table 1. Numerical illustration to validate proposed measures

Proposed measure 1	Numerical value	Proposed measure 2	Numerical value
$S_{\mathrm{PFS}L1}(P, Q)$	0.789507	$S_{\mathrm{PFS}L2}(P, Q)$	0.79403
$S_{\mathrm{PFS}L1}(P, R)$	0.672299	$S_{\mathrm{PFS}L2}(P, R)$	0.67774
$S_{\mathrm{PFS}L1}(Q, R)$	0.861912	$S_{\mathrm{PFS}L2}(Q, R)$	0.863838
Proposed measure 3	Numerical value	Proposed measure 4	Numerical value
$S_{\mathrm{WPFS}L1}(P, Q)$	0.92749	$S_{\mathrm{WPFS}L2}(P, Q)$	0.925726
$S_{\mathrm{WPFS}L1}(P, R)$	0.889423	$S_{\mathrm{WPFS}L2}(P, R)$	0.886281
$S_{\mathrm{WPFS}L1}(Q, R)$	0.954391	$S_{\mathrm{WPFS}L2}(Q, R)$	0.952944

Numerical justification. From the above computations, it supports that

P1. $0 \le S_{\mathrm{PFS}Lj}(P, Q),\ 0 \le S_{\mathrm{WPFS}Lj}(P, Q) \le 1,\ j = 1, 2.$

P2. $S_{\mathrm{PFS}Lj}(P, Q),\ S_{\mathrm{WPFS}Lj}(P, Q) = 1 \Leftrightarrow P = Q,\ j = 1, 2.$

P3. It follows that

$S_{\mathrm{PFS}Lj}(P, Q) = S_{\mathrm{PFS}Lj}(Q, P)$ and $S_{\mathrm{WPFS}Lj}(P, Q) = S_{WPFSLj}(Q,P)\ \ j = 1, 2.$

(use of square and absolute value)

P4. $S_{\mathrm{PFS}Lj}(P, R) \le S_{\mathrm{PFS}Tj}(P, Q)$ and $S_{\mathrm{PFS}Lj}(P, R) \le S_{\mathrm{PFS}Lj}(Q, R)$. Also,

$$S_{\mathrm{WPFS}Lj}(P, R) \leq S_{\mathrm{WPFS}Lj}(P, Q) \text{ and } S_{WPFSLj}(P, R) \leq S_{WPFSLj}(Q, R) \ \ \forall \ j = 1, 2.$$

5. Case study

Similarity analysis has been studied and applied widely in a variety of domains. This section demonstrates the validity of the similarity measures for PFSs provided in Section 4 by presenting a specific branch of engineering approach to decision-making.

Example 2. Suppose there exists a set of candidates C = {Alia (C_1), Priyanka (C_2), Salman (C_3), Deepika (C_4), Madhuri (C_5), Ranbir (C_6)}, S = {Mathematics (S_1), Physics (S_2), Chemistry (S_3)} be the set of subjects related to the performance of subject in 12th class and entrance examination, and suppose that there is a sample to be recognised as major engineering branches in India as B = {Computer Science Engineering – CSE (B_1), Information Technology – IT (B_2), Electronics and Communication Engineering – ECE (B_3), Electrical and Electronics Engineering – EEE (B_4), Mechanical Engineering/Civil Engineering – ME/CE (B_5). Further, we consider the weights of these subjects as 0.5, 0.3 and 0.2, respectively.

Let R_1 be the relation between applicants and subjects where represents the degree to which the candidate C clears the related subject prerequisites, whereas ζ represents the degree to which the candidate C does not clear the related subject prerequisites. Similarly, let R_2 be the relation between branches of engineering and subjects, where δ represents the degree to which the concerned subject prerequisites (S) which decide the branch of engineering (B) and ζ represents the degree to which the concerned subject prerequisites (S) which do not decide the branch of engineering (B). In the PFSs context, a simple and useful algorithm is proposed as follows:

Step 1. Represent the expert's viewpoints about the subjects and candidates such that the first entries are membership values, which represent the Pythagorean fuzzy values of the marks assigned to the questions that the applicants answered, and the second entries are non-membership values which represent the Pythagorean fuzzy values of the marks assigned to the questions that the applicants failed. To make computations easier, convert these values to PFSs. Here, C = {Alia (C_1), Priyanka (C_2), Salman (C_3), Deepika (C_4), Madhuri (C_5), Ranbir (C_6)}, is the set of candidates and S = {Mathematics (S_1), Physics (S_2), Chemistry (S_3)} the set of subjects related to the performance of subject in 12th class.

Step 2. Construct the relation between various branches of engineering available and subjects based on prerequisites where δ represents the degree to which the concerned subject S decides the branch of engineering, whereas ζ represents the degree to which the concerned subject S does not decide the branch of engineering.

Step 3. Construct the similarity relation candidate and branches for $S_{\mathrm{PFS}L1}$, $S_{\mathrm{PFS}L2}$, S_{WPFSL1}, and S_{WPFS2} using equations (6)–(9).

Step 4. Validate the results through comparative analysis of the proposed logarithmic similarity with the existing measures. The data are shown in Tables 2 and 3.

Table 2. The relation R_1 between candidates and their subjects

R_1	S_1	S_2	S_3
C_1	(0.5, 0.4)	(0.5, 0.3)	(0.5, 0.5)
C_2	(0.6, 0.3)	(0.4, 0.5)	(0.7, 0.2)
C_3	(0.7, 0.2)	(0.5, 0.4)	(0.4, 0.5)
C_4	(0.8, 0.2)	(0.6, 0.3)	(0.5, 0.3)
C_5	(0.7, 0.2)	(0.7, 0.1)	(0.6, 0.1)
C_6	(0.2, 0.3)	(0.4, 0.1)	(0.5, 0.2)

Table 3. The R_3 relation between subjects and the branches (disciplines)

R_2	B_1	B_2	B_3	B_4	B_5
S_1	(0.7, 0.2)	(0.8, 0.1)	(0.5, 0.3)	(0.5, 0.4)	(0.5, 0.3)
S_2	(0.6, 0.3)	(0.5, 0.2)	(0.5, 0.4)	(0.6, 0.3)	(0.6, 0.2)
S_3	(0.8, 0.2)	(0.7, 0.2)	(0.7, 0.3)	(0.8, 0.2)	(0.7, 0.2)

We now determine the degree of similarity between C and B using logarithmic similarity measures suggested in equations (6)–(9). The obtained measure values are presented in Tables 4–7.

Table 4. The relation between the candidate and the branches for $S_{\mathrm{PFS}L1}(P, Q)$

$S_{PFSL1}(P, Q)$	C_1	C_2	C_3	C_4	C_5	C_6
B_1	0.7709	0.8457	0.8185	0.8754	0.8856	0.7574
B_2	0.7779\	0.8555	0.8176	0.8926	0.8639	0.7889
B_3	0.8800	0.9217	0.8322	0.8160	0.8033	0.8363
B_4	0.8498	0.8455	0.7395	0.8026	0.8066	0.7963
B_5	0.8493	0.8852	0.7697	0.8339	0.8635	0.8514

Table 5. The relation between the candidate and the branches for $S_{\mathrm{PFS}L2}(P, Q)$

$S_{\mathrm{PFS}L2}(P, Q)$	C_1	C_2	C_3	C_4	C_5	C_6
B_1	0.7858	0.8561	0.8344	0.8476	0.8726	0.6919
B_2	0.8049	0.8563	0.8239	0.8719	0.8367	0.73955
B_3	0.8860	0.9232	0.8351	0.7896	0.7966	0.7850
B_4	0.8566	0.8620	0.7636	0.7832	0.8018	0.7323
B_5	0.8746	0.9051	0.7988	0.8066	0.8420	0.8027

Table 6. The relation between the candidate and the branches for $S_{\mathrm{WPFS}L1}(P, Q)$

$S_{\mathrm{WPFS}L1}(P, Q)$	C_1	C_2	C_3	C_4	C_5	C_6
B_1	0.9279	0.9488	0.9595	0.9650	0.9723	0.9149
B_2	0.9205	0.9411	0.9485	0.9727	0.9546	0.9155
B_3	0.9695	0.9726	0.9474	0.9317	0.9330	0.9480
B_4	0.9674	0.9487	0.9200	0.9286	0.9328	0.9344
B_5	0.9613	0.9605	0.9296	0.9383	0.9500	0.9514

Table 7. The relation between the candidate and the branches for $S_{\mathrm{WPFS}L2}(P, Q)$

$S_{\mathrm{WPFS}L2}(P, Q)$	C_1	C_2	C_3	C_4	C_5	C_6
B_1	0.9325	0.9533	0.9634	0.9560	0.9705	0.8938
B_2	0.9265	0.9406	0.9475	0.9667	0.9467	0.8977
B_3	0.9685	0.9718	0.9459	0.9222	0.9311	0.9316
B_4	0.9679	0.9563	0.9280	0.9238	0.9351	0.9140
B_5	0.9650	0.9648	0.9350	0.9275	0.9433	0.9355

• Taking into account the numerical computations of the tables above for the logarithmic similarity measures, and for 1, 2, ..., 6 = 1, 2, ..., 5, Alia, Priyanka, and Salman opted for ECE branch; Deepika opted for IT branch; Madhuri-CSE, however, Ranvir opted for ME/CE (Tables 4 and 5).

• For the measures and for 1, 2, ..., 6 = 1, 2, ..., 5, Alia and Priyanka opted for ECE branch, Salman and Madhuri – CSE; Deepika – IT and Ranvir opted for ME/CE branch (Tables 6 and 7).

This analysis is done because the higher value of the candidates against every similarity measure demonstrates the greater likelihood of having the option to choose the branch.

6. Comparative study

To demonstrate the dominance of the proposed logarithmic similarity measures, a comparison following Ullah [51] between the proposed similarity measures and the existing similarity measures is conducted, as based on the suggested numerical cases. We first demonstrate some existing similarity measures for the sake of comparison as shown in Table 8. The comparison demonstratedcan also be expressed graphically (Fig. 2).

Table 8. Similarity measures proposed by various authors

<table>
<tr><th>Measure</th><th>Reference</th></tr>
<tr><td>$\mathrm{Sim}^1(P,Q)=1-\frac{1}{n}\sum_{i=1}^{n}\omega_i\left(\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|\vee\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|\right)$</td><td>[39]</td></tr>
<tr><td>$\mathrm{Sim}^2(P,Q)=\frac{1}{n}\sum_{i=1}^{n}\cos\left(\frac{\pi}{2}\left(\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|\vee\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|\right)\right)$
$\mathrm{Sim}^3(P,Q)=\frac{1}{n}\sum_{i=1}^{n}\cos\left(\frac{\pi}{4}\left(\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|+\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|\right)\right)$
$\mathrm{Sim}^4(P,Q)=\frac{1}{n}\sum_{i=1}^{n}w_i\cos\left(\frac{\pi}{2}\left(\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|\vee\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|\right)\right)$
$\mathrm{Sim}^5(P,Q)=\frac{1}{n}\sum_{i=1}^{n}w_i\cos\left(\frac{\pi}{4}\left(\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|+\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|\right)\right)$</td><td>[54]</td></tr>
<tr><td>$\mathrm{Sim}^9(P,Q)=\frac{1}{n}\sum_{i=1}^{n}\left[2^{1-\left(\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|\vee\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|\right)}-1\right]$
$\mathrm{Sim}^{10}(P,Q)=\frac{1}{n}\sum_{i=1}^{n}\left[2^{1-\frac{1}{2}\left(\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|+\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|\right)}-1\right]$
$\mathrm{Sim}^{11}(P,Q)=\frac{1}{n}\sum_{i=1}^{n}\left[2^{1-\left(\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|\vee\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|\vee\left|\eta_P^2(x_i)-\eta_Q^2(x_i)\right|\right)}-1\right]$
$\mathrm{Sim}^{12}(P,Q)=\frac{1}{n}\sum_{i=1}^{n}\left[2^{1-\frac{1}{2}\left(\left|\delta_P^2(x_i)-\delta_Q^2(x_i)\right|+\left|\zeta_P^2(x_i)-\zeta_Q^2(x_i)\right|+\left|\eta_P^2(x_i)-\eta_Q^2(x_i)\right|\right)}-1\right]$</td><td>[68]</td></tr>
</table>

Table 9. Comparison of existing measures
with the proposed similarity measures

<table>
<tr><th>Sim</th><th>C_1</th><th>C_2</th><th>C_3</th><th>C_4</th><th>C_5</th><th>C_6</th></tr>
<tr><td>$\mathrm{Sim}^1(P,Q)$</td><td colspan="2" rowspan="12">ECE</td><td>ECE</td><td rowspan="12">IT</td><td>CSE</td><td>ME/CE</td></tr>
<tr><td>$\mathrm{Sim}^2(P,Q)$</td><td>IT</td><td>ME/CE</td><td>ECE</td></tr>
<tr><td>$\mathrm{Sim}^3(P,Q)$</td><td>ECE</td><td>IT</td><td>ME/CE</td></tr>
<tr><td>$\mathrm{Sim}^4(P,Q)$</td><td rowspan="2">IT</td><td rowspan="3">CSE</td><td>ECE</td></tr>
<tr><td>$\mathrm{Sim}^5(P,Q)$</td><td rowspan="4">ME/CE</td></tr>
<tr><td>$\mathrm{Sim}^6(P,Q)$</td><td rowspan="3">ECE</td></tr>
<tr><td>$\mathrm{Sim}^7(P,Q)$</td><td>ME/CE</td></tr>
<tr><td>$\mathrm{Sim}^8(P,Q)$</td><td rowspan="5">CSE</td></tr>
<tr><td>$\mathrm{Sim}^9(P,Q)$</td><td>CSE</td><td>ECE</td></tr>
<tr><td>$\mathrm{Sim}^{10}(P,Q)$</td><td>ECE</td><td rowspan="3">ME/CE</td></tr>
<tr><td>$\mathrm{Sim}^{11}(P,Q)$</td><td rowspan="2">CSE</td></tr>
<tr><td>$\mathrm{Sim}^{12}(P,Q)$</td></tr>
</table>

Table 9. Comparison of existing measures
with the proposed similarity measures

<table>
<tr><th>Sim</th><th>C_1</th><th>C_2</th><th>C_3</th><th>C_4</th><th>C_5</th><th>C_6</th></tr>
<tr><td>$S_{\mathrm{PFS}L1}(P,Q)$</td><td colspan="2" rowspan="4">ECE</td><td rowspan="2">ECE</td><td rowspan="4">IT</td><td rowspan="4">CSE</td><td rowspan="4">ME/CE</td></tr>
<tr><td>$S_{\mathrm{PFS}L2}(P,Q)$</td></tr>
<tr><td>$S_{\mathrm{WPFS}L1}(P,Q)$</td><td rowspan="2">CSE</td></tr>
<tr><td>$S_{\mathrm{WPFS}L2}(P,Q)$</td></tr>
</table>

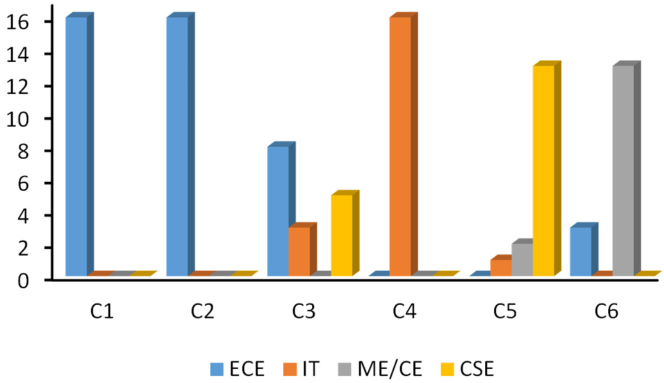

Fig. 2. Comparison of existing similarity measures with the proposed measures

Table 9 represents a comprehensive evaluation of the logarithmic similarity measures for PFSs on some common data sets displayed in Table 2 and Table 3. From the numerical results presented in Table 9, a comparison is done between the similarity measures proposed by the authors shown in Table 8 and the results attained using our proposed similarity measures for PFSs. It is noticed that the results obtained by using our proposed similarity measures are analogous with the existing measures.

7. Conclusion

In the concept of PFSs, how to measure the similarity precisely and accurately is still an open issue, which may lead to chaos in decision-making. In recent times, numerous similarity measures have been established for measuring the level of similarity between PFSs. Nevertheless, it appears that there have been no examinations of similarity measures based of logarithmic function for PFSs. To solve this problem, in this article, a completely new method of logarithmic similarity measures and weighted similarity measures which comply with the conventional parameters of PFSs that meets all the axioms of similarity requirements is proposed. The key feature of the newly proposed method is that it considers the index of hesitance and distributes its mass among membership and non-membership in a suitable manner, hence enhancing the function of membership and non-membership in producing similarity between PFSs. In addition, the decline in the hesitancy grade is critical in reducing the ambiguity in PFSs. We confirmed the credibility of the proposed similarity measures through numerical computations as well. Further, we employed these similarity measures for the application of career growth in engineering under the university admission procedure. Recommended PFSs for similarity measures are a significant device to address the vulnerabilities in the data in a more productive way when contrasted with the other existing sets. The proposed method yields far more realistic findings than some earlier methods, as it is more practical and corresponds to intuitive judgments. When compared to other methods, the suggested method determines the effectiveness following those generated by other methods that validate the newly proposed method in practical applications. It has been seen that the proposed similarity measures cannot be used to problems when information is in a T-spherical fuzzy environment. Therefore, these intended measures can be applied to medical diagnosis, complex decision-making, T-Spherical fuzzy environment, Neutrosophic fuzzy sets, and risk analysis in the future course of action.

Acknowledgements

The authors express their gratitude to the reviewers for their valuable suggestions which helped us in improving the quality of the paper to a great extent.

References

[1] ATANASSOV K.T., *Intuitionistic fuzzy sets*, Fuzzy Sets Syst., 1986, 20 (1), 87–96.
[2] ATANASSOV K.T., *More on intuitionistic fuzzy sets*, Fuzzy Sets Syst., 1989, 33 (1), 37–45.
[3] BADI I., PAMUCAR D., *Supplier selection for steelmaking company by using combined Grey–Marcos methods*, Dec. Mak. Appl. Manage. Eng., 2020, 3 (2), 37–48.
[4] CHEN S.-M., CHENG S.-H., LAN T.-C., *Multicriteria decision-making based on the TOPSIS method and similarity measures between intuitionistic fuzzy values*, Inf. Sci., 2016, 367–368, 279–295.
[5] EJEGWA P.A., *Intuitionistic fuzzy sets approach in appointment of positions in an organization via max–min–max rule*, Glob. J. Sci. Front. Res. Math. Dec. Sci., 2015, 15 (6), 1–6.
[6] EJEGWA P.A., *Distance and similarity measures for Pythagorean fuzzy sets*, Gran. Comp., 2020, 2, 1–17.
[7] EJEGWA P.A., *New similarity measures for Pythagorean fuzzy sets with applications*, Int. J. Fuzzy Comp. Model., 2020, 3 (1), 1–17.
[8] EJEGWA P.A., AKUBO A.J., JOSHUA O.M., *Intuitionistic fuzzzy sets in career determination*, J. Inf. Comp. Sci., 2014, 9 (4), 285–288.
[9] EJEGWA P.A., *Pythagorean fuzzy set and its application in career placements based on academic performance using max-min–max composition*, Comp. Int. Syst., 2019, 5 (2), 165–175.
[10] EJEGWA P.A, AWOLOLA J.A., *Novel distance measures for Pythagorean fuzzy sets with applications to pattern recognition problems*, Granul Comput., 2021, 6 (1), 181–189.

[11] EJEGWA P.A., FENG Y., WEN S., ZHANG W., *Determination of pattern recognition problems based on a pythagorean fuzzy correlation measure from statistical viewpoint*, 13th Int. Conf. Advanced Computational Intelligence (ICACI), 2021, 2021, 132–139.

[12] EJEGWA P., WEN S., FENG Y., ZHANG W., TANG N., *Novel Pythagorean fuzzy correlation measures via Pythagorean fuzzy deviation, variance and covariance with applications to pattern recognition and career placement*, IEEE Transactions on Fuzzy Systems (early access), DOI: 10.1109/TFUZZ.2021.3063794.

[13] EJEGWA P.A., WEN S., FENG Y., ZHANG W., CHEN J., *Some new Pythagorean fuzzy correlation techniques via statistical viewpoint with applications to decision-making problems*, J. Int. Fuzzy Syst., 2021, 40 (5), 9873–9886.

[14] GOU X., XU Z., REN P., *The properties of continuous Pythagorean fuzzy information*, Int. J. Int. Syst., 2016, 31 (5), 401–424.

[15] GARG H., *A new generalized Pythagorean fuzzy information aggregation using Einstein operations and its application to decision-making*, Int. J. Int. Syst., 2016, 31 (9), 886–920.

[16] GARG H., *Generalized Pythagorean fuzzy geometric aggregation operators using Einstein t-norm and t-conorm for multicriteria decision-making process*, Int. J. Int. Syst., 2017, 32 (6), 597–630.

[17] GARG H., *A new exponential operational law and their aggregation operators of interval-valued Pythagorean fuzzy information*, Int. J. Int. Syst., 2018, 33 (3), 653–683.

[18] GARG H., KUMAR K., *An advance study on the similarity measures of intuitionistic fuzzy sets based on the set pair analysis theory and their application in decision-making*, Soft Comp., 2018, 22 (15), 4959–4970.

[19] GERGIN R.E., PEKER İ., GÖK KISA A.C., *Supplier selection by integrated IFDEMATEL-IFTOPSIS method. A case study of automotive supply industry*, Dec. Mak. Appl. Manage. Eng., 2021, DOI: 10.31181/dmame211221075g.

[20] HUNG K.-C., WANG P.-K., *An integrated intuitionistic fuzzy similarity measures for medical problems*, Int. J. Comp. Int. Syst., 2014, 7 (2), 327–343.

[21] HUSSIAN Z., YANG M.S., *Distance and similarity measures of Pythagorean fuzzy sets based on the Hausdorff metric with application to fuzzy TOPSIS*, Int. J. Int. Syst., 2019, 34 (10), 2633–2654.

[22] HWANG C.-M., YANG M.-S., HUNG W.-L., LEE M.-G., *A similarity measure of intuitionistic fuzzy sets based on the Sugeno integral with its application to pattern recognition*, Inf. Sci., 2012, 189, 93–109.

[23] HWANG C.M., YANG M.S., HUNG W.L., *New similarity measures of intuitionistic fuzzy sets based on the Jaccard index with its application to clustering*, Int. J. Int. Syst., 2018, 33 (8), 1672–1688.

[24] JAN N., ZEDAM L., MAHMOOD T., ULLAH K., ALI Z., *Multiple attribute decision-making method under linguistic cubic information*, J. Int. Fuzzy Syst., 2018, 36 (1), 253–269.

[25] KHAN M.S.A., ABDULLAH S., ALI A., AMIN F., *Pythagorean fuzzy prioritized aggregation operators and their application to multi attribute group decision-making*, Gran. Comp., 2019, 4 (2), 249–263.

[26] DENGFENG L., CHUNTIAN C., *New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions*, Patt. Rec. Lett., 2002, 23 (1–3), 221–225.

[27] LIANG D., XU Z., *The new extension of TOPSIS method for multiple criteria-decision-making with hesitant Pythagorean fuzzy sets*, Appl. Soft Comp., 2017, 60, 167–179.

[28] MAHMOOD T., ULLAH K., ULLAH M., JAN N., DELI I., *Some aggregation operators for bipolar-valued hesitant fuzzy information based on einstein operational laws*, J. Eng. Appl. Sci., 2017, 36 (2), 63–72.

[29] MAHMOOD T., ULLAH K., KHAN Q., *Some aggregation operators for bipolar-valued hesitant fuzzy information*, J. Fund. Appl. Sci., 2018, 10 (4S), 240–245.

[30] MAHMOOD T., ULLAH K., KHAN Q., JAN N., *An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets*, Neural Comp. Appl., 2019, 31, 7041–7053.

[31] MOHD W.R.W., ABDULLAH L., *Similarity measures of Pythagorean fuzzy sets based on combination of cosine similarity measure and Euclidean distance measure*, AIP Conf Proc., 2018, 1974, PMID 030017-1-030017-7.

[32] MUTHUKUMAR P., SAI SUNDARA KRISHNAN G.S.S., *A similarity measure of intuitionistic fuzzy soft sets and its application in medical diagnosis*, Appl. Soft Comp., 2016, 41, 148–156.
[33] NGUYEN X.T., NGUYEN V.D., NGUYEN V.H., GARG H., *Exponential similarity measures for Pythagorean fuzzy sets and their applications to pattern recognition and decision-making process*, Comp. Int. Syst., 2019, 5 (2), 217–228.
[34] PENG X., *New similarity measure and distance measure for Pythagorean fuzzy set*, Comp. Int. Syst., 2019, 5 (2), 101–111.
[35] PENG X.D., GARG H., *Multiparametric similarity measures on Pythagorean fuzzy sets with applications to pattern recognition*, Appl. Int., 2019, 49 (12), 4058–4096.
[36] PENG X., SELVACHANDRAN G., *Pythagorean fuzzy set, state of the art and future directions*, Artif. Int. Rev., 2019, 52 (3), 1873–1927.
[37] PENG X., YANG Y., P*ythagorean fuzzy Choquet integral based MABAC method for multiple attribute group decision-making*, Int. J. Int. Syst., 2016, 31 (10), 989–1020.
[38] PENG X., YANG Y., *Some results for Pythagorean fuzzy sets*, Int. J. Int. Syst., 2015, 30 (11), 1133–1160.
[39] PENG X., YUAN H., YANG Y., *Pythagorean fuzzy information measures and their applications*, Int. J. Int. Syst., 2017, 32 (10), 991–1029.
[40] PETROVIC I., KANKARAS M., *A hybridized IT2FS-DEMATEL-AHPTOPSIS multicriteria decision-making approach, case study of selection and evaluation of criteria for determination of air traffic control radar position*, Dec. Mak. Appl. Manage. Eng., 2020, 3 (1), 134–152.
[41] RAHMAN K., ABDULLAH S., SHAKEEL M., KHAN M.S.A., ULLAH M., *Interval-valued Pythagorean fuzzy geometric aggregation operators and their application to group decision-making problem*, Cogent. Math., 2017, 4, 1–19.
[42] RAHMAN K., ABDULLAH S., *Generalized interval-valued Pythagorean fuzzy aggregation operators and their application to group decision-making*, Gran. Comp., 2019, 4 (1), 15–25.
[43] RAHMAN K., ALI A., ABDULLAH S, AMIN F., *Approaches to multi-attribute group decision-making based on induced interval-valued Pythagorean fuzzy Einstein aggregation operator*, New Math. Nat. Comp., 2018, 14 (3), 343–361.
[44] RAJARAJESWARI P., UMA N., *Intuitionistic fuzzy multi similarity measure based on cotangent function*, Int. J. Eng. Res. Techn., 2013, 2 (11), 1323–1329.
[45] REN P., XU Z., GOU X., *Pythagorean fuzzy TODIM approach to multi-criteria decision-making*, Appl. Soft. Comp., 2016, 42, 246–259.
[46] SON L.H., PHONG P.H., *On the performance evaluation of intuitionistic vector similarity measures for medical diagnosis*, J. Int. Fuzzy Syst., 2016, 31 (3), 1597–1608.
[47] SZMIDT E., KACPRZYK J., *Intuitionistic fuzzy sets in some medical applications*, Note IFS., 2001, 7 (4), 58–64.
[48] TARUNA, ARORA, H.D., KUMAR V., *Study of fuzzy distance measure and its applications to medical diagnosis*, Infor., 2021, 45 (1), 143–148.
[49] TIAN M.Y., *A new fuzzy similarity based on cotangent function for medical diagnosis*, Adv. Model. Optim., 2013, 15 (2), 151–156.
[50] THAO N.X., ALI M., SMARANDACHE F., *An intuitionistic fuzzy clustering algorithm based on a new correlation coefficient with application in medical diagnosis*, J. Int. Fuzzy Syst., 2019, 36 (1), 189–198.
[51] ULLAH K., *Picture fuzzy MacLaurin symmetric mean operators and their applications in solving multiattribute decision-making problems*, Math. Probl. Eng., 2021, Art. ID 1098631.
[52] ULLAH K., MAHMOOD T., JAN N., *Similarity measures for T-spherical fuzzy sets with applications in pattern recognition*, Symmetry, 2018, 10 (6), 193.
[53] ULLAH K., MAHMOOD T., JAN N., BROUMI S., KHAN Q., *On bipolarvalued hesitant fuzzy sets and their applications in multi-attribute decision-making*, Nucleus, 2018, 55 (2), 85–93.

[54] WEI G., WEI Y., *Similarity measures of Pythagorean fuzzy sets based on the cosine function and their applications*, Int. J. Int. Syst., 2018, 33 (3), 634–652.
[55] XIAO F., DING W., *Divergence measure of Pythagorean fuzzy sets and its application in medical diagnosis*, Appl. Soft. Comp., 2019, 79, 254–267.
[56] XU Z., *Some similarity measures of intuitionistic fuzzy sets and their applications to multiple attribute decision-making*, Fuzzy Opt. Dec. Mak., 2007, 6 (2), 109–121.
[57] XU X., ZHANG L., WAN Q., *A variation coefficient similarity measure and its application in emergency group decision-making*, Syst. Eng. Proc., 2012, 5, 119–124.
[58] YAGER R.R., *Pythagorean fuzzy subsets*, Proc. Joint IFSA World Congress and NAFIPS Annual Meeting. Edmonton, Canada, 2013, 57–61.
[59] YAGER R.R., *Pythagorean membership grades in multicriteria decision-making*, Technical report MII-3301, New Rochelle, Machine Intigence Institute, Iona College, 2013.
[60] YAGER R.R., *Pythagorean membership grades in multicriteria decision-making*, IEEE Trans. Fuzzy Syst., 2014, 22 (4), 958–965.
[61] YAGER R.R., ABBASOV A.M., *Pythagorean membership grades, complex numbers and decision-making*, Int. J. Int. Syst., 2013, 28 (5), 436–452.
[62] YANG M.S., HUSSAIN Z., *Fuzzy entropy for Pythagorean fuzzy sets with application to multicriterion decision-making*, Compl., 2018, 2018, 1–14.
[63] ZADEH L.A., *Fuzzy sets*, Inf. Cont., 1965, 8 (3), 338–353.
[64] ZENG W., LI D., YIN Q., *Distance and similarity measures of Pythagorean fuzzy sets and their applications to multiple criteria group decision-making*, Int. J. Int. Syst., 2018, 33 (11), 2236–2254.
[65] ZHANG X.L., *A novel approach based on similarity measure for Pythagorean fuzzy multiple criteria group decision-making*, Int. J. Int. Syst., 2016, 31 (6), 593–611.
[66] ZHANG X.L., XU Z.S., *Extension of TOPSIS to multi-criteria decision-making with Pythagorean fuzzy sets*, Int. J. Int. Syst., 2014, 29 (12), 1061–1078.
[67] ZHANG Q.S., YAO H.X., ZHANG Z.H., *Some similarity measures of interval-valued intuitionistic fuzzy sets and application to pattern recognition*, Appl. Mech. Mater., 2010, 44–47, 3888–3892.
[68] ZHANG Q., HU J., FENG J., LIU A., LI Y., *New similarity measures of Pythagorean fuzzy sets and their applications*, IEEE Access, 2019, 7, 138192–138202, DOI: 10.1109/ACCESS.2019.2942766.