ON TRANSACTION COSTS IN STOCK TRADING

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Abstract: Liquidity is an important characteristic of a stock traded on the stock exchange. The expected value of transaction costs, which takes into account the transaction's volume and duration, may be a considered as an important measure of a liquidity of a traded stock. In this paper the formulas for expected transaction cost, caused by bid-ask spread and market impact are presented. Moreover, in this article, the problem of determining a duration of a transaction of a stock sale which minimizes the transaction cost and takes into account the forecast of the expected stock price on the stock exchange, is considered.

Keywords: liquidity, transaction cost, bid-ask spread, market impact, trade duration

INTRODUCTION

Liquidity is an important issue in stock markets. In fact, a liquidity of a stock traded on the stock exchange is measured by the cost of its trading. For the purposes of market participants, the correct way to view liquidity should imply the possibility of sufficiently accurate forecasting the stock price change caused by the trade initiator and estimating the transaction cost. Transaction costs (trading costs) are widely recognized as an important factor which determines the financial investment performance. Their understanding and assessment is important for economic theory and participants of financial markets. Transaction costs are substantial component of realistic models of the stock market microstructure. In case of the stock transaction between two parties: buyer and seller, transaction costs refer to costs paid by one party of the transaction and not received by the second transaction party. The trading costs can be explicit or implicit. The major sources of transaction costs usually considered in financial investment are:

commissions (and similar payments), bid-ask spreads and market impact [Elton et al. 1999].

In this article the analysis of the dependence of the transaction cost on the bid-ask spread and the market impact is presented. Moreover, the formula for the average transaction cost is applied to determine the strategy which maximizes the expected amount of money received for selling the stock shares. One of the reason of planning the sale of the shares of the stock may be the forecast of a decrease in the future stock price. In this article, the model of the stock price process is proposed, with the possibility of the shorter duration of the negative drift in the stock price process, than the time for the investor's stock sale. The explicit method for determination of the strategy minimizing the expected transaction cost of selling X shares of the stock, is applied to the numerical computation of the duration of the stock sale which minimizes the cost of trading.

ON SOURCES OF TRANSACTION COSTS

Broker commissions are explicit costs of trading. They are usually easy to evaluate (as percentage of the transaction value) before the start of the trade and therefore they are not the source of the financial risk. In this article, the commissions and similar explicit transaction costs paid by the investor are not taken into account in calculating the average transaction cost of purchase or sale of the shares of the stock.

The bid-ask spread is defined as the difference between the stock's highest bid S^{bid} and the lowest S^{ask} ask prices of one share of the stock in the stock exchange. The average of S^{bid} and S^{ask} may be considered as the market price of the stock. The half of the bid-ask spread is the cost of trading one share of the stock.

Market impact (also called price impact) can be defined as a change in the stock price with respect to a reference price, caused by the transaction. This change is disadvantageous to the initiator of this transaction and therefore the market impact is a source of the trading cost. In theory of finance some distinguish between temporary and permanent market impact. The temporary price impact is considered as the cost of providing lacking liquidity to execute the trade in short time. It affects only a single trade and is assumed not to change the market value of the traded stock. Such impact is caused by supply and demand imbalance. Permanent price impact is considered as the change in the market value of the stock due to the transaction, which remains at least to the completion of this transaction. In case of the stock, a buy transaction signals that the stock may be undervalued and a sell transaction is a signal to the market that the stock may be overvalued. Therefore, the permanent market impact is perceived as a result of an adjustment of the market to the information content of the trade.

MARKET IMPACT FORMULAS AND TRADING COST

In theoretical finance price impact formulas can be used to build more realistic market models and explain the empirical phenomena which seem to contradict the market efficiency [Czekaj et. al. 2001]. In recent years, there has been observed a trend toward the applying of electronic trading algorithms based on a market impact model [Schied and Slynko 2011]. The well accommodated to the real financial market price impact model could help the market participants in pre-trade assessment of the performance of their trading strategies. In practice of financial management it is important to check whether the coefficients and even the functional form of the price impact formula reflect the recent stock market data.

The transaction of the stock purchase or sale, which is the source of market impact, may have a complex structure: it can be fragmented and executed incrementally by the sequences of single financial orders.

A popular formula for market impact, defined as the expected average price return between the beginning and the end of the stock transaction is given as follows:

$$MI = \pm\kappa\sigma\left(\frac{X}{V}\right)^{\delta} \tag{1}$$

where σ is the stock's daily volatility, X is the volume of the executed transaction, V denotes the average daily number of the traded shares of the stock, κ and δ are the numerical constants that can be estimated from the sample of historical transactions. The constant κ is usually described to be of order unity and it seems that it just means that is approximately equal to 1. The constant δ does not exceed 1, is typically found to be approximately 1/2 [Donier J., Bonart J. 2014] and is usually singly estimated for an entire stock market . It is not obvious that κ does not vary stock by stock. The positive and negative sign of MI respectively corresponds to the stock purchase and selling transaction.

The variant of the equation (1) with δ equal to 1, which means that market impact is linear in the traded volume, is used, for example, in [DeMiguel et al. 2014]. The linear dependence of market impact on the transaction volume can be partly justified in financial market microstructure theory by the Kyle model [Bouchaud 2009].

The formula (1) measures, in fact, the difference in the prices of the stock transaction. However, the equation (1) can be used to obtain the average cost of the transaction.

Consider the transaction of purchase of X shares of the stock, which starts at time 0 and ends at time T. Let S_0 denote the market price of one share of the stock just before the transaction and let TC denote the expected transaction cost,

per unit of the stock, paid by the initiator of this purchase transaction. The amount of money paid for the stock purchased is $S_0 X(1+TC)$.

It can be calculated that:

$$TC = \frac{s}{2} + \frac{1}{\delta+1}\sigma\left(\frac{X}{\overline{V}}\right)^{\delta} \tag{2}$$

In case of a transaction of stock selling the amount of money received by the stock seller is $S_0 X(1-TC)$ and the transaction costs TC is also given by the equality (2).

In formula (1) the algorithm of the transaction execution, applied by a market participant , is not taken explicitly into account. In practice there are many types of static and dynamic trading strategies. However, for a given transaction volume, the trading strategy seems to be roughly characterized by the transaction's duration which measures how long the transaction lasts. The duration of the transaction is determined by the speed of execution (trading rate) and the transaction volume. The omitting in (1) of explicit dependence of market impact on the stock trade duration may mean that the influence of the stock trading speed on price impact exists but is significantly smaller than the dependence of the price impact on the number of traded shares of the stock.

In [Almgren et al. 2005] the volume time is defined as the fraction of a stock average day's volume that has been traded up to clock time t. Under assumption that the stock's daily volume is independent of a trading day and the speed of trading of the stock's daily volume in the market is constant during a trading day the volume time coincides with physical time.

In [Almgren et al. 2005] the two variables I and J are defined. I denotes the permanent market impact and J is the transaction cost per unit of the stock. Let $\overline{I}$ and $\overline{J}$ respectively denote the average values of I and J. The formulas for $\overline{I}$ and $\overline{J}$ can be calculated as follows:

$$\overline{I} = \gamma\sigma\frac{X}{V}\left(\frac{\Theta}{V}\right)^{\frac{1}{4}} \tag{3}$$

and

$$\overline{J} = \frac{I}{2} + \mathrm{sgn}(X)\eta\sigma\left|\frac{X}{VT}\right|^{\frac{3}{5}} \tag{4}$$

where γ and η are numerical constants, X denotes the number of traded shares of the stock, V is the average daily volume of the stock, Θ denotes the number of outstanding stock's shares, T is a trade duration and σ is the daily volatility of the stock.

If J is negative, it means that the it was calculated for the stock sale transaction and then $|J|$ is the cost of selling per unit of the stock. The values of γ and η were determined in [Almgren et al. 2005] by linear regression: $\gamma = 0,314 \pm 0,041$ and $\eta = 0,142 \pm 0,0062$.

The price impact model described in [Almgren et al. 2005], refers to the transactions completed by uniform rate of trading over their volume time interval and takes into account the trade duration, which means the investor can apply this model in analysing the effect of the speed of his trading on the price impact. In model presented in [Almgren et al. 2005], the bid-ask spread cost is a part of the transaction cost.

THE STRATEGY OF SELLING STOCK WITH TRANSACTION COSTS

Consider the following formula for the expected transaction cost implied by the bid-ask spread and the market impact, which includes the stock's transaction duration and volume (as fraction of the average daily number of the traded shares of the stock):

$$\sigma a \theta + \sigma b \left(\frac{\theta}{T}\right)^c \tag{5}$$

where σ is the stock's daily volatility, θ denotes the volume of the investor's stock and T is the trade duration. Moreover, a, b and c denote the numerical coefficients and $0 < c < 1$.

Consider an investor who, at time 0, holds X units of the stock and expects that the trading activity of other market participants in time interval $[0, T_1]$ will cause a linear, negative drift μ in the process of the stock's market price. He wants to sell his stock's shares up to time T_2 where $T_2 \geq T_1$. The drift μ can be interpreted as the reaction of the market on the information announced at time 0, concerning the financial forecast of the company which issued the stock. For example, the negative trend μ might be generated by the information that the stock dividend would be less than the market participants expected. It can be assumed that the investor assesses the trend duration on the basis of his past experiences with the market reactions on the announcements concerning the financial situation of this company or other companies which issued stocks traded on the stock exchange. Since the investor does not forecast a direction of the stock price movements, which are independent of his trading, after time T_1 therefore he assumes a drift equal to 0 in the market price of the stock, in the interval $[T_1, T_2]$.

Let S_t denote the expected price of the stock at time t. For $[0,T_1]$, S_t is determined by the equality:

$$S_t = \begin{cases} (1+\mu t)S_0 \text{ if } t < T_1 \\ (1+\mu T_1)S_0 \quad \text{if } t \in [T_1,T_2] \end{cases} \tag{6}$$

Figure 1. The example of investor's views of the expected market price of the stock's share up to time T_2, due to the stock trading of other market participants
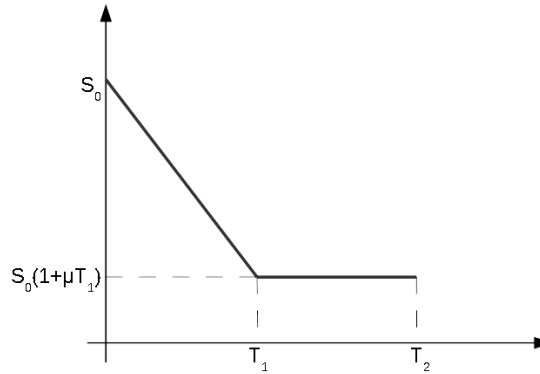
Source: own preparation

Under assumption of the constant speed of selling the investor's stock in the interval $[0,T]$, it can be calculated that the average transaction cost caused by the drift is given by the formula:

$$TC(\mu) = \begin{cases} \dfrac{1}{2}\mu T \text{ if } T < T_1 \\ \left(1-\dfrac{T_1}{2T}\right)\mu T_1 \text{ if } t \in [T_1,T_2] \end{cases} \tag{7}$$

The objective of the investor is to maximize the amount of money received from the sale of his X shares of the stock. In the choice of the duration T^* which maximizes the amount of money obtained for selling the stock's volume X, the investor takes into account that decreasing the speed of the execution of selling the stock reduces the trading cost of market impact but also, in time interval $[0,T_1]$, increases the cost caused by the drift μ.

By (5) and (7) the expected cost of the investor's transaction is

$$
TC(T) = \begin{cases} \sigma a\theta + \sigma b\left(\dfrac{\theta}{T}\right)^{c} + \dfrac{1}{2}\mu T & \text{for } T \le T_1 \\[3mm] \sigma a\theta + \sigma b\left(\dfrac{\theta}{T}\right)^{c} + \left(1 - \dfrac{T_1}{2T}\right)\mu T_1 & \text{for } T_1 < T \le T_2 \end{cases} \tag{8}
$$

For $T \le T_1$ the derivative of (8) with respect to T equals 0 for $T = \left(\dfrac{2\sigma bc\theta^{c}}{\mu}\right)^{\frac{1}{1+c}}$ and $TC(T)$ is convex on the interval $(0, T_1]$. Thus, the minimum of (8) on the set $(0, T_1]$ is obtained for

$$
T^{*}_{(0,T_1]} = \begin{cases} \left(\dfrac{2\sigma bc\theta^{c}}{\mu}\right)^{\frac{1}{1+c}} & \text{if } \left(\dfrac{2\sigma bc\theta^{c}}{\mu}\right)^{\frac{1}{1+c}} \le T_1 \\[5mm] T_1 & \text{if } \left(\dfrac{2\sigma bc\theta^{c}}{\mu}\right)^{\frac{1}{1+c}} > T_1 \end{cases} \tag{9}
$$

For $0 < c < 1$, $TC(T)$ is concave on the interval $[T_1, T_2]$. Therefore, the minimum of (8) on the set $[T_1, T_2]$ is obtained for

$$
T^{*}_{[T_1,T_2]} = \begin{cases} T_1 & \text{if } TC(T_1) \le TC(T_2) \\ T_2 & \text{if } TC(T_1) > TC(T_2) \end{cases} \tag{10}
$$

In consequence, by (9) and (10) it follows that

$$
T^{*} = \begin{cases} T^{*}_{(0,T_1]} & \text{if } TC\left(T^{*}_{[0,T_1]}\right) \le TC\left(T^{*}_{[T_1,T_2]}\right) \\ T^{*}_{[T_1,T_2]} & \text{if } TC\left(T^{*}_{[0,T_1]}\right) > TC\left(T^{*}_{[T_1,T_2]}\right) \end{cases} \tag{11}
$$

Numerical example

Consider an investor who intends to sell $\theta = 25\%$ of the stock's average daily number of the traded shares of the stock. In this numerical example of the formula (5) application, the values of a is set to 1, b is set to 0.142 and c is set to 0.6. For these values of the coefficients a, b and c, the dependence of the average transaction cost on the transaction volume in (5) is the same as the dependence of the expected cost of trading on the number of stock's shares in the linear version of (1) and the dependence of the average transaction cost on the trading rate in (5) is such as the dependence of the average transaction cost on the speed of trading in (4). Moreover, the values of T_1 and T_2 are set to 0.5 and 1, respectively.

Table 1 presents the values of the trade duration T^* as the function of the stock's price drift and the average daily volatility of the stock price.

Table 1. T^* as function of μ and σ

		σ						
		2.50%	**7.50%**	**12.50%**	**17.50%**	**22.50%**	**27.50%**	**32.50%**
μ	**2.00%**	0.226	0.449	1.000	1.000	1.000	1.000	1.000
	4.00%	0.147	0.291	0.401	1.000	1.000	1.000	1.000
	6.00%	0.114	0.226	0.311	0.384	0.449	1.000	1.000
	8.00%	0.095	0.189	0.260	0.321	0.375	0.426	1.000
	10.00%	0.083	0.164	0.226	0.279	0.327	0.370	0.411
	12.00%	0.074	0.147	0.202	0.249	0.291	0.330	0.367
	14.00%	0.067	0.133	0.183	0.226	0.265	0.300	0.333
	16.00%	0.062	0.123	0.169	0.208	0.243	0.276	0.306
	18.00%	0.057	0.114	0.157	0.193	0.226	0.256	0.285
	20.00%	0.054	0.107	0.147	0.181	0.212	0.240	0.266

Source: own computation

Table 2 show dependence of the value of the expected transaction cost on the drift of the stock and the stock's price.

Table 2. The value of average transaction cost depending on the on μ and σ

		σ						
		2.50%	**7.50%**	**12.50%**	**17.50%**	**22.50%**	**27.50%**	**32.50%**
μ	**2.00%**	1.23%	3.07%	4.65%	6.21%	7.77%	9.32%	10.88%
	4.00%	1.41%	3.43%	5.26%	6.96%	8.52%	10.07%	11.63%
	6.00%	1.54%	3.68%	5.62%	7.45%	9.22%	10.82%	12.38%
	8.00%	1.64%	3.89%	5.90%	7.80%	9.63%	11.42%	13.13%
	10.00%	1.73%	4.07%	6.14%	8.10%	9.98%	11.81%	13.60%
	12.00%	1.81%	4.22%	6.35%	8.36%	10.29%	12.16%	13.99%
	14.00%	1.88%	4.36%	6.55%	8.60%	10.57%	12.48%	14.34%
	16.00%	1.94%	4.49%	6.72%	8.81%	10.82%	12.76%	14.66%
	18.00%	2.00%	4.61%	6.88%	9.01%	11.05%	13.03%	14.96%
	20.00%	2.06%	4.72%	7.04%	9.20%	11.27%	13.28%	15.23%

Source: own computation

Figure 2 shows graphically how the expected transaction cost depends on the stock's price drift μ and the average daily volatility σ of the stock.

Figure 2. The example of the average cost of selling the stock as the function of μ and σ
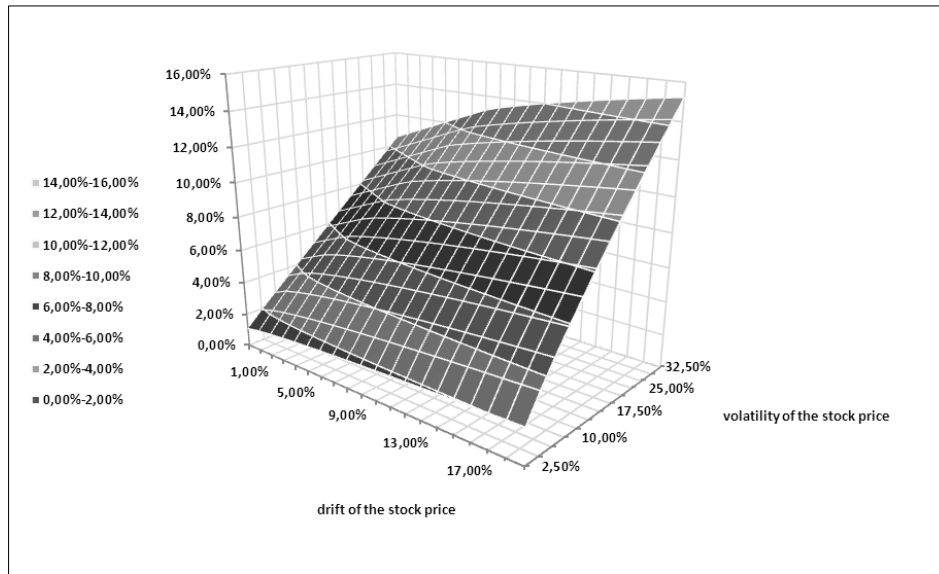
Source: own preparation

CONCLUSION

In this article the problem of determining the trading rate which minimizes the average cost of selling the investor's stock is explicitly solved in a framework of the model ,which can be applied in the case when the negative drift in the stock price lasts less than the investor's stock selling. The numerical example included in the paper shows that the solution of the problem of the expected transaction costs minimization in case of selling the stock, may be significantly affected by the volatility of the stock and the drift in the stock price process.

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