

# HIGH VOLATILITY ELIMINATES THE DISPOSITION EFFECT IN A MARKET CRISIS<sup>1</sup>

**Raymond Dacey<sup>2</sup>**  
**University of Idaho**

**Piotr Zielonka<sup>3</sup>**  
**Warsaw University of Life Sciences SGGW**

**Abstract:** *The disposition effect is an effect whereby investors tend to sell winning stocks and tend to hold losing stocks. This inclination is detrimental for investment results. Dacey and Zielonka (2008) showed the impact of the probability of further stock price rise under low stock price volatility on the disposition effect. Specifically, they showed that under low volatility, in the case of a gain, the investor is more likely to sell the winner even if the probability of the further gain is high, whereas in the case of a loss, the investor is more likely to hold the loser even when the probability of a further gain is small. In this paper we examined the disposition effect under high volatility. The general conclusion is that under high volatility, in the case of a gain, the investor behaves in the same way as for low volatility, whereas in the case of a loss, the investor is less and less likely to hold the loser as volatility increases. Thus, in the case of a loss under high volatility, the investor acts contrary to the disposition effect. This result explains the panic selling of stocks during a market collapse.*

**Key words:** *disposition effect, high volatility, panic selling, crisis selling.*

<sup>1</sup> Proper citation: Dacey, R. & Zielonka, P. (2013). High Volatility Eliminates the Disposition Effect in a Market Crisis. *Decyzje* 20: 5-20.

<sup>2</sup> Raymond Dacey, College of Business and Economics, University of Idaho, Moscow, Idaho, USA;  
e-mail: rdacey@uidaho.edu

<sup>3</sup> Piotr Zielonka, Department of Physics, Warsaw University of Life Sciences SGGW, Poland;  
e-mail: piotr\_zielonka@sggw.pl

As in Baudelaire's sad poem about the albatross, what is made to fly will not do well trapped on the ground, where it is forced to traipse. And it is quite fitting that "volatility" comes from *volare*, "to fly" in Latin. Depriving political (and other) systems of volatility harms them, causing eventually greater volatility of the cascading type.

Nassim Nicholas Taleb, *Antifragile: Things That Gain from Disorder*, p. 81.

## 1. Introduction

The disposition effect has been characterized in various ways: the "effect, whereby investors are anxious to sell their winners, but reluctant to sell their losers" (Shefrin 2005: 419); "the tendency to hold losers too long and sell winners too soon" (Odean 1998: 1775); and the effect, whereby investors "sell winners more readily than losers" (Odean 1998: 1779). A winner is defined to be a stock the value of which has increased since the time of purchase, whereas a loser is defined to be a stock the value of which has decreased since the time of purchase. Recent research provides evidence that people tend to sell winners more readily than losers even when transactions are made very quickly through the Internet (Lee, et al., 2008).

The disposition effect is common to various kinds of investors, including households, governments, and financial institutions (Grinblatt and Keloharju, 2001) and results in a less-than-optimal profit and greater-than-necessary-loss (Odean, 1998). The basic empirical evidence for the detrimental results of the disposition effect was given by Odean (1998). Odean analyzed trading records from 1987 to 1993 for ten thousand investment accounts. The median holding period for losing stocks was 124 days but 104 days for winning stocks. The losing stocks, which were held by investors, gave a 5% return in the subsequent year, whereas the winning stocks, which were sold, gained over 11% in the subsequent year. The disposition effect will not always produce a decrease in investors' profit, but its impact on the investment results seems to be very large (Lee, et al., 2008).

As the disposition effect has undesirable consequences for both individual and institutional investors, a detailed understanding of its underlying mechanisms and a determination of its boundary conditions are of great importance (Lee, et al., 2008). A deeper understanding of the disposition effect may help to explain some anomalies of the financial market, such as higher turnover for winners than for losers, and the momentum effect (Kaustia, 2010). However, there are market phenomena that are inconsistent with the disposition effect. For instance, during an abrupt market crisis, investors quickly sell losing stocks, which further deepens the crisis.

The standard account of the disposition effect is based on prospect theory (Kahneman and Tversky 1979; Tversky and Kahneman 1992) and employs the concavity of the value function over gains and the convexity of the value function over losses to explain selling winners and holding losers, respectively. The standard account is presented in Shefrin and Statman (1985) and Goldberg and von Nitzsch (2001). The standard account presumes that an individual purchases a share of stock and enjoys an initial gain or suffers an initial loss in the share price. Furthermore, the standard account presumes there are small, equally likely incremental changes in the share price. Small changes are modeled by the incremental gain or loss being less than the initial gain or loss in the share price. We will call this the low volatility case. A detailed account of the disposition effect is presented in Dacey and Zielonka (2008), where the authors show the dependence of the disposition effect on the probability of the incremental stock price raise. Dacey and Zielonka (2008) retain the presumption of small incremental changes but allow for unequally likely incremental gains and losses. They show that in the case of an initial gain, the investor is more likely to sell the winner even if the probability of the further gain is relatively high, and in the case of an initial loss, the investor is more likely to hold the loser even when the probability of a further gain is relatively small.

The foregoing results run counter to what we observe in turbulent markets. In particular, investors commonly hold stocks after enjoying sizable gains and often sell stocks after suffering sizable losses. Here we allow the incremental gain or loss to be greater than the initial gain or loss. We call this the high volatility case and we address the following question: "Does the disposition effect fail to hold under high volatility?" This paper presents an answer to this question. To do so, we relax both of the assumptions of the standard account of the disposition effect. First, we relax the assumption that the incremental changes are small, and we examine the high volatility case wherein the incremental changes are greater than the initial gain or loss in the share price. Second, we relax the standard assumption of equally likely incremental changes in the share price. Additionally, we relax the (unstated) assumption of emotional neutrality and examine the influence of fear on investor behavior given an initial loss.

The analysis presented here provides interesting results. First, under the assumptions of high volatility and equally likely incremental changes, the analysis shows that the proclivity to sell a winner holds unconditionally, but the proclivity to hold a loser does not hold unconditionally. Second, under the assumptions of high volatility and unequally likely incremental changes, the analysis shows that neither the proclivity to sell a winner nor the proclivity to hold a loser holds unconditionally. Third, under the assumptions of fear and low volatility, the analysis shows that the proclivity to hold a loser fails to hold if the probability of an incremental gain is reasonably less than  $\frac{1}{2}$ , and that under the assumptions of fear and high volatility, the analysis shows that the proclivity to hold a loser fails to hold if the probability of an incremental gain is less than  $\frac{1}{2}$ .

## 2. The Disposition Effect

The standard account of the disposition effect involves two decision problems. The first decision problem involves holding or selling a share of a stock that has had an initial increase in value, and may or may not increase further in value. Specifically, the first decision problem involves the decision to hold or sell a share of a stock that has increased to the value  $x_0$  and has equal chances of increasing or decreasing by an amount  $h$ . This decision problem is given in Table 1. We refer to this decision problem as the Gains Case.

**Table 1. The Gains Case**

Gains Case	Sell	Hold	Probability
incremental gain (h)	$x_0$	$x_0 + h$	1/2
incremental loss (-h)	$x_0$	$x_0 - h$	1/2

The second decision problem involves holding or selling a share of a stock that has had an initial decrease in value, and may or may not increase further in value. Specifically, the second decision problem involves the decision to hold or sell a share of a stock that has decreased to the value  $-x_0$  and has equal chances of increasing or decreasing by an amount  $h$ . This decision problem is given in Table 2. We refer to this decision problem as the Losses Case.

**Table 2. The Losses Case.**

Losses Case	Sell	Hold	Probability
incremental gain (h)	$-x_0$	$-x_0 + h$	1/2
incremental loss (-h)	$-x_0$	$-x_0 - h$	1/2

Note that in both the gains case and the losses case, low volatility is defined as  $h < x_0$  and high volatility is defined as  $h > x_0$ .

The standard account of the disposition effect presumes the investor behaves in accordance with prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992). Prospect theory posits a value function that represents risk aversion over gains, risk seeking over losses, and loss aversion. Following Tversky-Kahneman (1992: 309), the value function  $v$  is a piecewise-defined function, as follows:

$$v(x) = \begin{cases} x^\alpha & x \geq 0 \\ -\lambda(-x)^\beta & x \leq 0 \end{cases} \quad (1)$$

where  $x$  is the payoff, as a change in wealth,  $0 < a, b < 1$ , and  $\lambda > 1$ . The parameters  $a$  and  $b$  represent risk tolerance, and the parameter  $\lambda$  represents loss aversion. Prospect theory also posits probability weighting functions, for gains and losses, that represent over-valuation of low probabilities and under-valuation of medium and high probabilities. Following Tversky and Kahneman (1992: 309), the probability weighting functions for gains and losses are

$$w^+(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{\frac{1}{\gamma}}} \quad \text{and} \quad w^-(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{\frac{1}{\delta}}}, \quad (2)$$

respectively. In  $w^+$ ,  $p$  is the probability of an incremental gain  $h$  given an initial gain  $x_0$ , and in  $w^-$ ,  $p$  is the probability of an incremental gain  $h$  given an initial loss  $-x_0$ , and where  $0 < \gamma, \delta < 1$ . Tversky and Kahneman (1992: 311-312) provide parameter estimates as follows:  $\alpha = \beta = 0.88$ ,  $\lambda = 2.25$ ,  $\gamma = .61$ , and  $\delta = .69$ . We employ the Kahneman-Tversky  $v$ -function and  $w$ -functions, with the stated parameter estimates, for developing the diagrams presented here. In adopting the Kahneman-Tversky value function, we are representing the unstated assumption of the standard account whereby the investor is presumed to be emotionally neutral. We later employ an extension of the Kahneman-Tversky value function for the analysis of the fearful investor. We are aware that the adoption of these value functions restricts our results to the Kahneman-Tversky investor. However, the analysis presented here is an extension of the standard account of the disposition effect, which itself is based upon the Kahneman-Tversky investor.

### 3. The Gains Case and Volatility

Consider the decision problem given a gain  $x_0 > 0$ . Under prospect theory, the investor prefers hold to sell if and only if

$$w^+(p)v(x_0 + h) + w^+(1-p)v(x_0 - h) > v(x_0) \quad (3)$$

where  $x_0$  is the initial change in stock prices,  $h$  is the incremental change in stock price, and  $p$  is the probability of  $h$ . We model volatility by setting  $h = nx_0$ , so that low volatility, i.e.,  $h < x_0$ , occurs if  $0 < n < 1$  and high volatility, i.e.,  $h > x_0$ , occurs if  $n > 1$ . The standard account and the formal account presented in Dacey and Zielonka (2008) have shown that:

under low volatility, if the incremental gain and loss are equally likely,  
then the investor sells a winner and the disposition effect holds;

under low volatility, if the incremental gain and loss are not equally likely, then the investor may or may not sell a winner and the disposition effect fails to hold .

We now show that:

under high volatility, if the incremental gain and loss are equally likely, then the investor always sells a winner and the disposition effect holds;

under high volatility, if the incremental gain and loss are not equally likely, then the investor may or may not sell a winner, and the disposition effect holds or fails to hold, respectively.

The former claim is a general claim and requires a proof whereas the latter claim is existential claim and requires only an example. We begin with the proof of the former claim.

The basic components of the proof are straightforward. Substituting (1) and (2) into (3), and letting  $p = 1/2$  and  $h = nx_0$ , we have

$$w^+(1/2)(x_0 + nx_0)^\alpha + w^+(1/2)(-\lambda)(-x_0 - nx_0)^\beta > (x_0)^\alpha \tag{4}$$

We can factor  $x_0$  out of (4), whereupon we have the following: given an initial gain  $x_0 > 0$ , the investor prefers hold to sell if and only if

$$w^+(1/2)(n + 1)^\alpha + w^+(1/2)[- \lambda x_0^{\beta-\alpha} (n - 1)^\beta] > 1 \tag{5}$$

Assuming high volatility (i.e.,  $n > 1$ ) and equally likely incremental changes in the share price (i.e.,  $p = 1/2$ ), equation (5) becomes equation (6):

$$(n + 1)^\alpha - \lambda x_0^{\beta-\alpha} (n - 1)^\beta > \frac{1}{w^+(1/2)} \tag{6}$$

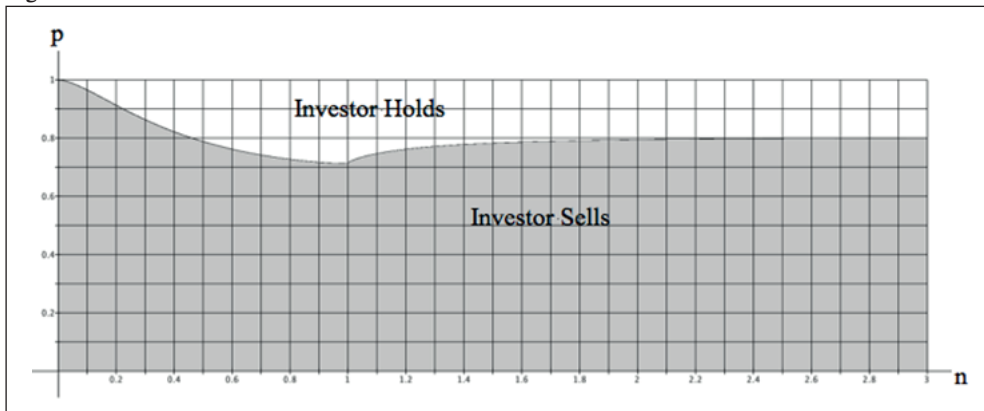
The claim that under high volatility, if the incremental gain and loss are equally likely, then the investor sells a winner, and the disposition effect holds, is proven by showing that the condition specified in equation (6) is met. The proof is presented in Appendix A.

As noted, we only require an example to show that under high volatility, if the incremental gain and loss are or are not equally likely, then the investor does not always sell a winner, and the disposition effect fails to hold. We do so via the equation that specifies indifference between holding and selling. Figure 1 presents the graph of the indifference equation

$$w^+(p)v(x_0 + nx_0) + w^+(1 - p)v(x_0 - nx_0) - v(x_0) = 0 \tag{7}$$

where  $w^+(p)$  is the probability weighting function for gains advanced in Tversky and Kahneman (1992, 310), with  $\gamma = 0.61$ ,  $n = h/x_0$ , and  $p$  is the probability of the incremental gain  $h$ . The lower region of Figure 1, shaded in grey, is the set of points  $\langle n, p \rangle$  at which the investor decides to sell, i.e., the set of points  $\langle n, p \rangle$  for which equation (4) holds, whereas the upper region of Figure 1 is the set of points  $\langle n, p \rangle$  at which the investor decides to hold, i.e., the set of points  $\langle n, p \rangle$  for which the reverse of the inequality in equation (4) holds. Figure 1 reveals that there are values of  $n$  greater than 1 and values of  $p$  other than  $1/2$  for which the investor chooses to sell a winner. However, Figure 1 also reveals that there are values of  $n$  greater than or equal to 1, and values of  $p$  other than  $1/2$ , for which the investor chooses to hold a winner. It is in this latter region that we lose the result that the investor always sells a winner, and the disposition effect disappears.

**Figure 1. The General Model of the Gains Case**



Note that Figure 1 also presents the results that the investor sells a winner under low volatility and equally likely incremental gains and losses, as per the standard account of the disposition effect. However, Figure 1 also reveals that the investor does not always sell a winner under low volatility and unequally likely incremental gains and losses, as per Dacey and Zielonka (2008).

Put simply, the answer to the question posed earlier, “Does the disposition effect fail to hold under high volatility (given an initial gain  $x_0 > 0$ )?”, is “Yes”.

#### 4. The Losses Case and Volatility

Consider the decision problem given an initial loss  $-x_0 < 0$ . Under prospect theory, the investor prefers hold to sell if and only if

$$w^-(p)v(-x_0 + h) + w^-(1-p)v(-x_0 - h) > v(-x_0) \quad (8)$$

Again, we model volatility by setting  $h = nx_0$ , so that low volatility occurs if  $0 < n < 1$ , and high volatility occurs if  $n > 1$ . The standard account and Dacey and Zielonka (2008), respectively, have shown that:

- under low volatility, if the incremental gain and loss are equally likely, then the investor holds a loser, and the disposition effect holds;
- under low volatility, if the incremental gain and loss are not equally likely, then the investor does not always hold a loser, and the disposition effect fails to hold.

We now show that:

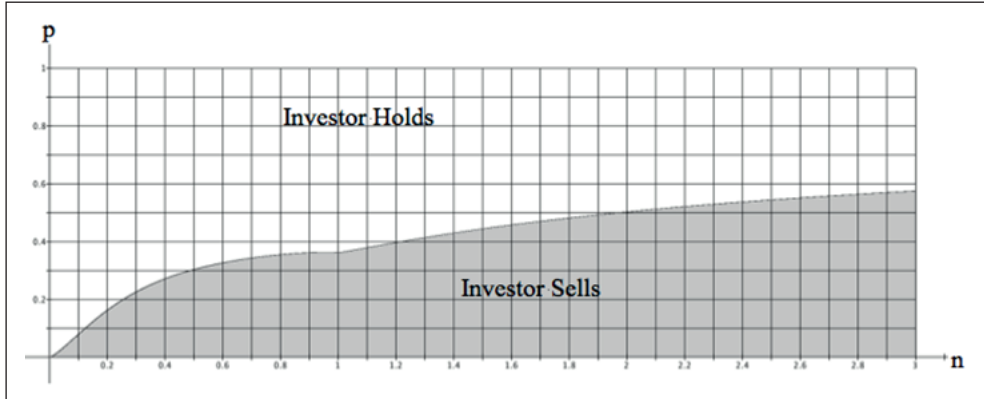
- under high volatility, if the incremental gain and loss are equally likely, then the investor does not always hold a loser, and the disposition effect fails to hold;
- under high volatility, if the incremental gain and loss are not equally likely, then the investor does not always hold a loser, and the disposition effect fails to hold.

In order to show that the investor does not always hold a loser given high volatility and equally or unequally likely incremental changes in the share price, we only require an example. We do so in Figure 2 below. Figure 2 presents the graph of the indifference equation

$$w^-(p)v(-x_0 + nx_0) + w^-(1-p)v(-x_0 - nx_0) - v(-x_0) = 0 \quad (9)$$

where  $w(p)$  is the probability weighting function advanced in Tversky and Kahneman (1992), with  $\gamma = 0.69$ ,  $n = h/x_0$ , and  $p$  is the probability of incremental gain  $h$ . The lower region of Figure 2, shaded in grey, represents the investor's decision to sell, whereas the upper region represents the investor's decision to hold. Figure 2 reveals that there are values of  $n$  greater than 1, and values of  $p$  equal to  $1/2$  and unequal to  $1/2$ , for which the investor chooses to hold a loser. However, Figure 2 also reveals that there are values of  $n$  greater than or equal to 1, and values of  $p$  equal to  $1/2$  and unequal to  $1/2$ , for which the investor chooses to sell a loser.



**Figure 2. The General Model of the Losses Case**

Note that Figure 2 also reveals that if  $h$  is only modestly larger than  $x_0$  (i.e., if  $x_0 < h < 2x_0$  so that  $1 < n < 2$ ), then the investor holds a loser, as in the standard account of the disposition effect. However, if  $h$  is very much larger than  $x_0$  (i.e., if  $h > 2x_0$ , so that  $n > 2$ ), then the investor sells a loser. Therefore, we have the result that in periods of very high volatility, the investor behaves correctly and sells a loser.

Figure 2 shows that for an initial loss  $-x_0 < 0$ , both the volatility and the probability of an incremental gain have a decisive impact on the investor's decision to hold or sell. When volatility is low, the investor prefers to hold a loser for almost all values of the probability of the incremental gain. However, as volatility increases, the probability of the incremental gain must also increase in order for the investor to choose to hold a loser. This yields the very strong conclusion that for high values of volatility, there is no disposition effect after an initial loss.

Put simply, the answer to the question posed earlier, "Does the disposition effect fail to hold under high volatility (given an initial loss  $-x_0 > 0$ )?", is "Yes".

## 5. The Losses Case and Fear

The disposition effect may be associated with emotional aspects of decision making, which play an important role in realizing gains or losses. For instance, selling a losing stock may result in a feeling of an immediate, irreversible loss, whereas holding it opens a possibility of reverse direction. Fogel and Berry (2006) examine the disposition effect under regret and find that despite the fact that investors exhibit greater regret about holding a losing stock too long than about selling a winning stock too soon, during an abrupt market crisis, investors quickly

sell losing stocks, which further deepens the crisis. Here, we examine the disposition effect under the fear induced by a serious market decline.

Here we acknowledge that a striking market decline can induce fear, and we employ the results advanced by Lerner and Keltner (2000, 2001) showing that fear induces risk aversion across frames and pessimism. In this section of the paper we consider the behavior of a fearful investor with an initial loss  $-x_0 < 0$ .

We model the fearful investor via a value function displaying risk aversion over losses and we model pessimism as a probability of an incremental gain that is less than  $1/2$ . We represent risk aversion across frames via an adaptation of the value functions used to date. Specifically, we let

$$v(x) = (x + T)^{.88} - T^{.88} \quad (10)$$

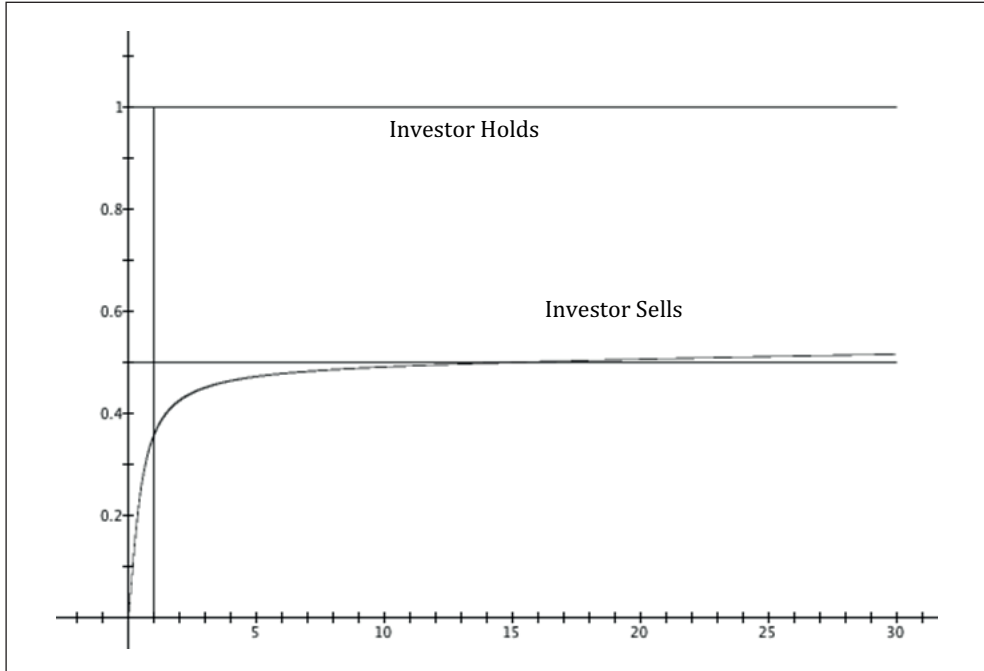
for  $T$  a large positive number greater than the absolute value of  $-x_0 - h$ , i.e., greater than  $x_0 + h$ . This specification of the value function is needed because the power functions presented in equation (1) have infinite slopes at the origin, and therefore cannot span both the losses and gains frames as a smooth function. Since  $-x_0 + h$  is in the gains frame under high volatility, we require a value function displaying risk aversion that can span both the losses and gains frames. For ease of exposition, we employ the value function specified in equation (10) with  $T = 500$ . We model pessimism as a less than even chance of an incremental gain, i.e., as  $p < 1/2$ . (This is consistent with the formal analysis of pessimism used in prospect theory. See, e.g., Wakker (2010: 172-176).)

As before, the investor is indifferent between holding and selling a loser if and only if equation (9) is satisfied. Also, as before, we model volatility by setting  $h = nx_0$ . Given risk aversion induced by fear, equation (9) then becomes the indifference equation

$$w^-(p)[(-x_0 + nx_0 + T)^{.88} - T^{.88}] + w^-(p)[(-x_0 - nx_0 + T)^{.88} - T^{.88}] - [(-x_0 + T)^{.88} - T^{.88}] = 0 \quad (11)$$

The graph of equation (11) is shown in Figure 3.

The investor holds at points above the curve and sells at points below the curve. Given pessimism, i.e., for values of  $p$  less than  $1/2$ , clearly, the fearful investor does not always hold a loser. Note that, under low volatility, if  $p$  is reasonably close to  $1/2$ , then the fearful investor does hold the loser and the disposition effect does not disappear. However, under low volatility, if the investor is very pessimistic, i.e., if  $p$  is well below  $1/2$ , then the fearful investor does sell the loser and the disposition effect disappears even under low volatility.

**Figure 3. The General Model of the Losses Case Under Fear**

Under high volatility and pessimism, the fearful investor holds the loser if  $p$  is close to  $\frac{1}{2}$  and the level of volatility is relatively low. However, if  $p$  is well below  $\frac{1}{2}$ , and/or the level of volatility is relatively or absolutely high, then the fearful investor does sell the loser and the disposition effect disappears under high volatility.

Put simply, the answer to the question posed earlier, “Does the disposition effect fail to hold under high volatility (given an initial loss  $-x_0 < 0$  and fear)?”, is “Yes”.

## 6. Conclusion

The present paper determines boundary conditions for the occurrence of the disposition effect in terms of the volatility of the price of the stock, the probability of a further rise in the price of the stock, and the influence of fear. We found that under some circumstances the disposition effect disappears. The standard account of the disposition effect assumes volatility is low, that the incremental gains and losses are equally likely, and that the investor is emotionally neutral. The standard account finds that the investor always sells a winner and always holds a loser.

The present analysis of the disposition effect refines and extends these results. Given unequally likely incremental gains and losses and high market volatility, there exist circumstances where the individual holds a winner and sells a loser. Whenever the level of volatility is high, after the initial loss the investor chooses to sell a stock, and this behavior is contrary to the standard account of the disposition effect. The investor is inclined to sell a winner, regardless of the level of volatility, unless the probability of the further stock price rise is very high (Figure 1), in accordance with the standard account of the disposition effect. However, the greater the level of volatility, the greater the tendency for the investor is to sell a loser (Figure 2). Finally, a fearful investor will sell a loser if the probability of an incremental gain is low, and/or the level of volatility is high. The foregoing results on losses explain the panic selling of stocks during a crisis, i.e., a market collapse. It is important to underline that such panic selling cannot be explained by the standard account of the disposition effect. What is more, since high volatility has played a major role in recent years (see Appendix B), an understanding of the boundary conditions for the disposition effect under high volatility seems particularly desirable and important.

Our approach reveals that three factors influence the investor's decision to hold or sell a stock. The factors are the difference between the purchase price of the stock and the current price of the stock, the probability of an incremental gain or loss in the current price of the stock, and the size of the incremental gain or loss in the current price of the stock. The first factor is the basis of the traditional prospect-theoretic account of the disposition effect. In particular, the traditional account of the disposition effects rests solely on the prospect theory result that the investor is risk averse over gains and risk seeking over losses.

Our extension of the traditional account of the disposition effect also reveals that the investor's decision whether to hold or sell the stock depends upon the investor's subjective probability distribution over the possible incremental changes in the current price of the stock. In the investment setting, the subjective probability distribution can be based on many sources, including the investor's personal experience, the content of technical analyses, and the published opinions of stock market analysts. (See Jeffrey (2004) for a formal account of subjective probability, and Kahneman and Tversky (1972) for an account of subjective probability related to prospect theory.)

The impact of the third factor – the volatility of the stock price – seems to be the most striking result of our approach. The decision whether to hold or sell a stock is determined by the combination of the aforementioned factors together with the level of volatility of the stock price. However, the impact of price volatility on the decision whether to sell or hold a stock seems to override the influence of the other two factors. Furthermore, the impact of volatility is asymmetric with respect to gains and losses.

The three factors considered here take the traditional account of the disposition effect to its limits. Specifically, we have shown that the simple application of the S-shaped value function of prospect theory, as employed in the traditional account of the disposition effect, is not sufficient to account for observed investor behavior. Specifically, we have shown that the three-factor extension of the traditional account of the disposition effect can explain the observed behavior of selling into a market decline.

## APPENDIX A

Here we prove that under high volatility, if the incremental gain and loss are equally likely, then the gains case of the disposition effect holds, i.e., the investor sells the winner. The proof is direct.

The investor prefers sell to hold if and only if

$$w^+(p)v(x_0 + h) + w^+(1-p)v(x_0 - h) < v(x_0)$$

High volatility implies  $h = nx_0$  where  $n > 1$ , and equally likely incremental gains and losses implies  $p = 1/2$ , so that, for the Kahneman-Tversky value function, the foregoing equation becomes

$$w^+(1/2)(x_0 + nx_0)^\alpha + w^+(1/2)(-\lambda)(-x_0 - nx_0)^\beta < (x_0)^\alpha$$

since  $x_0 + nx_0 > 0$  and  $x_0 - nx_0 < 0$ . Factoring yields

$$w^+(1/2)(x_0(1+n))^\alpha + w^+(1/2)(-\lambda)(x_0(n-1))^\beta < (x_0)^\alpha$$

Expanding, eliminating  $x_0$ , and factoring yields

$$w^+(1/2)\left[(1+n)^\alpha + (-\lambda)x_0^{\beta-\alpha}(n-1)^\beta\right] < 1$$

Therefore, we can show that, given high volatility and equally likely incremental gains and losses, the investor prefers sell to hold by showing that

$$\left[(1+n)^\alpha + (-\lambda)x_0^{\beta-\alpha}(n-1)^\beta\right] < \frac{1}{w^+(1/2)}$$

Let  $f(n)$  denote the left hand side of the foregoing equation, so that

$$f(n) = (n+1)^\alpha - \lambda x_0^{\beta-\alpha} (n-1)^\beta$$

We must show that  $f(n) < \frac{1}{w^+(1/2)}$  for all  $n \geq 1$ . Since  $f(1) = 2^\alpha$  and since  $2^\alpha < 1$  because  $0 < \alpha < 1$ , we can show that  $f(n) < 2$  for all  $n > 1$  by showing that the first

derivative of  $f(n)$  is negative, i.e., that  $\frac{df(n)}{dn} < 0$  for all  $n > 1$ . Furthermore, since  $2 < \frac{1}{w^+(1/2)}$ , to show that  $f(n) < 2$  is to show that  $f(n) < \frac{1}{w^+(1/2)}$ . The proof that

$\frac{df(n)}{dn} < 0$  for all  $n > 1$  is as follows:

$$f(n) = (n+1)^\alpha - \lambda x_0^{\beta-\alpha} (n-1)^\beta \text{ by definition;}$$

$$\frac{df(n)}{dn} = \alpha(n+1)^{\alpha-1} - \lambda x_0^{\beta-\alpha} \beta(n-1)^{\beta-1} \text{ by differentiation;}$$

$$\frac{df(n)}{dn} = \frac{\alpha}{(n+1)^{1-\alpha}} - \frac{\lambda x_0^{\beta-\alpha} \beta}{(n-1)^{1-\beta}} \text{ by rearrangement; and}$$

$$\frac{df(n)}{dn} = \frac{\alpha(n-1)^{1-\beta} - \lambda x_0^{\beta-\alpha} \beta(n+1)^{1-\alpha}}{(n+1)^{1-\alpha} (n-1)^{1-\beta}} \text{ by factoring.}$$

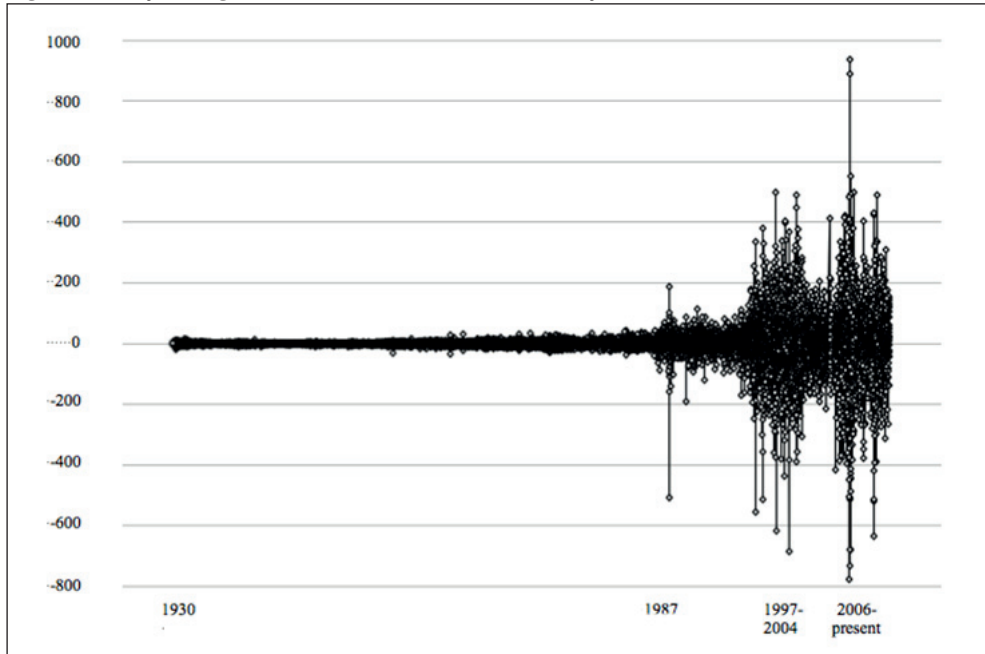
Presuming  $\alpha = \beta$  (as per Tversky and Kahneman, 1992, 311), so that  $1-\alpha = 1-\beta$ , and since  $n-1 < n+1$ , we have  $\alpha(n-1)^{1-\beta} < \beta(n+1)^{1-\alpha}$ . Since  $\lambda > 1$  and, for  $\alpha = \beta$ ,  $x_0^{\beta-\alpha} > 1$ , we have  $\alpha(n-1)^{1-\beta} < \lambda x_0^{\beta-\alpha} \beta(n+1)^{1-\alpha}$ . Hence, the numerator of  $\frac{df(n)}{dn}$  is negative. Since  $n > 1$  and since  $0 < 1-a < 1$  and  $0 < 1-b < 1$ , the denominator of  $\frac{df(n)}{dn}$  is positive. Thus, we have shown that  $\frac{df(n)}{dn} < 0$  for all  $n > 1$ , and thereby we have shown that  $f(n) < 2$  for all  $n \geq 1$ . Therefore, since  $2 < \frac{1}{w^+(1/2)}$ , we have shown that  $f(n) < \frac{1}{w^+(1/2)}$  for all  $n \geq 1$ , and thereby we have proven that given high volatility and equally likely incremental gains and losses, the investor holds a winner for all levels of volatility.

We presumed  $\alpha = \beta$ , as per the estimates advanced by Tversky and Kahneman (1992: 311). However, Tversky and Kahneman (1992: 312) also note, "Accordingly, risk aversion for gains is more pronounced than risk seeking for losses, for moderate and high probabilities ...." This implies that  $\alpha < \beta$ . If we allow  $\alpha < \beta$ , then the foregoing proof holds only if we also presume  $x_0 > 1$ , so that  $x_0^{\beta-\alpha} > 1$ . Note that the assumption that  $x_0 > 1$  amounts to assuming that the stock is not a penny stock. Thus, what we have proven is that given high volatility and equally likely incremental gains and losses, the investor holds a non-penny stock winner for all levels of volatility.

**APPENDIX B**

The claim that volatility has played a major role in recent years is easily established via an observation of almost any major stock index. We display below daily changes in the Dow Jones Index (DJI) from February 3, 1930 to December 30, 2013. As is readily apparent from Figure 4, volatility is strikingly high in recent years.

**Figure 4. Daily Changes in the Dow Jones Index, February 3, 1930 to December 30, 2013**



The major fluctuations come in three segments. The first segment is the single downturn of October 19, 1987; the middle segment runs from late 1997 to late 2004; the third segment runs from middle 2006 to the present. A similar analysis is presented in Mandelbrot and Hudson (2004, 88-96) and covers the first segment and most of the second segment. As the graph shows, the most recent segment provides the greatest fluctuations.

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