

## ANALYSIS OF TAIL DEPENDENCE STRUCTURE IN GLOBAL FINANCIAL MARKETS

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**Abstract:** The identification of tail (in)dependencies has drawn major attention in empirical financial studies. We concern on the structure of dependence which refers to dependence as symmetric or asymmetric, tail-dependent or tail-independent. We present the proper procedure of analysis dependence structure between some financial instruments. Our empirical results demonstrate different tail dependence structures underlying various global financial markets.

**Keywords:** tail-dependence, extremes, extreme value theory, copula

### INTRODUCTION

Dependencies between financial asset-returns have significantly increased during recent time periods in almost all international markets. This phenomenon is a direct consequence of globalization and relaxed market regulation in finance and insurance industry. Especially during bear markets many empirical surveys like Karolyi and Stulz (1996), Longin and Solnik (2001), Campbell, Koedijk and Kofman (2002) show evidence of increasing dependencies between financial asset-returns.

When investors and/or risk managers would have a better knowledge of the dependence during crises periods, they are able to make better allocation decisions and they can get a clearer view of the risks they are bearing. Estimating dependence between risky asset returns is the cornerstone of portfolio theory and many other finance applications. Common dependence measures such as Pearson's correlation coefficient are not always suited for a proper understanding of dependencies in financial markets [Embrechts et al. 2002]. In particular, dependencies between extreme events such as extreme negative stock returns or

large portfolio losses cause the need for alternative dependence measures. Several empirical surveys such as Ané, Kharoubi (2003) and Malevergne, Sornette (2004) exhibited that the concept of tail dependence is a useful tool to describe the dependence between extremal data. Tail dependence is described via the tail-dependence coefficient introduced by Sibuya (1960).

Investigating stock markets is relevant, because institutional investors (for example pension funds) often allocate more than 50% of their portfolios to stocks. So correct understanding of the dependence of asset process is important for proper risk measurement and portfolio diversification.

Motivated by these considerations in this paper we perform empirical analysis of extreme dependence between selected indices from Central and East Europe stock exchange markets, namely Polish WIG20, Hungarian BUX, Russian RTS, Czech PX50. Extreme dependence is defined as the dependence between extremely large returns. Central and Eastern European markets can become a very attractive option for global investors who want to diversify their portfolios internationally. We concern on the structure of dependence. Structure refers to dependence as symmetric or asymmetric, tail-dependent or tail-independent.

One objective of this paper is to present the proper procedure of analysis dependence structure between some financial instruments. Another objective is to test asymmetric tail dependence between Polish WIG20, Hungarian BUX, Russian RTS and Czech PX50 (if exists) is statistically significant.

The paper is organized as follows. Section 1 describes briefly the most important properties of extremes in financial stock returns. Section 2 discusses tail dependence concept and outlines the estimation method. Section 3 describe concept of tail dependence. Section 4 discusses the empirical results.

## EXTREMES IN FINANCIAL ASSET- RETURNS

Stock prices can be used to gain significant insight into corporations. For observable asset prices we use daily data on the log-returns. An extreme return is a return that exceeds a certain preestablished threshold (normally, a high order (95% or 99%) conditional quantile, i.e. a value of return that is exceeded with low probability: 5% or 1%).

The most important properties of stock returns are:

- gain-loss asymmetry: rises are less than falls,
- volatility clusters: returns of high volatility keep together.

Frequently, single extreme events like extremely large negative asset-returns (for example during a market crash or bear markets) account for most of companies. For investors building their portfolios globally the main goal is *“not to predict what or when – but instead be prepared and able to respond in an informed and planned manner to minimize the impact of a disruption”* [Steven Culp, Global Managing Director, Accenture Risk Management].

Extreme value theory is the natural choice for inferences on extreme values. The classical extreme bivariate theory is concerned with the limit behaviour of  $(M_n(X), M_n(Y)) = (\max_{i=1, \dots, n} X_i, \max_{i=1, \dots, n} Y_i)$  as  $n \rightarrow \infty$ . Because of the definition, the marginals of  $(M_n(X), M_n(Y))$  belong to the generalized extreme value (GEV) distribution family. The general form of a generalized extreme value GEV distribution is

$$GEV_{\mu, \sigma, \xi}(x) = \exp(-[1 + \xi \frac{x - \mu}{\sigma}]^{-1/\xi})$$

with  $\mu \in R$ ,  $\sigma > 0$ ,  $\xi \in R$  (Coles, 2001). To simplify the presentation, Coles (2001) assumes without loss of generality that  $F_X \equiv F_Y \equiv F$ , where  $F(\cdot)$  is the unit Frechet distribution. The following theorem (de Haan and Resnick, 1977) characterizes the limit joint distribution of  $(M_n(X), M_n(Y))$ :

$$\text{If } P(M_n(X) \leq nx, M_n(Y) \leq ny) \xrightarrow{n \rightarrow \infty} G(x, y)$$

where  $G$  is a non-degenerate distribution function, then  $G(\cdot, \cdot)$  takes the form  $G(x, y) = \exp(-V(x, y))$  with

$V(x, y) = 2 \int_0^1 \max(\omega/x, (1-\omega)/y) dH(\omega)$  and  $H$  is a distribution on  $[0, 1]$  with mean  $1/2$ .

## CONCEPT OF TAIL DEPENDENCE AND COPULA

The dependence between asset returns typically has pronounced nonlinear and time-varying features. In particular, the co-movement of asset prices tends to be stronger when returns are negative or when financial markets are more volatile (see [Longin and Solnik 2001; Ang and Chen 2002; Ang and Bekaert 2002; Capiello et al. 2006]). Also, the dependence does not disappear when returns take extreme (negative) values (see [Longin and Solnik 2001; Butler and Joaquin 2002; Hartmann et al. 2004]).

These properties of asymmetric dependence and (lower) tail dependence invalidate the use of the Pearson's correlation coefficient as a measure of dependence. For the same reason the multivariate normal distribution is inappropriate for asset returns, as it implies symmetric dependence and tail independence (Embrechts et al. 2002).

The tail dependence coefficient is the probability that a random variable exceeds a certain threshold given that another random variable has already exceeded that threshold. The following approach, Sibuya (1960) and Joe (1997) among others, represents the most common definition of tail dependence. Let

$(X, Y)$  be a random pair with joint cumulative distribution function  $F$  and marginals  $F_x$  and  $F_y$ . The quantity

$$\lambda_u = \lim_{v \rightarrow 1^-} P(X > F_x^{-1}(v) | Y > F_y^{-1}(v))$$

is the upper tail-dependence coefficient (upper TDC), provided the limit exists.

We say that  $(X, Y)$  is upper tail dependent if  $\lambda_u > 0$  and upper tail independent if  $\lambda_u < 0$ . Similarly, we define the lower tail-dependence coefficient  $\lambda_L$ .

Frahm et al. (2005) give estimators for the TDC under different assumptions: using a specific distribution (e.g. t-distribution), within a class of distributions (e.g. elliptically contoured distributions), using a specific copula (e.g. Gumbel), within a class of copulae (e.g. Archimedean) or a nonparametric estimation (without any parametric assumption). The authors compare the performance of the different estimators for different cases: whether the assumption is true or wrong and whether there is tail dependence or not. It turns out that some of the estimators perform well if there is tail dependence but bad if there is not. In practical applications, one will never know which copula model is the correct one. The estimation can only be under misspecification.

In recent years, copula functions have become a popular tool for describing nonlinear dependence between asset returns. Copulas separate the dependence structure from the marginal distributions and allow for a great deal of flexibility in the construction of an appropriate multivariate distribution for returns. So now we write the TDC via the notion of copula, introduced by Sklar (1959).

A copula  $C$  is a cumulative distribution function whose margins are uniformly distributed on  $[0, 1]$ . The joint distribution function  $F$  of any random pair  $(X, Y)$  can be represented as (refer to [Joe 1997]) for more information on copulas)

$$F(x) = C(F_x(x), F_y(y))$$

The coefficient of upper tail dependence can be written in terms of copula

$$\lambda_U = \lim_{v \rightarrow 1^-} \frac{1 - 2v + C(v, v)}{1 - v}$$

Analogously, we have

$$\lambda_L = \lim_{v \rightarrow 0^+} \frac{C(v, v)}{v}$$

A copula is useful because it can be used to analyze the dependence structure of variables in a multivariate distribution.

Some commonly used copulas in economics and finance include: the bivariate Gaussian copula, the student-t copula, the Gumbel copula, the Clayton

copula and the Symmetrized Joe-Clayton (SJC) copula. Difficulties in selecting a copula model, brings us to the important issue of testing for tail dependence.

## TESTING FOR TAIL DEPENDENCE

The concept of tail dependence represents the current standard to describe the amount of extremal dependence. While Extreme Value Theory allows for constructing estimators of the tail dependence coefficient, tests for tail independence are indispensable when working with tail dependence, since all estimators of the *tail dependence coefficient* are strongly misleading when the data does not stem from a tail dependence setting.

One of the most interesting approach for testing for tail dependence is given in Falk and Michel (2006). They prove the following theorem:

With  $c \rightarrow 0$ , we have uniformly for  $t \in [0,1]$ :

$$P(X + Y > ct \mid X + Y > c) = \begin{cases} t^2; & \text{there is no tail dependence} \\ t; & \text{else} \end{cases}$$

Using this theorem, Falk and Michel proposed four different tests for tail dependence, which can be grouped into two different classes: a Neymann-Pearson test (NP) and three goodness of fit tests: Fisher's  $\kappa$ , Kolmogorov-Smirnov and  $\chi^2$

In the latter class, the Komolgorov-Smirnov-test (KS) turns out to be the best in the simulation study by Falk and Michel (2006). An examination of the power of the extreme-value dependence tests was made by Trzpiot and Majewska (2011). In order to examine this issue they carried out Monte Carlo experiments. Results showed the highest power of Neyman-Pearson and Kolmogorov-Smirnov (KS) tests and the lowest power of the chi-square test. Therefore, in the following, only KS tests is described.

Let's define, conditional on  $K(n) = m$ :

$$U_i = F_c(C_i / c) = (1 - (1 - C_i) \exp(C_i)) / (1 - (1 - c) \exp c), \quad \forall i \in \{1, \dots, m\}.$$

Denote  $\hat{F}_m(t) = \frac{1}{m} \sum_{I_{[0,t]}} C_i$  the ecdf of  $U_i$ ,  $i = 1, \dots, m$ .

The Kolmogorov test statistic is then:

$$T_{KS} := \frac{1}{m} \sup_{t \in [0,1]} |\hat{F}_m(t) - t|.$$

The approximate  $p$ -value is  $p_{KS} = 1 - K(T_{KS})$ , where  $K$  is the cdf of the Kolmogorov distribution.

According to a rule of thumb given by the authors: for  $m > 30$ , tail independence is rejected if  $T_{KS} > c_{0.05} = 1.36$

## RESULTS FOR THE DEPENDENCE STRUCTURE. EMPIRICAL STRUCTURE

For the empirical analysis we use daily series of four indices from Central and East Europe stock exchange markets, namely Polish WIG20, Hungarian BUX, Russian RTS, Czech PX50. Data for the period April 6th 2011 to March 29th 2012 come from the stock exchanges websites.

Dependence structure examination procedure is preceded in the four steps. First look at dependence (with conventional measure). Now we consider the results of dependence between each pair of stock returns. The well known Spearman's  $\rho$  and Kendall's  $\tau$  rank correlation coefficients provide alternative nonparametric measures of dependence between variables that, unlike the simple correlation coefficient, do not require a linear relationship between the variables. For this reason they are commonly studied with copula models.

According to the conventional measure of dependence – Kendall's rank correlation the weakness rank correlation is between RTX and PX50.

### Sense of dependence structure

In order to get a sense of the dependence structure in the data, following Knight, Lizieri and Satchel (2005), we calculate an empirical copula table. To do this, we first rank the pairs of return series in ascending order and then we divide each series evenly into 6 bins. Bin 1 includes the observations with the lowest values and bin 6 includes observations with the highest values. We want to know how the values of one series are associated with the values of the other series, especially whether lower returns in - for example – WIG20 are associated with lower returns in the PX50. Thus, we count the numbers of observations that are in cell  $(i, j)$ .

The dependence information we can obtain from the frequency table is as follows:

- if the two series are perfectly positively correlated, most observations lie on the diagonal,
- if they are independent, then we would expect that the numbers in each cell are about the same,
- if the series are perfectly negatively correlated, most observations should lie on the diagonal connecting the upper-right corner and the lower- left corner,
- if there is positive lower tail dependence between the two series, we would expect that more observations in cell (1,1),
- if positive upper tail dependence exists, we would expect large number in cell (6,6).

Table 1 shows the dependence structure for real returns between WIG20 and PX50, as an example. Cell (1,1) has a joint frequency of 61, which means that out of 186 observations, there are 61 occurrences when both the FedEx and UPS

returns lie in their respective lowest 6th percentiles ( $1/6^{\text{th}}$  quantile). This number is the largest among all cells, and it is much bigger than numbers in other cells, pointing to evidence of lower tail dependence. There are 43 occurrences in cell (6,6), which is not apparently larger than other cells, indicating no or not strong evidence of upper tail dependence. Clearly, the table shows evidence of asymmetric tail dependence.

Table 1. Joint frequency table for pair of stock return: WIG20 and PX50

|   | 1  | 2  | 3  | 4  | 5  | 6  |
|---|----|----|----|----|----|----|
| 1 | 61 | 36 | 31 | 28 | 19 | 11 |
| 2 | 41 | 54 | 34 | 29 | 23 | 5  |
| 3 | 26 | 27 | 21 | 43 | 17 | 44 |
| 4 | 19 | 34 | 38 | 20 | 47 | 28 |
| 5 | 22 | 25 | 41 | 23 | 34 | 41 |
| 6 | 17 | 10 | 21 | 43 | 39 | 43 |

Source: own calculation

### Testing significance of tail dependence

We estimate and test for asymmetric tail dependence between all pairs returns. First, we select a flexible copula function to model the joint distribution of the each pair of returns. Because of empirical evidence of asymmetric tail proper selection is SJC copula (Symmetrized Joe-Clayton). SJC copula allows for both asymmetric upper and lower tail dependence and symmetric dependence as a special case. It is defined as:

$$C_{SJC}(u, v | \lambda_u, \lambda_l) = 0,5 \cdot (C_{JC}(u, v | \lambda_u, \lambda_l) + (C_{JC}(1-u, 1-v | \lambda_u, \lambda_l) + u + v + 1))$$

where

$$C_{JC}(u, v | \lambda_u, \lambda_l) = 1 - 1 - \{ [1 - (1-u)^k]^{-r} + [1 - (1-v)^k]^{-r} - 1 \}^{-1/r} \}^{1/k}$$

and

$$k = 1/\log_2(2 - \lambda_u), \quad r = -1/\log_2(\lambda_l), \quad \lambda_l \in (0,1), \quad \lambda_u \in (0,1).$$

Then we compute for all  $(i, j)$  ( $i, j = 1, \dots, 7$ ) pairs of returns the upper and lower tail dependence coefficients using the copulas parameters estimates (Table 2 and Table 3). We also employed KS test for tail dependence.

The most important conclusions are:

- existence of dependence between Poland-Czech, Poland-Hungary stock markets (the strongest between Poland-Czech),
- Polish, Czech and Hungarian equity markets are dependent on the Russian market (as the largest financial market in consideration).

- significant lower and upper tail dependence between indexes listed on European stock exchanges,
- insignificant upper tail dependence between Polish and Russian indexes this implies that the these indexes are more dependent during extreme downturns than during extreme upturns of the these markets.

Table 2. Lower Tail Dependence Coefficients

|       | WIG20          | BUX            | RTS     | PX50 |
|-------|----------------|----------------|---------|------|
| WIG20 |                |                |         |      |
| BUX   | <b>0.2972*</b> |                |         |      |
| RTS   | 0.1399*        | <b>0.3248*</b> |         |      |
| PX50  | <b>0.5010*</b> | 0.1355*        | 0.1173* |      |

Note: \* indicates significance at 5% level

Source: own calculation

Table 3. Upper Tail Dependence Coefficients

|       | WIG20          | BUX            | RTS     | PX50 |
|-------|----------------|----------------|---------|------|
| WIG20 |                |                |         |      |
| BUX   | 0.1594*        |                |         |      |
| RTS   | 0.0502         | <b>0.2276*</b> |         |      |
| PX50  | <b>0.4932*</b> | 0.1156*        | 0.1265* |      |

Note: \* indicates significance at 5% level

Source: own calculation

## CONCLUSION

In this paper, we examined the extreme co-movements between stock returns of indexes from Central and East Europe stock exchange markets using the copula approach. This method of studying dependence is useful because it can be used to study not only the degree of dependence among random variables, such as asset prices, but also their structure of dependence, including asymmetric dependence in the tails of their joint distribution. Our empirical results point to strongest and significant asymmetric tail dependence between stock returns in Europe with the lower tail dependence being significantly greater than upper tail dependence. Our results insignificant upper tail dependence between Polish and Russian indexes.

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