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REVIEW OF VALUE AT RISK ESTIMATION METHODS

PRZEGLĄD METOD SZACOWANIA WARTOŚCI ZAGROŻONEJ (VALUE AT RISK)

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Abstract: On a daily basis, managers in risk management teams use a number of methods to manage various types of risk. One of the most popular methods of measuring market risk is Value at Risk. Estimation of Value at Risk gives a possibility to determine a loss, which can occur or can be exceeded with a given probability and tolerance level. Moreover, this measure of risk shows in just one number entire risk of the portfolio. In addition, various methods and probability distributions can be used to estimate Value at Risk. A goal of this paper is the evaluation of Value at Risk estimation methods on the basis of backtesting results. In the empirical part, the data for 4 investment portfolios was used. The portfolios were diversified in terms of geographic location of firms that were taken into consideration.

Keywords: Value at Risk, estimation, backtesting, investment portfolio.

Streszczenie: Menedżerowie w pionie zarządzania ryzykiem używają wielu metod, by zarządzać różnego rodzaju ryzykiem. Jedną z najpopularniejszych metod zarządzania ryzykiem rynkowym jest szacowanie wartości zagrożonej (VaR). Obliczenie wartości zagrożonej daje możliwość oszacowania wartości straty, która może zostać osiągnięta lub przekroczona z danym prawdopodobieństwem. Co ważne, narzędzie, jakim jest VaR, daje możliwość oszacowania ryzyka całkowitego dla analizowanego portfela. Niemniej jednak wiele metod oraz rozkładów prawdopodobieństwa może zostać użytych do oszacowania wartości zagrożonej. Celem niniejszej pracy jest ocena metod szacowania VaR za pomocą modeli testów wstecznych oraz wyciągnięcie wniosków na ich podstawie. W części empirycznej zostały użyte dane dla czterech portfeli inwestycyjnych. Zostały one zdywersyfikowane wg kryterium położenia geograficznego firm, których akcje zostały wzięte do analizy.

Słowa kluczowe: wartość zagrożona, estymacja, testy wsteczne, ryzyko całkowite, portfel inwestycyjny.

1. Introduction

Every day, risk managers use various methods to estimate several types of risk. Value at Risk became one of the most popular methods of market risk estimation. According to Hull, VaR is a tool which gives a possibility to estimate the total risk of the portfolio [2011, p. 219].

Total risk is a sum of specific risk and systematic risk. Specific risk is a type of risk that can be diversified. It depends on the business, and a particular equity. Systematic risk is the type of risk, which cannot be diversified and depends on the economic cycle, inflation, politics and the stock market.

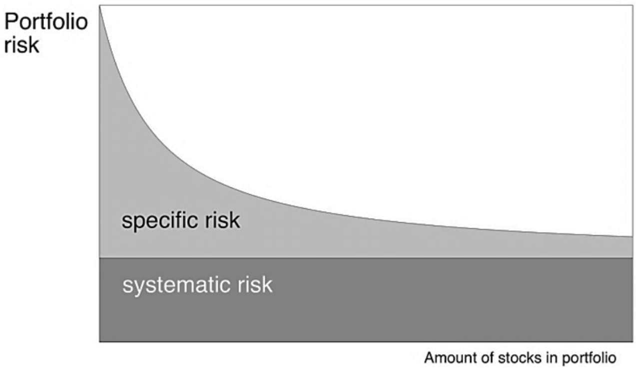

Fig. 1. Entire risk of portfolio

Source: own study based on [Jajuga, Jajuga 2006, p. 246].

In author's opinion, after the financial crisis in 2007-2009, the influence of systematic risk on total – risk is still increasing. This can be especially observed on the quotes of assets that are included in Dow Jones Industrial Average Index. Recently, bigger falls in value of assets have been caused by political issues rather than the worse performance of a company. Nevertheless, in the market risk management process usually total risk is taken into consideration and as Hull states, proper VaR estimation is very important in finance nowadays [2011, p. 219]. In consequence, the aim of the paper is the evaluation of VaR estimation methods, and conclusions resulting from the backtesting.

Value at Risk can be estimated using various methods, but the most common are [Jajuga, in Jajuga 2007, p. 102; Piontek 2010, p. 468]:

- historical simulation;
- Monte Carlo simulation;
- unconditional variance-covariance approach with normal and Student's-t distributions;
- AR-GARCH approach with normal and Student's-t conditional distributions (specific variance-covariance approach);

- group of methods using the quantile of non-normal distribution;
- extreme-value-theory approach.

However, methods that are used to estimate VaR should be graded, as grading approaches give a possibility to ascertain whether the model should be rejected or not in the risk management process. Validation of models may be conducted by using several backtesting tests such as:

- proportion of failures test [Kupiec 1995],
- independence test [Christoffersen 1996],
- loss functions [Lopez 1998; Sarma et al. 2003].

In the empirical part of that paper, the first four VaR estimation methods and all pointed out backtesting tests are analysed.

2. Value at Risk

Before a presentation of particular VaR estimation approaches, definition of Value at Risk should be mentioned. "Value at Risk is such a loss in market value of a portfolio that probability it will occur or even will be exceeded in a given period of time is equal to the predefined tolerance level" [Jajuga, in Jajuga 2007, p. 99].

Mathematically, it can be expressed as follows:

$$P[V \leq V_0 - VaR] = \alpha, \tag{1}$$

where: V – value of portfolio at the end of the considered period, V_0 – value of a portfolio at the beginning of the considered period, VaR – estimated Value at Risk, α – predefined tolerance level.

Since the paper is focused on the research of Value at Risk based on the rates of return, the next definition better fits the study. In particular, VaR can be defined as the quantile of the unconditional or forecasted conditional distribution of the rates of return.

$$P\left(r_t \leq F_{r,t}^{-1}(\alpha)\right) = \alpha, \tag{2}$$

$$VaR_{r,t}(\alpha) = F_{r,t}^{-1}(\alpha), \tag{3}$$

where: r_t is the rate of return on the portfolio (in that paper only logarithmic rates of return are considered) and $F_{r,t}^{-1}(\alpha)$ is a quantile of loss distribution related to the probability of 1–α.

In the empirical part of the paper all methods are estimated with 1% and 5% tolerance level. Parameters for all methods are estimated using the rolling window procedure with 1,000 consecutive observations, apart from the Monte Carlo simulation method. In this particular case 255 observations are used. Step in rolling window (from one estimation of parameters to another and from one prognosis of VaR to another) is equal to one.

2.1. Value at Risk estimation methods – simulation methods

Despite the fact that two methods which are considered in the empirical part of the paper contain the word *simulation* in their names, the former – historical simulation – is the simplest method among the considered and is not actually connected with simulation. This method is based on the determination of the quantile of distribution of historical rates of return on a portfolio [Jajuga, in Jajuga 2007, p. 103].

The second one – Monte Carlo simulation – is a more advanced method. It uses Geometric Brownian motion with drift to simulate the path of the analysed process. This particular method can be described as follows:

1) evaluating the expected value (arithmetic mean) μ and variance σ^2 of 255 historical rates of return of the portfolio;

2) generating 10,000 semi-random variables from the N(0,1) distribution for each period that is considered;

3) creating antithetic variables;

4) changing all the variables (now there are 20,000 for each period analysed) into the simulated distribution of the rates of return with the following equation:

$$r_t = exp\left[\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \left(\sigma\sqrt{\Delta t}\right)\varepsilon\right], \tag{4}$$

5) computing given quantiles of the simulated distribution of the rates of return.

2.2. Value at Risk estimation methods – variance covariance approach

The most popular and at the same time most basic Value at Risk estimation method among variance-covariance approaches, is the approach with multidimensional unconditional normal distribution with arithmetic means (expected values) and covariance matrix. Variance of the portfolio is computed from the formula [Xu, Chen 2012]:

$$\sigma_p = w' \sum w, \tag{5}$$

where, $\sum$ is a symmetric, positive-definite matrix formulated as:

$$\sum = \begin{pmatrix} VAR(R_1) & \cdots & Cov(R_1R_n) \\ \vdots & \ddots & \vdots \\ Cov(R_1R_n) & \cdots & VAR(R_n) \end{pmatrix}, \tag{6}$$

where R_n – the rates of return from the n^{th} component of the portfolio and w is a vector of shares of particular components in the portfolio:

$$w = (w_1, \ldots, w_n)\ . \tag{7}$$

Value at Risk is computed according to the formula:

$$VaR_\alpha = -\mu + \sqrt{\sigma_p}N_\alpha, \tag{8}$$

where: N_α – the quantile of normal distribution N(0,1) and

$$\mu = w\begin{pmatrix} \frac{R_1}{1000} \\ \vdots \\ \frac{R_n}{1000} \end{pmatrix}. \tag{9}$$

Estimation of the covariance matrix can be also performed under the assumption that the rates of return follow Student's t-distribution. Random variable under Student's t-distribution and v degrees of freedom takes a form of [Mercik 2013]:

$$t = \frac{U}{\sqrt{Z}}\sqrt{v}\,, \tag{10}$$

where: U is a random variable from N(0,1) distribution, Z is a variable from the χ^2 distribution with v degrees of freedom, U and Z are uncorrelated random variables.

Using Student's t-distribution to estimate Value at Risk may be a good approach, especially when 1% VaR is estimated. Models connected with normal distribution tend to underestimate VaR under 1% tolerance level. This is based on the fact that the distributions of the rates of return are leptokurtic (they have a higher value of kurtosis than the normal distribution), thus, the distributions of returns have fatter tails. In other words, there is a higher probability of outliers.

The density function of Student's t-distribution is as follows [Mercik 2013]:

$$F(t) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\sqrt{v\pi}}\left(1+\frac{x^2}{v}\right)^{-\left(\frac{v+1}{2}\right)}, \tag{11}$$

where: Γ(z) – gamma function for parameter z, where Γ (z)= $\int_0^\infty x^{z-1}e^{-x}dx$.

For $v > 4$ kurtosis exists. Then kurtosis is equal to:

$$K = \frac{6}{v-4}. \tag{12}$$

For $v > 2$ variance exists. Then variance is equal to:

$$V(t) = \frac{v}{v-2}. \tag{13}$$

It should be highlighted that Student's t-distribution with $v \to \infty$ follows N(0,1). Value at Risk estimated using the variance-covariance method with unconditional Student's t distribution is evaluated using the formula:

$$VaRt_\alpha = -\mu + \sqrt{\sigma_p\frac{v-2}{v}}\,t_{v,\alpha}, \tag{14}$$

$t_{v,\alpha}$ – quantile of Student's t-distribution with v degrees of freedom (the number of degrees of freedom is estimated as the arithmetic mean of degrees of freedom of the fitted Student's t-distribution for each portfolio component).

Another approach of estimating VaR is the variance-covariance method. In this approach the conditional expected value and the conditional variance are forecasted with AR-GARCH models. This type of an approach has an advantage in comparison to models with unconditional EV and VAR, because conditional models give a possibility to describe such features of time series as:

- the occurrence of autocorrelation in the rates of return;
- the heteroscedasticity of variance;
- the leptokurtosis of the rates of return (similarly to unconditional approach, but there is no need to estimate such a high number of coefficients).

AR models were limited to the 5th lag to describe the changing autocorrelation (from one trading week) of returns. GARCH order was limited to GARCH(1,1) as this is the most popular lag used in GARCH modelling – for a particular time series there is no possibility to sustain stationarity of the process with higher lags. Value at Risk according to AR-GARCH model in this particular case is as follows [Bollerslev 1986; Doman, Doman 2009; Piontek 2002]:

$$P\left(r_t \leq \mu_t + \sqrt{h_t} D_c^{-1}(\alpha)\right) = \alpha, \tag{15}$$

where:

$$\mu_t = \phi_0 + \phi_1 r_{t-1} + \phi_2 r_{t-2} + \phi_3 r_{t-3} + \phi_4 r_{t-4} + \phi_5 r_{t-5}, \tag{16}$$

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}, \tag{17}$$

$$\varepsilon^2 = \sqrt{h_t} c_t \text{ and } \alpha + \beta < 1^1 \tag{18}$$

c_t~IID(0,1), $D_c^{-1}(\alpha)$ is a quantile of conditional distribution for a given tolerance level.

In the paper two conditional distributions of variable c_t are be considered:

- Normal distribution N(0,1).

$$f_n(\varepsilon_t, h_t, \theta_N) = \frac{1}{\sqrt{2\pi h_t}} exp\left\{-\frac{\varepsilon^2}{h_t}\right\}. \tag{19}$$

- Student's t distribution St's t $(0,1,\nu)$.

$$f_S(\varepsilon_t, h_t, \theta_S) = \frac{\Gamma\left(\frac{\nu+1}{2}\right) h_t^{-1/2}}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{(\nu-2)\pi}} \left(1 + \frac{\varepsilon_t^2}{(\nu-2)h_t}\right)^{-\frac{\nu+1}{2}}. \tag{20}$$

[1] The condition of stationarity of the process.

3. Empirical research

3.1. Backtesting

Backtesting, as it was mentioned before, is based on three tests where one of them has two variations. It should be highlighted that in the case of loss functions, only the VaR values that are not rejected by PoF and Ind tests are taken into consideration[2]. Proportion of Failures test and Independence test are strictly connected with failure process $[I_t(q)]_{t=1}^{t=T}$, where T is the number of units, from which the back-tested period consists. The failure function is defined as follows [Lopez 1995]:

$$I_t(q) = \begin{cases} 1; & r_{p,t} \leq F_{rp,t}^{-1}(q), if\ failure\ occurs \\ 0; & r_{p,t} \geq F_{rp,t}^{-1}(q), if\ no\ failure\ occurs \end{cases}.$$

PoF test proposed by Kupiec is the test, which examines whether the amount of failures of VaR model is significant. The null hypothesis for that test is that the empirically determined probability matches the given tolerance level of VaR [Lopez 1995]:

$$H_0: \hat{q} = q.$$

The test statistic is based on the likelihood ratio by the following formula and it is asymptotically chi-square distributed with one degree of freedom:

$$LR_{POF} = -2ln\left(\frac{(1-q)^{T_0}q^{T_1}}{(1-\hat{q})^{T_0}\hat{q}^{T_1}}\right) \sim \chi_1^2, \tag{21}$$

where:

$$\hat{q} = \frac{T_1}{T_0+T_1}, \qquad T_1 = \sum_{t=1}^{T} I_t(q), \qquad T_0 = T - T_1$$

The VaR estimation method is reliable if amount of failures is within the non-critical area.

Independence test proposed by Christoffersen, considers the dependency of exceedances of VaR made by the rates of return. Test statistic is given by the formula [Christoffersen 1998]:

$$LR_{ind} = -2ln\left(\frac{(1-\bar{q})^{T_{00}+T_{10}}\bar{q}^{T_{01}+T_{11}}}{(1-\hat{q}_{01})^{T_{00}}\hat{q}_{01}(1-\hat{q}_{11})^{T_{10}}\hat{q}_{11}^{T_{11}}}\right), \tag{22}$$

where:

$$\hat{q}_{ij} = \frac{T_{ij}}{T_{i0}+T_{i1}}, \qquad \bar{q} = \frac{T_{01}+T_{11}}{T_{00}+T_{01}+T_{10}+T_{11}},$$

and T_{ij} is a number of periods in which $I_t = j$, if $I_{t-1} = i$.

[2] Mix tests were not used. When one of these two tests rejected the particular method, it was not considered in further study.

Lopez proposed another approach of validation of VaR estimation methods and modification of this method was proposed by Sarma, Thomas and Shah. For each period which is analysed, a loss function is computed on the basis of the rate of returns accordingly to the formula described below [Lopez 1995; Sarma et al. 2003; Piontek 2007]:

$$L\left(VaR_{r,t}(q), r_{t+1}\right) = \begin{cases} f\left(VaR_{r,t}(q), r_{t+1}\right) & r_{t+1} \leq VaR_{r,t}(q) \\ g\left(VaR_{r,t}(q), r_{t+1}\right) & r_{t+1} \geq VaR_{r,t}(q) \end{cases}, \qquad (23)$$

where:

$f(VaR_{r,t}(q), r_{t+1}) = 1 + \left(r_{t+1} + VaR_{r,t}(q)\right)^2$ – Lopez's proposition,

$f(VaR_{r,t}(q), r_{t+1}) = \left(r_{t+1} + VaR_{r,t}(q)\right)^2$ – Sarma-Thomas-Shah's proposition,

$g\left(VaR_{r,t}(q), r_{t+1}\right) = 0$ – Lopez's proposition,

$g\left(VaR_{r,t}(q), r_{t+1}\right) = \varphi VaR_{r,t}$ – Sarma-Thomas-Shah's proposition.

In both propositions models are penalised by rates of return exceeding the VaR level, but in STS they are penalised for the overestimation of the VaR, too. To determine the best VaR estimation, one needs to consider the loss function which has the lowest value. In the empirical research φ coefficient is equal to 0.6.

3.2. Research sample and empirical results

The research sample consists of 4 portfolios, which contain 4 equities with the highest market capitalization in particular indices (2017-08-11). Portfolios are diversified geographically to increase the independence of the results and the influence of systematic risk[3] on the final results. All the time series have a length of 1814 observations, but the first estimation is based on the first 1000 observations. Then the rolling window procedure is followed.

Components of the portfolios are as follows:

- WIG20 (PKN Orlen, PKO BP, PEKAO SA, PZU);
- Nikkei225 (Toyota Motor Corporation, Mitsubishi UFJ Financial Group, Docomo; Soft Bank Group);
- DJIA (Microsoft, Exxon, Apple, Johnson and Johnson);
- FTSE100 (HSBC, British American Tobacco, BP, Unilever).

All the components have equal share in the portfolio.

PoF test and IND test were conducted at the 5% significance level.

[3] Level of systematic risk is followed by changing location.

Table 1. Results of the PoF test (WIG20)

Model	WIG20											
	Historical simulation		Student's t varcov		Norm. varcov		AR-GARCH Student's t		Monte Carlo		AR-GARCH Norm.	
% VaR	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05
rejection	No	No	No	Yes	No	Yes	Yes	Yes	No	No	No	No

Source: own study.

Table 2. Results of the Independence test (NIKKEI225)

Model	NIKKEI 225											
	Historical simulation		Student's t varcov		Norm. varcov		AR-GARCH Student's t		Monte Carlo		AR-GARCH Norm.	
% VaR	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05
rejection	Yes	Yes	Yes	Yes	Yes	Yes	No	No	Yes	Yes	No	No

Source: own study.

Table 3. Results of Lopez loss function test (DJIA)

Model	DJIA											
	Historical simulation		Student's t varcov		Norm. varcov		AR-GARCH Student's t		Monte Carlo		AR-GARCH Norm.	
%VaR	1	5	1	5	1	5	1	5	1	5	1	5
Value of loss function	0.98%	5.41%	–	–	1.48%	4.18%	0.86%	3.08%	1.60%	4.80%	–	4.31%

Source: own study.

Table 4. Results of STS loss function test (FTSE100)

Model	FTSE100											
	Historical simulation		Student's t varcov		Norm. varcov		AR-GARCH Student's t		Monte Carlo		AR-GARCH Norm.	
%VaR	1	5	1	5	1	5	1	5	1	5	1	5
Value of loss function	–	0.81%	–	0.81%	–	0.84%	1.56%	0.99%	–	0.93%	–	0,90%

Source: own study.

Results of all backtesting procedures are presented next. As far as the PoF and Ind Test results are concerned, WIG20 – 1%, 5% Historical simulation, 1% and 5% Student's t varcov, 5% Normal varcov, 1% and 5% AR-GARCH Student's t and 5% Monte Carlo models were rejected. Similarly, DJIA – 1%, 5% AR-GARCH Student's t and 1% AR-GARCH Normal models were also rejected. FTSE100 – 1% Historical simulation, 1% Student's t varcov, 1% Normal varcov and 1% Monte Carlo and 1% AR-GARCH Normal models were rejected as well. Only for

NIKKEI225, and only the AR-GARCH models were not rejected at any of the significance levels.

In case of the Lopez loss function is concerned, the results are as follows:

- WIG20 – 1% varcov Normal and 1% Monte Carlo were the best models among considered; the value of the loss function was equal (in both cases) 1.35%;
- NIKKEI225 – 1% AR-GARCH Student's t model outperformed rest of models with the value of the loss function equal to 0.98%;
- DJIA – 1% Historical simulation outperformed rest of models with the value of loss function equal to 0,98%;
- FTSE100 – 1% AR-GARCH Student's t model outperformed rest of models with value of the loss function equal to 1.35%.

Finally, for the STS loss function, WIG20 – 5% AR-GARCH Normal model outperformed the rest of models with the value of the loss function equal to 1.12%. For NIKKEI225 – 5%, the AR-GARCH Normal was the best model with the value of the loss function equal to 1.34%. For the DJIA – 5% Student's t varcov model was the best with the value of the loss function equal to 0.82%. Whereas for FTSE100, the 5% Student's t varcov model was also the best, with a similar value of the loss function equal to 0.80%.

4. Conclusions

The aim of the paper was to evaluate VaR estimation methods and draw conclusions based on results of backtesting. Due to the fact that obtained results are ambiguous, there is no possibility to make a very sound conclusion. In case of the Lopez Loss function, it can be concluded that methods based on conditional models with Student's t distribution were the best for two portfolios. In case of STS Loss function, AR-GARCH models with conditional Normal distribution had the advantage in VaR estimation for two out of four portfolios. Almost in all cases, the methods based on AR-GARCH approach were not rejected based on the Independence test[4]. It can be concluded that for the considered samples GARCH approach was the most universal one. All in all, VaR estimation methods should be fitted for a particular portfolio. Financial institutions should choose their own methods for VaR estimation, due to the fact that various portfolios have different particular and numerous properties.

[4] In a few cases there were no dependencies.

References

Bollerslev T., 1986, *Generalized autoregressive conditional heteroscedasticity*, Journal of Econometrics, no. 31, p. 307-327.

Christoffersen P.F., 1998, *Evaluating interval forecasts*, International Economic Review, vol. 39, no. 4, p. 841-862.

Doman M., Doman R., 2009, *Modelowanie zmienności i ryzyka. Metody ekonometrii finansowej*, Wolters Kluwer Business, Kraków.

Hull J.C., 2011, *Zarządzanie ryzykiem instytucji finansowych*, Wydawnictwo PWN, Warszawa.

Jajuga K., 2007, *Teoretyczne podstawy zarządzania ryzykiem*, [in]: Jajuga K. (ed.), *Zarządzanie ryzykiem,* Wydawnictwo Naukowe PWN, Warszawa, pp. 11-170.

Jajuga K., Jajuga, T., 2006, *Inwestycje*, Wydawnictwo Naukowe PWN, Warszawa.

Kupiec P., 1995, *Techniques for Verifying the Accuracy of Risk Management Models*, Journal of Derivatives, Vol. 3, p. 73-84.

Lopez J.A., 1998, *Methods for evaluating Value-at-Risk. Estimates*, Economic Policy Review, vol. 4, no. 3, p. 199-124.

Mercik A.R., 2013, *Wykorzystanie rozkładu t-Studenta do szacowania wartości zagrożonej*, [in:] Jajuga K., Ronka-Chmielowiec W. (eds.), *Inwestycje finansowe i ubezpieczenia – tendencje światowe a rynek polski*, Wydawnictwo Uniwersytetu Ekonomicznego we Wrocławiu, Wrocław, p. 202-211.

Piontek K., 2002, *Pomiar ryzyka metodą VaR a modele AR-GARCH ze składnikiem losowym o warunkowym rozkładzie z "grubymi ogonami",* [in:] W. Tarczyński (ed.), *Rynek kapitałowy: skuteczne isnwestowanie: konferencja naukowa zorganizowana przez Katedrę Ekonometrii i Statystyki Wydziału Nauk Ekonomicznych i Zarządzania, Międzyzdroje, 8-10 września 2002 r. Część 2.,* Wydawnictwo Naukowe Uniwersytetu Szczecińskiego, Szczecin, p. 467-484.

Piontek K., 2007, *Przegląd i porównanie metod oceny modeli VaR*, [in:] P. Chrzan (ed.), *Matematyczne i ekonometryczne metody oceny ryzyka finansowego*, Prace Naukowe Akademii Ekonomicznej w Katowicach, Katowice, p. 113-124.

Piontek K., 2010, *The analysis of power for some chosen VaR backtesting procedures – simulation approach*, [in:] Fink A., Lausen B., Seidel W., Ultsch A. (eds.), *Advances in Data Analysis, Data Handling and Business Intelligence, Studies in Classification, Data Analysis and Knowledge Organization*, Springer-Verlag, Heidelberg, p. 481-490.

Sarma M., Thomas S., Shah A., 2003, *Selection of Value-at-Risk Models*, Journal of Forecasting, p. 337-358.

Xu C., Chen H., 2012. *Measuring Portfolio Value at Risk*, LUP Student Papers, https://lup.lub.lu.se/student-papers/search/publication/2628584 (27.08.2017).