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INTERNATIONAL DIVERSIFICATION IN THE PERIODS OF MAJOR ECONOMIC CHANGES

MIĘDZYNARODOWA DYWERSYFIKACJA W OKRESACH DUŻYCH ZMIAN EKONOMICZNYCH

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Abstract: Diversification is one of the most important elements considered in the process of the construction of investment portfolios. A special role is attributed to the diversification in periods of rapid changes in the financial markets. In the article, the problem of international diversification was analysed on the example of selected European, American and Asian markets. The level of diversification was assessed by various measures: Portfolio Diversification Index, Rao's Quadratic Entropy and Diversification Ratio. In the study diversified portfolios were compared. These portfolios were constructed for the data from the periods before, during and after the last economic crisis. The conducted research showed that European markets were most diversified, regardless of the nature of the analysed period. The study also showed that strongly interrelated countries did not necessarily have strong influence on diversification.

Keywords: diversification, Rao's Quadratic Entropy, Portfolio Diversification Index, Most Diversified Portfolio.

Streszczenie: Dywersyfikacja jest jednym z ważniejszych elementów w procesie konstrukcji portfela inwestycyjnego. Szczególną rolę przypisuje się jej w okresach gwałtownych zmian zachodzących na rynkach finansowych. W artykule analizowano problem dywersyfikacji międzynarodowej na przykładzie wybranych rynków europejskich, amerykańskich i azjatyckich. Poziom dywersyfikacji oceniany był za pomocą następujących miar: indeks dywersyfikacji portfela, kwadratowa entropia Rao czy współczynnik dywersyfikacji. W badaniach porównano portfele zdywersyfikowane, konstruowane dla danych pochodzących z okresu przed, w trakcie oraz po ostatnim kryzysie ekonomicznym. Przeprowadzone badania wykazały, że bez względu na charakter badanego okresu najbardziej zdywersyfikowane są rynki europejskie. Ponadto, stwierdzono, że na poziom dywersyfikacji niekoniecznie wpływ mają rynki wykazujące silną zależność z innymi krajami.

Słowa kluczowe: dywersyfikacja, kwadratowa entropia Rao, indeks dywersyfikacji portfela, portfele najbardziej zdywersyfikowane.

1. Introduction

Diversification is one of the fundamental concepts in the modern portfolio theory. The uncertainty in the financial markets makes portfolio diversification a unique tool for managing the investment in the difficult market conditions. One of the diversification types is the international diversification, which is the main object of the presented study. Various empirical research has shown that bad assumptions in portfolio construction can lead to bad results, especially in times of sudden changes on the investment market. The recent economic crisis has been the beginning of a new trend in portfolio research, where the assumption of portfolio rates of return started to be omitted. In the literature the optimization models which allow the construction of well-diversified portfolios are proposed. Examples of such portfolios are the Most Diversified Portfolios (*MDP*) or optimal portfolios in the sense of Rao's Quadratic Entropy (*RQE* portfolios).

The strong relations between portfolio components (stocks, markets) are an important element influencing portfolio analysis. It is well-known that such relationships intensify during or after the periods of economic crisis. So, one can state two questions: whether diversification intensifies during the economic crisis and how the existence of these strong relationships between components affects diversification.

The article is divided into two parts. In the first part selected methods of diversified portfolios construction are presented. Also, the measures to quantify the level of diversification are discussed. The second part of the article is the presentation of the results of empirical research. The main goal of the study was the analysis of diversification for selected world markets. Diversification was compared for the European, Asian and American markets. The level of diversification was measured by the Diversification Ratio, Rao's Quadratic Entropy and the Portfolio Diversification Index. The relations between the analysed markets and their connection with the diversification were also considered. Presented study is new in the financial and investment field. None of the mentioned methods had been applied for the international markets so far. These methods had also not been compared with each other.

2. Definitions of selected diversification measures

One of the diversification measures used in the research is Diversification Ratio (*DR*) proposed by Cheng and Roulac [2007]. The authors of this measure argued that the essence of diversification lies in the differences between the risk of portfolio and the weighted sum of the risks of individual components. Firstly, Diversification Ratio was applied to measure the geographical diversification. However, Choueifaty and Coignard [2008] proposed this measure for portfolio

analysis. Diversification Ratio can be formulated for different measures of risk. Most often, the *DR* is defined as the quotient of the weighted sum of standard deviations of components and the standard deviation of the entire portfolio. The mathematical definition of *DR* is the following:

$$DR = \frac{\sigma_a}{\sigma_p} = \frac{\sum_{i=1}^{N} \sigma_i x_i}{\sigma_p}, \tag{1}$$

where: N – the number of stocks in portfolio, σ_p – the standard deviation of the portfolio, σ_a – the weighted sum of standard deviations for components, σ_i – the standard deviation of i-th stock in the portfolio, x_i – the share of the i-th stock in the portfolio (for $i = 1, 2, \ldots, N$).

A higher value of *DR* indicates a higher level of diversification. The values of *DR* do not determine what part of risk can be diversified, because the values of the *DR* are higher than 1. *DR* equal to 1 means a portfolio that is fully concentrated in one asset. Applying *DR* as an objective function with which one can construct the most diversified portfolio (*MDP*). The optimization model which can be used to construct the well-diversified portfolio is the following:

$$\begin{aligned} &DR \rightarrow max, \\ &\sum_{i=1}^{N} x_i = 1, \\ &x_i \geq 0. \end{aligned} \tag{2}$$

The result of using the above model is a portfolio that maximizes the distance between two definitions of portfolio volatility – the weighted sum of volatility of assets of the portfolio and the total volatility of the portfolio [Cheng, Roulac 2007].

Another measure used to construct a diversified portfolio was Rao's Quadratic Entropy (*RQE*). Rao's Quadratic Entropy [Rao 1982a, 1982b] was introduced as a measure of diversity. Mainly it was used in statistics and in ecology. However, Carmicheal et al. [2015] presented this entropy as a measure of portfolio diversification. The Rao's Quadratic Entropy for portfolio is defined as:

$$RQE = \frac{1}{2} \sum_{i,j=1}^{N} d_{ij} x_i x_j, \tag{3}$$

where $D = \left[d_{ij} \right]_{i,j=1}^{N}$ – the function of dissimilarity.

The function of dissimilarity measures the differences between any two components of the portfolio. As a dissimilarity function one can assume any function of two arguments, satisfying the following conditions:

- $d_{ij} \geq 0$ for all $i, j = 1, 2, \ldots, N$;
- $d_{ij} = d_{ji}$ for all $i, j = 1, 2, \ldots, N$;
- $d_{ii} = 0$ for all $i = 1, 2, \ldots, N$.

In the literature, the function of dissimilarity is defined among others for the Kronecker delta, the covariance matrix of rates of return or the correlation matrix. In all these cases the *RQE* is a generalization of another measure of diversification. Using the correlation matrix in the definition of dissimilarity function, the results are very similar to the results obtained for *MDP*. In the presented research the function D was defined for the covariance matrix. The values of d_{ij} were calculated according to the following formula [Carmicheal et al. 2015]:

$$d_{ij} = \sigma_i^2 + \sigma_j^2 - 2\sigma_{ij}, \tag{4}$$

where σ_i^2 is the variance of i-th stock ($i = 1, 2, \ldots, N$) and σ_{ij} is the covariance between i-th and j-th stocks ($i, j = 1, 2, ..., N$). Thus, the Rao's Quadratic Entropy for the portfolio was calculated as:

$$RQE = \frac{1}{2}\sum_{i,j=1}^{N}\left(\sigma_i^2 + \sigma_j^2 - 2\sigma_{ij}\right)x_i x_j. \tag{5}$$

In this case the RQE portfolio is an equivalent to the diversification return [Booth, Fama 1992; Willenbrock 2011]. The optimal *RQE* portfolios were selected by using the following model:

$$\begin{aligned} &RQE \rightarrow max, \\ &\sum_{i=1}^{N} x_i = 1, \\ &x_i \geq 0. \end{aligned} \tag{6}$$

The higher value of *RQE*, the higher level of portfolio diversification. In this case, as a result we receive a portfolio with minimum concentration of information. These portfolios are also called the portfolios with maximum effective number of independent risk factors [Carmicheal et al. 2015].

The ideal situation on the stock market is when the rates of return of all stocks are uncorrelated. Unfortunately, this situation is only hypothetical. In the research, very often, various methods are used to transform the set of correlated stocks into the set of independent variables. Also, the problem of diversification can be considered in this context. One of the most popular methods to create a set of uncorrelated data is Principal Component Analysis (*PCA*). In the presented research the approach of *PCA* was used both to assess the level of diversification, as well as to select markets that were strongly interrelated.

Let Σ denote – the covariance matrix of the rates of return of stocks. This matrix can be transformed to the form:

$$\Sigma = E\Delta E^T , \tag{7}$$

where E is the square matrix of degree N composed of eigenvectors (e_i, $i = 1, 2, \ldots, N$) of covariance matrix Σ and Δ is the diagonal matrix of degree N which elements are eigenvalues (λ_i, $i = 1, 2, \ldots, N$) of the matrix Σ. The set of eigenvectors defines the set of N uncorrelated portfolios (called principal portfolios). Rates of return of these portfolios are responsible for randomness of the market.

Using the eigenvalues of covariance matrix, Rudin and Morgan [2006] defined the Portfolio Diversification Index in the form:

$$PDI = 2\sum_{k=1}^{N} \frac{k\lambda_k}{\sum_{i=1}^{N} \lambda_i} - 1 . \tag{8}$$

The most important properties of *PDI* index are the following:

- the higher value of *PDI*, the higher the level of diversification,
- $PDI = 1$ for a portfolio dominated by one component,
- $PDI = N$ for a fully diversified portfolio only if all the components are uncorrelated and all the shares are equal to $1/N$,
- $PDI < N$ reflects the interaction in different assets (more variability of rates of return is explained by the few first principal components).

The principal component analysis can be also used to determine the existence of strong relationships between the markets. In this case the procedure is the following:

- the principal component analysis is applied to the set of the analysed stocks,
- the eigenvalues higher than 1 indicate the significant principal components,
- the varimax rotation is used to the factors of principal components,
- in the significant principal components, factors with the absolute values higher than 0.7 are selected (the highest absolute values of factors of principal component indicate the stocks (markets) most represented by the principal component – between these markets strong relations exist).

All methods presented above were used in the study to compare the level of diversification for the selected European, Asian and American markets in the pre- and post-crisis periods. The results are presented in the next section.

3. Empirical analysis of the diversification level for selected world markets in the period 2005-2016

The problem of international diversification was analysed for selected world markets. The first group consisted of 20 European markets, in the second group 13 markets (1 Australian and 12 Asian) were studied, and in the third group – 6 American countries were considered. Markets were analysed on the basis of selected indices which represented given countries. The full list of the analysed markets is as follows:

- group I – European markets: Belgium, Bulgaria, Czech Republic, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Italy, Latvia, Lithuania, The Netherlands, Norway, Poland, Portugal, Romania, Russia, Slovakia, Spain, Sweden, Switzerland, Turkey, Ukraine, United Kingdom;
- group II – Asian and Australian markets: Australia, China, Hong Kong, India, Indonesia, Japan, Malaysia, New Zealand, Philippines, Singapore, South Korea, Taiwan, Thailand;
- group III – American markets: Argentina, Brazil, Canada, Chile, Mexico, USA.

The selection of the analysed indices was connected with the data availability and completeness. Calculations were carried out on the basis of daily rates of return for selected markets. All markets were analysed in the period 2005-2016. Additionally, on the basis of quotations of the WIG20 index, these 4 following sub-periods were established:

- period I: January 2005 – June 2007 (long-term increases of quotations),
- period II: July 2007 – February 2009 (log-term decreases of quotations),
- period III: March 2009 – March 2011 (renewed increases of quotations),
- period IV: April 2011 – December 2016 (low fluctuations of quotations).

Firstly, all markets were analysed according to the level of diversification measured by using three discussed methods (Tables 1-3). For the *DR* and *RQE* indices, their optimal values (obtained as a solution of models (2) and (6)) are presented.

Table 1. *PDI* index for the analysed markets in a given period

Market	Period I	Period II	Period III	Period IV	2005-16
European	11.09	7.57	8.99	10.09	9.76
Asian	7.48	5.19	6.23	7.00	6.38
American	3.19	2.60	2.73	3.56	3.12

Source: own study.

Table 2. *RQE* values for the analysed markets in a given period

Market	Period I	Period II	Period III	Period IV	2005-16
European	0.000310	0.001415	0.000546	0.000461	0.000281
Asian	0.000284	0.000715	0.000256	0.000196	0.000283
American	0.000067	0.000184	0.000070	0.000114	0.000103

Source: own study.

Table 3. *DR* values for the analysed markets in a given period

Market	Period I	Period II	Period III	Period IV	2005-16
European	36.84	23.58	29.67	39.00	30.85
Asian	25.93	19.31	31.40	35.54	29.73
American	60.68	32.54	53.74	47.22	48.90

Source: own study.

According to *PDI* the most diversified was the European market, both for all sub-periods and also for the whole period 2005-2016. In all cases, the lowest diversification was obtained for the American market. Similar results were received for the optimal values of Rao's Quadratic Entropy. The only difference was for the whole period 2005-2016, then the Asian market was a little bit more diversified then the European market. However, this difference was not very significant. We can state that the *PDI* and *RQE* are consistent in their assessment in the sense that they order markets (sets of stocks) in a similar way.

The analysis of markets assessment on the basis of *DR* indicates that the American market was the most diversified in all periods. Except for the post-crisis period, in all other periods the least diversified market was the Asian market. In the sub-period III, the lowest level of *DR* was received for the European market.

We can observe that for the *PDI* and *DR* the level of diversification in the period of crisis was lower than in the pre- and post-crisis periods. For *RQE* the dependency was opposite. In the second sub-period the level of diversification increased and then decreased.

The comparison of *RQE* and *MDP* was extended by the analysis of composition of both types of portfolios. In Tables 4-6 the markets rankings according to shares in the *MDP* and *RQE* portfolios were presented. The values of shares were obtained as a result of solution of models (2) and (6). Value 1 means the market with the highest share and the symbol "–" indicates the market with zero share in the portfolio. On the basis of the presented results it is easy to state that portfolios *RQE* and *MDP* were totally different in composition. Generally, the *RQE* portfolios consisted of smaller number of components and also the differences between the values of shares in both portfolios were significant.

From the European markets, only four were not a part of any *RQE* portfolios. These were: Belgium, Germany, United Kingdom and Switzerland. Three markets: Iceland, Russia and Ukraine had influence on the diversification in every analysed period. The strongest influence (measured by the size of share) on the diversification for the given period had the following countries: in the pre-crisis period – Romania, Turkey, Ukraine; in the period of crisis – Iceland, Russia, Hungary; in the post-crisis period – Greece, Ukraine, Latvia; in the sub-period IV – Greece, Ukraine and Russia; in the long period 2005-2016 – The Netherlands, Ukraine and Latvia. We could also observe big differences between the shares for some markets in the subsequent periods, for example: Iceland (12^{th} place in the sub-period I, 1^{st} place in the sub-period II and 11^{th} place in the sub-period III) or Greece (zero-share in the first two periods and 1^{st} place in the sub-periods III and IV).

Generally, the *MDP* portfolios for the European markets were a little bit more different – they had more components than *RQE* portfolios in the same period. Countries such as The Netherlands, France, Germany, Finland and Sweden did not have any influence on the level of diversification (all of them were zero-share

markets). Romania, Iceland, Latvia, Estonia, Lithuania, Slovakia, Switzerland, Bulgaria, Ukraine and Poland – these markets were a part of the *MDP* in all periods. The highest shares had mainly the markets of Iceland, Latvia, Slovakia and Bulgaria. It should be also noted that for the *MDP* there were not so big differences between the positions in rankings for a given market.

Table 4. Rankings of the European countries according to shares in *RQE* and *MDP* portfolios

Country	*RQE* portfolios					*MDP* portfolios				
	I	II	III	IV	05-16	I	II	III	IV	05-16
Belgium	–	–	–	–	–	–	3	–	–	–
Bulgaria	8	7	12	8	–	3	5	4	1	4
Czech Rep.	11	–	–	–	–	9	–	–	–	–
Estonia	14	–	8	–	–	5	6	6	9	6
Finland	16	–	–	–	–	–	–	–	–	–
France	17	–	–	–	–	–	–	–	–	–
Germany	–	–	–	–	–	–	–	–	–	–
Greece	–	–	1	1	–	–	–	8	10	9
Hungary	6	3	6	12	–	–	10	15	11	12
Iceland	12	1	11	10	5	4	4	1	2	2
Italy	–	–	15	6	–	11	–	–	–	–
Latvia	10	–	3	7	3	1	2	3	4	3
Lithuania	13	–	13	–	–	7	12	12	5	10
The Netherlands	–	–	–	–	1	–	–	–	–	–
Norway	9	–	–	–	–	12	–	–	–	–
Poland	5	–	10	–	–	14	9	9	12	13
Portugal	–	–	–	11	–	6	–	10	14	14
Romania	1	6	7	–	–	10	8	13	15	11
Russia	4	2	5	3	4	–	–	14	13	–
Slovakia	7	–	4	4	–	2	1	2	3	1
Spain	–	–	14	9	–	–	–	–	16	–
Sweden	15	–	–	–	–	–	–	–	–	–
Switzerland	–	–	–	–	–	15	11	7	8	5
Turkey	2	5	9	5	–	13	–	5	6	8
Ukraine	3	4	2	2	2	8	7	11	7	7
United Kingdom	–	–	–	–	–	16	–	–	–	–

Source: own study.

Only two of the Asian markets were a part of RQE portfolios in every period. These were China and India – both countries were not related to other Asian countries. Three Asian markets – Japan, Philippines and Thailand had a zero share only in one of the analysed periods. Countries such as Malaysia, Singapore and Taiwan in every case had a share equal to zero. Only two Asian markets – Hong Kong and Singapore – were not a component in any MDP portfolio. New Zealand, China and Thailand had the biggest influence on diversification.

Table 5. Rankings of the Asian countries according to shares in *RQE* and *MDP* portfolios

Country	*RQE* portfolios					*MDP* portfolios				
	I	II	III	IV	05-16	I	II	III	IV	05-16
Australia	–	3	–	–	7	–	5	10	10	8
China	1	1	1	1	1	4	2	4	3	2
Hong Kong	–	4	–	–	–	–	–	–	–	–
India	2	2	2	6	2	6	3	5	4	4
Indonesia	–	–	3	4	6	10	–	7	7	10
Japan	–	5	4	2	3	8	7	8	9	6
Malaysia	–	–	–	–	–	7	–	–	5	9
New Zealand	–	6	–	–	–	1	1	1	1	1
Philippines	4	–	5	5	5	3	8	3	6	5
Singapore	–	–	–	–	–	–	–	–	–	12
South Korea	5	–	–	–	–	9	–	6	11	11
Taiwan	–	–	–	–	–	5	6	9	8	7
Thailand	3	–	6	3	4	2	4	3	2	3

Source: own study.

Table 6. Rankings of the American countries according to shares in *RQE* and *MDP* portfolios

Country	*RQE* portfolios					*MDP* portfolios				
	I	II	III	IV	05-16	I	II	III	IV	05-16
Argentina	1	3	1	1	1	4	3	3	1	3
Brazil	2	1	3	2	2	6	–	4	5	6
Canada	–	2	2	3	3	2	2	2	4	2
Chile	3	4	4	4	4	1	1	1	2	1
Mexico	4	–	–	5	–	5	–	6	3	5
USA	–	5	–	–	–	3	4	5	6	4

Source: own study.

Two American markets in every analysed period had the lowest share or the share was equal to zero. These were USA and Mexico. Argentina was a country with the highest share, except for the second period when Brazil was on the top position. It should be noted that Canada for most periods had a high share in the *RQE* portfolios (2^{nd}-3^{rd} position). Almost all countries were the components of the most diversified portfolios. Only in the portfolio for the data from the period of the crisis, two countries (Brazil and Mexico) had the shares equal to zero.

RQE and *MDP* portfolios were also compared according to the level of diversification measured in the classical way – by the number of components in the portfolio (the highest values in rankings). This comparison indicates that under different conditions the *MDP* portfolios were more diversified than the *RQE* portfolios.

Table 7. Groups of strongly related markets

Period	European markets
I	The Netherlands, Belgium, France, Germany, Italy, Spain, Switzerland
II	The Netherlands, Belgium, Hungary, France, Germany, Italy, Spain, Switzerland, Poland, Turkey
III	The Netherlands, Belgium, France, Germany, Italy, Spain, Switzerland
IV	The Netherlands, Belgium, France, Germany, Italy, Finland, Spain, Sweden, Switzerland
2005-2016	The Netherlands, Belgium, France, Germany, Italy, Finland, Spain, Switzerland
	American markets
I	Brazil, USA, Mexico, Argentina, Canada
II	Brazil, USA, Mexico, Chile, Argentina
III	Brazil, USA, Mexico, Chile, Argentina
IV	Brazil, USA, Mexico
2005-2016	Brazil, USA, Mexico, Chile, Argentina
	Asian markets
I	Hong Kong, South Korea, Japan, Singapore, Taiwan
II	Hong Kong, Indonesia, Malaysia, South Korea, Thailand, India, Singapore, Taiwan
III	Hong Kong, Malaysia, South Korea, Singapore, Taiwan
IV	Hong Kong, South Korea, Singapore
2005-2016	Hong Kong, South Korea, Singapore, Taiwan

Source: own study.

Groups of countries responsible for the diversification were also compared with the group of countries strongly related in the given period. To determine the strong relationships between the markets the *PCA* was applied (Table 7).

For each sub-period few European countries repeated in the first factors: The Netherlands, Belgium, France, Germany, Italy, Spain and Switzerland. It means that regardless of the nature of volatility of quotations, there were strong relationships between these markets. The biggest group of strongly related countries was reported for the period of the crisis. In this period the first principal component was represented, besides countries mentioned above, by the following countries: Hungary, Poland and Turkey. In the IV sub-period in the group of countries with the strongest relations Finland and Sweden additionally appeared.

For the whole period 2005-2016 the analysis indicated that in the first principal component there were 9 markets: The Netherlands, Belgium, France, Germany, Italy, Finland, Spain, Sweden and Switzerland. Almost all of these countries had a zero-share or very low share in the *RQE* and *MDP* portfolios. The exception is Switzerland which was a part of *MDP* portfolios in every period. Three Asian markets repeated in every sub-period as strongly related. These countries were: Hong-Kong, South Korea and Singapore. Only South Korea had a non-zero share in the *MDP* portfolios in most periods. The other two countries had a weak influence on the diversification in single cases. In all the analysed periods, three American markets repeated: Brazil, USA, Mexico. It should be noted that this is the only group where all markets were strongly related at least in one period.

4. Conclusion

The research carried out indicates that, in general, the most diversified markets were European markets. This property is independent of the conditions on the financial markets. On the basis of the presented results one can state that the *PDI* and *RQE* (optimal values of these indexes) order the set of stocks in the similar way. The assessments according to the *DR* were totally different. The previous research indicated that the *RQE* defined by the correlation matrix provides similar results to the *MDP*. However, when we use the covariance matrix to define the dissimilarity function, the *RQE* and *MDP* are portfolios that are different in composition. The conducted research provides also the confirmation that the countries influencing diversification are different than the markets which are strongly related to each other.

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