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EFFECTIVENESS OF HYBRID OPTIMIZATION METHODS IN SOLVING TEST PROBLEMS AND PRACTICAL ISSUES

Summary: This paper shows the results of analyzing the effectiveness and efficiency of a hybrid approach against a variety of optimization problems. An application with a serial-optimization structure, consisting of several methods with different characteristics, is presented. In order to evaluate the initial effectiveness of the hybrid optimization method, a number of test tasks, represented by complex functions with many variables, were examined. Additionally, a real-life case, determining an optimal product variety in a supermarket environment, regarding the highest rate of return for given conditions and limitations, is presented as an instance of the practical use of a hybrid algorithm. The research shows that the results achieved by the hybrid-optimization method are highly satisfactory, both in terms of efficiency as well as effectiveness.

Keywords: hybrid, optimization, decision problem.

1. Introduction

Development of decision problem classes induces a higher demand for effective methods regarding decision support in many areas of human activity. Therefore, making a proper business decision requires a sophisticated analysis performed on large data sets.

The more elaborate the decision-making process, the higher computational complexity required to solve these tasks. On the other hand, new emerging optimization methods are usually dedicated for selected classes of tasks.

It should be noticed that in most cases, the decisions undertaken by a manager do not have to fulfill the strict meaning of the word “optimality”. Solving real-life problems usually take place in a dynamic environment, often with the absence of a full set of parameters - these only roughly map reality. Thus, business decisions supported by a management support system should lead to achieving far better results than decisions made without such a system.

The hybrid approach presented in this paper, fulfills those requirements by eradicating an area of a multidimensional function. Moreover, it offers a sub-optimal solution, which can satisfy a decision-maker.

2. Hybrid optimization methods

The essence of hybrid optimization methods is to construct a hybrid that uses components with different characteristics. This approach takes the advantages of each integrated method while eliminating or limiting the influence of their shortcomings.

The hybrid presented in this paper has a serial structure. It consists in combining several methods that follow each other – the current method transmits data to the next one for further analysis.

The aim of developing such a hybrid method was to obtain the most flexible tool, which could solve a wide range of tasks (supporting manager’s decisions) by using different computing characteristics. By its very nature, a hybrid method does not require the user to adapt its structure to the problem. Actually, management problems are characterized by a high degree of structure complexity and the number of existing restrictions. These problems are often described by functions that are complex, discontinuous, not differentiable and containing internal constraints like equations and/or inequalities.

For this reason, a solution of this type of tasks is not feasible by using classical optimization methods, such as analytical ones. Thus, in this case, the application of a hybrid method containing simple components can lead to satisfying results.

The hybrid variant of the optimization method presented in this paper is the simplest possible model that uses a hybrid approach. It consists of only two components:

1. Initial method − Monte Carlo method
2. Finalizing method − Hooke-Jeeves’ method (see [Findeisen, Szymanowski, Wierzbicki 1980]).

It should be noted that even such a simplified model of a hybrid method, without intermediate layers, is able to successfully solve tasks which are problematic even for more complex dedicated optimization methods (see [Bersini 1996)]. Moreover, it can solve a wide range of problems where analytical methods are useless.

3. Effectiveness of a hybrid method in solving sample tasks.

For an initial evaluation of the proposed hybrid approach, regarding its effectiveness, we have used a sample benchmark functions commonly used to evaluate optimization algorithms (ICEO). The hybrid method was implemented in a novel application and tested across a set of benchmark functions.

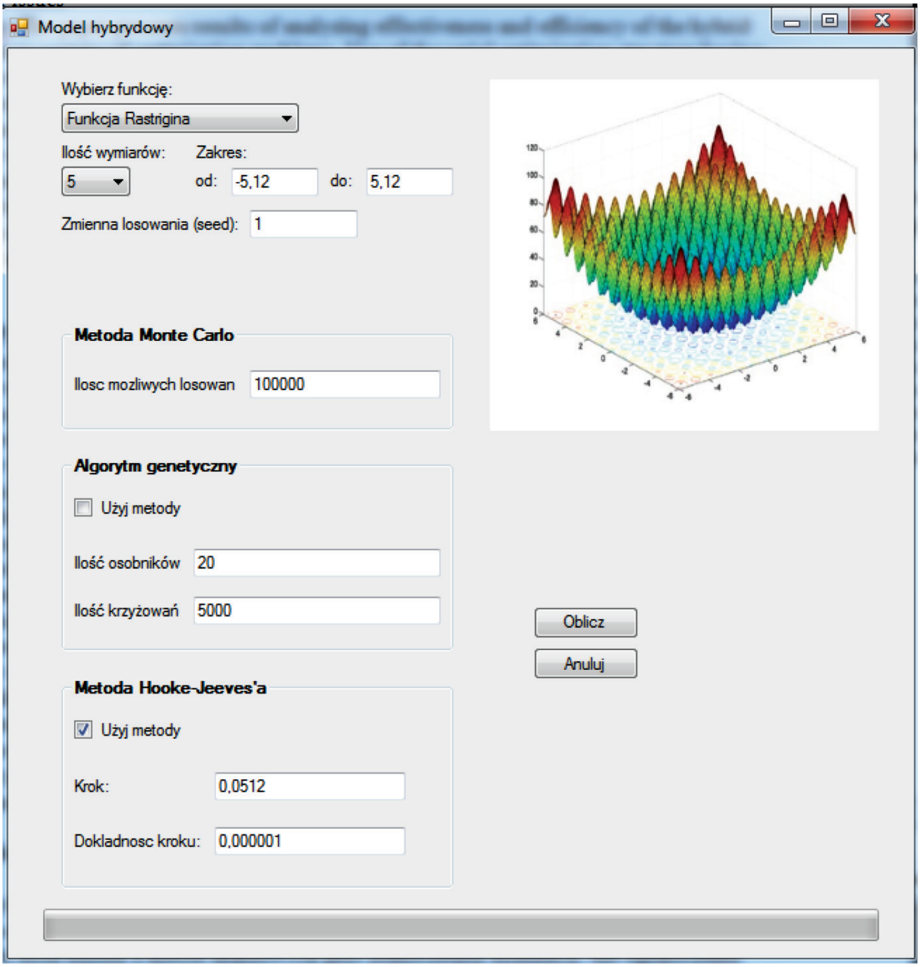


Figure 1. Sample view of an application containing hybrid-optimization algorithm

Source: own elaboration.

In order to preserve the same conditions for all the experiments, analyses were conducted in a five-dimensional domain. The sample benchmark functions optimization results , with different hybrid optimization parameters, are presented below:

Quadratic function

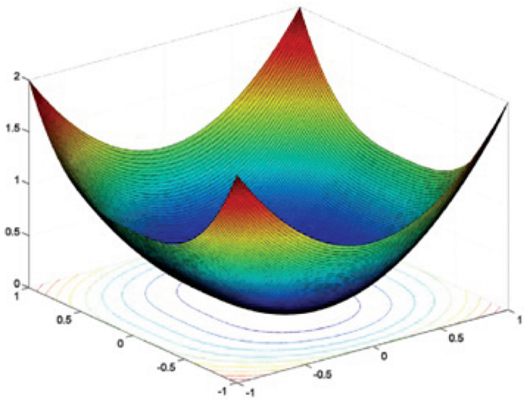
This is the simplest function required for testing the correctness of an algorithm implementation.

Moreover, this function is unimodal, continuous and differentiable throughout its domain.

Tables' description:

Monte Carlo iterations – iterations number of the initial optimization method, before moving to a finalizing one.

HJ Step − initial step length of the Hooke Jeeves method (1% of the optimized function's domain range)



Monte Carlo iterations	**1**
HJ step	0,01
demanded precision	0,000001
f(x):	0,0000000000004265
x[0]:	–0,0000004781701891
x[1]:	–0,0000000827473173
x[2]:	0,0000003636507418
x[3]:	0,0000002362314647
x[4]:	–0,0000000542239110

Monte Carlo iterations	**100000**
HJ step	0,01
demanded precision	0,000001
f(x):	0,0000000000006675
x[0]:	–0,0000005976625059
x[1]:	–0,0000000110329467
x[2]:	–0,0000003687359482
x[3]:	–0,0000003981855510
x[4]:	0,0000001249424112

Monte Carlo iterations	**100**
HJ step	0,01
demanded precision	0,000001
f(x):	0,0000000000010856
x[0]:	0,0000004405393636
x[1]:	0,0000003806155748
x[2]:	0,0000004372314445
x[3]:	–0,0000005707616539
x[4]:	0,0000004793097260

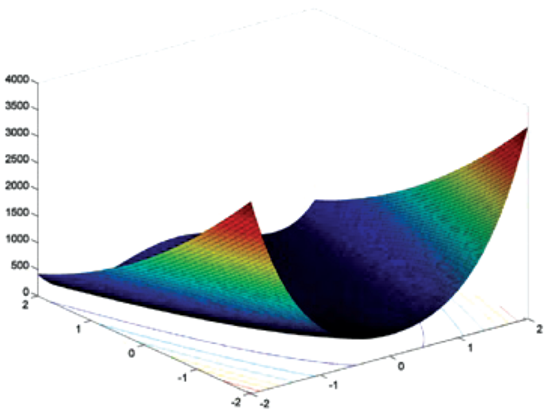
Figure 2. Quadratic function graph

Source: own elaboration.

Our results confirmed the correctness of the Hooke Jeeves algorithm implementation. The second phase of the hybrid optimization, regardless of the iteration number in the initial method (Monte Carlo), proceeds to a minimum point. The minimum point was reached in each case with the required accuracy.

Rosenbrock's "saddle"

This is a unimodal function, continuous and differentiable throughout its domain. Due to a specific shape of a "slowly descending valley", the function is commonly used to test the optimization algorithms.



Monte Carlo iterations	**1**
HJ step	0,02048
demanded precision	0,000001
f(x):	0,0000002315654134
x[0]:	0,9999475000000410
x[1]:	0,9998953146712670
x[2]:	0,9997911437147140
x[3]:	0,9995826642599060
x[4]:	0,9991651298215450

Monte Carlo iterations	**100**
HJ step	0,02048
demanded precision	0,000001
f(x):	0,0000001516622176
x[0]:	0,9999574396044130
x[1]:	0,9999155700808250
x[2]:	0,9998310474287460
x[3]:	0,9996621724436370
x[4]:	0,9993243462452360

Monte Carlo iterations	**100000**
HJ step	0,02048
demanded precision	0,000001
f(x):	0,0000002248331728
x[0]:	0,9999488387442550
x[1]:	0,9998970487679580
x[2]:	0,9997941696131100
x[3]:	0,9995886744069800
x[4]:	0,9991771568910590

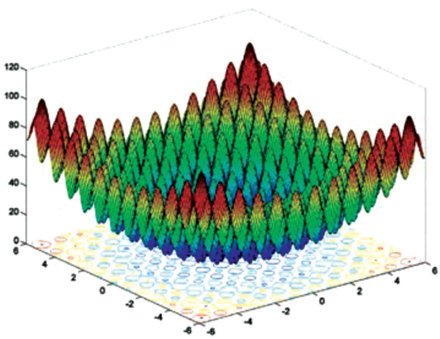
Figure 3. "Rosenbrock's saddle" function graph

Source: own elaboration.

In the case of the Rosenbrock function optimization, due to its specific shape, the first phase of the hybrid method (Monte Carlo) slightly reduced the number of steps in the second phase. The starting point was closer to optimum. Moreover, this specific shape of the Rosenbrock function allows us to notice a vulnerability of the Hooke-Jeeves method to some particular shape conditions of the optimized function. In the neighborhood of point (–1,1,1,1,1), due to a gradient direction of a Rozenbrock's bottom slope, the Hooke-Jeeves method incorrectly shows a value of (4.0) as a minimum of the function (explorations are carried out in orthogonal directions and their combinations with the same step length for all coordinates). This problem, termed as a "problem of an ostensible minimum" was described in more detail in previous papers (see [Twardochleb, Rychcicki 2009]). However, an application of the Monte Carlo method in the initial phase of the hybrid optimization avoids this problem. Moreover, a simple modification of the Hooke Jeeves algorithm, which introduces modifiable steps' length to the orthogonal directions of our research, allows the method to avoid getting stuck at the "ostensible minimum" point.

Rastrigin function

This is a function containing multiple local minimums. With an increase of dimensionality, the number of local minimums increases 11 times with each additional dimension of the Rastrigin function. For instance, in our sample, the function of five variables has a number of 161 051 local minimums.



Monte Carlo iterations	**1**
HJ step	0,0512
demanded precision	0,000001
f(x):	37,8082774660949000
x[0]:	–2,9848555732313700
x[1]:	–3,9797832361662700
x[2]:	0,0000002993917980
x[3]:	2,9848558970051000
x[4]:	1,9899122223735800

Monte Carlo iterations	**100**
HJ step	0,0512
demanded precision	0,000001
f(x):	20,8940948133116000
x[0]:	–1,9899129484467300
x[1]:	1,9899124119551700
x[2]:	0,0000002030256161
x[3]:	–2,9848561013587100
x[4]:	1,9899119579980400

Monte Carlo iterations	**100000**
HJ step	0,0512
demanded precision	0,000001
f(x):	7,9596674189774800
x[0]:	–0,9949585755372250
x[1]:	–1,9899118279518600
x[2]:	0,9949587108177000
x[3]:	–0,9949584175126010
x[4]:	–0,9949588144169410

Monte Carlo iterations	1	100	100000
Euclidean distance	6,16	4,58	2,83
„Manhattan" 100 distance	12	9	6
Function value at a point	37,8082774660949000	20,8940948133116000	7,9596674189774800

Figure 4. Rastrigin's function graph

Source: own elaboration.

Therefore the results allow us to consider the hybrid method is a sufficient one to solve this type of tasks. It should be noticed that in real-life cases it is more important to obtain a satisfactory solution in a short operating time than a perfect solution within a long period.

4. Application of hybrid optimization method for solving real tasks of business decisions

In order to investigate the usefulness of the presented hybrid approach for an optimization, a research was also conducted for the sample tasks of management decisions.

For instance, one of the cases depends on optimizing the model of sales assortment taking into account expert knowledge regarding basic parameters. In Table 1 modeled rates of return for several product groups are presented according to their percentage involvement. The figures based on the experts' opinions are discrete − values of the function (rate of return for each class of goods) are given at selected points which determine the value of invested capital in that group. Hence, from a decision-maker's perspective, it is important to select such a range of goods where the total rate of return generated by the group of products is maximal.

Table 1. Rates of return of a sample group of goods

Product group (*n*)	i	1	2	3	4	5	6	…
1	Involved capital (C)	30 000 zł	40 000 zł	50 000 zł	60 000 zł	70 000 zł	80 000 zł	…
	Expected rate of return (ROE)	200%	210%	230%	235%	240%	240%	…
2	Involved capital (C)	20 000 zł	30 000 zł	40 000 zł	50 000 zł	60 000 zł	70 000 zł	…
	Expected rate of return (ROE)	90%	95%	95%	95%	100%	100%	…
…								

Source: own elaboration.

For each of the n groups of products, an expert has determined the expected rate of return (*ROE*) based on invested capital (*C*). This can be written as an ordered collection of pairs:

$$\{(K_{n1}, ROE_{n1}), \ldots, (K_{ni}, ROE_{ni})\},$$

where $K_{n1} < K_{n2} < \ldots < K_{ni.}$

In order to run an optimization process, it was decided to linearize the analyzed domain using a linear extrapolation of the function for every group in the set according to:

$$f_n(x_n) = \frac{ROE_{nk} - ROE_{nj}}{K_{nk} - K_{nj}}(x_n - K_{nj}),$$

where the indexes determine the adjacent ranges based on data provided by an expert. Therefore, the function for each group of products will take the form of a set of linear functions in each interval defined by specified parameters.

While in the example there are n product groups, an objective function takes the form:

$$f(x) = \sum f(x_n),$$

where x_n describes capital invested in the n-th group of products. The function returns an expected rate of return for the total invested capital.

It should be noted that in the presented example there is a restriction in the amount of invested capital (equation constraint) that can be formally written as

$$\sum x_n = K_{\max},$$

where K_{max} is total invested capital and there are inequality constraints for each group of products $x_i \geq 0$, which prevent investing a below-zero amount of capital in any group of products.

The final function is a composite of several partial functions, each of which is not differentiable in the whole domain – its characteristic depends on the parameters set by an expert (in a real-life situation, the parameters may dynamically change according to records from point of sales).Thus, it is almost impossible to assess the characteristics of an objective function before the optimization process begins. The optimization, because of the nature of the function and number of constraints, cannot be carried out using analytical tools.

Because of the equation constraints, a simple mechanism was used to return to the surface defined by this limitation:

If at the time t of the optimization

$$K_t = \sum x_i,\ K_t > K_{max},$$

then for each x_i

$$x_i = x_i \frac{K_{\max}}{K_t}.$$

For the presented example, assuming an invested capital (C_{max}) of 100 000 zł for five product groups, the results of the optimization using the hybrid method are presented in Table 2.

Table 2. Results of a hybrid optimization method against a different number of Monte Carlo iterations

Monte Carlo iterations	1
HJ step	1000
demanded precision	0,000001
f(x):	97 908,7830335271
x[0]:	7 655,67807476989
x[1]:	3 411,14554777864
x[2]:	29 999,99999873010
x[3]:	38 695,4183115572
x[4]:	20 237,7580671642

Monte Carlo iterations	100
HJ step	1000
demanded precision	0,000001
f(x):	170 773,428450695
x[0]:	393,818093903356
x[1]:	7 524,84881252449
x[2]:	79 999,9999996474
x[3]:	10 210,7723001275
x[4]:	1 870,56079379732

Monte Carlo iterations	10000
HJ step	1000
demanded precision	0,000001
f(x):	175 549,012692215
x[0]:	1 725,1121227414
x[1]:	1 709,51391654887
x[2]:	79 999,9999998976
x[3]:	16 513,4305384022
x[4]:	51,94342240992

Source: own elaboration.

The results show (Table 2) that the analyzed function is not a unimodal function. This is documented by the fact that the results differ in the initial phase of the optimization according to the number of Monte Carlo iterations. Moreover, it is possible to obtain a shape of the function only when the process of investigating a solution has been accomplished.

It is noticeable that every iteration of the Hooke Jeeves optimization method gives a better result. It is also worth noting that the method is resistant to the problems of the function's domain, not differentiability.

Finally, the hybrid method generates a satisfactory solution without an initial analysis of the surface's shape of the optimized functions.

5. Conclusions

The results of investigating the hybrid optimization method show the high efficiency of the proposed approach regarding different optimization tasks. The hybrid optimization method demonstrated a satisfactory effectiveness in solving a variety of tasks, even in the simplest version. Moreover, it was shown that the combination of simple optimization methods and the hybrid structure leads to a tool that features high versatility. The effectiveness of the hybrid optimization was satisfactory, both in terms of seeking a minimum of benchmarked functions, as well as solving the practical problem.

The managerial decision support case is an instance of a task where detailed characteristics of the mathematical problem are not known before the optimization process is accomplished. This is a common property of most real tasks, which require computing support due to the large amount of data that need to be analyzed before making a decision. These types of tasks, due to their specific characteristics, are usually not solvable with conventional optimization methods. The results obtained by the hybrid method lead to the conclusion that the proposed approach will be highly effective in solving other tasks of this type. Furthermore, the hybrid method is supposed to be an appropriate tool for building advanced managerial decision support systems.

It is recommended to continue the expansion of the structure of the hybrid method, for instance by including an intermediate layer (e.g. using genetic algorithms). This will further improve the proposed method. Currently, there have been developed successive variants of hybrid optimization methods to build tools of even higher efficiency and improved effectiveness in solving complex tasks.

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SKUTECZNOŚĆ HYBRYDOWYCH METOD OPTYMALIZACJI W ROZWIĄZYWANIU PROBLEMÓW TESTOWYCH I ZASTOSOWANIACH PRAKTYCZNYCH

Streszczenie: W artykule przedstawiono wyniki badań nad skutecznością i efektywnością hybrydowej metody optymalizacji w rozwiązywaniu różnorodnych zadań optymalizacyjnych. Zaprezentowano szeregową strukturę hybrydową, składającą się z kilku metod składowych o odmiennej charakterystyce. Skuteczność optymalizacji hybrydowej wykazano poprzez rozwiązywanie przykładowych zadań testowych (funkcje benchmarkowe). Dodatkowo zaprezentowano skuteczność optymalizacji hybrydowej w rozwiązywaniu praktycznych problemów decyzyjnych na przykładzie zadania ustalenia optymalnego doboru asortymentu w sklepie wielkopowierzchniowym w celu uzyskania maksymalnej stopy zwrotu. Zastosowanie hybrydowej metody optymalizacji pozwoliło na znalezienie zadowalającego rozwiązania zarówno dla funkcji testowych, jak i dla zadania z obszaru wspomagania decyzji menedżerskich.

Słowa kluczowe: hybrydowe metody optymalizacji, wspomaganie decyzji, funkcje benchmarkowe.