A C T A   U N I V E R S I T A T I S   L O D Z I E N S I S
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EVALUATION OF GOLDFELD-QUANDT HOMOSCEDASTICITY TESTS POWER FOR LINEAR TREND CASE

1. INTRODUCTION

Goldfeld and Quandt (1965, 1967) presented two proposals of random term homoscedasticity tests in a single-equation linear econometric model. They concluded on the base of a Monte-Carlo experiment that the power of both tests is acceptable for practice. However, this claim can rise doubts. Looking at the estimated values of the test power one can see that the nonparametric test (peak test) is significantly worse than the parametric one. On the other hand, the comparision of Goldfeld-Quandt parametric test power with BLUS test (Theil 1965) and Harvey-Philips test performed by Harvey and Philips (1974), using exact numerical computations, proved that the Goldfeld-Quandt proposal is at least as good as the competitors.

The aim of this paper is to present some further results concerning the Goldfeld-Quandt homoscedasticity tests.

Goldfeld and Quandt drew 100 samples. In order to obtain more accurate results, our study is based on 10 000 repetitions. In the same time we limit it to the linear trend model.

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2. GOLDFELD-QUANDT HOMOSCEDASTICITY TESTS

Let us consider a linear econometric model

$$y = \mathbf{X}\alpha + \varepsilon \tag{1}$$

with a possibility of heteroscedasticity of the random term while the other classical assumptions hold (see i.e. Theil 1971, section 3.2). Thus the random term covariance matrix can be expressed as follows

$$\text{var } \varepsilon = \Omega = \begin{bmatrix} \sigma_1^2 & 0 & \dots & 0 \\ 0 & \sigma_2^2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \sigma_n^2 \end{bmatrix}$$

Let us assume that we verify the hypothesis

$$H_0\colon \sigma_1^2 = \sigma_2^2 = \dots = \sigma_n^2 \tag{2}$$

against a general monotonic heteroscedasticity

$$H_1\colon \sigma_1^2 \le \sigma_2^2 \le \dots \le \sigma_n^2 \tag{3}$$

Let

$$e = y - \mathbf{X}a$$

be the ordinary least squares residuals vector, and:

$$a = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T y.$$

2.1. THE PEAK TEST

We say that t-th residual $(t = 2, 3, \dots, n)$ creates a "peak" if $|e_t| > |e_u|$ for each $u = 1, \dots, t - 1$. The statistic of Goldfeld-Quandt nonparametric test is the number of peaks obtained for the residuals - the elements of vector e:

$$G = \text{card}\{t\colon 2 \le t \le n, \underset{u=1,\dots,t-1}{\forall} |e_t| > |e_u|\} \tag{4}$$

2.2. THE GOLDFELD-QUANDT PARAMETRIC TEST

Let us express the matrix X and the vectors y and e in the form

$$X = \begin{bmatrix} X_1 \\ X_c \\ X_2 \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_c \\ y_2 \end{bmatrix}, \quad e = \begin{bmatrix} e_1 \\ e_c \\ e_2 \end{bmatrix},$$

where X_1, y_1, e_1 contain n_1 rows, X_2, y_2, e_2 - n_2 rows, and

$$n_1 + n_2 \leq n.$$

The Goldfeld-Quandt's idea consists in applying the classical F test against group heteroscedasticity while $n_c = n - n_1 - n_2$ central observations are removed. Let

$$e_1 = y_1 - X_1a_1 \quad \text{where} \quad a_1 = (X_1^TX_1)^{-1}X_1^Ty_1,$$

$$e_2 = y_2 - X_2a_2 \quad \text{where} \quad a_2 = (X_2^TX_2)^{-1}X_2^Ty_2.$$

The statistic

$$F_c = \frac{e_1^Te_1/(n_1 - k)}{e_2^Te_2/(n_2 - k)} \tag{5}$$

(k denotes the number of columns in matrix X) has the F distribution with $(n_1 - k, n_2 - k)$ degrees of freedom.

3. THE PROBLEM

The aim of this paper is evaluation of Goldfeld-Quandt tests power. Our results are based on a Monte-Carlo experiment.

In our previous work (see T o m a s z e w i c z 1987, M i e d z i ń s k a and T o m a s z e w i c z 1989) we took into account following single-parameter heteroscedasticity models:

- linear heteroscedasticity:

$$\sigma_t^2 = \sigma_0^2\phi_L(\beta, t) = \sigma_0^2(1 + \beta\frac{t}{n}) \tag{6}$$

- parabolic heteroscedasticity:

$$\sigma_t^2 = \sigma_0^2 \phi_K(\beta, t) = \sigma_0^2 \left[1 + \beta\left(\frac{t-1}{n}\right)^2\right] \tag{7}$$

- exponential heteroscedasticity:

$$\sigma_t^2 = \sigma_0^2 \phi_W(\beta, t) = \sigma_0^2 \, e^{\beta t/n} \tag{8}$$

- group heteroscedasticity:

$$\sigma_0^2 = \sigma_0^2 \phi_G(\beta, t) = \sigma_0^2 \begin{cases} 1 & \text{for } t \le n/2 \\ 1 + \beta & \text{for } t > n/2 \end{cases} \tag{9}$$

Obviously, values of β parameter implying $\sigma_t^2 > 0$ for all $t = 1, 2, \ldots, n$ can be accepted.

To compare different models the common heteroscedasticity measure is needed. The most natural one seems to be the coefficient

$$v = \frac{\sqrt{\frac{1}{n} \sum_t (\sigma_t^2 - \bar{\sigma}^2)^2}}{\bar{\sigma}^2},$$

where

$$\bar{\sigma}^2 = \frac{1}{n} \sum_t \sigma_t^2.$$

In this paper we consider the same heteroscedasticity models. Analogously as before we limit our study to linear model (1) where

$$X = \begin{bmatrix} 1 & 1 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ 1 & n \end{bmatrix}. \tag{10}$$

Basing on their results G o l d f e l d and Q u a n d t (1965) suggested that the number of rejected observations n_c should be ca. 30% of total and $n_1 = n_2$. This postulate should not be accepted as general one, because the optimal number n_c depends on the structure of matrix X, and the type of heteroscedasticity. Anyway, for any analysis of the Goldfeld-Quandt parametric test, the numbers n_c or n_1 must be fixed. In B a l c e r a k and T o m a s z e w i c z (1990) we applied a heuristic procedure aiming at the estimation of the optimal number n_1 as a function of n. The procedure was based on a special Monte--Carlo experiment. For

- 3 models of heteroscedasticity (6)-(8),
- 5 values of v ($v \neq 0$),

q = 100 samples were drawn. In each sample the empirical power of the Goldfeld-Quandt parametric test

$$\lambda(n, \alpha, n_1, v, f)$$

was calculated, dependently from

- number of observations $n_1 = n_2$,
- significance level $\alpha = 0.05$, $\alpha = 0.10$.

As a measure of the test power the following Kendall statistic T_n

$$\psi(n, n_1) = \sum_{\alpha} \sum_{k=3}^{n/2} \sum_{v} \sum_{f} \mathrm{sign}(\lambda(n, \alpha, n_1, v, f) - \lambda(n, \alpha, k, v, f))$$

was taken.

For fixed n, sequences

$$\psi(n_1) = \psi(n, n_1)$$

were smoothed by means of parabola

$$\psi(n, n_1) = a_0(n) + a_1(n)n_1 + a_2(n)n_1^2$$

(according to the OLS method). As optimal value n_1

$$n_1^*(n) = - \frac{a_1(n)}{2Q_2 n}$$

was chosen, when $\psi(n, n_1)$ has maximum. Supposing that $n_1^*(n)$ sequence should be smooth, as optimal n_1 we didn´t take $n_1^*(n)$ but their approximations obtained according to OLS method also by means of parabola

$$n_1^{**}(n)/n = b_0 + b_1 n + b_2 n^2$$

with additional condition

$$b_0 = 5 - 10b_1 + 100b_2$$

which means that

$$n_1^{**}(10) = 5.$$

Finally values $\hat{n}_1(n)$ equal to $n_1^{**}(n)$ rounded to the nearest integer values were taken as optimal values n_1. The results are given in Table 1.

Table 1

Estimated optimal values of n_1

n	n_1	n	n_1	n	n_1	n	n_1	n	n_1	n	n_1	n	n_1	n	n_1
10	5	17	8	24	10	31	12	38	14	45	16	52	19	59	22
11	5	18	8	25	10	32	12	39	14	46	16	53	19	60	22
12	6	19	8	26	11	33	13	40	15	47	17	54	19		
13	6	20	9	27	11	34	13	41	15	48	17	55	20		
14	7	21	9	28	11	35	13	42	15	49	17	56	20		
15	7	22	9	29	11	36	13	43	15	50	18	57	21		
16	7	23	10	30	12	37	14	44	16	51	18	58	21		

Source: The author's calculations.

4. EVALUATION OF TEST POWER

The range of our experiment was as follows. For
- four heteroscedasticity models (5)-(8);
- 6 values of coefficient v (see Table 2);
- 6 sample sizes n = 10, 20, 30, 40, 50, 60;

Table 2

Heteroscedasticity measure - coefficient v

Type of heter.	v_0	v_1	v_2	v_3	v_4	v_5
Linear	0.0	0.1	0.2	0.3	0.4	0.5
Parabolic	0.0	0.1	0.3	0.5	0.7	0.9
Exponential	0.0	0.1	0.3	0.5	0.7	0.9
Group	0.0	0.2	0.4	0.6	0.8	1.0

Source: As Table 1.

q = 10 000 samples were drawn (i.e. q vectors y were generated). In each sample

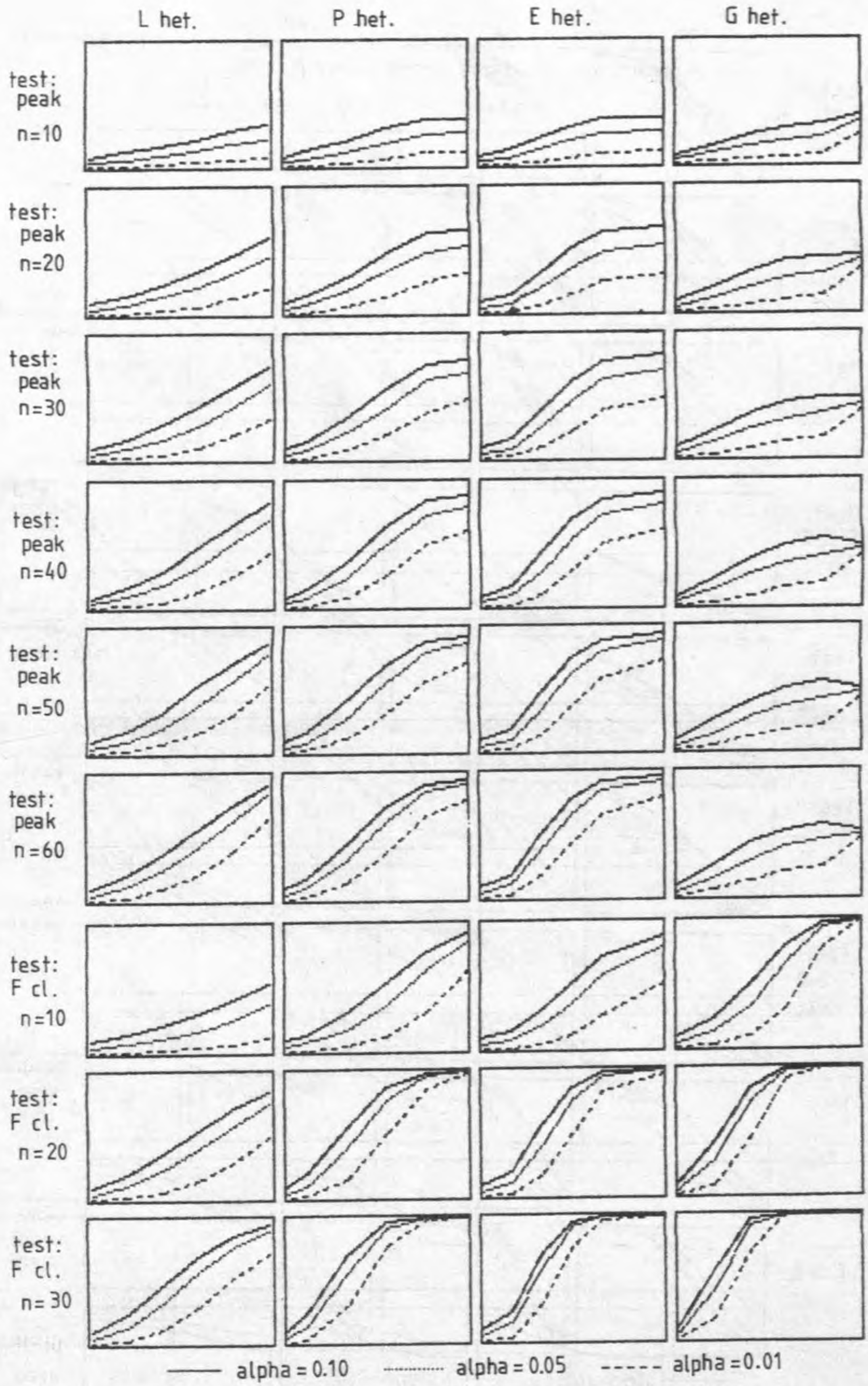

Fig. 1. Homoscedasticity tests power as a function of v

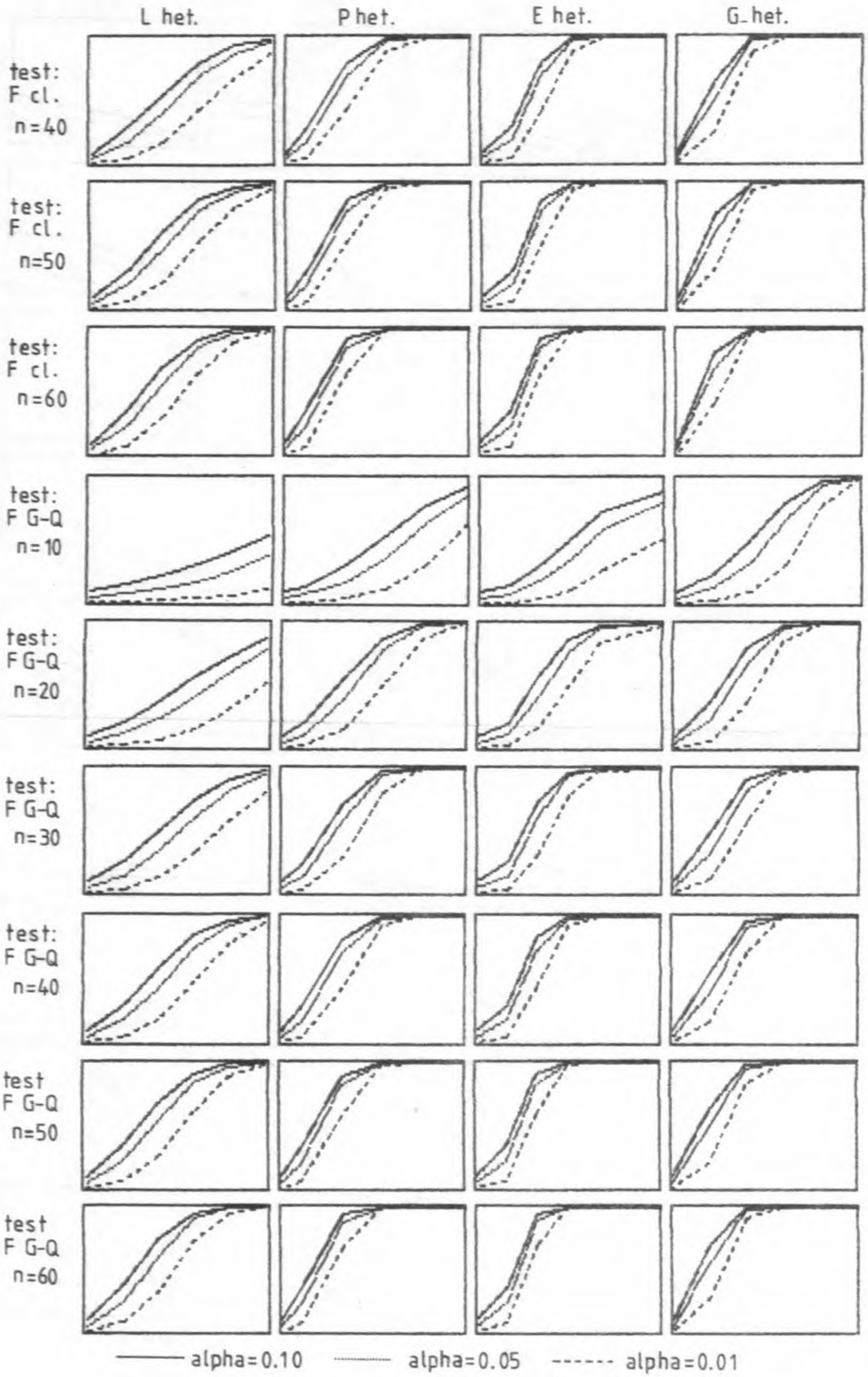

Fig. 1.1. Homoscedasticity tests power as a function of v

- the value of nonparametric Goldfeld-Quandt test statistic G,
- the value of parametric Goldfeld-Quandt test statistic F_c (n_1 was taken according to Table 1),
- the value of classical F test statistic (variant F_c when $n_1 = n_2 = n/2$, i.e. $n_c = 0$)

were computed. These values are compared with critical values of the tests at three significance levels

$\alpha = 0.10,\ 0.05,\ 0.01.$

The randomized Goldfeld-Quandt peak test was applied. The interpolated quantiles were computed according to the approximation formula

$$g(n, \alpha) = \delta_{-2}(\alpha)n^{-2} + \delta_{-1}(\alpha)n^{-1} + \delta_0(\alpha) + \delta_1(\alpha)n + \delta_2(\alpha)n^2$$

- see Balcerak and Tomaszewicz (1990). The coefficients $\delta_j(\alpha)$ for chosen α are presented in Table 3.

Table 3

The coefficients for approximation of interpolated quantiles

α	δ_{-2}	δ_{-1}	δ_0	δ_1	δ_2
0.10	10.50	-12.526	4.8981	0.04635	-0.000247
0.05	20.04	-16.027	5.5490	0.05400	-0.000348
0.01	17.76	-16.902	6.3512	0.07571	-0.000579

Source: As Table 1.

Figure 1 presents curves of the power of the test (range $0 \leq m \leq 1$ on the ordinate) with respect to the v parameter of heteroscedasticity. The range on the abscissa depends on the model of heteroscedasticity:

$v_0 \leq v \leq v_5$,

especially

$0 \leq v \leq 0.5$ for linear heteroscedasticity,

$0 \leq v \leq 0.9$ for parabolic heteroscedasticity,

$0 \leq v \leq 0.9$ for exponential heteroscedasticity,

$0 \leq v \leq 1.0$ for group heteroscedasticity.

The results prove considerable superiority of parametric tests over the peak test. Differences between powers of F_c Goldfeld-

-Quandt test and classical F test seem to be small: classical F test predominates for group heteroscedasticity, in other cases - F_c test. These differences are better visible on Figure 2. Range of v on the abscissa is the same as on Figure 1, and on the ordinate

$$-0.15 \leq m \leq 0.10.$$

The more polarized are $\sigma_1, \dots, \sigma_n$ variances of the distribution, i.e. the more heteroscedasticity model differs from the linear one, the smaller the Goldfeld-Quandt test power is. Differences between powers slighty depend on the choice of significance level.

Of course, the larger significance level, the larger the power of test is. However, for significance levels considered in the experiment the shapes of test power curves are similar. It can be observed on Figure 1.

Special attention should be paid to the fact that in the group heteroscedasticity case, the power of the peak test does not have to be increasing function of the heteroscedasticity parameter v. For large values of v probability of inequality

$$|e_t| < |e_u|$$

for all $t = 1, \dots, n_1$ and $u = n_1 + 1, \dots, n$ is close to 1. So, with probability close to 1, statistic G_n (4) is the sum of two independent random variables

$$G_n = G_{n_1} + G_{n_2} + 1$$

(increased by 1 because e_{n_1+1} always creates the peak), distribution of which is the number of peaks distribution in the homoscedasticity case (variances σ_t^2 are constant in each group of observations $t = 1, \dots, n_1$ and $u = n_1 + 1, \dots, n$). Maximum is clearly visible for larger n and α (see Figure 1).

Tests power depending on number of observations n is shown on Figure 3. The power is quickly increasing with the growth of n (for $v > 0$). The exception is the peak test power for group heteroscedasticity, when the increase is rather small.

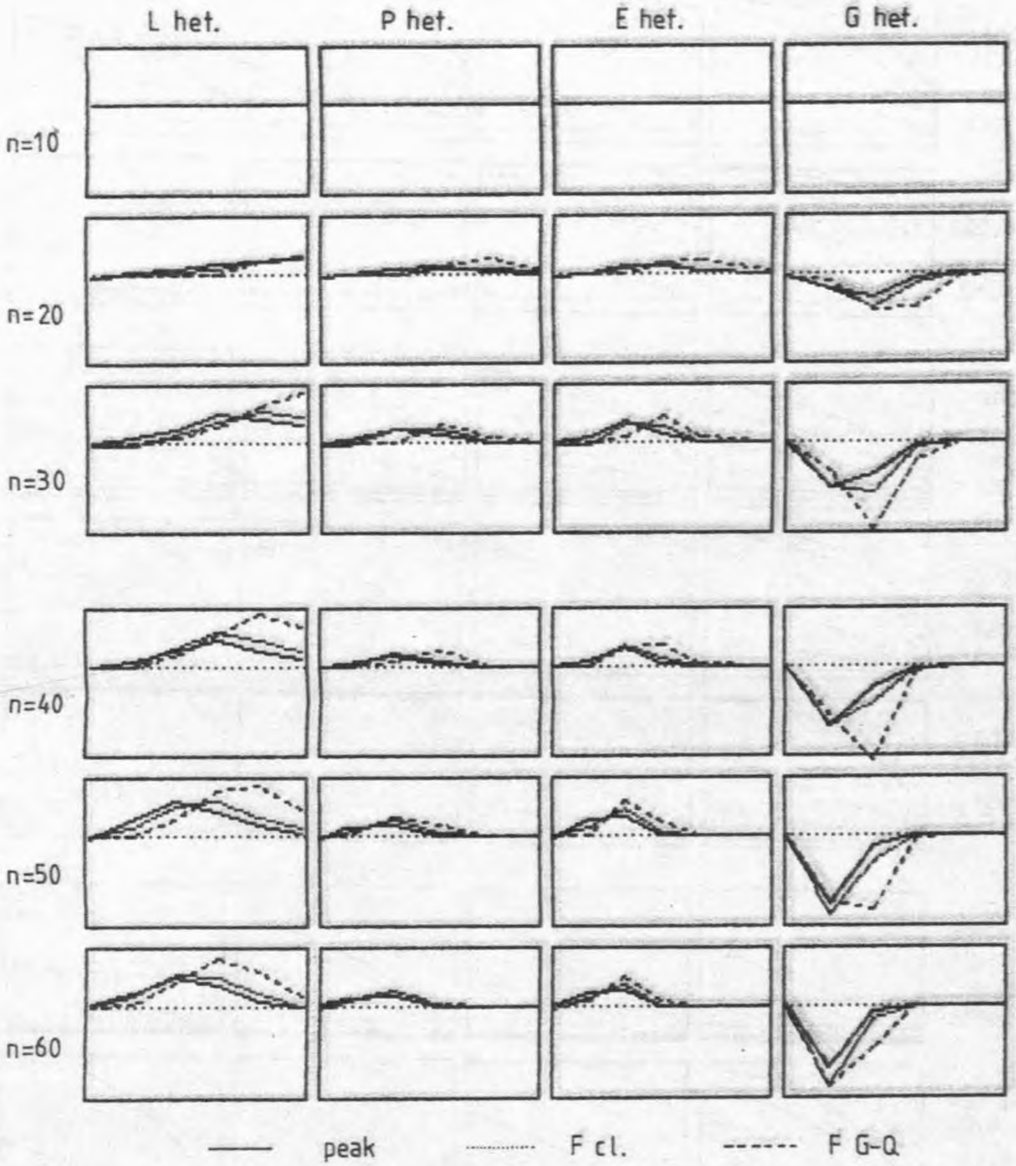

Fig. 2. Difference between F_c and F tests power as a function of v

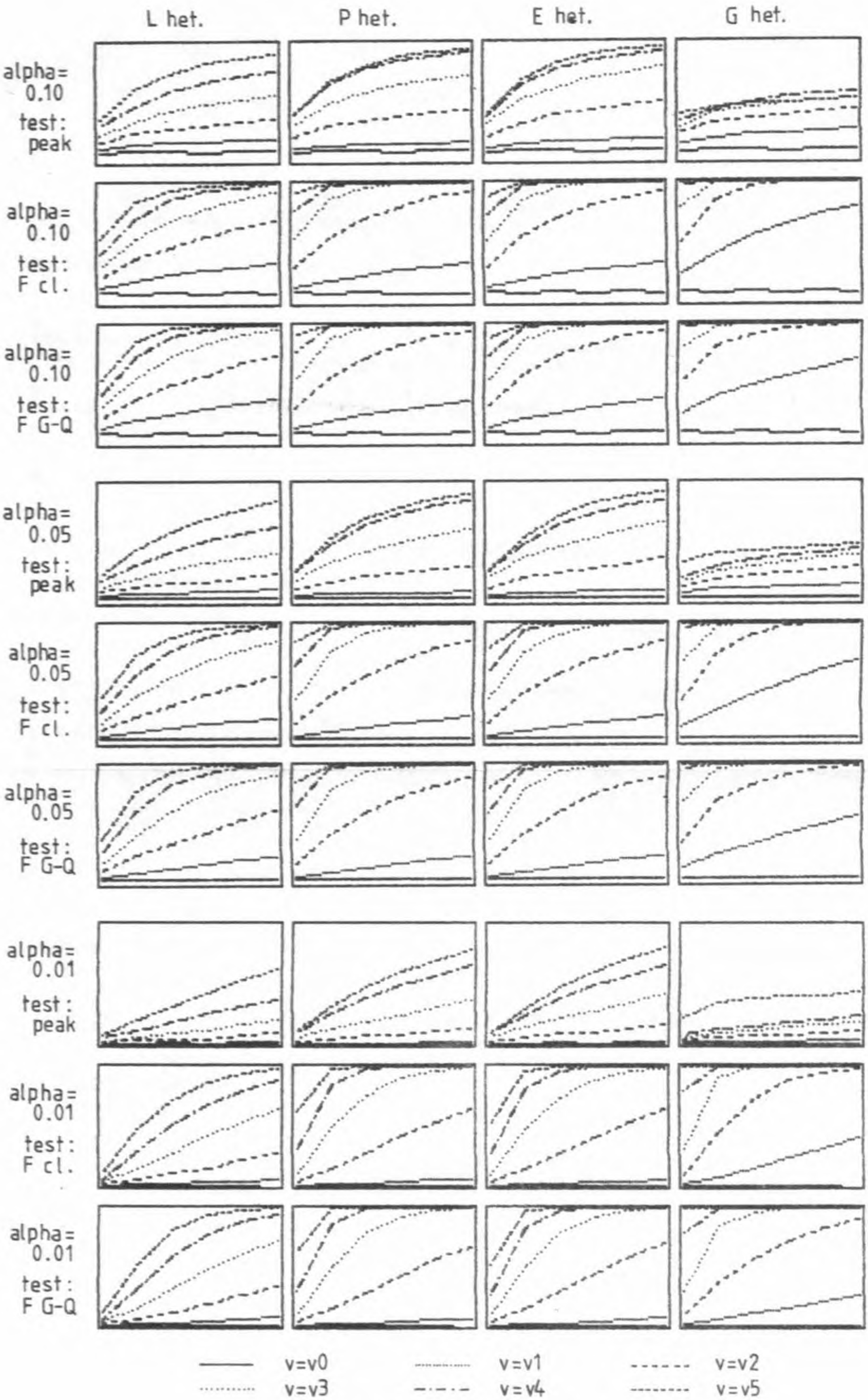

Fig. 3. Homoscedasticity tests power as a function of n

5. FINAL CONCLUSIONS

Basing on the results of the experiment two conclusions seem to be indisputable.

1. The peak test power is clearly smaller than F test power, so the only (rather weak) argument for this test is the simplicity of calculations.

2. Removing central observations while using F test causes growth of power. The more steady the random term variance, the faster the growth of power is. When the increase of variance is abrupt (group heteroscedasticity) power may decrease. So, the parametric Goldfeld-Quandt test is worth using when there are clear reasons to suppose a smooth growth of variance. In the opposite case, it is better to keep the classical F test.

Attention should be paid to the fact that the experiment was performed in circumstances "more advantegous" for F test, especially under condition of the normality of the random term distribution. It is possible that if distribution of ε is not normal or if other classical assumptions do not hold F test advantage over peak test (nonparametric) is significantly smaller. This hypothesis is based on intuition only and its accepting or rejecting needs more detailed research.

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OCENA MOCY TESTÓW HOMOSKEDASTYCZNOŚCI GOLDFELDA-QUANDTA DLA LINIOWEGO TRENDU

Celem tego artykułu jest ocena mocy testów homoskedastyczności Goldfelda-Quandta - testu szczytów i testu parametrycznego. Badania opierają się na wynikach eksperymentu Monte-Carlo dla 4 modeli heteroskedastyczności (liniowej, kwadratowej, wykładniczej i grupowej), 6 wartości współczynnika zmienności, 6 liczebności próby i 3 poziomów istotności. Obydwa testy porównano z klasycznym testem F przeciw heteroskedastyczności grupowej.