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ESTABLISHING THE BANK RATES WITH USING OF STATISTICAL METHODS

Abstract. In our article we would like to propose some statistical solution to the problem of the changing the rates of bank products (deposits and loans) after appearing market destabilisation factor such as changing the Central Bank loan rates.

Due to the complexity of the problem we will focus on two alternative ways: application of game theory and application of Analytic Hierarchy Process (AHP). We will treat the described problem as a game with nature – with a goal of finding an optimal solution. Showing the application of the AHP method we will propose the use of a procedure based on two semi-dependant AHP models.

Key words: Analytic Hierarchy Process, games with nature.

1. ESTABLISHING THE PROBLEM

Our problem is how the bank should react after occurring the market destabilisation factor. The are many such factors:
– a change of Central Bank base rates;
– a change in methodology (or rates) of compulsory deposit reserves;
– an introduction a tax on interest incomes for depositors.

The simplest solution is to follow the market by making the same change in rates as competitors (or Central Bank) or waiting for decisions of main competitors. However, it is not the way to find an optimal solution from profitability point of view. Therefore, we propose two methods for reaching an optimal solution:
– games theory
– Analytic Hierarchy Process method.

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The approach to the problem will always be as follow:

1. The outernal factor.

2. Creating a model due to the chosen method.

3. Finding an optimal solution (for example: increasing loan rates without changing deposit rates).

Due to simplification of the models we will limit our consideration to average deposit and loan rates. Also we will use the following notions:

– unidirectional change – in case of increasing Central Bank base rates it means increasing a rate, in case of decreasing Central Bank base rates it means decreasing a rate;

– anisotropic change – in case of increasing Central Bank base rates it means decreasing a rate, in case of decreasing Central Bank base rates it means increasing a rate.

2. APPLICATION OF THE GAME THEORY IN ESTABLISHING BANK RATES

2.1. Games with nature

A game is a contest involving two or more decision makers, each of whom wants to win. The game theory searches for optimal strategies. Game models are classified by the number of players, sum of all payoffs and the number of strategies employed.

Let us consider two-person zero-sum game without full information in which the second person (player) disappears. The second player may exist – however, because of unknown reasons he does not calculate in his strategies the existence of the second player. The consequence of that is casual (unexpected) behaviour of the second player. In that case wins of the first player are not considered as losses to the other. Also losses of the first player are not wins for the second player. That is why we can call strategies of the second player the outernal factors. Such a problem is for us a **game with nature**.

In a game with nature a player can choose some decisions and his opponent – the nature makes one of possible market situation. It is not a conflict game and it ends for the player as a win or a loss due to the state of nature.

In payoff matrix for the game with nature **W** (Table 1) rows describe strategies and columns describe possible states of nature. Elements $\mathbf{w}_{ij}$ of this matrix are a measurable win or loss (negative win) due to a chosen strategy under happening one of state of nature. We can find optimal solution using one of the following criteria:

- The maximin rule;
- The Savage rule;
- The Hurwicz rule;
- The Bayes-Laplace rule.

Table 1

A payoff matrix for the game with nature

Strategies	State of nature (market situation)			
	S_1	S_2	...	S_m
A_1	W_{11}	W_{12}	...	W_{1m}
A_2	W_{21}	W_{22}	...	W_{2m}
...	...	...	...	...
...	...	...	...	...
A_n	W_{n1}	W_{n2}	...	W_{nm}

2.2. Application of games with nature

In Table 2 we present the payoff matrix for the game with nature leading to an optimal solution in view of changing Central Bank base rates.

Table 2

A payoff matrix for the game with nature leading to an optimal solution in view of changing Central Bank base rates

Strategies	State of nature (market situation)			
	S_1	S_2	...	S_9
A_1	W_{11}	W_{21}	...	W_{19}
A_2	W_{21}	W_{22}	...	W_{29}
...	...	...	...	...
A_9	W_{91}	W_{92}	...	W_{99}

where:

A_1 – no change;

A_2 – unidirectional change of the average deposit rate with no change of the average loan rate;

A_3 – anisotropic change the average deposit rate with no change of the average loan rate;

A_4 – no change of the average deposit rate with unidirectional change of the average loan rate;

A_5 – unidirectional change of the average deposit and loan rates;

A_6 – anisotropic change the average deposit rate with unidirectional change of the average loan rate;

A_7 – no change of the average deposit rate with anisotropic change the average loan rate;

A_8 – unidirectional change of the average deposit rate with anisotropic change the average loan rate;

A_9 – anisotropic change of the average deposit and loan rates;

S_1 – no change of rates in competitive banks;

S_2 – unidirectional change of the average deposit with no change of the average loan rate in competitive banks;

S_3 – anisotropic change of the average deposit with no change of the average loan rate in competitive banks;

S_4 – no change of the average deposit with unidirectional change of the average loan rate in competitive banks;

S_5 – unidirectional change of the average deposit and loan rates in competitive banks;

S_6 – anisotropic change of the average deposit with unidirectional change of the average loan rate in competitive banks;

S_7 – no change of the average deposit with anisotropic change the average loan rate in competitive banks;

S_8 – unidirectional change of the average deposit with anisotropic change the average loan rate in competitive banks;

S_9 – anisotropic change of the average deposit and loan rates in competitive banks;

W_{ij} – net interest income as the function of the average deposit rate, the average loan rate, deposits and loans of the bank.

The simplest solution for building the function of the net interest income is to calculate it as a difference between interest income (a product of the average loan rate and number of loans) and interest expense (a product of the average deposit rate and number of deposits). Another solution might follow the described model:

1. Creating matrix **D** (with elements d_{ij}) in which number of deposits depends on strategies and states of nature.

2. Creating matrix **K** (with elements k_{ij}) in which number of loans depends on strategies and states of nature.

3. Making an assumption that 100% other liabilities covers other assets, we can build balance equations (to assure that total assets will equal total liabilities in our model):

$$cr_{ij} + k_{ij} + lu_{ij} = d_{ij} + lz_{ij},$$

$$cr_{ij} = r_E{}^* d_{ij},$$

$$lu_{ij} = \max(0;\ d_{ij} - (cr_{ij} + k_{ij})),$$

$$lz_{ij} = \max(0;\ (cr_{ij} + k_{ij}) - d_{ij}),$$

where:

cr_{ij} – cash and cash equivalent due to elements of matrix **D**;

r_E – average relation of cash and cash equivalent to deposits;

lu_{ij} – placement with other banks due a combination of a strategy i and state of nature j;

lz_{ij} – deposits from other banks due a combination of a strategy i and state of nature j.

4. Making a payoff matrix in which the function of the net interest income is described by the equation:

$$W_{ij} = (k_{ij}{}^* rk_i + lu_{ij}{}^* lu_R) - (d_{ij}{}^* rd_i + lz_{ij}{}^* lz_R)$$

where:

rk_i – the average loan rate due to strategy i;

rd_i – the average deposit rate due to strategy i;

lu_R – the average rate of placement with other banks;

lz_R – the average rate of deposits from other banks.

Having the payoff matrix we should decide according to which rule find an optimal solution. It will always depend on number of information that we have, however, it is most likely that the most often used solutions will be Hurwicz and maximin rules.

3. APPLICATION OF THE AHP IN ESTABLISHING BANK RATES

3.1. The analytic hierarchy process

The AHP is a multicriteria decision support method created by Thomas L. Saaty. It provides an objective way for reaching an optimal decision for both individual and group decision makers. The AHP is designed to select the best from a number of alternatives evaluated with respect to several criteria. It is taken by carrying out pairwise comparison judgements which are used to develop overall priorities for ranking the alternatives. This method allows for some level of inconsistency in judgements (that is unavoidable in practice) and provides some measures for limiting that.
In the AHP process there are four main stages:

1. Building a hierarchy model – the basic AHP model consists of three levels: goal, criteria level and alternatives. Depending on complexity of the problem it is possible to add as many as necessary levels of subcriteria.

2. Identifying the preferences of decision makers – in AHP it is done by collecting information about pairwise judgements due to a goal (for criteria), a specified criterion (for alternatives or subcriteria) or a subcriterion (for alternatives) [classical Saaty solution in Domański, Kondrasiuk (2000), Saaty (1986), Saaty (1994)].

3. Synthesis – it is obtained by a process of weighting and adding down the hierarchy leading to multilinear form in two possible modes:

– the distributive mode in which the principal eigenvector is normalized to yield a unique estimate of ratio scale underlying the judgements;

– the ideal mode in which the normalized values of alternatives for each criterion are divided by the value of the highest rate alternative.

4. Sensitivity analysis that gives an answer to a question whether the alternative chosen as the best would be changed in case of modifying criteria/subcriteria preferences.

3.2. Application of the AHP

After occurring a market destabilisation factor we should build two AHP models. The first model will lead us to an optimal solution for changing the average deposit rate and the second one will give us an answer to the question how to change the average loan rate.

3.2.1. Establishing the price of the Bank Deposits

Following AHP methodology we have structured the AHP hierarchy presented in figure 1.

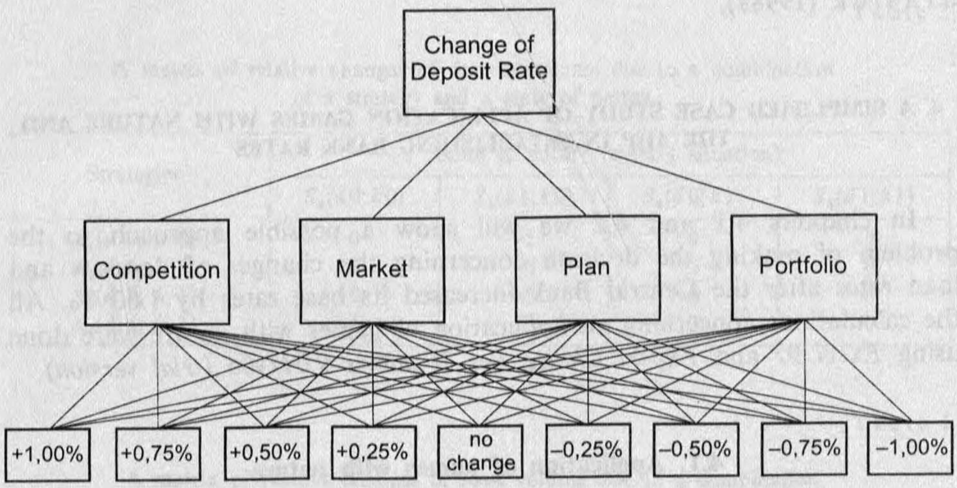

Fig. 1. The three level hierarchy used for changing deposit rate of the bank Domański and Kondrasiuk (2000)

The final version of the model uses four criteria:

– COMPETITION – marketing point of view on pricing deposits according to deposit rates of competitive banks;

– MARKET – treasury point of view, including possible buying bank deposits (and alternative costs);

– PLAN – financial planning and prognosis of future benefits and costs of the bank;

– PORTFOLIO – present assets portfolio of the bank as the measure of efficiency already acquired deposits.

Due to simplification of the model, we have decided to limit possible alternatives to changes of the average deposit rate from increasing to decreasing the rate by 1.00% with 0.25% step Domański and Kondrasiuk (2000).

3.2.2. Establishing the base loan rate of the bank

The base structure uses the following criteria:

– COMPETITION – loan rates of competitive banks;

– DEMAND – present and possible market share;

– DEPOSITS – the source of the money converting into loans;

– INTERBANK MONEY MARKET – as an alternative source of the money converting into loans.

The possible alternatives are also limited from increasing to decreasing the average loan rate by 1.00% with 0.25% step Domański and Kondrasiuk (1998a).

4. A SIMPLIFIED CASE STUDY OF APPLICATION GAMES WITH NATURE AND THE AHP IN ESTABLISHING BANK RATES

In chapters 4.1 and 4.2 we will show a possible approach to the problem of making the decision concerning the changes of deposits and loan rates after the Central Bank increased its base rates by 1.00 %. All the calculations concerning an application of games with nature were done using *Excel 97* and *Expert Choice For Windows 9.047v06* (*trial version*).

4.1. Application of games with nature

Due to the model described in chapter 2.2 we may assume the relation between chosen strategies and relative changes of deposits volume (Table 3) and loan volumes (Table 4). Furthermore, we assume:

– deposits volume (before destabilisation factor) – PLN 650.00 million

– loans volume (before destabilisation factor) – PLN 615.25 million

– average deposit rate (before destabilisation factor) – 14.00%

– average loan rate (before destabilisation factor) – 19.00%

– average relation of cash and cash equivalent to deposits – 5.50%

– the average rate of deposits from other banks – 16.15%

– the average rate of placement with other banks – 15.85%

and we will consider 4 strategies:

A_1 – no change;

A_2 – increase of the average deposit rate by 1.00% with no change of the average loan rate;

A_4 – no change of the average deposit rate with increase of the average loan rate by 1.00%;

A_5 – increase by 1.00% both the average deposit and loan rates; accompanied by 4 state of nature:

S_1 – no change of rates in competitive banks;

S_2 – increase of the average deposit rate by 1.00% with no change of the average loan rate in competitive banks;

S_4 – no change of the average deposit with increase of the average loan rate by 1.00% in competitive banks;

S_5 – increase by 1.00% both the average deposit and loan rates in competitive banks.

Table 3

A matrix of relative changes of deposit volume due to a combination of a strategy and a state of nature

Strategies	State of nature (market situation)			
	$S_1(d\,0; k\,0)$	$S_2(d\,1; k\,0)$	$S_4(d\,0; k\,1)$	$S_5(d\,1; k\,1)$
$A_1(d\ 0;\ k\ 0)$	0	–5	0	–5
$A_2(d\ 1;\ k\ 0)$	5	0	5	0
$A_4(d\ 0;\ k\ 1)$	0	–5	0	–5
$A_5(d\ 1;\ k\ 1)$	5	0	5	0

Table 4

A matrix of relative changes of loan volume due to a combination of a strategy and a state of nature

Strategies	State of nature (market situation)			
	$S_1(d\,0; k\,0)$	$S_2(d\,1; k\,0)$	$S_4(d\,0; k\,1)$	$S_5(d\,1; k\,1)$
$A_1(d\ 0;\ k\ 0)$	0	0	5	5
$A_2(d\ 1;\ k\ 0)$	0	0	5	5
$A_4(d\ 0;\ k\ 1)$	–5	–5	0	0
$A_5(d\ 1;\ k\ 1)$	–5	–5	0	0

Table 5

A payoff matrix **W** of the net interest profit (PLN million)

Strategies	State of nature (market situation)			
	$S_1(d\,0; k\,0)$	$S_2(d\,1; k\,0)$	$S_4(d\,0; k\,1)$	$S_5(d\,1; k\,1)$
$A_1(d\ 0;\ k\ 0)$	2.14	2.11	2.22	2.18
$A_2(d\ 1;\ k\ 0)$	1.60	1.60	1.68	1.67
$A_4(d\ 0;\ k\ 1)$	2.55	2.52	2.65	2.62
$A_5(d\ 1;\ k\ 1)$	2.01	2.01	2.11	2.11

Table 6

The maximin rule (PLN million)

Strategies	Row minimums of matrix **W**	
$A_1(d\ 0;\ k\ 0)$	$\min_1 w_{i1}$	2.11
$A_2(d\ 1;\ k\ 0)$	$\min_2 w_{i2}$	1.60
$A_4(d\ 0;\ k\ 1)$	$\min_4 w_{i4}$	2.52
$A_5(d\ 1;\ k\ 1)$	$\min_5 w_{i5}$	2.01
$W = \max_i \min_j w_{ij}$		2.52

Due to the maximin rule an optimal strategy is to increase the average loan rate without changing the average rate of deposits (Table 6).

Applying the Savage rule results in conclusion that an optimal strategy is to increase the average loan rate without changing the average rate of deposits (Table 8).

Table 7

A matrix of relative losses **R** (PLN million)

Strategies	State of nature (market situation)			
	$S_1(d\,0; k\,0)$	$S_2(d\,1; k\,0)$	$S_4(d\,0; k\,1)$	$S_5(d\,1; k\,1)$
$A_1(d\ 0;\ k\ 0)$	0.41	0.41	0.44	0.44
$A_2(d\ 1;\ k\ 0)$	0.95	0.92	0.97	0.95
$A_4(d\ 0;\ k\ 1)$	0.00	0.00	0.00	0.00
$A_5(d\ 1;\ k\ 1)$	0.54	0.52	0.54	0.51

Table 8

The Savage rule (PLN million)

Strategies	Row minimums of matrix **R**	
$A_1(d\ 0;\ k\ 0)$	$\max_1 r_{i1}$	0.44
$A_2(d\ 1;\ k\ 0)$	$\max_2 r_{i2}$	0.97
$A_4(d\ 0;\ k\ 1)$	$\max_4 r_{i4}$	0.00
$A_5(d\ 1;\ k\ 1)$	$\max_5 r_{i5}$	0.54
$W = \min_i \max_j r_{ij}$		0.00

Table 9

The Hurwicz rule (PLN million)

Strategies	$w_j = \min\limits_j w_{ij}$	$W_j = \max\limits_j w_{ij}$	$\vartheta \cdot w_j + (1-\vartheta) \cdot W_j$ for $\vartheta = 0.50$
$A_1(d\ 0;\ k\ 0)$	2.11	2.22	2.16
$A_2(d\ 1;\ k\ 0)$	1.60	1.68	1.64
$A_4(d\ 0;\ k\ 1)$	2.52	2.65	2.59
$A_5(d\ 1;\ k\ 1)$	2.01	2.11	2.06
$W = \max\limits_i\{\vartheta \cdot w_j + (1-\vartheta) \cdot W_j\} = \max\limits_i a_i(\vartheta)$.			**2.59**

Using the Hurwicz rule leads to an increase in the average loan rate without changing the average rate of deposits as an optimal solution (Table 9).

An optimal strategy due to the Bayes-Laplace rule is to increase the average loan rate without changing the average rate of deposits (Table 10).

All the rules in this case have led to the strategy of increasing the average loan rate by 1.00% without increasing deposit rates.

Table 10

The Bayes-Laplace rule (PLN million) with addition assumption of equal probability of all states of nature

Strategies	Expected value E_i
$A_1(d\ 0;\ k\ 0)$	2.16
$A_2(d\ 1;\ k\ 0)$	1.64
$A_4(d\ 0;\ k\ 1)$	2.59
$A_5(d\ 1;\ k\ 1)$	2.06
$W = \max\limits_i E_i$	2.59

4.2. APPLICATION OF THE AHP

The following Figures 2–11 show the most crucial steps of solving the AHP model described in chapter 3.2.1 and 3.2.2 (in a very compressed way) with *Expert Choice For Windows 9.047v06*. Both the models are limited to alternatives from: no increase to increase by 1.00% rate with 0.25% step. Due to information collected from decision makers with AHP method an optimal solution is 1.00% increase of loan rates with no change of deposits rates.

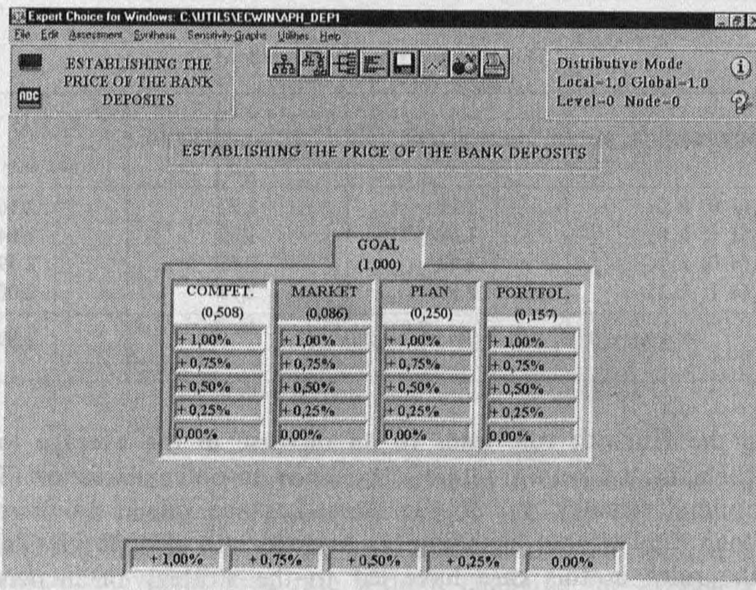

Fig. 2. The general model for establishing the price of the bank deposits

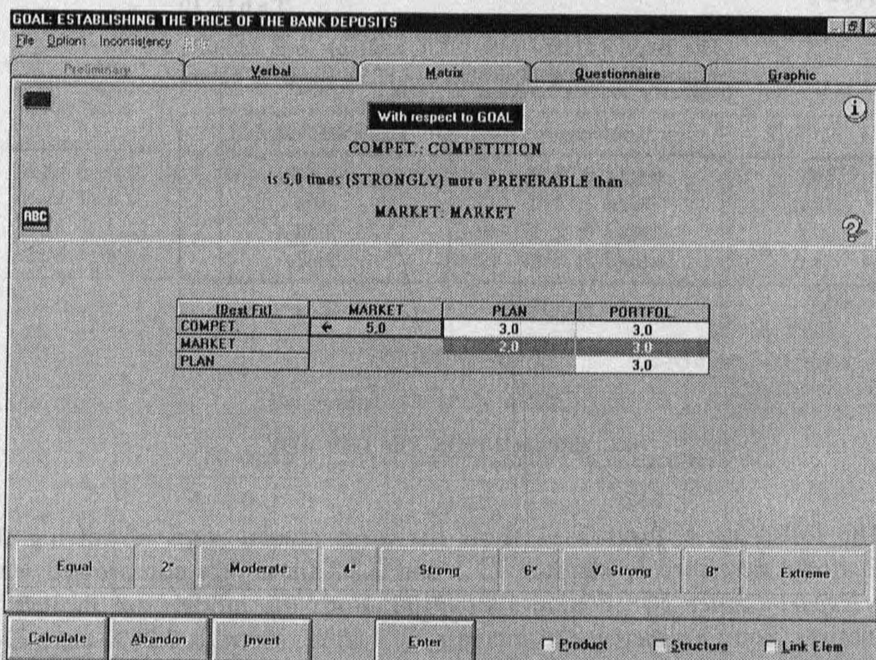

Fig. 3. The pairwise comparison of criteria due to a goal

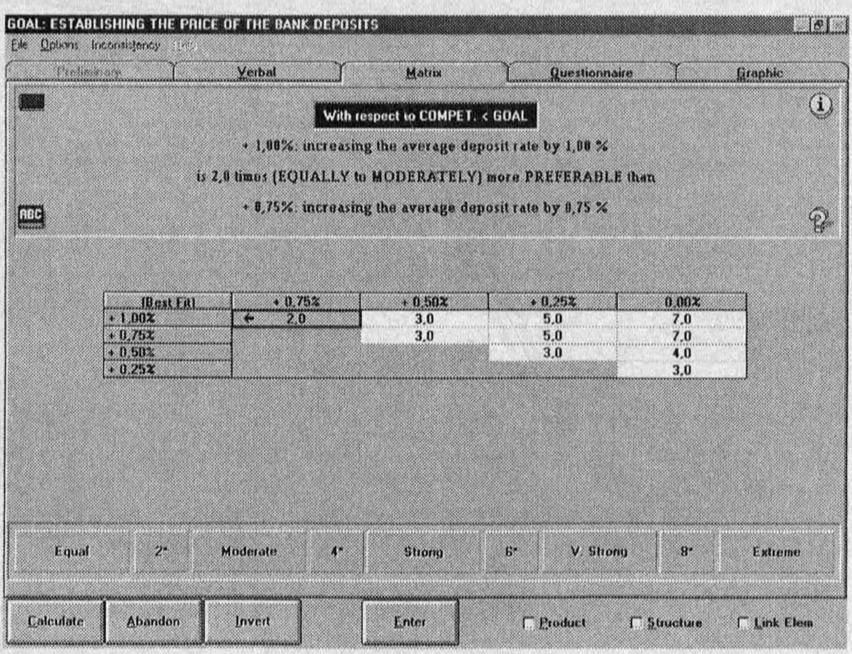

Fig. 4. The pairwise comparison of alternatives due to a COMPETION criterion

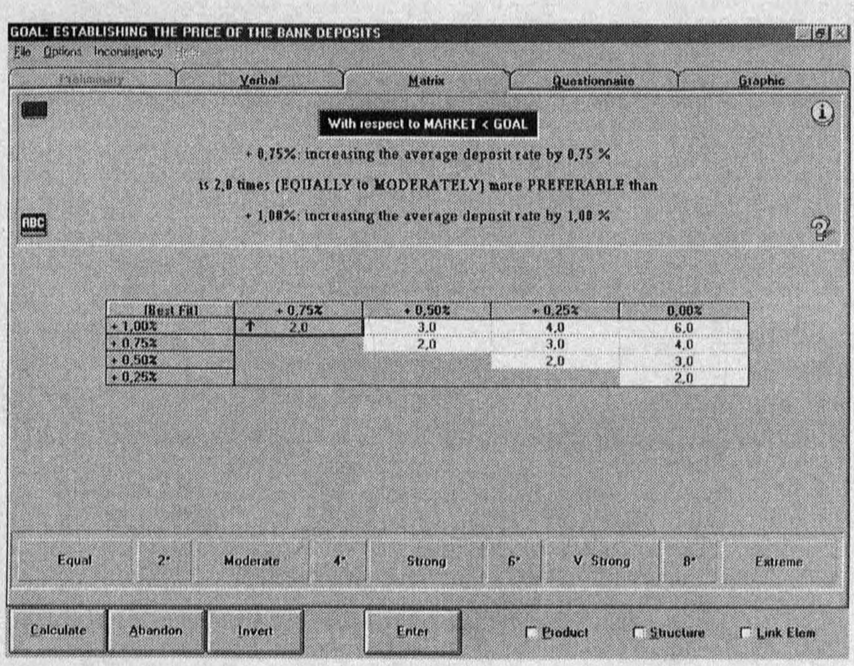

Fig. 5. The pairwise comparison of alternatives due to a MARKET criterion

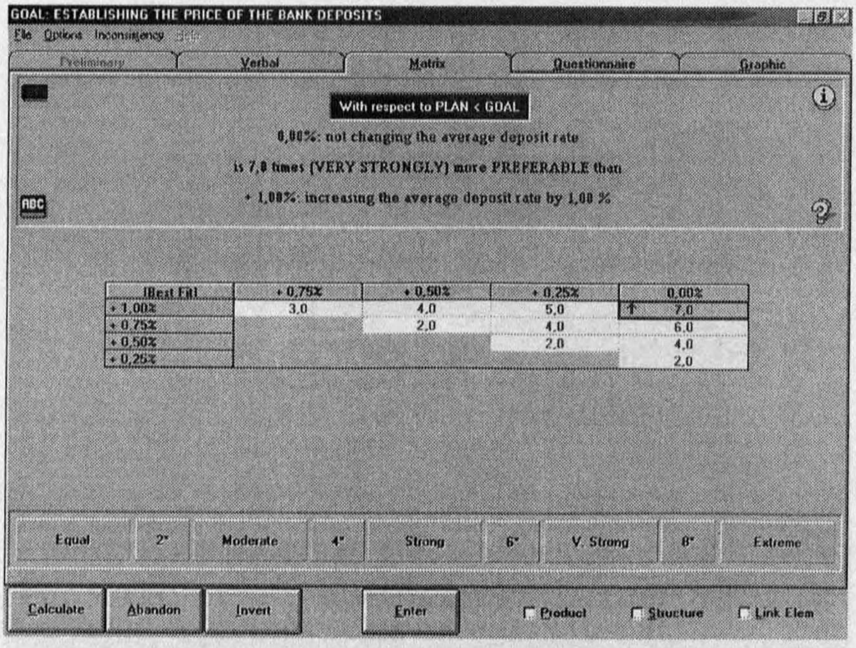

Fig. 6. The pairwise comparison of alternatives due to a PLAN criterion

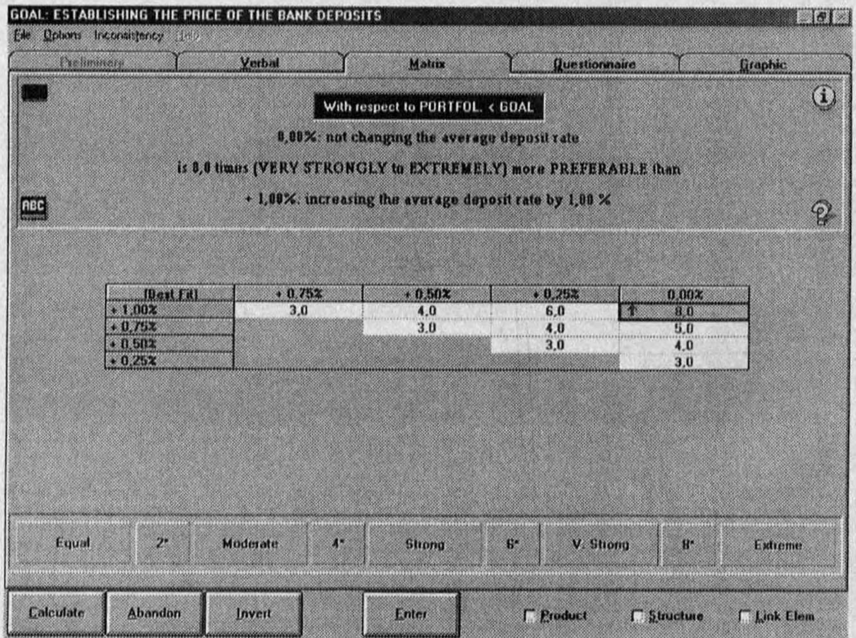

Fig. 7. The pairwise comparison of alternatives due to a PORTFOLIO criterion

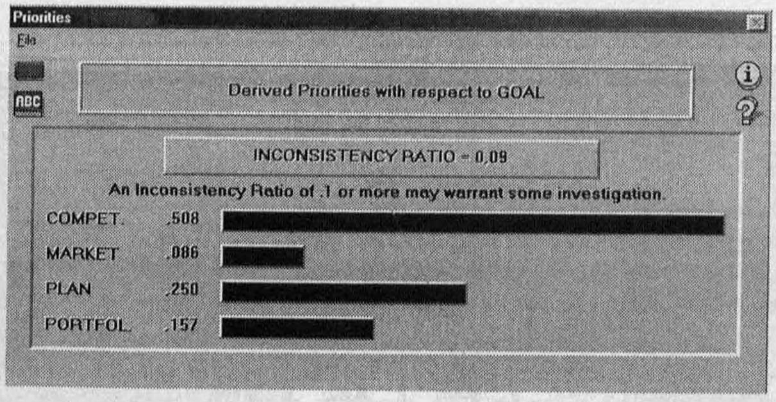

Fig. 8. Inconsistency ratio calculated for criteria

ESTABLISHING THE PRICE OF THE BANK DEPOSITS

Synthesis of Leaf Nodes with respect to GOAL

Distributive Mode

OVERALL INCONSISTENCY INDEX = 0,06

LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	LEVEL 5
COMPET. =,508				
	+ 1,00% =,212			
	+ 0,75% =,160			
	+ 0,50% =,077			
	+ 0,25% =,038			
	0,00% =,020			
PLAN =,250				
	0,00% =,116			
	+ 0,25% =,065			
	+ 0,50% =,036			
	+ 0,75% =,021			
	+ 1,00% =,011			
PORTFOL.=,157				
	0,00% =,076			
	+ 0,25% =,041			
	+ 0,50% =,022			
	+ 0,75% =,012			
	+ 1,00% =,006			
MARKET =,086				
	0,00% =,037			
	+ 0,25% =,022			
	+ 0,50% =,014			
	+ 0,75% =,008			
	+ 1,00% =,005			

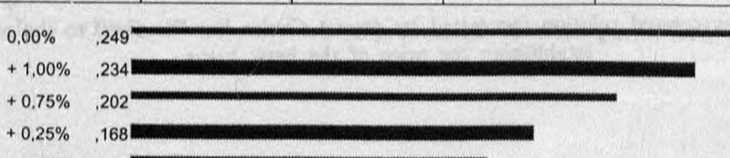

Fig. 9. A solution (generated by *Expert Choice For Windows*) to the model for establishing the price of the bank deposits

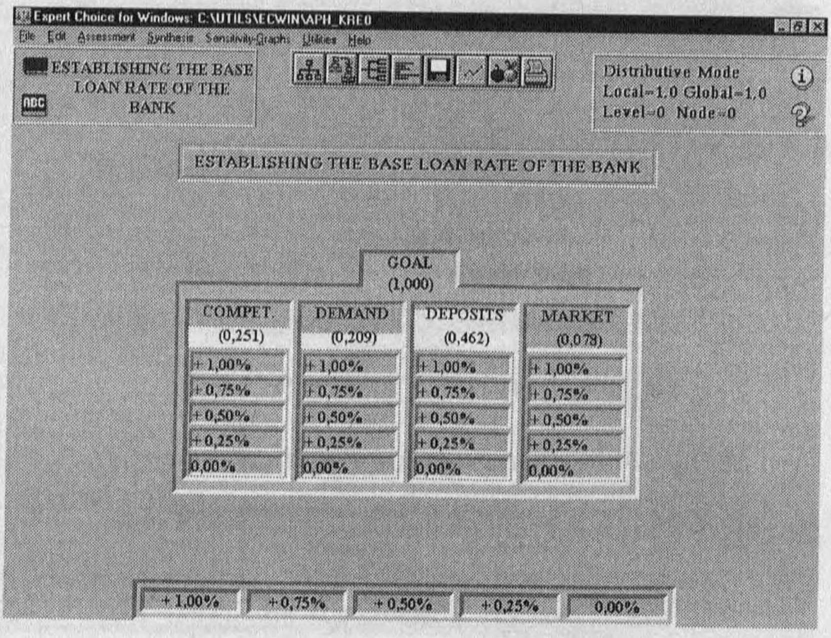

Fig. 10. The general model for establishing the price of the bank loans

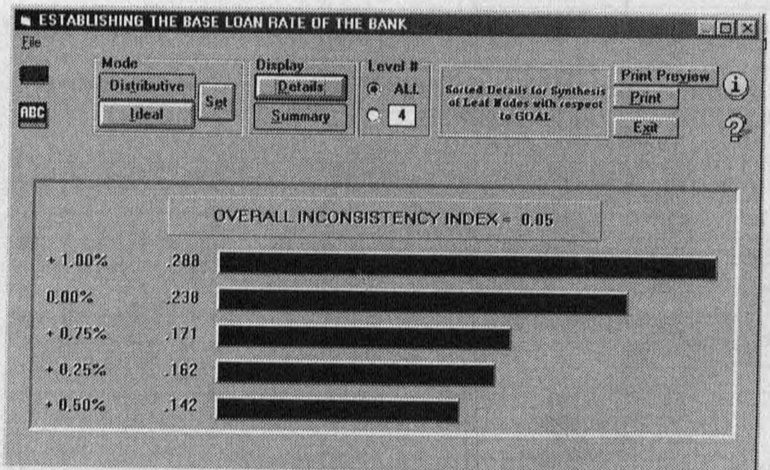

Fig. 11. An aggregated solution (generated by *Expert Choice For Windows*) to the model for establishing the price of the bank loans

5. SUMMARY

The proposed approach to the problem of changing the bank rates in view of market destabilisation factor leads to conclusion that depending on number of information we can use both the game with nature and the AHP method. Games with nature give decision makers more precise solution when we can build a satisfactory model for calculating possible wins and losses. In view of lack of some information the AHP method is more useful due to its formalisation for decision makers preferences.

In practise we can use both methods as a support for decision makers.

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USTALANIE BANKOWYCH STÓP PROCENTOWYCH Z WYKORZYSTANIEM METOD STATYSTYCZNYCH

(Streszczenie)

W pracy rozważany jest problem ustalania w sposób optymalny bankowych stóp procentowych. Do tego celu wykorzystano metodę optymalizacji wielokryterialnej (Analityczny Proces Hierarchiczny) oraz niektóre procedury teorii gier.