

# Multiplication of Air Accidents Frequency and Hazard Desirability Coefficients for ICAO Safety Risk Tolerability Matrix Solution

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Using mathematical method of priorities arrangement, also known as problem “about leader”, air accidents frequency and hazard desirability coefficients were found. Those coefficients are proposed by ICAO for safety risk tolerability matrix solution. Integral index of mentioned parameters combinations hazard is found with help of Harrington desirability function. Risk levels determination index is introduced for researched matrix.

**Keywords:** flight safety, tolerability matrix, frequency and hazard event desirability coefficients, Harrington function, risk level determination criteria.

## 1. PROBLEM STATEMENT

Civil aviation (CA) plays great role in the world of economy and it is one of constantly and dynamically developing areas. Also one of the key CA viability support components is safe, protected, efficient, and ecology balanced aircrafts flight conditions maintenance along with their technical reliability on global, regional and national levels. That was the reason for ICAO to develop Global Aviation Safety Plan as strategy document that contains planning methods dedicated to reach global harmony in flight safety (FS) area [1].

According to ICAO definition FS is «expected characteristic» that is first of all based on the estimation and management of different risks appearing in the aviation transport system. In the context of current paper researches risk will be understood as “chance of unwanted situation happening” [2; 3]. Also ICAO definitely stated that risk can be controlled, i.e. different means for risk situations prediction and risk level decrease [4] may be applied.

Obviously it is impossible to solve risk management tasks without some qualitative and quantitative indexes, that is especially urgent when human factor (HF) influence upon FS should be taken into account.

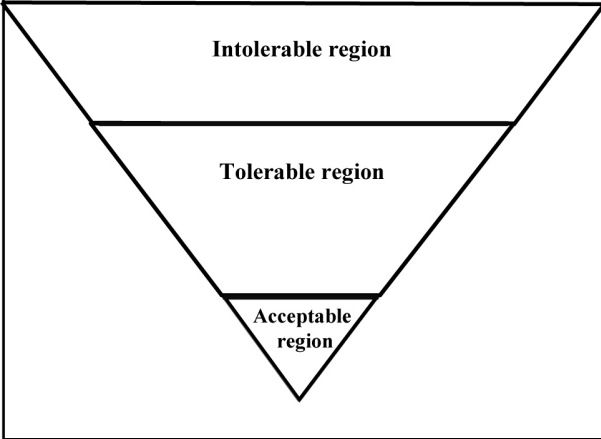
## 2. PREVIOUS RESEARCHES ANALYSIS

Paying attention to the importance of FS management (FSM), especially the proactive one, ICAO has developed special Safety Management Manual (SMM) where implemented “risk tolerability matrix” for aviation hazards estimation (table 1), where correspondent levels were stated: intolerable, tolerable, acceptable. But in the first edition of SMM (2006) there were no recommendations proposed for solution of that matrix.

Nevertheless, some activities on that topic were made in the same year [5,6]. It concerns ability of finding key points in estimated benefit-safety functions (EBSF) for calculating risk levels. Mentioned functions are built with help of limited point number and are used as indexes of aviation operators professional activities in case of decision taking (DT) task.

Next SMM edition (2009) contained five differential indexes for likelihood and severity of unwanted events [4], that may be presented as term-multitude (TM) of correspondent linguistic variables (LV) from fuzzy mathematics [7, 8].

Table 1. ICAO recommendations about solving “Tolerability matrix”.

Proposed criteria	Risk estimation index	Risk acceptability
1	2	3
	<p><b>5A, 5B, 5C,</b> <b>4A, 4B, 3A</b></p>	Unacceptable under current circumstances
	<p><b>5D, 5E, 4C, 4D,</b> <b>4E, 3B, 3C, 3D,</b> <b>2A, 2B, 2C</b></p>	Relatively acceptable in case of risk decreasing procedures appliance. Manager approve may be required
	<p><b>3E, 2D, 2E, 1A,</b> <b>1B, 1C, 1D, 1E</b></p>	Acceptable

Thus it is:

– TM of LV «event danger (ED)»:

$$T^M(ED) = \begin{matrix} \tilde{R}_a & \tilde{R}_b \\ \tilde{R}_c & \tilde{R}_d & \tilde{R}_e \\ + \text{major} + \text{minor} + \text{negligible} \end{matrix} \quad (1)$$

– TM of LV «event frequency (EF)»:

$$T^M(EF) = \begin{matrix} \tilde{R}_5 & \tilde{R}_4 \\ \tilde{R}_3 & \tilde{R}_2 & \tilde{R}_1 \\ + \text{remote} + \text{improbable} + \text{extremeley improbable} \end{matrix} \quad (2)$$

Implementing tools (risk matrix, - RM), based on different combinations of EF and ED linguistic parameters (table. 1), ICAO seems to found integral solution IKAO of risk management problem (table. 1), that has underlined uncertainty. Really, 24% of option are stated to be unacceptable, 44% – limitedly acceptable, 32% - acceptable (fig. 1 a).

Though groundings for those definitions seems to be empiric. Really, 1A option related to “extremely improbable catastrophic risk” of the original ICAO matrix belongs to acceptable group along with 5E option that is related to “frequent negligible risk”. But the fact is that according to modern understanding of risks – those that are accumulated during long period of time may be even greater reason for air accident. Also the resonance question [9], along with hidden hazards and defects [4; 10] makes those risks unacceptable even in case of “risk decreasing means“ (table. 1). Spreading ICAO flight safety management (FSM) system, FAA printed special circular [11],

where in observed RM 76% of components are stated to be acceptable (fig. 1 b). But some changes was made comparing to initial ICAO recommendations. Acceptable options number was increased up to 40% and relatively acceptable options number was decreased to 36%.

In paper [12] it was proposed to define risk level *R* as ED i EF indexes product:

$$R = P \cdot S \quad (3)$$

where

*P* – likelihood index;

*S* – severity index.

Assuming that almost impossible events and minor consequences events has correspondent values of *P*=0 and *S*=0, and mostly probable event along with catastrophic consequences event has values of *P*=1 and *S*=1, it is obvious that these parameters range is:  $P = \overline{0,1}$ ,  $S = \overline{0,1}$ .

Parameters ranges are divided into five areas that allowed to use multiplicative approach for indexes combinations according to formula (3). Every RM option has received ED and EF combined value which led to uncertainty. For example *R*=0,48 belongs to unacceptable options 5C, 4B, 3A and on the other hand it belongs to relatively acceptable risk options 4B, 3C.

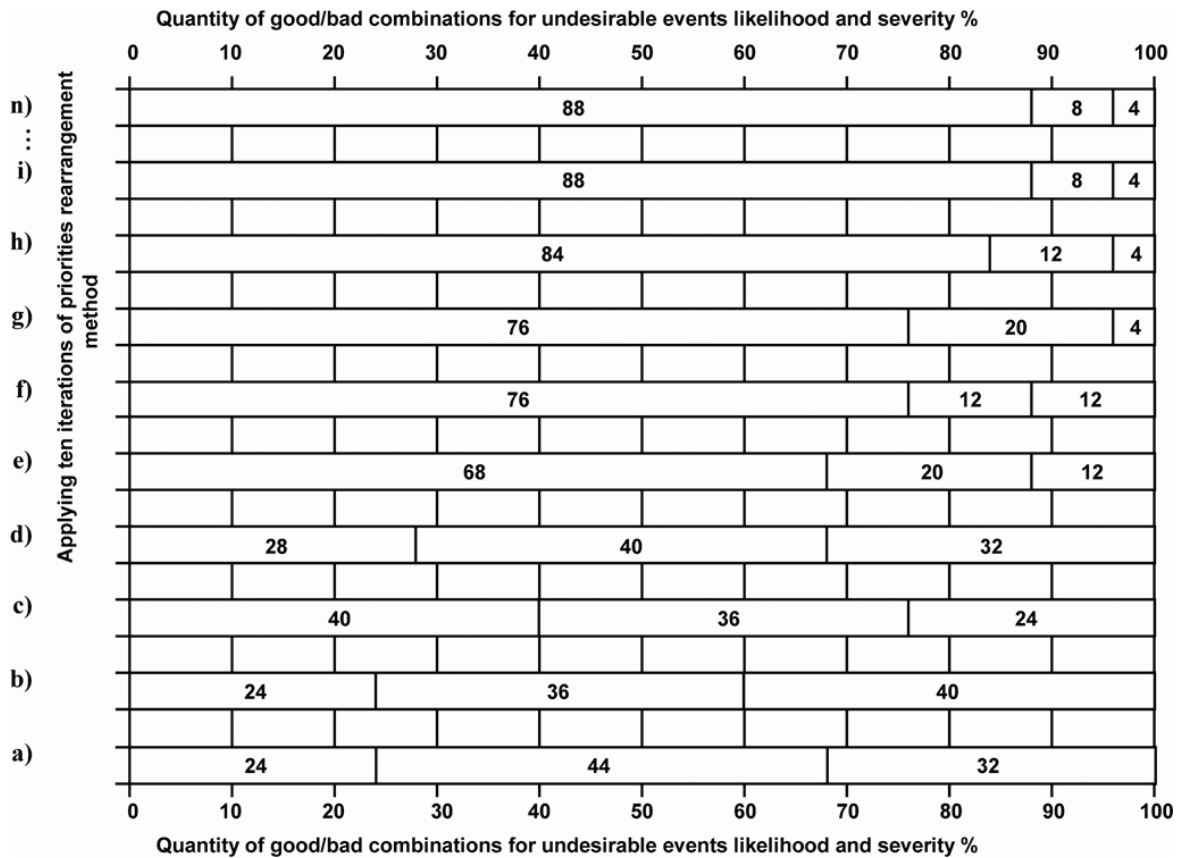


Fig. 1. Risk tolerability matrix solution with help of different analytical methods (in every row the most left part relates to intolerable risk, central part relates to tolerable risk, the most right part relates to acceptable risk): a) ICAO approach; b) FAA approach; c) Harrington coefficients; d) – j) priority rearrangement method results, ten iterations.

One reason of these results may be subjective approach to values arrangement. Another reason may be necessity of vector multiplication instead of scalar one [13]:

$$\vec{R} = \vec{P} \times \vec{S} \tag{4}$$

Well-grounded RM research was made in paper [14], where matrices of sizes 2x2 up to 5x5 were analyzed concerning different human activities areas. “Weak consistency” concept is proposed that must be present in matrix. Shortly this concept could be described in following way: quantitative representation of unacceptable risk part must be less then acceptable one. “Weak consistency” is obligatory for risk matrices. Two important theorems were also proven

1. ED and EF combinations, that corresponds to unacceptable and acceptable risks should have no common borders.
2. Increase of intermediate groups number (“relatively acceptable” group for 5x5 matrix) gives no increase in matrix efficiency.

We may conclude that RM allows to differentiate only acceptable and unacceptable risks. Moreover in paper [14] it is stated that only 10% of hazards may be compared with help of RM. Even taking into account necessity of further researches in this area it is obvious that RM should be used with caution. Also it clear that efficient FSM DT can’t be grounded only on the RM. Taking into account the above mentioned aspects, it is important to point at propositions of paper [15] about introduction of additional parameters during risks estimations, analysis and management.

### 3. PROBLEM STATEMENT

From the analysis, **the purpose** of this paper is firstly ICAO “tolerability matrix” solution with help of desirability coefficients and qualitative indexes of aviation events frequency and hazard aggregated by Harrington function. Secondly **the purpose** is the research on new approaches, methods and models of “tolerability matrix” solution from first-line aviation operators (air traffic controllers (A/C) and aircraft crew) point of view.

4. UNWANTED EVENT DESIRABILITY AND DANGER COEFFICIENTS

It should be noted that stated purpose could also be reached with help of Harrington desirability coefficients used exactly for qualitative, linguistic scale with dimension similar to dimensions of scales under study (1) and (2).

Then integrated risk index may be easily received with help of multiplicative Harrington desirability function when calculation is reduced to determination of event frequency and hazard coefficients geometrical average [16].

In this case (from fig. 1 c) quantity of generally acceptable combinations of ED and EF is reduced down to 60% comparing to ICAO and FAA recommendations. This can be explained by non-linearity of Harrington desirability coefficients. Such approach is totally correct taking into account nonlinearity of ED and EF perception and it corresponds to FS support requirements.

From expressions (1) and (2) comes obvious ranking of unwanted events qualitative frequency and hazard characteristics:

- for ED indexes:

$$\tilde{R}_e \succ \tilde{R}_d \succ \tilde{R}_c \succ \tilde{R}_b \succ \tilde{R}_a; \tag{5}$$

- for EF indexes:

$$\tilde{R}_1 \succ \tilde{R}_2 \succ \tilde{R}_3 \succ \tilde{R}_4 \succ \tilde{R}_5 \tag{6}$$

This leads to opportunity of qualitative indexes defuzzyfication with help of mathematical method of priority arrangement (MPA) (also known as the problem “about leader”) basing on the researches experience [17, 18, 19]. MPA use square adjacent matrix  $C = \|c_{ij}\|$  for qualitative indexes estimates, that are defined with help of ranking (5) (6):

$$c_{ij} = \begin{cases} 2 \text{ if } \tilde{R}_i \text{ is more desired than } \tilde{R}_j : \tilde{R}_i \succ \tilde{R}_j \\ 0 \text{ if vice versa } \tilde{R}_i \prec \tilde{R}_j \end{cases}$$

If matrix  $C$  is not decomposable, there are procedures that, according to Perron–Frobenius theorem, leads to maximal eigen value with correspondent eigenvector. This allows to prove that normalized iterative desirability calculation process of qualitative index  $\tilde{R}_j$  is convergent.

As it comes from [17; 19], using MPA gives researcher additional opportunity to take into

account side (indirect) advantages of one qualitative index value upon another. Correspondent calculations results are given in table 2.

5. INTEGRATED INDEX DETERMINATION AND “TOLERABILITY MATRIX” SOLUTION

To receive integrated index of unwanted event risk let us use multiplicative approach for different events desirability and frequency coefficients aggregation. As experience states [16, 20] mostly approbated in this area is Harrington desirability function, presented in context of our research as follows:

$$\alpha R_i = \sqrt{\alpha_j^{ED} \alpha_k^{EF}}, \tag{7}$$

where  $\alpha R_i$  – integral quantitative index of i-th desirability combinations combination ( $i = \overline{1,25}$ ) ED  $\alpha_j^{ED}$ ,  $j = \overline{1,5}$ , and EF  $\alpha_k^{EF}$ ,  $k = \overline{1,5}$ .

Unlike more widespread additive approach from formula (7) we may conclude opportunity for relative compensation of low desirability coefficients indexes. Besides using data from table 2 useful nomogram (fig. 2) could be used, leading to more narrow RM risks analysis.

Fig. 1 represents dynamics of hazard and frequency coefficients combinations acceptability, received according to third iteration of MPA, compared with ICAO and FAA propositions and Harrington desirability coefficient.

Table 2. Priority rearrangement method results for desirability indexes definition.

Index		Iteration where corresponding coefficients were received										
ED	EF	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
1	2	3	4	5	6	7	8	9	10	11	12	13
E	1	0,36	0,4824	0,5733	0,6407	0,6914	0,7303	0,7610	0,7856	0,8057	0,8225	0,8367
D	2	0,28	0,2941	0,2800	0,2575	0,2345	0,2136	0,1953	0,1793	0,1656	0,1536	0,1431
C	3	0,20	0,1529	0,1111	0,0818	0,0619	0,0482	0,0384	0,0312	0,0259	0,0217	0,0185
B	4	0,12	0,0588	0,0311	0,0180	0,0112	0,0074	0,0051	0,0037	0,0027	0,0021	0,0016
A	5	0,04	0,0118	0,0044	0,0020	0,0010	0,0006	0,0003	0,0002	0,0001	0,0001	0,0001
Σ		1	1	1	1	1	1	1	1	1	1	1

NOTE: ED – event danger; EF – event frequency.

First MPA iteration gives general number of acceptable options equal to 72% that is closest result to ICAO proposition. Although it is obvious from table 2 that in this case relation is linear, which is inappropriate in case of human factor. Common sense states that coefficients can't be of linear nature when research concerns human operator. This is partially proved by fuzzy operations of concentration and dilation [7, 8] used for linguistic scales. Non-linearity starts from second MPA iteration. As a result most of combinations (68%) are not allowable concerning proposed method of their acceptability determination that seems reasonable in case of FS.

During further MPA method appliance, significant acceptable combinations number decrease is observed. Starting from sixth iteration only two combinations will correspond to partially acceptable risk level and only one will correspond

to acceptable. This results in certain doubts about proposed groups proportions in ICAO “tolerability matrix”.

### 6. NEW METHODS OF “TOLERABILITY MATRIX” SOLUTION CONCERNING HUMAN FACTOR

Described opportunities of analytical researches may be used for risk analysis and FSM in aviation institutions. On the other side relatively to AO they may be applied only at theoretical part of their training. At the same time it is important to point at new methods and trends of proactive HF researches that may be used as for FSM (i.e. RM solution), as for person-oriented AO training.

In paper [21], examining mentioned hazard and frequency indexes of unwanted events as LV and basing on their TM it was possible to build

Table 3. ICAO tolerability matrix solution example using event danger/frequency desirability coefficients along with multiplicative approach at first iteration of priority rearrangement method.

Risk frequency	Risk danger									
	A		B		C		D		E	
1	2	3	4	5	6	7	8	9	10	11
5	<b>5A</b>	<b>0,04</b>	<b>5B</b>	<b>0,0693</b>	<b>5C</b>	<b>0,0894</b>	5D	<b>0,1058</b>	5E	0,12
4	<b>4A</b>	<b>0,0693</b>	<b>4B</b>	0,12	4C	0,1549	4D	0,1833	4E	0,2078
3	<b>3A</b>	<b>0,0894</b>	3B	0,1549	3C	0,20	3D	<i>0,2366</i>	<i>3E</i>	<i>0,2683</i>
2	2A	<b>0,1058</b>	2B	0,1833	2C	<i>0,2366</i>	<i>2D</i>	<i>0,28</i>	<i>2E</i>	<i>0,3175</i>
1	<i>1A</i>	0,12	<i>1B</i>	0,2078	<i>1C</i>	<i>0,2683</i>	<i>1D</i>	<i>0,3175</i>	<i>1E</i>	<i>0,36</i>

NOTE: boxes № 2, 4, 6, 8, 10 shows ICAO recommended solution;  
boxes № 3, 5, 7, 9, 11 – desirability coefficients solution;  
**bold** represents unacceptable solutions, *italic* represents partially acceptable results.

corresponding membership functions (MF) that being united with help of Dubois-Prade method [8, 22] it leads to integral risk estimate. For now propositions for practical implementation are not proved and physical sense of argument that will be base for MF is not defined.

Second problem was removed in paper [23] where LV MF were built for clear quantitative argument – flight level norm. This gave opportunity to set quantitative intervals of flight level norm violation in correspondence to linguistic estimates. However some researches concerning “undesired event frequency” LV MF that would lead to “tolerability matrix” solution wasn’t held yet.

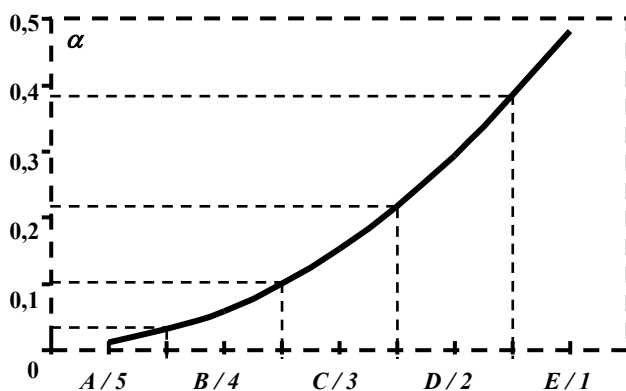


Fig. 2. Nomogram of event danger/frequency desirability coefficients from first iteration of priority rearrangement method.

“Tolerability matrix” solution with help of unlimited number of A/C key points and indexes of DT problem in professional activities also was continued in paper [24].

## 7. CONCLUSIONS

Summarizing scientific results presented in this paper we may state that:

1. ICAO “Tolerability matrix” solution was proposed based on the multiplicative composition of undesired events hazard and frequency indexes.
2. Mentioned coefficients are received with help of MPA for TM LV «ED» «EF» terms defuzzification, that allows to support quantitative nonlinearity depending on FS requirements.
3. It was found that nonlinearity leads to significant decrease of acceptable options.

Further researches should be directed in:

- better research of «ED» an «EF» TM LV as basic one for “tolerability matrix solution”;
- composition of «ED» and «EF» MF LV for clear arguments indexes along with development of methods for integral risk determination;
- building estimation functions of usefulness-safety for DT tasks.

Shortly all that concerns HF in FSM.

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