Transportation Needs of Entrepreneurs in Wroclaw Agglomeration – Study Results

Yesser Yedes, Anis Chelbi
École Supérieure des Sciences et Techniques de Tunis, Tunisia
Nidhal Rezeg
Université de Metz, France

This paper deals with the integrated supply chain management problem in the context of a single vendor-single buyer system for which the production unit is assumed to randomly shift from an in-control to an out-of-control state. Two different strategies, integrating production, shipment and maintenance policies, are proposed and compared to satisfy the buyers order at a minimum integrated total cost rate. The first strategy is based on a classical production policy for which the buyer's order of size nQ is manufactured continuously and shipped by lots of size Q. The second strategy suggests that the same buyer's order should be produced and shipped separately by equal sized lots Q. For both strategies, a corrective or preventive maintenance action is performed at the end of each production cycle, depending on the state of the production unit, and a new setup is carried out. The total integrated average cost per time unit is considered as the performance criterion allowing choosing the best policy for any given situation.
Key words: supply chain management, production, inventory, maintenance, quality, single-vendor single-buyer

1. INTRODUCTION

The Economic Ordering Lot sizing (or Economic Production Lot sizing) problem has been widely and differently treated in the literature in both single and joint context. The models proposed, since Harris's classic square root economic order quantity (EOQ) model, have been improved relaxing assumptions and/or taking into account more factors and parameters.

A multitude of individual models relates to the case of imperfect production process. For example, Rosenblatt and Lee [8], Cheng [14] and Khouja and Mehrez [11] developed models assuming that the production process shifts from an in-control to an out-of-control state after a random period of time. They tried to optimize the total cost but didn't suggest any solution to counter the unreliability problem. On the other hand, many authors have integrated different maintenance actions in order to stop or minimize the production of non-conforming items, such as Lee and Srinivason [5], Ben-Daya and Khursheed [10], Ben-Daya [9] and Aghezzaf et al. [4]. Recently, Yang et al. [17] proposed a new method for scheduling of maintenance operations in a manufacturing system using the continuous assessment and prediction of the level of performance degradation of manufacturing equipment. Chelbi et al [1] have modelled an integrated production-maintenance strategy for unreliable production systems producing conforming and non-conforming items. The related optimal solutions, minimising the total average cost per time unit, correspond to the optimal values of the lot size and the system age at which preventive maintenance must be performed. Radhoui et al. [7] presented a joint strategy of quality control and maintenance for an imperfect production process. They developed a simulation model to determine the optimal threshold value of the proportion of nonconforming items, allowing either to perform or not to perform a preventive or a corrective maintenance action, and the optimal size of a buffer stock built to cover the demand during the random period of time necessary to carry out the maintenance action.

Regarding the Joint Economic Lot sizing Problem (*JELP*) with imperfect production or process unreliability consideration, a great number of related studies integrate the cost of nonconforming items in the expected total cost and determine the optimal production-shipment policy. One could say that the proportion of nonconforming items seems to represent a constraint for the main *JELP* problem but not a problem in itself. Indeed, the proposed models don't try to resolve the problem at the origin (the source of nonconformity) but search only to go around it in order to limit damages. For example, we cite Huang [3] who considered a defective process, in a single-vendor-single-buyer supply chain, which produces a random percentage of imperfect items. He derived an analytic solution for a particular situation where the delivered quantity is the same at each replenishment. Previously, Goyal et al. [13] treated the same Huang's [3] problem, but assumed that items of imperfect quality are sold as a single batch at discounted price at the end of the screening period.

Moreover, strategies proposing different policies of quality inspection, such as Ritvirool and Ferrell [2] or Chung and Wee [12], don't offer real solutions to the problem, particularly if applied at the reception of products in the buyer's warehouse. Such policies allow limiting (sampling inspection) or stopping (100% inspection) the propagation of the problem from the buyer to the final customer.

The few works providing effective solutions, such as Affisco et al. [6] or Liu and Sila [15], suggest integrating an investment cost in quality improvement, depending on the improvement rate, but without specifying methods or means to make such an improvement.

Frequently, imperfect quality is associated with process unreliability, which is identified in many situations as production unit (machine or installation) unreliability. Consequently, maintenance actions could constitute in many cases a solution to stop or to prevent producing nonconforming items. As mentioned above, it has already been suggested for many years in the literature to consider only the vendor side. Yedes et al. [16] is one of the rare studies which tried to extend this idea to the context of supply chain. The authors proposed and compared two joint single-vendor single buyer strategies integrating production, inventory and maintenance policies. They supposed that the production process is imperfect and may shift randomly from an in-control to an out-of-control state characterized by a fixed production rate of imperfect items which is assumed to be inferior to the vendor's inventory accumulation rate (the difference between the production rate and the demand rate).

In this paper, we propose to extend Yedes et al.'s [16] model considering the case where the fixed imperfect production rate, related to the out-of-control state, could exceed the vendor's inventory accumulation rate. As for Yedes et al. [16], two joint production-inventory-maintenance strategies are considered. The problem will be formulated in next section and the mathematical model will be developed in section 3. Section 4 is dedicated to solving numerical procedure. Section 5 presents an illustrative example with the obtained results. Finally, some concluding remarks are summarized in section 6.

2. PROBLEM FORMULATION

This paper develops two integrated vendor–buyer production, inventory and maintenance strategies in the context of an imperfect production process that may shift randomly to an out-of-control state characterized by the production of non-conforming units at a fixed rate. We develop a framework allowing choosing one of two proposed strategies in order to minimize the total integrated average cost rate for any given situation.

We assume that the production cycle starts with a new system in an in-control state producing items of acceptable quality at a rate P greater than the average demand rate D. After a random period of time τ, the production system (considered as a single unit) shifts to an out-of-control state producing non-conforming units, at a fixed rate α, which are instantaneously detected and rejected thanks to a 100% screening process. At the end of each production cycle, a maintenance action and a new setup are performed. The maintenance action could be either preventive in case the system has not shifted to the out-of-control state, or corrective (overhaul) in case such a shift has occurred. Both types of maintenance actions allow restoring the system to an as good as new condition before the next production cycle starts.

In such a context of imperfect production process, the two proposed integrated vendor-buyer production-inventory policies are the following.

The first one, we call continuous production strategy, suggests that the buyer orders batches of size nQ every time his on hand inventory reaches the reorder point s after the reception of all the last ordered quantity. The vendor manufactures the quantity nQ continuously but delivers periodically by lot of size Q every Q/D time units. At the end of each production cycle (nQ/D time units), a preventive or a corrective maintenance action is undertaken depending on whether the production unit has shifted or not to the out-of-control state generating non-conforming rejected items. In case the shift has occurred, the quantity shipped at each of the shipment dates could be inferior to Q and then the buyer would incur a shortage cost since he would not be able to satisfy his customer's orders.

The second policy, called lot-for-lot strategy, consists in producing and delivering the ordered quantity nQ in smaller batches of size Q separately. A preventive maintenance action is performed immediately after the production of each lot (i.e. every cycle of Q/D time units) in case the system has not shifted to the out-of-control state, in order to restore the system to the as good as new condition before launching the production of the next lot. In case the shift to the out-of-control state occurs, only the quantity in the vendor's stock at the date Q/D (inferior to Q) will be delivered before undertaking a corrective maintenance action. Each item non-chipped on time won't be replaced and a related shortage cost will be incurred by the buyer.

The total integrated average cost per time unit corresponding to each strategy is considered as the performance criterion. The mathematical expressions of this cost rate are developed for each policy and a computational procedure is used to find the best choice (n^*,Q^*) for any given situation with given costs related to inventory (held by the buyer and the vendor), maintenance and quality; and given the probability distribution associated to the time to shift to the out-of-control state.

We adopt the following notation and assumptions to formulate the proposed model. Some additional notations and assumptions will be listed where used.

Notation:

D - average demand rate in units per unit time

P - production rate in units per unit time, $P > D$

α - production rate of non-conforming units

τ - time to shift to the out-of-control state (random variable)

n - number of lots ordered by the buyer from the vendor

Q - elementary lot size

c_t - the capacity of the transport equipment

K - setup cost for the vendor

A - ordering cost for each order of size nQ

F - transportation cost for each shipment

h_v - holding cost per unit per unit time for the vendor

h_b - holding cost per unit per unit time for the buyer

$ETC^{(1)}$ expected total cost per unit time for the continuous production strategy

$ETC^{(2)}$ expected total cost per unit time for the lot-for-lot strategy

C_{cm} - Corrective maintenance action cost

C_{pm} - Preventive maintenance action cost

C_s - Shortage cost per non delivered item

C_{cq} - Quality control cost per unit

C_{nq} - Incurred cost per non-conforming unit

$f(\tau)$ - probability density function associated to the time to shift to the out-of control state

Assumptions:

1. The time τ to shift to the out-of-control state is a random variable and follows a general distribution.
2. The shift to the out-of-control state is instantaneously detected.
3. While in the out-of-control state, the system produces non-conforming items at a constant rate α.
4. All non-conforming items produced are detected and automatically rejected.
5. Maintenance actions take negligible durations and restore the system to the as-good-as-new state.
6. The production system is set up after every maintenance action.
7. Shortages are allowed with no possible replacement.

3. MODELS DEVELOPMENT

As mentioned above, the case corresponding to an expected imperfect production rate $\alpha \le P\text{-}D$ has been tackled by Yedes et al. [16]. The related model will be summarized for the two considered strategies (continuous and lot-for-lot). Obviously, we will focus on the difference between this case and the one corresponding to $\alpha > P\text{-}D$ which mainly concerns the continuous production policy.

The expected total integrated cost per time unit for both strategies is expressed as follows:

$$ETC^{(i)}(n,Q)=TC_b^{(i)}+TC_v^{(i)} \quad (1)$$

TC_b and TC_v are the expected total costs respectively for the buyer and the vendor. TC_b corresponds to the sum of the ordering cost (EC_O), the transportation cost (EC_T), the inventory holding cost (EC_{sb}) and the shortage cost (EC_P). TC_v is composed of the setup cost (EC_K), the maintenance cost (EC_M), the inventory holding cost (EC_{sv}), the cost of non-conforming items (EC_{NQ}) and the quality control cost (EC_{CQ}).

$$TC_b^{(i)}= EC_O^{(i)} + EC_T^{(i)} + EC_{sb}^{(i)} + EC_P^{(i)} \quad (2)$$

$$TC_v^{(i)}= EC_K^{(i)} +EC_{sv}^{(i)}+EC_{NQ}^{(i)}+EC_{CQ}^{(i)}+EC_M^{(i)} \quad (3)$$

Let's detail all components to make up the expected total integrated cost rate expression for each policy.

3.1 CONTINUOUS PRODUCTION STRATEGY

Case $\alpha \le P\text{-}D$* (Yedes et al. [16])*:

In this case, shortage would occur only at the first shipment date (*Q/D* or *Q/P* time units after the production launch) if $\tau < Q/P$ (figure 1b). Even if $\tau = 0$ and the vendor's inventory is accumulated at a rate D (i.e. $\alpha_{max}=P\text{-}D$) the inventory level at each of the following shipment dates would be equal to Q. Otherwise, the vendor will be able to satisfy the entire buyer's order and to deliver a lot at each of the n shipment dates, as planed.

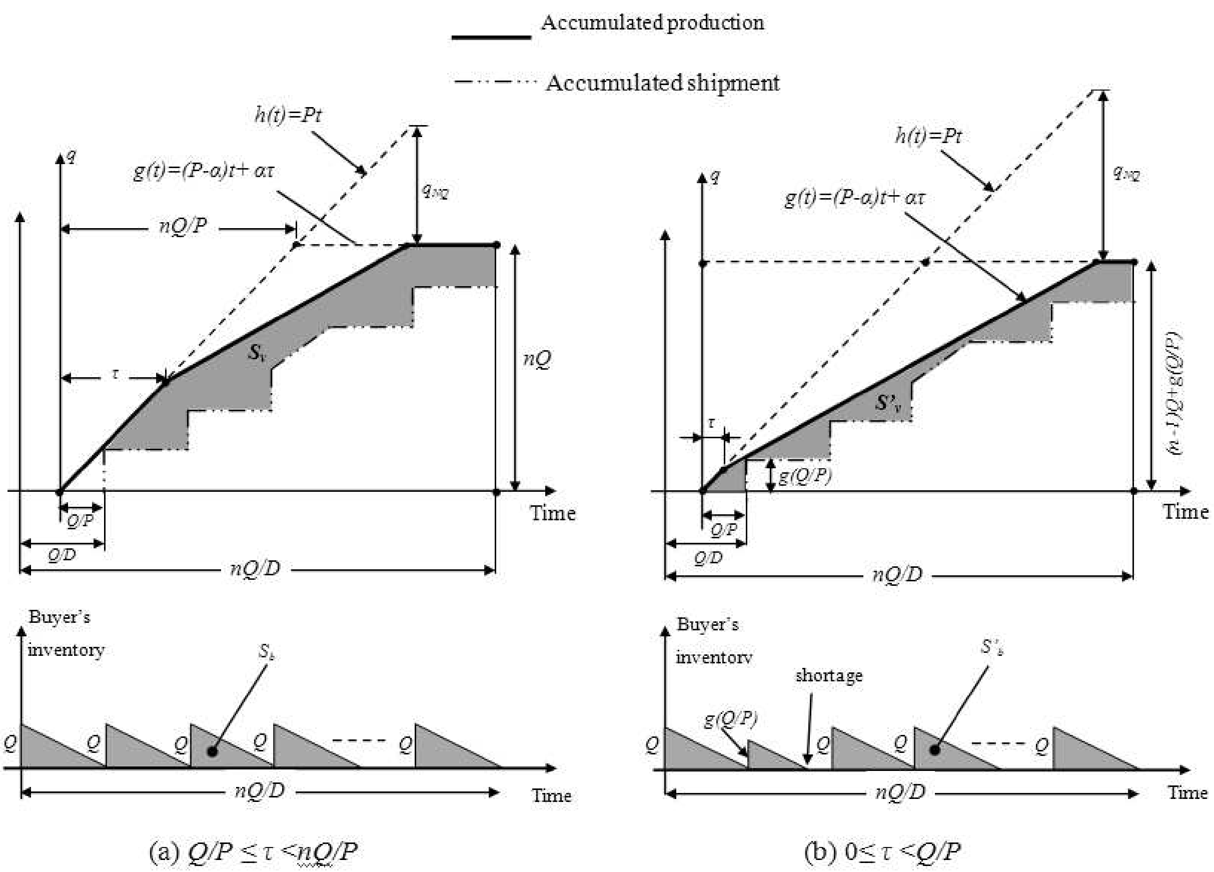

Fig.1. Vendor's accumulated production and shipment, and buyer's inventory variations for the continuous production strategy

The expected total cost rate in this case is expressed as:

$$\begin{aligned}ETC^{(1)}(n,Q) &= \frac{D}{Q}\left(\frac{A+K+C_{pm}}{n}+F\right)+\frac{Q}{2}\left[h_b+h_v\left[n\left(1-\frac{D}{P}\right)-1+2\frac{D}{P}\right]\right]+C_{cq}D\\
&+\frac{\alpha D}{nP}\left[C_S-\frac{P}{P-\alpha}C_{cq}-\frac{\alpha}{P-\alpha}C_{nq}+h_b\frac{Q}{D}\left(\frac{\alpha}{2P}-1\right)-h_vQ\left(\frac{\alpha}{2P^2}-\frac{n-1}{P}-\frac{\alpha n}{P(P-\alpha)}+\frac{\alpha^2}{2P^2(P-\alpha)}\right)\right]\times\int_0^{Q/P}f(\tau)d\tau\\
&+\frac{\alpha D}{P-\alpha}\left[C_{cq}+C_{nq}+\frac{P-\alpha}{n\alpha Q}(C_{cm}-C_{pm})-\frac{nQ}{2P}h_v\right]\times\int_0^{nQ/P}f(\tau)d\tau\\
&+\frac{\alpha D}{n(P-\alpha)}\left[nh_v-\frac{P}{Q}C_{nq}\right]\times\int_0^{nQ/P}\tau f(\tau)d\tau-\frac{\alpha PD}{nQ(P-\alpha)}C_{cq}\int_{Q/P}^{nQ/P}\tau f(\tau)d\tau\\
&+\frac{\alpha D}{n}\left[\frac{\alpha}{(P-\alpha)Q}C_{nq}-\frac{C_S}{Q}+\frac{h_b}{D}\left(1-\frac{\alpha}{P}\right)-\frac{(n-1)P+\alpha}{P(P-\alpha)}h_v\right]\times\int_0^{Q/P}\tau f(\tau)d\tau\\
&+\frac{\alpha^2}{2nQ}\left[h_b+\frac{D}{(P-\alpha)}h_v\right]\times\int_0^{Q/P}\tau^2 f(\tau)d\tau-\frac{\alpha PD}{2nQ(P-\alpha)}h_v\int_0^{nQ/P}\tau^2 f(\tau)d\tau \qquad (4)\end{aligned}$$

Case α > P-D:

Compared to the preceding case, where the imperfect production rate doesn't exceed the inventory accumulation rate in the system, the expressions of the expected reorder, transportation, setup and maintenance costs don't change. Indeed, only one setup is made to manufacture the entire buyer's order of size nQ which will be transferred periodically in n shipments (every lQ/D time units, l varies from 1 to n). In addition, a maintenance action whose type depends on the state of the production unit, will be performed at the end of the cycle.

Hence, the expected ordering, transportation, setup and maintenance costs correspond respectively to:

$$EC_O^{(1)}=\frac{A}{\frac{nQ}{D}}=\frac{AD}{nQ} \qquad (5)$$

$$EC_T^{(1)}=\frac{nF}{\frac{nQ}{D}}=\frac{FD}{Q} \qquad (6)$$

$$EC_K^{(1)}=\frac{K}{\frac{nQ}{D}}=\frac{KD}{nQ} \qquad (7)$$

$$EC_M^{(1)}=\frac{D}{nQ}\left[C_{cm}\int_0^{nQ/P}f(\tau)d\tau+C_{pm}\int_{nQ/P}^{+\infty}f(\tau)d\tau\right]=$$

$$=\frac{C_{pm}D}{nQ}+(C_{cm}-C_{pm})\frac{D}{nQ}\int_0^{nQ/P}f(\tau)d\tau \qquad (8)$$

The calculation of the rest of the components change as detailed below.

In the beginning, let τ_i be the instant defined by:

$$\forall\, 1\le i\le n,\qquad g_{\tau_i}\left(\frac{Q}{P}+(i-1)\frac{Q}{D}\right)=$$

$$=iQ \Leftrightarrow (P-\alpha)\times\left(\frac{Q}{P}+(i-1)\frac{Q}{D}\right)+\alpha\tau_i=iQ$$

where g represents the variation of the accumulated production after the shift to the out-of-control state (figure2).

That is: $\tau_i=\frac{iQ}{\alpha}-\frac{P-\alpha}{\alpha}\left(\frac{Q}{P}+(i-1)\frac{Q}{D}\right)$ for i = 1 à n, and $\tau_0 = 0$.

The shortage cost corresponds to:

$$EC_p^{(1)}=\frac{C_S D}{nQ}\int_0^{+\infty}q_S(\tau)f(\tau)d\tau \qquad (9)$$

The shortage quantity, q_S, represents all non-shipped products during a cycle of length nQ/D. According to figure 2, the ordered quantity will be totally shipped only if the shift to the out-of-control state occurs after the instant τ_n (i.e. if $\tau \ge \tau_n$). Thus, whatever $0 \le \tau < \tau_n$:

$$q_S(\tau) = nQ - g\left(\frac{Q}{P} + (n-1)\frac{Q}{D}\right) = nQ\left(1 - \frac{P-\alpha}{D}\right) + (P-\alpha)\left(\frac{1}{D} - \frac{1}{P}\right)Q - \alpha\tau \quad (10)$$

Consequently, the shortage cost can be written as:

$$EC_p^{(1)} = \frac{C_S D}{nQ}\int_0^{\tau_n} q_S(\tau) f(\tau) d\tau = C_S\left[\left[(D-P+\alpha) + \frac{P-\alpha}{n}\left(1-\frac{D}{P}\right)\right] \times \int_0^{\tau_n} f(\tau)d\tau - \frac{\alpha D}{nQ}\int_0^{\tau_n} \tau f(\tau) d\tau\right] \quad (11)$$

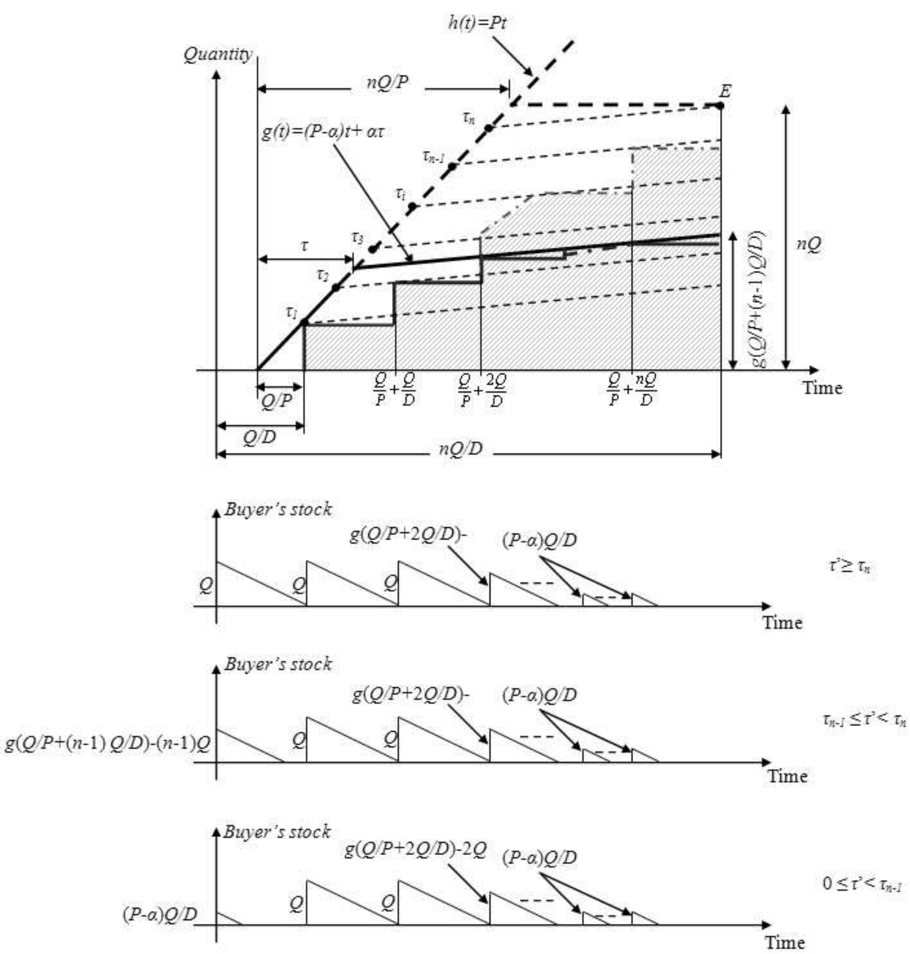

Fig.2. Accumulated production, accumulated shipment and buyer's inventory variations for α > *P-D*

- (case $\tau_2 \le \tau < \tau_3$)

The cost of non-conforming items is expressed as:

$$EC_{NQ}^{(1)} = \frac{C_{nq} D}{nQ}\int_0^{+\infty} q_{NQ}(\tau) f(\tau) d\tau \quad (12)$$

Where, q_{NQ} corresponds to the quantity of non-conforming items depending on τ and is differently calculated for the three following cases:

- Case $0 \le \tau < \tau_n$:

$$q_{NQ}(\tau) = h(t_E) - g(t_E) = h\left(\frac{Q}{P} + (n-1)\frac{Q}{D}\right) - g\left(\frac{Q}{P} + (n-1)\frac{Q}{D}\right)$$

$$= P\left(\frac{Q}{P} + (n-1)\frac{Q}{D}\right) - (P-\alpha)\left(\frac{1}{D} - \frac{1}{P}\right)Q - \alpha\tau = \alpha Q\left(\frac{1}{P} + \frac{n-1}{D}\right) - \alpha\tau \quad (13)$$

- Case $\tau_n \le \tau < nQ/P$:

$$q_{NQ}(\tau) = h(t_D) - nQ = P \times \frac{nQ - \alpha\tau}{P-\alpha} - nQ = \frac{\alpha nQ}{P-\alpha} - \frac{\alpha P}{P-\alpha}\tau \quad (14)$$

D is the point characterized by $g(t_D) = nQ$, and $h(t)$ represents the accumulated production at rate P.

- Cas $\tau \ge nQ/P$:

$$q_{NQ} = 0$$

Hence, the expression of de $EC_{NQ}^{(1)}$ is given by :

$$EC_{NQ}^{(1)} = \frac{C_{nq} D}{nQ} \int_0^{\tau_n} \left[\alpha Q\left(\frac{1}{P} + \frac{n-1}{D}\right) - \alpha\tau\right] f(\tau)d\tau + \int_{\tau_n}^{nQ/P} \left(\frac{\alpha nQ}{P-\alpha} - \frac{\alpha P}{P-\alpha}\tau\right) f(\tau)d\tau$$

$$= \frac{\alpha C_{nq} D}{P-\alpha} \times \left[\int_0^{nQ/P} f(\tau)d\tau - \int_0^{\tau_n} f(\tau)d\tau + \frac{\alpha}{nQ}\int_0^{\tau_n} \tau f(\tau)d\tau - \frac{P}{nQ}\int_0^{nQ/P} \tau f(\tau)d\tau\right] + \frac{\alpha C_{nq} D}{n} \times \left(\frac{1}{P} + \frac{n-1}{D}\right) \times \int_0^{\tau_n} f(\tau)d\tau \quad (15)$$

The quality control cost is obtained by :

$$EC_{CQ}^{(1)} = \frac{C_{cq} D}{nQ} \int_0^{+\infty} q_{CQ}(\tau) f(\tau)d\tau \quad (16)$$

q_{CQ} is the whole produced quantity (conforming and non-conforming items), and it depends on τ:

- Case $0 \le \tau < \tau_n$:

$$q_{CQ}(\tau) = h(t_E) = h\left(\frac{Q}{P} + (n-1)\frac{Q}{D}\right) = P \times \left(\frac{Q}{P} + (n-1)\frac{Q}{D}\right) = Q\left[1 + \frac{(n-1)P}{D}\right] \quad (17)$$

- Case $\tau_n \le \tau < nQ/P$:

$$q_{CQ}(\tau) = h(t_D) = P \cdot t_D = P\frac{nQ - \alpha\tau}{P-\alpha} = \frac{nPQ}{P-\alpha} - \frac{\alpha P}{P-\alpha}\tau \quad (18)$$

- Case $\tau \ge nQ/P$:

$$q_{CQ} = nQ \quad (19)$$

$EC_{CQ}^{(1)}$ can be written as:

$$EC_{CQ}^{(1)} = \frac{C_{cq} D}{nQ}\left[\int_0^{\tau_n} Q\left(1 + \frac{(n-1)P}{D}\right) f(\tau)d\tau + \int_{\tau_n}^{nQ/P} \frac{nPQ}{P-\alpha} - \frac{\alpha P}{P-\alpha}\tau f(\tau)d\tau + \int_{nQ/P}^{+\infty} nQ f(\tau)d\tau\right]$$

$$= C_{cq} D + C_{cq} D \times \left[\frac{n-1}{n}\left(\frac{P}{D} - 1\right)\int_0^{\tau_n} f(\tau)d\tau + \frac{\alpha}{P-\alpha}\int_{\tau_n}^{nQ/P} f(\tau)d\tau - \frac{\alpha P}{nQ(P-\alpha)}\int_{\tau_n}^{nQ/P} \tau f(\tau)d\tau\right] \quad (20)$$

To calculate the vendor's inventory cost, $EC_{SV}^{(1)}$, it is necessary to determine the related expected stock which can be obtained from the surface between the accumulated production curve and the accumulated shipment one. The accumulated production varies according to $h(t)$ and $g(t)$ respectively before and after the shift to the out-of-control state (figure 2). Concerning the accumulated shipment, it strongly depends on τ:

- if $\tau_i \leq \tau < \tau_{i+1}$ (for i = 0 to n-1) the sizes of the n shipments will be distributed in the following way:
  - the i^{th} first lots of size Q
  - the size of the (i+1)th lot corresponds to $g\left(\frac{Q}{P}+i\frac{Q}{D}\right)-iQ$
  - the $(n-i-1)^{th}$ remaining lots of size $(P-\alpha)\frac{Q}{D}$
- if $\tau \geq \tau_n$, there will be only equal sized shipments Q (no possible shortage)
  Therefore, $EC_{SV}^{(1)}$ can be expressed as :

$$EC_{SV}^{(1)}=\frac{h_v D}{nQ}\int_0^{+\infty} S^V(\tau)f(\tau)d\tau \tag{21}$$

Where, S^V is the expected surface between the accumulated production and shipment curves depending on τ. Taking into account the different shipments sizes, S^V is written as:

- Case $\tau_i \leq \tau < \tau_{i+1}$: (with i = 0 to n-1)

$$S_i^V(\tau)=\frac{P\tau^2}{2}+P\tau\left(\frac{Q}{P}+i\frac{Q}{D}-\tau\right)+\frac{P-\alpha}{2}\left(\frac{Q}{P}+i\frac{Q}{D}-\tau\right)^2-\frac{i(i+1)}{2}\times\frac{Q^2}{D}+\frac{(n-i-1)(P-\alpha)}{2}\times\frac{Q^2}{D^2} \tag{22}$$

- Case $\tau_n \leq \tau < nQ/P$:

$$S_n^V(\tau)=\frac{nQ^2}{2D}\left[n\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]-\frac{\alpha n^2Q^2}{2P(P-\alpha)}+\frac{\alpha nQ}{P-\alpha}\tau-\frac{\alpha P}{2(P-\alpha)}\tau^2 \tag{23}$$

- Case $\tau \geq nQ/P$: (no shift to the o*ut-of-control* state during the production cycle)

$$S_{n+1}^V(\tau)=\frac{nQ^2}{2D}\left[n\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right] \tag{24}$$

Consequently, $EC_{SV}^{(1)}$ corresponds to :

$$\begin{aligned}EC_{SV}^{(1)}=\frac{h_v D}{nQ}\Bigg[&\sum_{i=0}^{n-1}\int_{\tau_i}^{\tau_{i+1}} S_i^V(\tau)f(\tau)d\tau+\frac{nQ^2}{2D}\left[n\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]\times\int_{\tau_n}^{+\infty} f(\tau)d\tau\\ &-\frac{\alpha n^2Q^2}{2P(P-\alpha)}\int_{\tau_n}^{nQ/P} f(\tau)d\tau+\frac{\alpha nQ}{P-\alpha}\int_{\tau_n}^{nQ/P}\tau f(\tau)d\tau-\frac{\alpha P}{2(P-\alpha)}\int_{\tau_n}^{nQ/P}\tau^2 f(\tau)d\tau\Bigg]\end{aligned} \tag{25}$$

The buyer's inventory cost depends on τ (the instant to shift to the out-of-control during the current cycle) and τ' (the instant to shift to the out-of-control during the previous cycle as it can be noticed for the three possible scenarios in figure 2). We can express it as:

$$EC_{SB}^{(1)}=\frac{h_b D}{nQ}\int_0^{+\infty} S^B(\tau)f(\tau)d\tau \tag{26}$$

with S^B the expected surface delimited by the buyer's inventory variation curve and the time axis (figure 2), depending on τ and τ':

- Case $\tau_i \leq \tau' < \tau_{i+1}$: for i = 0 to n-2 (the case n =1 is excluded)

$$S_i^B(\tau) = \frac{(P-\alpha)^2 Q^2}{2D^3} \times \int_0^{\tau_{n-1}} f(\tau')d\tau' + \frac{1}{2D} \int_{\tau_{n-1}}^{\tau_n} \left[g\left(\frac{Q}{P} + (n-1)\frac{Q}{D}\right) - (n-1)Q \right]^2 f(\tau')d\tau' + \frac{Q^2}{2D} \int_{\tau_n}^{+\infty} f(\tau')d\tau'$$

$$+\frac{iQ^2}{2D} + \frac{1}{2D} \times \left[g\left(\frac{Q}{P} + i\frac{Q}{D}\right) - iQ \right]^2 + \frac{(n-2-i)(P-\alpha)^2}{2D^3} Q^2 \tag{27}$$

- Case $\tau' \geq \tau_{n-1}$:

$$S_{n-1}^B(\tau) = \frac{(P-\alpha)^2 Q^2}{2D^3} \times \int_0^{\tau_{n-1}} f(\tau')d\tau' + \frac{1}{2D} \int_{\tau_{n-1}}^{\tau_n} \left[g\left(\frac{Q}{P} + (n-1)\frac{Q}{D}\right) - (n-1)Q \right]^2 f(\tau')d\tau' + \frac{Q^2}{2D} \int_{\tau_n}^{+\infty} f(\tau')d\tau' + \frac{(n-1)Q^2}{2D} \tag{28}$$

So, $EC_{SB}^{(1)}$ corresponds to :

$$EC_{SB}^{(1)} = \frac{h_b D}{nQ} \left[\sum_{i=0}^{n-2} \int_{\tau_i}^{\tau_{i+1}} S_i^B(\tau) f(\tau) d\tau + \int_{\tau_{n-1}}^{+\infty} S_{n-1}^B(\tau) f(\tau) d\tau \right] \tag{29}$$

Finally, using equations (5), (6), (7), (8), (11), (15), (20), (25) and (29), we can conclude that the expression of the total expected integrated cost can be written as:

$$\begin{aligned} ETC^{(1)}(n,Q) = {} & \frac{D}{Q}\left(\frac{A+K+C_{pm}}{n} + F\right) + C_{cq}D + D \times \left[\frac{C_{cm}-C_{pm}}{nQ} + \frac{C_{nq}\alpha}{P-\alpha}\right] \times \int_0^{nQ/P} f(\tau)d\tau \\ & + \left[C_S \times \left[(D-P-\alpha) + \frac{P-\alpha}{n}\left(1-\frac{D}{P}\right)\right] + C_{nq}\alpha D\left[\frac{1}{n}\left(\frac{1}{P}+\frac{n-1}{D}\right) - \frac{1}{P-\alpha}\right] + C_{cq} \times (P-D) \times \left(\frac{n-1}{n}\right)\right] \times \int_0^{\tau_n} f(\tau)d\tau \\ & + \frac{D}{P-\alpha}\left[C_{cq}\alpha - \frac{nQ\alpha}{2P}h_v\right] \times \int_{\tau_n}^{nQ/P} f(\tau)d\tau + \frac{h_v Q}{2}\left[n\left(1-\frac{D}{P}\right) - 1 + \frac{2D}{P}\right] \times \int_{\tau_n}^{+\infty} f(\tau)d\tau \\ & + \frac{\alpha D}{nQ}\left[\frac{C_{nq}}{P-\alpha} \times (\alpha - P) - C_S\right] \times \int_0^{\tau_n} \tau f(\tau)d\tau + \frac{\alpha D}{P-\alpha}\left(h_v - \frac{P}{nQ}(C_{cq}+C_{nq})\right) \int_{\tau_n}^{nQ/P} \tau f(\tau)d\tau - \frac{h_v \alpha P D}{2nQ(P-\alpha)} \times \int_{\tau_n}^{nQ/P} \tau^2 f(\tau)d\tau \\ & + \frac{h_v D}{nQ} \sum_{i=0}^{n-1} \int_{\tau_i}^{\tau_{i+1}} S_i^V(\tau) f(\tau)d\tau + \frac{h_b D}{nQ} \sum_{i=0}^{n-2} \int_{\tau_i}^{\tau_{i+1}} S_i^B(\tau) f(\tau)d\tau + \frac{h_b D}{nQ} \times \int_{\tau_{n-1i}}^{+\infty} S_{n-1}^B(\tau) f(\tau)d\tau \end{aligned} \tag{30}$$

With S_i^V, S_i^B and S_{n-1}^B corresponding respectively to equations (22), (27) and (28).

3.2 LOT-FOR-LOT STRATEGY

As it can be noticed in figure 3, the shipment is periodic (every *Q/D* time units), but the delivered quantity depends on the production system state at every shipment date. Every time the shift to the *out-of-control* state occurs during one of the *n* production cycles of length *Q/D*, the delivered quantity next date will be inferior to *Q* and then the accumulated shipment at the end of the order cycle (*nQ/D*) will be less than the buyer's ordered quantity. However, we assume that such shift doesn't affect in anyway the production system behaviour during the next production cycles since a maintenance action, which is supposed to restore it in an as good as new condition, will be carried out immediately after every shipment.

In addition, the expected total integrated cost corresponding to this strategy can be formulated similarly for the two cases $\alpha \leq P\text{-}D$ and $\alpha > P\text{-}D$ still because of the independency between the production cycles (*Q/P*). The model developed by Yedes et al. (2010) is given by:

$$ETC^{(2)}(n,Q)=\frac{D}{Q}\left(\frac{A}{n}+K+C_{pm}+F\right)+\frac{Q}{2}\left(h_b+h_v\frac{D}{P}\right)+C_{cq}D$$
$$+\frac{\alpha D}{P}\left[C_S+C_{nq}+\frac{P}{\alpha Q}(C_{cm}-C_{pm})-\frac{h_vQ}{2nP}+\frac{h_bQ}{D}\left(\frac{\alpha}{2P}-1\right)\right]\times\int_0^{Q/P}f(\tau)d\tau$$
$$+\alpha D\left[\frac{h_v}{nP}+\frac{h_b}{D}\left(1-\frac{\alpha}{P}\right)-\frac{C_{nq}}{Q}-\frac{C_S}{Q}\right]\times\int_0^{Q/P}\tau f(\tau)d\tau+\frac{\alpha}{2Q}\left[\alpha h_b-\frac{D}{n}h_v\right]\times\int_0^{Q/P}\tau^2 f(\tau)d\tau \quad (40)$$

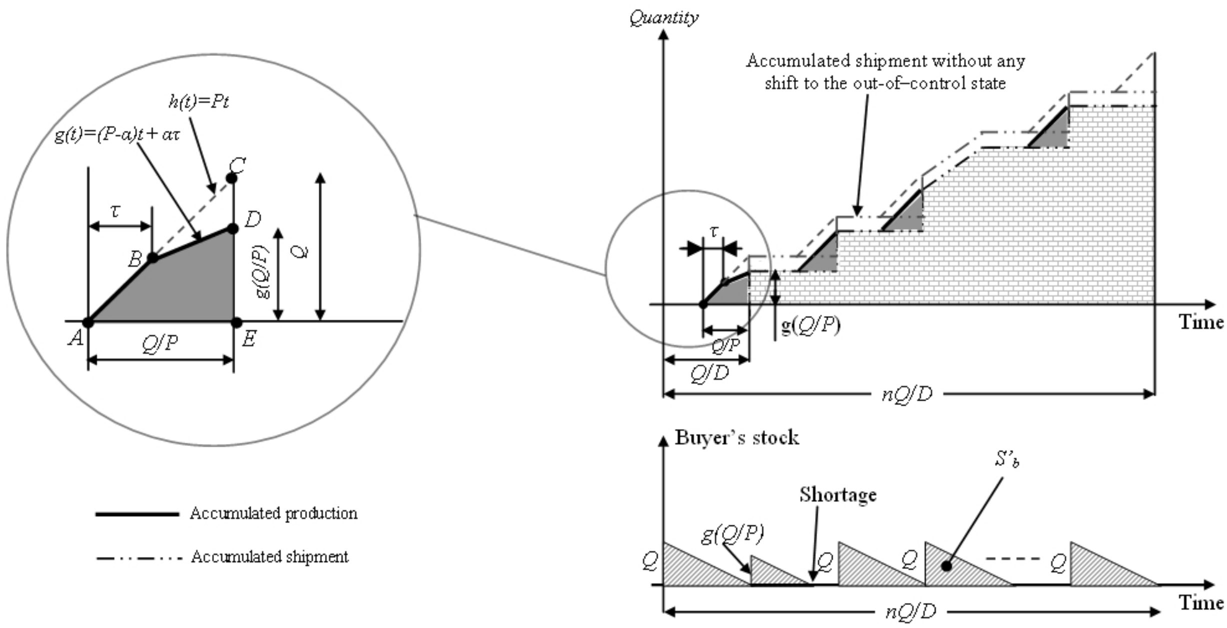

Fig.3. Vendor's accumulated production and shipment, and buyer's inventory variations for the lot-for-lot strategy (scenario corresponding to only one shift to the out-of-control state during the first production cycle)

4. NUMERICAL EXAMPLE

Due to the complexity of the models, we developed a numerical procedure to obtain approximate optimal solutions ($ETC^{(i)}*$, $n*$, $Q*$) for each of the proposed strategies for any given situation. The best strategy to adopt corresponds to the one yielding the lowest expected total integrated cost per time unit.

To illustrate our approach, we consider a situation with the following input data which have been arbitrarily chosen.

Input data:

- The distribution associated to the time to shift to the out-of-control state is a Weibull law with shape parameter θ =2.5 and scale parameter λ=2.
- P= 3200units/time unit, D=1000 units/time unit, α =2250units/time unit.
- K=50$, F= 25$, A=50$.
- h_b=8$/unit/time unit, h_v=5$/unit/time unit.
- C_{cm}=200$.
- C_{cq}= 0.5$/unit, C_{nq}= 20$/unit, C_S= 20$/unit.

The results shown in table1 demonstrate, as it has been stated by [16], that for $\alpha \le P\text{-}D$ the best approximate solution can be yielded by either one of both considered policies depending on the set of input parameters (i.e. the continuous production strategy has to be chosen for $C_{pm} \ge 110\$$, while the lot-for-lot strategy provides the best solution for relatively small values of C_{pm}: $C_{pm} \le 50\$$) and the lot-for-lot strategy gets more and more interesting with the increase of α. This statement is valid until a certain level of $\alpha > P\text{-}D$ beyond it the lot-for-lot strategy becomes the most economic even for great preventive maintenance costs. Indeed, as noted in [16], there is a certain threshold of the preventive maintenance cost $C_{pm}*$ under which the lot-for-lot strategy is the most economic. In the considered situation, this threshold remains between 50$ and

100$ when α increases from 1000 to 2000units/time unit ($\alpha < P\text{-}D$=2200). On the other hand, as soon as α exceeds *P-D*, C_{pm}* migrates to great values making larger the spectrum of C_{pm} favorable to the choice of the lot-for-lot strategy (for α=2250 units/time unit 110$<$C_{pm}$*<185$ and if α reaches 2900units/time unit C_{pm}* surpasses 185$). In addition, for the case $\alpha > P\text{-}D$, as α gets greater the profit yielded by the lot-for-lot strategy, compared to the continuous production one, becomes more and more important: i.e. for C_{pm}=5$, when α raises 1000units/time unit between 1000 and 2000units/time unit, the profits $ETC^{*(2)}$-$ETC^{*(1)}$ increases by 39.07$, while increasing 650units/time units (passing from 2250 to 2900 units/time unit) $ETC^{*(2)}$-$ETC^{*(1)}$ augments by 53.03$. This can be explained by the fact that the production of non-conforming items weighs down the expected total integrated cost much more for the case $\alpha>P\text{-}D$ and specially for the continuous production strategy. Indeed, with only one maintenance action at the end of the production cycle and a great non-conforming production rate, the production unit would generate an important number of non-conformities and shortages. Contrarily, the lot-for-lot strategy, with the possibility to restore the system in an as good as new condition at each shipment date, reduces the probability to shift to the out of control state and provides an expected gain on the non-quality and shortage costs which covers the loss on the maintenance cost even for great values of C_{pm}.

Table 1 Obtained results varying α and *Cpm*

		Lot-for-lot strategy			**Continuous strategy**			**$ETC^{*(2)}$- $ETC^{*(1)}$**
	Cpm	***n***	***Q***	***$ETC^{(2)}$***	***n***	***Q***	***$ETC^{(1)}$***	
α = 1000	5	379	127	1747,04	3	81	1937,41	-190,37
	50	**370**	**159**	**2060,86**	**3**	**90**	**2110,38**	**-49,52**
	110	**416**	**192**	**2401,53**	**4**	**82**	**2315,06**	**86,47**
	185	365	227	2757,51	4	90	2527 ,65	229,86
α = 1500	5	424	127	1750,01	2	102	1958,04	-208,03
	50	**502**	**157**	**2065,92**	**3**	**87**	**2138,02**	**-72,1**
	110	**542**	**190**	**2409,83**	**3**	**96**	**2352,82**	**57,01**
	185	480	224	2770,03	4	86	2583,47	186,56
α = 2000	5	517	126	1752,88	2	99	1982,32	-229,44
	50	**415**	**156**	**2071,24**	**3**	**82**	**2182,92**	**-111,68**
	110	**329**	**188**	**2418,41**	**3**	**91**	**2410,78**	**7,63**
	185	407	221	2782,43	3	100	2667,98	114,45
α = 2250	5	424	126	1754,56	2	96	2000,65	-246,09
	50	407	156	2073,83	3	79	2217,24	-143,41
	110	**375**	**187**	**2422,38**	**3**	**87**	**2454,67**	**-32,29**
	185	**407**	**220**	**2788,47**	**3**	**96**	**2723,63**	**64,84**
α = 2500	5	417	125	1756,07	2	94	2020,98	-264,91
	50	387	155	2076,40	2	105	2245,43	-169,03
	110	**304**	**186**	**2426,58**	**3**	**83**	**2506,73**	**-80,15**
	185	**374**	**219**	**2794,48**	**3**	**91**	**2789,11**	**5,37**
α = 2900	5	425	125	1758,44	2	90	2057,56	-299,12
	50	408	154	2080,39	2	100	2292,82	-212,43
	110	520	185	2432,56	2	111	2575,06	-142,5
	185	466	217	2803,75	2	122	2893,11	-89,36

We can conclude that globally, an increasing rate of non-conforming items would encourage the buyer and the vendor to choose the lot-for-lot strategy. This strategy reduces the period of time between successive maintenance actions (time to produce one lot of size *Q*) which allows decreasing the probability to shift to the out-of-control state. Consequently, the expected cost of non-conforming items and the expected shortage cost would be reduced. Contrarily, the continuous production strategy is more interesting for small production rates of non-conforming units. In this case, the incurred expected costs related to shortages and the rejection of non-conforming items would not be prevailing compared to the maintenance cost. That is, it would not be justified multiplying the number of maintenance actions since it would cost more than shortages and the production of non-conforming items.

.

5. CONCLUSION

In this paper, we treated an integrated single vendor single buyer supply chain optimisation problem in the context of an imperfect production process that may shift randomly to an out-of-control state. As Yedes et al [16], we proposed two management strategies considering simultaneously production, inventory and maintenance policies; but we extended their model to the case of any production rate of non-conforming items α. The main purpose being to demonstrate how much maintenance actions the total integrated cost could reduce by decreasing non-conformities and shortages, especially for great values of α exceeding the vendor's inventory accumulation rate P-D. Arbitrarily chosen numerical data have been used to illustrate our approach and to demonstrate how one or the other policy could turn out to be more cost-effective depending on the values of the preventive maintenance cost.

6. BIBLIOGRAPHY

[1] Anis Chelbi, Nidhal Rezg, Mahdi Radhoui. Simultaneous determination of production lot size and preventive maintenance schedule for unreliable production system. Journal of Quality in Maintenance Engineering, 14:161-176, 2008.

[2] Apichai Ritvirool, Wiliam G. Ferrell Jr.. The effect on inventory of cooperation in single-vendor, single-buyer systems with quality considerations. International Journal of Operational Research

[3] Chao-Kuei Huang. An optimal policy for a single-vendor single-buyer integrated production–inventory problem with process unreliability consideration. International Journal of Production Economics, 91:91-98.

[4] El-Houssaine Aghezzaf, Anouar Jamali, Daoud Ait-Kadi. An integrated production and preventive maintenance planning model. European Journal of Operational Research, 181: 679-685, 06.

[5] Hyo Lee, Mandyam M. Srinivason. A production/inventory policy for an unreliable machine. Rahim M.A. and Ben Daya M. (Ed.), Integrated models in production planning, inventory, quality, and maintenance, Kluwer Academic Publishers, 79-94, 01.

[6] John F. Affisco, M. Javad Paknejad, Farok Nasri. Quality improvement and setup reduction in the joint economic lot size model. European Journal of Operations Research,142: 497-508, 02.

[7] Mahdi Radhoui, Nidhal Rezg, Anis Chelbi. Joint quality control and preventive maintenance strategy for imperfect production process. Journal of Intelligent Manufacturing, 21:205-212, 08.

[8] Meir J. Rosenblatt, Hau L. Lee. Economic production cycles with imperfect production processes. IIE Transactions, 18:48-55, 86.

[9] Mohamed Ben-Daya. The economic production lot-sizing problem with imperfect production processes and imperfect maintenance. International Journal of Production Economics, 76:257-264,02.

[10] Mohamed Ben-Daya, Sayed Naveed Khursheed. The Economic Production Lot-Sizing Problem with Deteriorating Manufacturing Processes and Preventive maintenance. Vaxjo, Sweden, Proceedings IFRIM.

[11] Moutaz Khouja, Abraham Mehrez. An economic production lot size model with variable production rate and imperfect quality. Journal of the Operational Research Society, 45:1405-1417, 94.

[12] Po-Chung Yang, Hui-Ming Wee. A single-vendor and multiple-buyers production–inventory policy for a deteriorating item. European Journal of Operational Research, 143:570-581, 08.

[13] Suresh Kumar Goyal, Chao-Kuei Huang, Kuo-Chao Chen. A simple integrated production policy of an imperfect item for vendor and buyer. Production Planning & Control, 14:596 – 602, 03.

[14] Cheng, T. C. E.. An economic order quantity model with demand dependent unit production cost and imperfect production processes. IIE Transactions, 23 :23-28.

[15] Xing chu Liu, Çetinkaya Sila. A note on "quality improvement and setup reduction in the joint economic lot size model". European Journal of Operations Research, 182:194-204, 07.

[16] Yesser Yedes, Anis Chelbi, Nidhal Rezg. Quasi-optimal integrated production, inventory and maintenance policies for a single-vendor single-buyer system with imperfect production process. Journal of Intelligent Manufacturing, 2(4), DOI 10.1007, 10.

[17] Zimin Yang, Dragan Djurdjanovic, Jun Ni. Maintenance scheduling in manufacturing systems based on predicted machine degradation. Journal of Intelligent Manufacturing, 19:87-98, 08.