Central European Journal of Economic Modelling and Econometrics

Determinants of the VAT Gap in EU Member States from 2000 to 2016

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Submitted: 17.08.2022, Accepted: 18.11.2022

Abstract

The spectacular surge of Poland's VAT revenues after 2015 prompted a discussion about the role of the tax administration in collecting tax liabilities. Unfortunately, the scarcity of the available data prevents empirical studies from reaching reliable conclusions about the determinants of VAT revenues. Given that, this article presents a wider attempt at identifying the determinants of VAT revenues in the EU Member States. Using panel cointegration methods, several working hypotheses linking VAT gap to income factors, the business cycle, tax carousels, and an effectiveness of the government are evaluated. The results of the research provide evidence that the VAT gap in the EU countries is under a strong influence from variables approximating changes in *per capita* incomes, the business cycle, and the openness of an economy to intra-EU trade. The latter finding is a sufficient indication that the improvements made to Poland's tax system were both legitimate and effective.

Keywords: VAT revenues' determinants, econometric modelling, panel cointegration

JEL Classification: E61, H22, H26, C51

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1 Introduction

The spectacular surge in VAT revenues in Poland from 2017 to 2018 initiated a discussion about the effectiveness of the tax administration in collecting tax liabilities, the effectiveness of tax collection improvements, the sizes of tax gaps, and the possibility of maintaining fiscal stimulation through social and investment programmes. The review of studies, analyses, and comments shows a lack of consensus on almost all these issues (see, for instance, Bratkowski and Kotecki 2018, PIE 2018). The methods for measuring the VAT gap, including the widely used top-down method, also by the European Commission, are imprecise in many ways (*vide* European Commission 2013, 2014, 2015, 2016a, 2017, 2018a, 2019, 2020, 2021; for details, see for instance, Mazur et al. 2019, Reckon LLP 2009, European Commission 2016b, Hutton 2017). The discontinuity of VAT gap estimates resulting from a change in the approach to defining the macro-categories of the European System of National and Regional Accounts at the turn of 2011 is accentuated (the ESA'95 system was replaced then by ESA'2010), as well as the procyclicality of VAT revenues with respect to the tax base, which may distort the perception of the existing VAT gap. It is argued, therefore, that empirical inference about the size of VAT gap and the causes of its variability is so uncertain that the conclusions are always debatable.

Even so, studies are being undertaken, which seek possible relationships between VAT gap changes and the measures tightening of VAT collection after 2015. According to the report by the Polish Economic Institute (PIE 2018), the most important of those were legislative changes (introducing penal interests, VAT sanction, extended confiscation), upgrades to the VAT system (the fuel package, the Single Control File, split payment), and the reform of the tax administration, all of which contributed to a reduction in the VAT revenue gap.

To determine the validity of the above views, appropriate methods of quantitative analysis are necessary. The standard approach in econometric studies of factors influencing the VAT gap consists in calculating the annual estimates of potential VAT revenues and building panel data models where the absolute or relative VAT gap is assumed to depend on hypothetical macroeconomic, legal variables, and socio-economic variables, which are sometimes selected *ad-hoc* (see, e.g., Andreoni et al. 1998). The determinants of the VAT gap are then selected using statistical criteria: a variable's parameter value that is statistically significantly different from zero is assumed to indicate the variable has an effect on the VAT gap. However, in addition to the time series used in the studies being relatively short and only available for few countries, which makes them inappropriate for investigating reliably models with extensive sets of explanatory variables, it is also noteworthy that the common feature of all available studies is dismissal of the problems arising from the integration and cointegration of variables. The consequences of econometric investigations ignoring the danger of drawing conclusions from spurious regressions that invalidate standard statistical inference are hard to overestimate (Kao 1999, Baltagi and Kao 2001).

The above criticism against empirical investigations of the VAT gap determinants points to the need for studies free of such methodological flaws. This paper presents the results of a panel cointegration analysis of factors influencing the VAT revenue gap in the EU Member States and makes an attempt at a precise identification of factors, thus verifying other authors' findings. Its structure follows the sequence of research questions and answers thereto. Section 2 discusses the general concept of the causes of tax gaps and critically reviews most empirical works published on the topic so far. The research hypotheses and the strategy employed to verify them are presented in Section 3. Section 4 outlines the data properties and econometric methods used in the research. The results of empirical research using 2000-2016 data from 21 EU countries and conclusions are provided in Section 5. The paper closes with a summation.

2 A review of VAT gap studies

An inescapable conclusion from the review of studies on VAT gap determinants is that most empirical works draw on prediction from the deterrence model proposed by Allingham and Sandmo (1972), who built it on the standard assumption that a representative economic agent maximises utility represented by the return from assets held. Central to Allingham and Sandmo's model is their treatment of not reporting income for tax purposes as equivalent to purchasing risk-carrying assets, the risk of which increases with the probability of a tax audit and the severity of the penalty for tax evasion. Then, raising tax rates does not increase budgetary revenues as long as the increase in agents' incomes outweighs the cost of a penalty and the risk of a tax audit.

In their review of studies on the determinants of the VAT gap, Andreoni et al. (1998) presented the directions of generalisation of the deterrence model and emphasised the significance of behavioural, moral, and social aspects of tax-decision making by economic agents, which consider in this process individually defined moral principles, the fairness of the tax system and tax burdens, which ultimately allows them to endorse (or not) tax-funded government spending. Andreoni et al. (1998) have argued that a tax system and its gaps should be studied from several angles, such as (i) public finances and multiplier effects induced by the tax system, (ii) the enforcement of laws, (iii) the effectiveness of the tax administration, and (iv) ethical challenges which may disturb the structure of labour supply (e.g., Luttmer and Singhal 2014).

The above analytical scheme aligns with one of the first studies by Agha and Haughton (1996). Using cross-sectional data from 17 OECD countries, they confirmed that the VAT gap depended on the VAT base rate and the number of rates. The negative impact of an increasing base or weighted VAT rate on budgetary revenues was reported in later empirical analyses (Matthews 2003, 14 EU Member States, 1970-1999; Christie and Holzner 2004, 29 European countries, 2000-2003; European Commission 2013, 26 EU Member States). Although most empirical investigations accept conclusions from the deterrence model and – explicitly or implicitly – verify a hypothesis complementing the Allingham and Sandmo (1972) model about the presence in economies of mechanisms described by the Laffer curve (e.g., Mathews 2003), on factors contributing to the VAT gap otherwise use eclectic specifications of the empirical models. Christie and Holzner (2004) have analysed the effectiveness of VAT collection allowing for the effect of symptomatic variables such as moral norms, tax system fairness, taxpayers' satisfaction with public service quality, their level of risk aversion, the risk of tax administration imposing a penalty and its severity, the scale of income inequalities within and between countries (GDP *per capita*), the level of corruption, and the complexity of tax systems. Aizenman and Jinjirak (2005), in addition to assessing the VAT gap in terms of income inequalities quantified by Gini's index and GDP *per capita*, also used the agriculture share of GDP, the urbanisation rates, the stability indicators of the political system, and countries' openness to foreign trade as variables potentially influencing VAT revenues. The authors of empirical studies gradually extended the list of factors that might have some effect on the VAT gap (see, Reckon LLP 2009, European Commission 2013, 2018a, 2019, 2020, 2021, Zidkova 2014, Szczypińska 2018). The trend is well illustrated by the analysis conducted by Reckon LLP (2009), where 13 variables believed to be able to explain the VAT gap were considered. In these circumstances, it is hardly surprising that studies analysing alternative determinants of the VAT gap for different periods and panels of countries and using different data sets have produced different results. Moreover, the same studies sometimes present parameter estimates obtained with models that have significantly different specifications (*vide* Reckon LLP 2009, tables 33-36). Apart from the econometric and interpretational doubts that any closer analysis of the models' parameter estimates must raise, it is important to note that the main variables shaping the VAT gap are, according to Reckon LLP (2009), the corruption perception index (CPI) and the ratio between potential VAT revenues and the country's GDP. A European Commission (2013) study pointed to the importance of the pro-cyclicality of VAT revenues (represented by unemployment rate changes) and the VAT rate. Another European Commission (2018a) study concluded that procyclical changes in the VAT gap are a combined effect of the structure of a country's economy (specifically, retail, industry, and telecommunications shares of economic activity), the differentiation of VAT rates, the size of its population, and information technology and tax administration expenditure. Zidkova (2014) has considered 16 variables representing factors of potential influence on the VAT gap based on cross-sectional models constructed with 2002 and 2006 data from 24 EU countries. The parameter estimates showed the study's conclusions to be excessively dependent on the period of analysis and the analytical form of the model. A convincing influence on the VAT gap was found for household final consumption (increasing the VAT gap) and the ratio of VAT revenue to GDP (negatively related to the VAT gap). In the recent study, Szczypińska (2018) tested 13 variables such as the weighted average VAT rate, the number of VAT rates, VAT collection costs as a share of total VAT revenue, VAT compliance time (indicating the complexity of the VAT system), Gini's index, and the CPI (representing the quality of institutions) for their potential relationship with the VAT gap. The empirical confirmation of influence on the VAT gap was found for four variables, namely, the weighted average VAT rate, the CPI, and VAT collection costs.

European Commission's (2020, 2021) recent studies considered VAT gap sensitivity to 64 potential determinants divided into 4 groups, namely, (i) tax administration variables, (ii) macroeconomic variables, (iii) economic structure and institutional variables, and (iv) tax fraud proxies. They showed that the agriculture's share' of the economy and the imports of risky products (fraud proxy) increased the VAT gap, while GDP growth, general government surplus, communication, the financial sectors' share of the economy, and IT expenditures reduced it. These findings stand in obvious contrast with those published by the European Commission (2019), pointing to an unemployment rate, changes in countries' population, dispersion of tax rates in a country, and – somewhat surprisingly – the art sector's share of the economy as the variables that influenced the VAT gap.

3 Research hypotheses

The review of empirical studies on the VAT gap shows that they are problematic in many ways. Firstly, attention is drawn to the diversity of empirical models and conclusions drawn from them, which are presented by authors as optimal or acceptable. With slight simplification, the model with which the studies start is the following:

$$VGAP_{kt} = f(\mathbf{X}_{kt}, \mathbf{\Theta}_k, \varepsilon_{kt}), \tag{1}$$

where $VGAP_{kt}$ denotes the VAT gap in absolute terms ($VGAP_{kt} = VR_{kt} - VTTL_{kt}$) or relative terms ($VGAP_{kt} = (VR_{kt} - VTTL_{kt})/VTTL_{kt}$), VR_{kt} represents VAT revenues in country k in year t, $VTTL_{kt}$ – potential VAT revenues (VAT total tax liabilities), $\mathbf{X}_{kt}$ – vector of M VAT gap determinants, $\mathbf{\Theta}_k$ – the vector of the model parameters, $\varepsilon_{\mathrm{kt}}$ – a white-noise random term.

The eclecticism of research approaches and empirical results is largely determined by a lack of clear theoretical guidelines about relevant determinants and the fact that in many works the number of explanatory variables is disproportionately large compared with the time series dimension. As a result, it is difficult or even impossible to build a model using the 'from general-to-specific' approach, and random results can be expected. The range of VAT gap determinants used in studies is impressively wide, starting with the risk of penalization (approximated by the judicial effectiveness index), the fairness of the tax system, and the level of the VAT rate used by Christie and Holzner (2005), and ending with real GDP *per capita*, countries openness to foreign trade, the rate of urbanisation, the stability of the political system, Gini's index, and the agriculture share of GDP employed by Aizenman and Jinjirak (2005).

The second problem is that the VAT gap models are built assuming that variables are stationary. Consequently, there is the risk of inferring from spurious regressions, which has many well-known negative consequences (e.g. Klein et al. 1999). Models with variables integrated of order higher than zero require a full cointegration analysis to distinguish between long-term relationships, short-term dependencies, and short-term adjustments of VAT revenues to equilibrium.

Thirdly, ignoring the cointegration of variables prevents analysing the impact of different determinants on the VAT gap by time horizon. This comment is directly related to the criticism of the assumption made by almost all authors about VAT revenue unit elasticity with respect to potential VAT revenues ($VGAP = VR - VTTL$ for absolute revenues and $vgap = vr - vttl$ for relative ones; the small letters denote natural logarithms of the variables):

$$vr_{kt} = \theta_0 vttl_{kt} + \sum_{l=1}^{L} \theta_{kl} M_{klt} + \sum_{n=1}^{N} \gamma_{kn} S_{knt} + \varepsilon_{kt}, \tag{2}$$

where M_k and S_k are the medium- and short-term determinants of VAT revenues, respectively; to simplify the notation the deterministic variables are omitted in equation (2).

The thesis that potential VAT revenues determine actual VAT revenues in the long term and that short- and medium-term fluctuations in actual VAT revenues around their equilibrium path result from factors such as the level of the VAT rate, the phase of the economic cycle (represented, for instance, by employment rate changes), and changes in the tax collection system is not controversial. The rejection of the homogeneity hypothesis $\theta_0 = 1$ means that the list of short- and medium-term determinants of VAT revenues and the VAT gap is inappropriate. Moreover, ignoring this fact and imposing restriction $\theta_0 = 1$ results in an erroneous evaluation of the role of short- and medium-term factors, or in constructing models with artefact-type relationships. The latter occur in empirical models when the statistical confirmation of the presence of one of the explanatory variables in the model only arises from the presence of another variable, with neither of them being a real determinant of the dependent variable.

With the above sketchy criticism of the VAT gap models in mind, the empirical analysis presented below starts with model (2) with estimated parameter θ_0. Five hypotheses represented by the following research questions are considered:

1. Does the relationship between VAT revenues and potential VAT revenues stabilize in the long term?

   The simplest way to verify the hypothesis involves estimating the parameters of the model:

   $$vr_{kt} = \theta_0 vttl_{kt} + \varepsilon_{kt}, \tag{3}$$

   and then performing a formal test of restriction $\theta_0 = 1$. When the long-term homogeneity restriction cannot be rejected, the long-term VAT revenues remain in a constant ratio to potential revenues. The rejection of hypothesis $\theta_0 = 1$ means that only the model whose estimate of parameter $\theta_0 > 1$ points to the pro-cyclicity of the tax gap has an acceptable economic interpretation. In such a case, specification (3) needs to be extended by adding variables representing changes in the economic cycle.

A review of recent empirical studies pointed to the anti-cylicality of the VAT gap as the most frequent cause of discrepancies between VAT revenues and VAT liabilities (e.g., Sancak 2010, Ueda 2017, Boschi and d'Addona 2019, Durán-Cabré at al. 2020, Konopczak 2020, 2022). Hence, the empirical part of the paper presents parameter estimates for models using the rate of unemployment (U) as a business cycle proxy. Including the rate of unemployment in the model is justified by the fact that increasing unemployment indirectly deteriorates enterprises' financial performance, reduces their liquidity, and encourages some VAT payers to evade its payment and move businesses to the shadow economy. In the alternative specification of model (3), the rate of unemployment was replaced by a variable approximating the size of the shadow economy ($SHDEC$).

2. Does economic growth reduce the VAT gap?

   This question is answered by verifying an intuitive hypothesis tested in many earlier studies that wealthier societies are less inclined to avoid the payment of taxes. Hence:

$$vr_{kt} = \theta_0 vttl_{kt} + \theta_1 gdpc_{kt} + \varepsilon_{kt}, \tag{4}$$

   where *gdpc* denotes GDP *per capita* in real terms (a natural logarithm). A complex hypothesis $\theta_0 = 1$ and $\theta_1 > 0$ is tested.

3. Do carousel frauds have a significant effect on the size of the VAT gap?

   Earlier VAT gap studies (e.g., European Commission 2013) assumed that the mandatory registration of goods crossing borders made non-payment of the VAT less likely. Consequently, their authors argued that variables such as an economy's openness to foreign trade reduced its VAT gap. In time, however, the emergence of carousel frauds in intra-EU trade such as the MTIC fraud (Missing Trader Intra Community; see, European Commission 2018a, 2018b, PIE 2018) have weakened the intuitive relationship between an increase in a country's foreign trade volume and its VAT revenues. In the model:

$$vr_{kt} = \theta_0 vttl_{kt} + \theta_1 gdpc_{kt} + \theta_2 INTR_{kt} + \varepsilon_{kt}, \tag{5}$$

   where $INTR$ denotes a country's share of intra-EU trade and the estimate of parameter θ_2 depends on the impact of both aforementioned processes – its value is negative and statistically significantly different from zero when the impact of carousel frauds on VAT revenues is stronger than the revenues' increase driven by an expanding volume of intra-EU trade and positive and statistically significantly different from zero when VAT revenues from expanding foreign trade are greater than the effect of carousel frauds.

Empirical results confirming the effect of MTIC frauds on VAT revenues have been published by authors such as Liu et al. (2016), Carfora et al. (2020), and European Commission (2020, 2021). According to Carfora et al. (2020), the theoretical rationale for using intra-EU trade to approximate the MTIC mechanism can be found in the works by Marrelli (1984) and Marrelli and Martina (1984). Their analysis started with assumptions about (i) separability of a firm's production and evasion decisions, and (ii) risk-neutrality of the firm. A firm's expected profit is a function of probability τ of tax frauds being detected:

$$E(s) = \tau \left(p \cdot x - c(x) - t(x-f) - g(f)\right) + (1-\tau)\left(p \cdot x - c(x) - t(x-f)\right), \quad (6)$$

where: p – tax-inclusive price, x – quantity of production, f – scale of tax underreporting, $c(x)$ – cost function, $t(x-f)$ – taxes reported, $g(f)$ – penalty for detected tax underreporting. Then, $E(s)$ is only driven by firm's risk-aversion. However, in the case of a firm being part of a system of multiple, intense international trading activity, probability τ also becomes a function of the risk appetite of its trading partner; hence the conclusion that increasing openness to intra-EU trade can contribute to an increasing scale of tax frauds.

4. Do social inequalities and tensions contribute to the VAT gap?

Similarly to the case of openness to foreign trade, the effect of social inequalities on VAT revenues is ambiguous. When income disparities in an economy are not considerable and population incomes allow a satisfyingly high level of consumption, the incidence of tax frauds is likely to be relatively low. Then, however, the tax administration has a problem identifying taxpayer groups representing a higher risk of fraud and thus requiring more frequent audits. On the other hand, substantial income disparities may encourage less successful VAT payers to report revenues below the mandatory threshold for VAT registration and taxpayers generating substantial revenues to take the risk of being penalized for fax frauds or paying litigation costs.

Summing up, including Gini's index ($GINI$) measuring income inequalities into the set of variables influencing VAT revenues leads to the following model:

$$vr_{kt} = \theta_0 vttl_{kt} + \theta_1 gdpc_{kt} + \theta_2 GINI_{kt} + \varepsilon_{kt}, \quad (7)$$

which allows the economic interpretation of both positive and negative estimates of parameter θ_2 provided their values are statistically significantly different from zero.

5. Does legal strictness reduce the VAT gap?

   The hypothesis about the size of the VAT gap depending on the effectiveness of the government is straightforward to test with the following model:

$$vr_{kt} = \theta_0 vttl_{kt} + \theta_1 gdpc_{kt} + \theta_2 L_{kt} + \varepsilon_{kt}, \tag{8}$$

   where L is an index measuring the effectiveness of the government and its increasing value means that the effectiveness is improving, $\theta_2 > 0$.

   The empirical research in this study used four indices: the government effectiveness index (G_EFF), the regulatory quality index (G_QLT), the rule of law index (G_LAW), and the corruption perception index (CPI).

   Some of the variables used by other authors to study the VAT gap were also included, namely, the base VAT rate and final consumption taxation (RV and C_TAX, respectively; the reducing effect of both these on VAT revenues is explained by the Laffer curve), and the number of VAT rates, NRV, showing the complexity of the VAT system and the existence of 'inducements' for businesses to apply lower VAT rates than due. Because the empirical research failed to confirm the variables' influence on VAT revenues, their estimates and conclusions are not presented.

4 The data and econometric methodology

The study uses 2000-2016 annual data from EU Member States (including the United Kingdom). Initially, 24 EU countries (without Croatia, Cyprus, Malta, and Luxembourg) were examined. Croatia and Cyprus were excluded from analysis of the impossibility of creating a coherent database for these two countries. Malta and Luxembourg were removed because some legal and economic processes distinguished them from other countries: a large number of foreign companies seeking registration in Malta to take advantage of tax preferences (Malta) and the massive foreign trade (Luxembourg). Because of numerous outliers that the model variants proved unable to remove, at the second stage of research, also Bulgaria, Romania and Latvia were excluded from the sample. Ultimately, it included $K = 21$ countries and the time dimension of the study was $T = 17$.

VAT gap estimates were sourced from the Study to Quantify and Analyse the VAT Gap in the 28 EU Member States (European Commission 2015, 2016a, 2017, 2018a, 2019) periodically published by the EC and earlier European Commission publications (2013, 2014). The estimates are calculated using a top-down approach whose only limitation is the availability of appropriately disaggregated data series. VAT total tax liabilities ($VTTL$) were determined using the formula:

$$VTTL_t = \sum_{i=1}^{I} r_{it} C_{it} + \sum_{i=1}^{I} r_{it} Q_{it} \cdot \omega_{it} + \sum_{i=1}^{I} r_{it} J_{it} \cdot \omega_{it}, \tag{9}$$

where:
C_i – final consumption of goods and services originating from the i-th sector,
Q_i – intermediate consumption in the i-th sector,
J_i – gross fixed capital formation in the i-th sector,
r_i – the effective VAT rate in the i-th sector,
ω_i – the i-th sector's share of VAT-taxable consumption and/or production,
$i = 1, \ldots, I$.
The other time series used in the study were obtained from the Eurostat, European Commission (Taxation and Custom Union), and World Bank databases (data sources and the definitions of the variables are provided in the Appendix).
Before the estimation of the parameters of models (3)-(5) and (7) and their variants containing the rate of unemployment, U, the share of the shadow economy, $SHDEC$, the base VAT rate, RV, the number of VAT rates, NRV, and the taxation of final consumption, C_TAX, the variables were tested for order of integration using standard first-generation tests assuming a lack of cross-sectional correlation, LLC and IPS (Levin et al., 2002, Im et al. 2003), and the second generation PANIC tests allowing for cross-sectional dependence (Bai and Ng 2004). The results of the tests provided strong evidence that variables such as revenues (vr and $vttl$), GDP per capita ($gdpc$), the share of intra-community trade ($INTR$), governance effectiveness (G_EFF), and the share of the shadow economy ($SHDEC$) were integrated of order one. To avoid the risk of constructing spurious regressions and inferring from false premises requires performing variable cointegration tests and constructing models using the cointegration procedures.
Because of the limited length of time series and cross-sectional dimensions, standard residual-based cointegration tests assuming the presence of only one equilibrium relationship between variables in particular models were performed (Kao 1999, Pedroni 1999, 2001, 2004). The results of the tests were inconclusive. The Kao test showed that variables in all models were cointegrated, whereas the Pedroni tests utilising autoregressive coefficients PP_ρ and PG_ρ did not allow rejecting the null hypothesis that cointegration did not occur in any of the models. The results of other Pedroni cointegration tests (the variance ratio test PP_v, the tests based on the t-ratio of the autoregressive coefficient PP_t and PG_t, and the ADF-type tests PP_{DF} and PG_{DF}) are provided in Table 1. The final decision on which models should be subjected to cointegration analysis was made based on the conclusions presented by Wagner and Hlouskova (2010). The results of their large-scale simulation experiments pointed out that the smallest unit root tests' size distortions occur in the ADF-type tests PP_{DF} and PG_{DF} and that this property of the tests is insensitive to the presence of cross-sectional correlation, cross-cointegration, and more than one cointegration vector in the model. The findings are obvious. Therefore, models with using Gini's index ($GINI$) and the regulatory quality index (G_QLT) as regressors (lines 6 and 10 in Table 1) were removed from the analysis. Secondly, the presence of only 5 cases of probability values slightly exceeding a significance level of 0.10 in the other models

was deemed a sufficient argument for the models to be subjected to cointegration analysis.

Table 1: Pedroni cointegration tests

	$vr = \boldsymbol{\theta}^{'}\mathbf{x} + \ldots$	PP_v	PP_t	PP_{DF}	PG_t	PG_{DF}
1	$\mathbf{x}^{'} = [vttl]$	0,026	0,002	0,014	0,000	0,001
2	$\mathbf{x}^{'} = [vttl,\ gdpc]$	0,020	0,002	0,000	0,000	0,000
3	$\mathbf{x}^{'} = [vttl,\ gdpc,\ INTR]$	0,037	0,000	0,000	0,000	0,000
4	$\mathbf{x}^{'} = [vttl,\ gdpc,\ OPNS]$	0,207*	0,005	0,000	0,000	0,000
5	$\mathbf{x}^{'} = [vttl,\ gdpc,\ U]$	0,059	0,000	0,200*	0,000	0,047
6	$\mathbf{x}^{'} = [vttl,\ gdpc,\ GINI]$	0,178*	0,008	0,344*	0,000	0,241*
7	$\mathbf{x}^{'} = [vttl,\ gdpc,\ SHDEC]$	0,077	0,012	0,109*	0,000	0,045
8	$\mathbf{x}^{'} = [vttl,\ gdpc,\ CPI]$	0,012	0,006	0,167*	0,000	0,038
9	$\mathbf{x}^{'} = [vttl,\ gdpc,\ G\_EFF]$	0,152*	0,000	0,049	0,000	0,058
10	$\mathbf{x}^{'} = [vttl,\ gdpc,\ G\_QLT]$	0,190*	0,006	0,258*	0,000	0,103*
11	$\mathbf{x}^{'} = [vttl,\ gdpc,\ G\_LAW]$	0,103*	0,000	0,005	0,000	0,035
12	$\mathbf{x}^{'} = [vttl,\ gdpc,\ C\_TAX]$	0,034	0,000	0,004	0,000	0,013

Note: P-values in the table concern tests with the null hypothesis assuming a lack of cointegration; the asterisks mark the results of tests that provide no grounds to reject the null hypothesis. PP and PG denote tests where the alternative hypothesis is homo- and heterogeneous, respectively (for details, see Pedroni 1999, 2001, 2004).

The parameters of the selected models were estimated using three methods: the fully-modified ordinary least squares method (FMOLS, Phillips and Hansen 1990, Phillips and Moon 1999, Pedroni 1995, 2001), the dynamic ordinary least squares method (DOLS, Saikkonen 1991, 1992, Stock and Watson 1993, Kao and Chiang 2000, Mark and Sul 2003), and the pooled mean group-autoregressive distributed lag model (PMG-ARDL Pesaran et al. 1999; for overview see: Kębłowski 2009).
The construction of the pooled FMOLS estimator starts with an extension of the Engle and Granger approach:

$$\widetilde{y}_{it} = \alpha_i + \theta_i^{'}\widetilde{\mathbf{x}}_{it} + u_{it}, \tag{10}$$

$$\Delta\widetilde{\mathbf{x}}_{it} = \boldsymbol{\varepsilon}_{it}. \tag{11}$$

If the disturbances are stationary, $\left[u_{it},\ \varepsilon_{it}^{'}\right] \sim I(0)$, then there is the long-run covariance matrix:

$$\boldsymbol{\Omega}_i = \sum_{s=-\infty}^{\infty}\left(\left[u_{it},\ \boldsymbol{\varepsilon}_{it}^{'}\right]\left[u_{i,t+s},\ \boldsymbol{\varepsilon}_{i,t+s}^{'}\right]^{'}\right) = \begin{bmatrix} \omega_{ui} & \boldsymbol{\Omega}_{u\varepsilon i} \\ \boldsymbol{\Omega}_{u\varepsilon i}^{'} & \boldsymbol{\Omega}_{\varepsilon i} \end{bmatrix}, \tag{12}$$

whose element $\boldsymbol{\Omega}_{\varepsilon i}$ is of full rank. This means that regressors $\widetilde{\mathbf{x}}_{it}$ are not cointegrated and that it is possible to make a correction for endogeneity by replacing the observations of dependent variable y_{it} by its transformations:

$$y_{it}^{+} = y_{it} - \widehat{\boldsymbol{\Omega}}_{u\varepsilon i}\widehat{\boldsymbol{\Omega}}_{\varepsilon i}^{-1}\Delta\mathbf{x}_{it}, \tag{13}$$

where y_{it} denotes de-meaned $\widetilde{y}_{it}$ and the circumflexes denote estimates yielded by the OLS method. Correcting for serial correlation of the disturbances requires calculating the one-sided long run variance matrix:

$$\boldsymbol{\Lambda}_i = \sum_{s=0}^{\infty}\left(\left[u_{it},\ \boldsymbol{\varepsilon}_{it}^{'}\right]\left[u_{i,t-s},\ \boldsymbol{\varepsilon}_{i,t-s}^{'}\right]^{'}\right) = \begin{bmatrix}\Lambda_{ui} & \boldsymbol{\Lambda}_{u\varepsilon i}\\ \boldsymbol{\Lambda}_{u\varepsilon i}^{'} & \boldsymbol{\Lambda}_{\varepsilon i}\end{bmatrix}, \tag{14}$$

and then:

$$\widehat{\boldsymbol{\Lambda}}_{u\varepsilon i}^{+} = \widehat{\boldsymbol{\Lambda}}_{u\varepsilon i} - \widehat{\boldsymbol{\Omega}}_{u\varepsilon i}\widehat{\boldsymbol{\Omega}}_{\varepsilon i}^{-1}\widehat{\boldsymbol{\Lambda}}_{\varepsilon i}. \tag{15}$$

Assuming the long-run homogeneity of the equilibrium parameters $\boldsymbol{\theta}_i = \boldsymbol{\theta}$, the pooled FMOLS estimator is given by the formula:

$$\widehat{\boldsymbol{\theta}}_{PFMOLS} = \left(\sum_{i=1}^{I}\sum_{t=1}^{T}\mathbf{x}_{it}\mathbf{x}_{it}^{'}\right)^{-1}\left(\sum_{i=1}^{I}\sum_{t=1}^{T}\left(\mathbf{x}_{it}y_{it}^{+} - T(\widehat{\boldsymbol{\Lambda}}_{u\varepsilon i}^{+})\right)\right), \tag{16}$$

where $\mathbf{x}_{it}$ is de-meaned $\widetilde{\mathbf{x}}_{it}$.
Deriving the pooled DOLS estimator involves allowing for Saikonnen's (1991) decomposition of the u_{it} term, according to which, for the absolutely summable coefficients γ_{kis}, where $k = 1, \ldots, K$ represents the k-th regressor $\widetilde{x}_{kit}$ in $\widetilde{\mathbf{x}}_{it}$, there is:

$$u_{\mathrm{it}} = \sum_{s=-\infty}^{\infty}\boldsymbol{\gamma}_{is}^{'}\boldsymbol{\varepsilon}_{i,t+s} + v_{it}. \tag{17}$$

The error term v_{it} has a zero mean and is stationary and orthogonal with respect to disturbances $\boldsymbol{\varepsilon}_{i,t+s}$ for all lags and leads. Given (11), simple transformations allow the following equation to be written:

$$\widetilde{y}_{it} = \alpha_i + \boldsymbol{\theta}_i^{'}\widetilde{\mathbf{x}}_{it} + \sum_{s=-S}^{S}\boldsymbol{\gamma}_{is}^{'}\Delta\widetilde{\mathbf{x}}_{i,t+s} + \dot{v}_{it}, \tag{18}$$

where $\dot{v}_{\mathrm{it}} = v_{\mathrm{it}} + \sum_{|s|>S}\boldsymbol{\gamma}_{\mathrm{is}}^{'}\boldsymbol{\varepsilon}_{i,t+s}$.
For homogeneity $\boldsymbol{\theta}_i = \boldsymbol{\theta}$, the pooled DOLS estimator takes the following form:

$$\left[\widehat{\boldsymbol{\beta}}^{'}, \widehat{\boldsymbol{\gamma}}_1^{'}, \ldots, \widehat{\boldsymbol{\gamma}}_I^{'}\right]_{PDOLS}^{'} = \left(\sum_{i=1}^{I}\sum_{t=1}^{T}\mathbf{z}_{it}\mathbf{z}_{it}^{'}\right)^{-1}\left(\sum_{i=1}^{I}\sum_{t=1}^{T}(\mathbf{z}_{it}\widetilde{y}_{it})\right) \tag{19}$$

where $\mathbf{z}_{it} = [\widetilde{\mathbf{x}}_{it}^{'}, \mathbf{0}', \ldots, \mathbf{0}', \mathbf{w}_{it}^{'}, \mathbf{0}', \ldots, \mathbf{0}']$, $\mathbf{w}_{it}^{'} = [\Delta\widetilde{\mathbf{x}}_{i,t-S}^{'}, \ldots, \Delta\widetilde{\mathbf{x}}_{it}^{'}, \ldots, \Delta\widetilde{\mathbf{x}}_{i,t+S}^{'}]$, $\mathbf{w}_{jt}^{'} = \mathbf{0}'$ for $j = 1, \ldots, I$ and $j \neq i$.

The grouped counterparts of estimators (16) and (19), i.e., G-FMOLS and G-DOLS, can be found in Pedroni (2000) and Pedroni (2001), respectively.

Because the asymptotic distributions of estimators FMOLS and DOLS are identical, which one is chosen is formally irrelevant. Nonetheless, the experiments by Wagner and Hlouskova (2010) mentioned above have clearly shown that it is the DOLS that should be preferred over the FMOLS in empirical research (19) for its obviously superior properties in cases involving independent and dependent data panels, as well as cross-sectional cointegration. Wagner and Hlouskova (2010) attribute the superiority of the DOLS to the 'imprecise' estimates of the correction factors in equations (13) and (15). Last but not least, they point to the definitely inferior properties of the grouped estimators compared with their pooled counterparts, P-FMOLS and P-DOLS, especially for small T.

The third of the estimators used in the study was the PMG proposed by Pesaran et al. (1999). The PMG is the panel extension of the estimator derived in Pesaran and Shin (1998) for $I = 1$. The analysis starts with the standard autoregressive distributed lag model:

$$\widetilde{y}_{it} = \alpha_i + \sum_{s=1}^{S} \lambda_{is}\widetilde{y}_{i,t-s} + \sum_{q=0}^{Q} \boldsymbol{\delta}'_{iq}\widetilde{\mathbf{x}}_{i,t-q} + u_{it}, \tag{20}$$

which was transformed into the error correction model:

$$\Delta\widetilde{y}_{it} = \alpha_i + \widetilde{\lambda}_i\widetilde{y}_{i,t-1} + \boldsymbol{\beta}_i^{'}\widetilde{\mathbf{x}}_{it} + \sum_{s=1}^{S-1} \widetilde{\lambda}_{is}\Delta\widetilde{y}_{i,t-s} + \sum_{q=0}^{Q-1} \widetilde{\boldsymbol{\delta}}_{\mathrm{iq}}^{'}\Delta\widetilde{x}_{i,t-q} + \varepsilon_{it}, \tag{21}$$

where λ_{is} denotes autoregression coefficients and $\boldsymbol{\delta}_{iq}$ represents short-term parameters,

$$\widetilde{\lambda}_i = -(1 - \sum_{s=1}^{S} \lambda_{is}),$$

$$\boldsymbol{\beta}_i = \sum_{q=0}^{Q} \boldsymbol{\delta}_{iq},$$

$$\widetilde{\lambda}_{is} = -\sum_{j=s+1}^{S} \lambda_{ij} \text{ for } j = 1, 2, \ldots, S-1,$$

$$\widetilde{\boldsymbol{\delta}}_{ij}^{'} = -\sum_{l=q+1}^{Q} \delta_{il} \text{ for } l = 1, 2, \ldots, Q-1.$$

The long-run parameters are calculated in the usual way, i.e., $\boldsymbol{\theta}_i = \boldsymbol{\beta}_i\lambda_i$ for $i = 1, 2, \ldots, I$.

According to the assumption adopted by Pesaran et al. (1999), the long-run parameters meet the homogeneity condition $\boldsymbol{\theta}_i = \boldsymbol{\theta}$, and the error correction terms and the short-run parameters vary across sections. To deal with the non-linear relationships between $\boldsymbol{\theta}$ and the short-run parameters in (21), Pesaran et al. (1999) recommend using the ML methods with concentrated likelihood functions. Lastly, the overall error correction term and the short-run parameters are estimated by calculating the arithmetic means of the estimates for panel sections.
To the author's best knowledge, there are no simulation studies comparing the properties of the panel PMG-ARDL with the panel DOLS or the panel PMG-ARDL with the panel FMOLS estimators. Only indirect results are available. The results of the simulations conducted by Pesaran and Shin (1998) to compare the properties of the FMOLS (16) and the PMG-ARDL for $I = 1$ pointed to the superiority of the second estimator in the case of finite samples.

5 Empirical results

The limited time dimension of the panel and a large number of variables potentially influencing the VAT gap prevent using multi-equation models to estimate equilibrium relations and the testing of models with more than one cointegration vector, as well as the construction of dynamic single-equation models with a comprehensive dynamic structure and the use of a full-scale 'from-general-to-specific' modelling strategy. Hence, the solution adopted in empirical research consisted in constructing single-equation 'scenario' models with a maximum of 3 explanatory variables each and trying to answer mutually non-exclusive research questions. It was assumed that the identification of 'scenario' equilibrium relationships does not require deciding which one is more relevant when explanatory variables containing complementary information are used as the regressors. According to the second assumption, the results of the cointegration tests were not the only factor that decided whether or not the estimates of the cointegrating vectors were acceptable. The properties and quality of the tests are overly dependent on the yet unsolved problem of cross-correlation and cross-cointegration of explanatory variables. Therefore, the criterion for deciding the acceptability of the equilibrium parameter estimates was enhanced by including the evaluation of the values and precision of the estimates of the error correction terms (ECT), which show how fast VAT revenues return to their equilibrium trajectory. A relatively small value of the ECT, particularly in the model where VAT total tax liabilities are the only cause of changing VAT revenues, was assumed to symptomise puzzling long-lasting disequilibria that cannot be substantiated when the ratio between the VAT gap and potential VAT revenues is constant under equilibrium circumstances.
The 'scenario' models were estimated using all three methods mentioned above. The number of lags and leads in the DOLS estimator was determined based on the Schwarz criterion; the FMOLS was estimated using the Bartlett kernel and Newey-

West bandwidth; the lag adopted for the ARDL models was limited to two years; its optimal value was also established according to the Schwarz criterion.
The parameter estimates of model (3) linking VAT revenue (vr) to VAT total tax liabilities ($vttl$) and model (4) that additionally considers GDP *per capita* ($gdpc$) are presented in Table 2. The estimates show several regularities.

Table 2: Estimation results of the parameters of the models (3)-(4)

		LT		ST			Diagnostics			
	Estimator	$vttl$	$gdpc$	ECT	$\Delta vttl$	$\Delta gdpc$	$SC/\overline{R2}$	$JB(k)$	$JB(s)$	LR
1	PMG	1,040	–	-0,710	0,298	–	-3,099	6,91	-0.51	0,088
		(44,5)		(13,0)	(2,3)					
2		0,996	0,127	-0,703	0,122	0,676	-3,188	8,06	0,18	0,321
		(28,5)	(2,6)	(13,1)	(1,0)	(3,6)				
3*		1	0,060	-0,789	–	0,698	-3,281	7,43	0,09	–
			(2,0)	(16,8)		(3,8)				
4	DOLS	1,011	–	–	–	–	0,9992	6,24	-0,02	0,770
		(26,8)								
5		0,917	0,161	–	–	–	0,9994	6,21	0,16	0,121
		(17,3)	(2,6)							
6	FMOLS	1,040	–	–	–	–	0,9990	9,64	-0,66	0,270
		(28,6)								
7		0,918	0,180	–	–	–	0,9991	10,66	-0,71	0,142
		(16,4)	(2,8)							

Note: columns LT and ST contain the estimates of the long- and short-term parameters, respectively; the parenthesised values represent t-ratios. Diagnostics includes the Schwarz information criterion and adjusted coefficient of determination $\overline{R2}$; $JB(k)$ i $JB(s)$ stand for kurtosis and skewness in the Jarque-Bera test; the LR column contains p-values in the LR test of the homogeneity restriction.

Firstly, the long-run elasticity estimates in model (3) where VAT revenues vt only depend on VAT total tax liabilities $vttl$ are greater than 1, $\widehat{\theta}_0 > 1$, regardless of the estimation method used. According to the argumentation presented above, this result can be interpreted as resulting from the medium-run influence of cyclical or institutional factors. At the same time, it is possible in all cases to impose a long-run homogeneity restriction $\theta_0 = 1$ with a standard significance level of 0.05 (Table 1, rows 1, 4, and 6). Therefore, from a very formalistic perspective, the VAT gap can be considered a stationary variable, implying that the ratio between the level of VAT revenue losses and potential VAT revenues is constant in the long term. Accepting this result would have two consequences: an immediate 'solution' to the problem of modelling VAT gap changes and the dismissal of the procyclicality of vr with respect to $vttl$ or the procyclicality of $vttl$ with respect the VAT tax base widely observed in shorter time horizons. Summing up, accepting that $E\left(vr - vttl - \widehat{\alpha}\right) = 0$ does not explain why VAT revenues fluctuate.

Secondly, using a dynamic model and estimating its parameters by the PMG-ARDL method is more informative about the causes of VAT revenue changes and allows a richer interpretation of the estimates compared with the FMOLS and DOLS for a static relationship. The estimates in the first row of Table 2 clearly show the inert adjustment of VAT revenues to the equilibrium trajectory determined by changes in VAT total tax liabilities. The estimate of the error correction term indicates that the mechanisms balancing the $vr - vttl - \widehat{\alpha}$ system take 1 year to reduce the deviation of VAT revenues from their equilibrium path to 29%, meaning that 95% of the deviation will be reduced within around 3.5 years. This is a fairly substantial amount given that the modelled relationship is practically an identity.

Thirdly, adding GDP *per capita* to the model specification causes the DOLS and FMOLS to yield fundamentally different estimates. The estimate of the equilibrium parameter on *gdpc* turns out to be significantly different from zero at a 0.01 significance level, and the estimates of the parameters on *vttl* fall to 0.92. Moreover, the imposition of homogeneity restriction $\theta_0 = 1$ is no longer obvious. Different results are obtained with the PMG estimator. They show that the estimate of parameter θ_0 equals 1, the estimate of the equilibrium parameter on *gdpc* is different from zero at a significance level of 0.05, and the estimates of the parameters on $\Delta gdpc$ point out that in the short term, VAT revenues are strongly and positively related to changes in GDP *per capita*.

What needs to be remembered when analysing the parameter estimates of the models where VAT revenues *vr* are related to potential VAT revenues *vttl*, PKB *per capita*, *gdpc*, and countries' shares of intra-EU trade $INTR$ (model (5), see Table 3) is that the models in rows 2-4 are both special cases of the initial model (Table 3, row 1) and the extensions of the variant of the model that only includes *vr*, *vttl* and *gdpc* (Table 2, row 3).

The conclusions are the following. Firstly, the inclusion of variable $INTR$ reduces the estimate of the long-term parameter on VAT total tax liabilities *vttl* to a hardly interpretable value of 0.958, which suggests that there exists a mechanism that permanently increases the relative size of the VAT gap. The estimates of the other parameters are similar to those obtained from model (4); the long-term parameter on $INTR$ significantly differs from zero, but its short-term counterpart indicates that the effect of $INTR$ on VAT revenues is not significant.

Secondly, the imposition of the homogeneity restriction that is non-obvious given the LR test results leads to a model with a non-significant effect of $INTR$ on VAT revenues (Table 3, row 2), whereas an arbitrary imposition of a zero restriction on the long-term parameter on $INTR$ creates a variant of the model where elasticity θ_0 only slightly exceeds 1 and the imposition of the homogeneity restriction is fully acceptable (Table 3, row 3).

The simultaneous introduction of both these changes into the specification produces a satisfying result: VAT revenues, vr, are determined by potential VAT revenues, *vt,* and the share of intra-EU trade, $INTR$, in the long term, and by fluctuations

Table 3: Estimation results of the parameters of the model (5)

		LT			ST				Diagnostics			
	Estimator	$vttl$	$gdpc$	$INTR$	ECT	$\Delta vttl$	$\Delta gdpc$	$\Delta INTR$	$SC/\overline{R2}$	$JB(k)$	$JB(s)$	LR
1	PMG	0,958	0,125	-0,017	-0,712	0,073	0,746	-0,080	-2,977	6,27	0,07	0,176
		(30,6)	(2,7)	(4,3)	(10,7)	(0,7)	(3,7)	(1,2)				
2		1	0,044	-0,014	-0,802	–	0,706	-0,065	-3,036	5,10	-0,08	–
		–	(1,6)	(3,9)	(13,7)		(3,5)	(0,8)				
3		1,016	–	-0,015	-0,711	0,052	0,802	-0,091	-2,976	6,04	0,09	0,415
		(51,9)		(4,4)	(10,7)	(0,5)	(4,0)	(1,3)				
4*		1	–	-0,015	-0,786	–	0,731	-0,072	-3,047	4.96	-0,03	–
		–		(4,2)	(13,3)		(3,6)	(0,9)				
5	DOLS	0,931	0,144	-0,004	–	–	–	–	0,9993	5,67	0,26	0,227
		(16,5)	(2,1)	(0,1)								
6		1		0,005	–	–	–	–	0,8898	5,76	-0,02	–
				(0,5)								
7	FMOLS	0,921	0,179	-0,002	–	–	–	–	0,9991	10,67	-0,75	0,164
		(16,2)	(2,6)	(0,1)								
8		1		-0,001	–	–	–	–	0,8886	5,92	-0,23	–
				(0,1)								

Note: see Table 2.

in GDP per capita, *gdpc*, in the short-term, (Table 3, row 4). In all variants of the model estimated by the PMG-ARDL method, the estimates of the error correction term range between 0.7 and 0.8 and are similar to those obtained with models (3) and (4).

Thirdly, the use of estimators FMOLS and DOLS does not allow the dependency of vr on $INTR$ to be confirmed. Additionally, the imposition of restriction $\theta_0 = 1$ in both variants of the model is not obvious.

To answer the fourth research question, Gini's coefficient ($GINI$) and the rate of unemployment (U) which also represented changes in the economic cycle, were included in the model of VAT revenues. Confirming the possibility of Gini's index influencing VAT revenues, vr, proved infeasible because of the aforementioned impossibility of demonstrating the existence of an appropriate cointegrating relationship (Table 1, row 6). Model (4) containing the rate of unemployment was analysed in the same way as model (5). The results of the analysis are summarised in Table 4 and lead to similar conclusions. They confirm the probability of U influencing vr, but the relationship is observed in the long term only. According to the ECT estimate, 95% of disequilibrium is reduced over a period of 1.8 years. The value and accuracy of the estimate of the parameter on $\Delta gdpc$ are also smaller, which is

Table 4: Estimation results of the parameters of the model (4) extended by unemployment rate (U)

		LT			ST				Diagnostics			
	Estimator	$vttl$	$gdpc$	U	ECT	$\Delta vttl$	$\Delta gdpc$	ΔU	$SC/\overline{R2}$	$JB(k)$	$JB(s)$	LR
1	PMG	0,999	0,103	0,001	-0,716	0,100	0,379	-0,004	-2,969	6,39	-0,14	0,975
		(30,1)	(1,9)	(0,4)	(12,3)	(0,7)	(2,4)	(1,5)				
2		1	-0,008	-0,003	-0,803	–	0,537	0,000	-3,022	6,97	-0.15	–
		–	(0,2)	(2,9)	(17,8)		(3,4)	(0,0)				
3		1,030	–	-0,001	-0,727	0,080	0,408	-0,004	-2,977	6,38	-0,10	0.225
		(41,2)		(0,7)	(13,1)	(0,6)	(2,6)	(1,5)				
4*		1	–	-0,002	-0,804	–	0,536	0,000	-3,039	6,96	-0,16	–
		–		(3,3)	(17,8)		(3,4)	(0,0)				
5	DOLS	0,946	–	-0,004	–	–	–	–	0,9995	5,86	-0,20	0,122
		(27,1)		(4,3)								
6		1	–	-0,003	–	–	–	–	0,8993	6,25	-0,42	–
				(3,9)								
7	FMOLS	0,942	–	-0,003	–	–	–	–	0,9994	6,74	-0,36	0,111
		(25,9)		(3,1)								
8		1	–	-0,002	–	–	–	–	0,8936	6,53	-0,20	–
				(2,8)								

Note: see Table 2.

Table 5: Estimation results of the parameters of the model (8) with Corruption Perception Index (CPI)

		LT			ST				Diagnostics			
	Estimator	$vttl$	$gdpc$	CPI	ECT	$\Delta vttl$	$\Delta gdpc$	ΔCPI	$SC/\overline{R2}$	$JB(k)$	$JB(s)$	LR
1	PMG	0,963	0,142	0,001	-0,686	0,163	0,616	0,000	-2,925	4,79	0,15	0,294
		(27,6)	(2,8)	(1,4)	(12,0)	(1,3)	(3,7)	(0,2)				
2		1,043	–	0,002	-0,672	0,137	0,669	0,000	-2,922	4,78	0,18	0,060
		(45,5)		(3,1)	(11,6)	(1,1)	(4,0)	(0,0)				
3	DOLS	1,024	–	0,001	–	–	–	–	0,9994	4,88	0,28	0,475
		(30,5)		(1,1)								
4	FMOLS	1,013	–	0,000	–	–	–	–	0,9993	6,11	-0,27	0,700
		(30,8)		(0,0)								

Note: see Table 2.

unsurprising because U and *gdpc* fluctuation are partly driven by the same economic processes.
The testing of the hypotheses about a relationship between the quality of institutions and a smaller VAT gap yielded inconclusive results. The attempt to link VAT revenue fluctuations to the World Bank indexes (G_EFF, $\_EFF$, G_LAW) proved unsuccessful. The estimates of the parameters on these three variables were empirically indistinguishable from zero. Different results were only obtained for models with the CPI; it must be noted, however, that adding the index to the list of explanatory variables is empirically justifiable only when the elasticity of revenues (vr) with respect to potential VAT revenues ($vttl$) is markedly greater than one. In line with the reasoning presented in Section 3, the result may point to the model failing to sufficiently account for the procyclicality of VAT revenues.
Summing up, the estimates presented in Tables 2 through 5, their preliminary character must be emphasised. Because of the small-time dimension T, the purpose of constructing models (2) through (7) and their variants containing other explanatory variables was to find as many variables whose effect on VAT revenues is important for economic policy as possible, rather than the 'final' structure of the model and a full list of variables explaining VAT revenues. Accordingly, the last stage of the research involved estimating the parameters of models combining the proven mechanisms from the accepted partial models (marked with asterisks in Tables 2-5). The alternative specifications were created by combining explanatory variables *gdpc*, $INTR$, U, and CPI. Because of the precision of parameter estimates and coherent economic interpretation, the following models were recognised as optimal.

Model $INTR + U$
equilibrium:

$$vr_t = vttl_t - \underset{(3,6)}{0.014} \cdot INTR_t - \underset{(2,0)}{0.0016} \cdot U_t, \tag{22}$$

short-term:

$$\Delta vr_t = -\underset{(14.9)}{0.807} \cdot \mathrm{coi}_{t-1} + \Delta vttl_t + \underset{(3.9)}{0.556} \cdot \Delta gdpc_t. \tag{23}$$

Model $INTR + CPI$
equilibrium:

$$vr_t = vttl_t - \underset{(3,5)}{0,015} INTR_t, \tag{24}$$

short-term:

$$\Delta vr_t = -\underset{(12.5)}{0.795} coi_{t-1} + \Delta vttl_t + \underset{(3.1)}{0.706} \Delta gdpc_t + \underset{(1.8)}{0.0020} \Delta CPI_{t-1}. \tag{25}$$

Models (22) and (23) represent an attempt at combining conclusions derived from the partial models presented in Tables 3-4. The estimates of the equilibrium relationship confirm that a country's VAT revenues are influenced by its share of intra-EU trade; a negative estimate of the parameter on $INTR$ clearly indicates that tax carousels are more important for them compared with the tax base growth. At the same time, the procyclicality of VAT revenues is confirmed by the negative and significantly precise (p-value below 0.05) estimate of the long-term parameter by unemployment rate U. Indirect arguments in support of these observations are provided by a comparison of results obtained with model (22)-(23) and the partial models (Tables 3-4). Firstly, the estimate of the long-term parameter on $INTR$ in equation (22) is only slightly smaller compared with model (5), which seems to point to 'orthogonality' of the information contained in variables $INTR$ and U. Secondly, after including the rate of unemployment in the cointegration relations, the value of the error correction term increases from -0.711 to -0.807. This substantiates the need for VAT models to contain medium-term mechanisms contributing to the procyclicality of VAT revenues. Thirdly, the models of VAT revenues should consider the potential impact of the short-term mechanisms. In the case of static models estimated using the FMOLS and DOLS methods, the imposition of the homogeneity restriction $\theta_0 = 1$ proves problematic (Tables 3-4, rows 5 and 7). In system (22)-(23), however, where the estimate of parameter θ_0 is 0.991, p-value of 0.62 in the LR test justifies the imposition of homogeneity restriction $\theta_0 = 1$. The finding that GDP *per capita*, too, influences VAT revenues in the short-term can be interpreted as showing that effect of household incomes on VAT collectability. It is also noteworthy that including the rate of unemployment markedly lowers the estimate of the parameter on $\Delta gdpc$ (from 0.7-0.8 to 0.556 in model (5), see Table 3). This result provides a strong basis to conclude that a model with short-term fluctuations in GDP *per capita* accounts for income effects and the procyclicality of VAT revenues at the same time.

Model (24)-(25) is a case of a model with two explanatory variables that should operate in opposite directions: the effect of mechanisms causing leaks in the VAT collection system represented by $INTR$ should be offset by improvements to the legislative framework (*via* CPI). The estimation results do not provide convincing empirical evidence that such a relationship does exist. Using $INTR$ as an explanatory variable in the cointegrating relationship makes it impossible to find a stable dependence of vr on CPI. The mechanisms represented by the corruption perception index seem to influence VAT revenues in short term only. Moreover, the estimate of the parameter on ΔCPI in equation (25) is so imprecise that the thesis about legal changes having an effect on VAT collection effectiveness gives rise to justifiable doubts.

6 Conclusions

VAT gap studies have three easily identifiable characteristics. Firstly, they are based on panel data. Secondly, they usually consider large sets of regressors, which are frequently introduced into models on an *ad-hoc* basis to test research hypotheses. Thirdly, they dismiss the potential absence of the cointegration of variables. The presence of these three characteristics causes that the conclusions presented by the studies may appear unsatisfactory due to their provisional character. In particular, there is always uncertainty over whether conclusions would not have been different if still *other* regressors were considered; the impacts resulting from the dismissal of the cointegration of variables are not clear, either, likewise the usefulness of the conclusions for the economic policy of any of the panel countries.
The above criticism was one of the reasons for this study of VAT revenues and the VAT gap. As the small time dimension of the data panel used prevented the application of the traditional general-to-specific modelling strategy, a modified modelling procedure was employed. Its first step involved the formulation of working hypotheses assuming that the level of VAT revenues depended on a number of precisely defined socio-economic processes, each being possibly independent of its 'competitors'. With such a defined assumption about the 'near-orthogonality' of 'competitive' regressors, it was rational to consider partial models for examining VAT revenues with respect to (i) household incomes, (ii) the economic cycle, (iii) economies' growing openness to foreign trade, (iv) social inequalities, and (v) efficiency of governance and the legal system. Moreover, mechanisms implied by the Laffer curve were analysed. The results of the cointegration tests or the estimates of the cointegrating vectors showed that in most models, the effect of the majority of the regressors on VAT revenues was either none or not significant. However, the data analysis clearly confirmed (i) a shrinking VAT gap following the growth of household *per capita* incomes, (ii) the growth of the VAT gap in countries with an increasing share of intra-EU trade, and (iii) procyclicality of the VAT gap; the symptoms of a shrinking VAT gap were also confirmed for falling corruption levels. The prerequisite to accepting the partial models was the acceptance of the hypothesis about the cointegration of their variables and the hypothesis about the long-term homogeneity between VAT revenues and VAT total tax liabilities.
The second phase of modelling involved the construction of alternative variants of combined models. The optimal model summing up the results of the research confirmed that VAT revenues were directly and proportionally dependent in the long-term on VAT total tax liabilities and that in the medium-term, the size of the VAT gap was related to the phase of the business cycle and, more importantly, to the country's share of intra-EU trade. The negative and precise estimate of the parameter on the latter variable strongly supports the thesis that tax carousels reduce the tax revenues of a state.
Naturally, the above conclusions cannot be directly employed to create a complete

picture of the mechanisms determining the VAT gap in Poland. However, they provide a solid base for making several comments for use in future research. Firstly, the results presented in this paper clearly imply a need for a detailed analysis of the consequences of the changes to the VAT collection system introduced in Poland in late 2015 because a country's participation in intra-EU trade is the only variable whose presence in the equations describing medium-term VAT gap changes in the EU countries does not raise any doubts. The proposed direction of research appears almost obvious if we also assume that the variable's represent changes in the impact of tax carousels. One research approach that can be used involves the construction of smooth transition autoregressive models with endogenously identified changes in VAT collection regimes, in which case the thesis about the remedial effect of the improvements made to the Polish tax system beginning in 2015 becomes a fully falsifiable research hypothesis. Secondly, assuming that the only cause of changes in the VAT gap is its procyclicality is utterly wrong. While the results presented in this article confirm the possibility of obtaining expected estimates of equilibrium parameters on relevant regressors (unemployment rate and growth rate of GDP *per capita*), the low precision of the estimates or their exclusively short-term character indicates that the list of regressors should not be limited to business cycle approximations.

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Appendix

VR – VAT revenues
Data sources: European Commission (2013, 2015, 2016, 2017, 2018a)
Due to different estimation periods in European Commission reports and the presence of backward revisions in relation to previous reports, the data was obtained using the chain linking method.

$VTTL$ – VAT total tax liabilities
The relationship between $VTTL$ and VR was analyzed in Agha and Haughton (1996), Aizenmann and Jinjarak (2005), Christie and Holzner (2004), Reckon LLP (2009), Zidkova (2014), European Commission (2013, 2018a), Szczypińska (2018).
Data sources: European Commission (2013, 2015, 2016, 2017, 2018a)
Due to different estimation periods in European Commission reports and the presence of backward revisions in relation to previous reports, the data was obtained using the chain linking method.

$GDPC$ – gross domestic product *per capita*
The variable was considered in Aizenmann and Jinjarak (2005), Christie and Holzner (2004), Reckon LLP (2009), Zidkova (2014), European Commission (2013, 2018a).
Data sources: Eurostat, Main GDP aggregates per capita [nama_10_pc] (constant proces 2010)

$INTR$ – participation in intra-EU trade

$$INTR_k = (E_k + M_k)/(E_{EU} + M_{EU}), \quad k = 1, \ldots, K;$$

E_k, E_{EU} – intra-EU exports of the country k, total intra-EU exports EU,
M_k, M_{EU} – intra-EU imports of the country k, total intra-EU imports EU.
The variable was considered in Aizenmann and Jinjarak (2005), Zidkova (2014), Szczypińska (2018).
Data sources: Eurostat, Intra and Extra-EU trade by Member State and by product group [ext_lt_intratrd]; Eurostat, Main GDP aggregates per capita [nama_10_pc]

$OPNS$ – openness to foreign trade

$$INTR_k = (E_k + M_k)/GDP_k, \quad k = 1, \ldots, K;$$

GDP_k – GDP in country k.
The variable was considered in Aizenmann and Jinjarak (2005), Zidkova (2014), Szczypińska (2018).
Data sources: Eurostat, Intra and Extra-EU trade by Member State and by product group [ext_lt_intratrd]; Eurostat, Main GDP aggregates per capita [nama_10_pc]

U – unemployment rate
The variable was considered in Reckon LLP (2009), European Commission (2018a).
Data sources: Eurostat, Unemployment by sex and age – annual average [une_rt_a]

$GINI$ – Gini index
The variable was considered in Aizenmann and Jinjarak (2005), Christie and Holzner (2004), Reckon LLP (2009), Zidkova (2014), Szczypińska (2018).
Data sources: Eurostat, Gini coefficient of equivalised disposable income – EU-SILC survey [ilc_di12]

$SHDEC$ – size of shadow economy
The variable was considered in Zidkova (2014), Szczypińska (2018)
Data sources: IMF Working Paper, January 2018, Shadow Economies Around the World: What Did We Learn Over the Last 20 Years?

CPI – corruption perception index
The variable was considered in Christie and Holzner (2004), Reckon LLP (2009), Zidkova (2014), Szczypińska (2018).
Data sources: `www.transparency.org`

G_EFF – government effectiveness
The variable or it's counterparts were considered in Christie and Holzner (2004), Reckon LLP (2009), European Commission (2018a).
Data sources: World Governance Indicators, World Bank

G_QLT – regulatory quality
The variable or it's counterparts were considered in Christie and Holzner (2004), Reckon LLP (2009), European Commission (2018a).
Data sources: World Governance Indicators, World Bank

G_LAW – rule of law
The variable or it's counterparts were considered in Christie and Holzner (2004), Reckon LLP (2009), European Commission (2018a).
Data sources: World Governance Indicators, World Bank

C_TAX – taxation of final consumption
The variable was considered in Zidkova (2014).
Data sources: Taxation trends in the European Union 2014, Taxation and Customs Union; Taxation trends in the European Union 2016, Taxation and Customs Union

RV – standard/base VAT rate (VAT rate applied to most goods and services)
The variable was considered in Agha and Haughton (1996), Matthews (2003), Christie and Holzner (2004), Reckon LLP (2009), Zidkova (2014), Szczypińska (2018).
Data sources: Taxation trends in the European Union 2014, Taxation and Customs Union; Taxation trends in the European Union 2016, Taxation and Customs Union

NRV – number of VAT rates
The variable was considered in Agha and Haughton (1996), Christie and Holzner (2004), Zidkova (2014), Szczypińska (2018), European Commission (2018a).
Data sources: Taxation trends in the European Union 2014, Taxation and Customs Union; Taxation trends in the European Union 2016, Taxation and Customs Union