Milada HIRSCHOVÁ
Naďa VONDROVÁ
Charles University

DOI: 10.14746/bo.2023.1.4

Where Czech meets math: Implicative-causal relations in mathematical word problems

Keywords: implicative-causal relation; word problem as a text type; conversational implicature; formulaic stereotypy

Abstract

The paper deals with the functioning of *if*-clauses in mathematical word problems and with their equivalents. First, the nature of a word problem as a text type is shown. Further, the difference between complex sentences with proper implicative-causal relation and a lay use of conditional clauses is examined. As its main goal, the paper presents a comparison of various instances of conditional clauses in mathematics word problems. Also, it shows the role of formulaic stereotypy and conventional assumptions in word problem texts as an integral part of both the mathematical and the communicative competence.

1. Introductory remarks

1.1 The importance of linguistic factors in children´s problem solving and in the level of difficulties the pupils experience have been investigated in countless studies, among them Nesher & Teubal 1975; Lewis & Mayer 1987; Hembree 1992; Daroczy, Wolska, Meurers, & Nuerk 2015; Durand-Guerrier 2003; Martin & Bassok 2005; Vicente et al., 2008; Plath & Leiss 2018; Vondrová et al 2019. Word problems (WPs) have been an important component of mathematics teaching; mainly as a demonstration and an evidence of the fact that mathematical skills (coping with mathematical operations) can be a useful instrument for everyday activities and an important aid to assist anyone

in real-life situations (shopping, planning daily schedules, financial literacy etc.). At the same time, WPs represent an interdisciplinary phenomenon – they are texts requiring certain level of both mathematical and linguistic/communicative knowledge (competence). It is the applicability of mathematical notions and operations together with the choice of attractive "stories" of WPs which is mostly stressed and foregrounded (as a supporting/motivating factor) in mathematics teaching. On the other hand, in the last decades researchers started to doubt the role of WPs as a bridge between mathematical and real worlds, pointing to the fact that WPs as traditionally used in classes do not promote the development of pupils' disposition towards authentic mathematical modelling. (Cf. Verschaffel et al. 2010). WPs' long-term low popularity as well as the rather low rates of successful WP solving (at least in Czech elementary schools) keep inviting both the teachers and researchers to study the related issues.

1.2 In their summary of studies, Verschaffel et al. (2010) point to the efforts to reconceptualize WPs as genuine excercises in mathematical modelling or even to replace them by authentic real-world problems. In this study, however, we will focus only on the WPs as "verbal descriptions of problem situations wherein one or more questions are raised the answer to which can be obtained by the application of mathematical operations to numerical data available in the problem statement" (cf. Verschaffel, Greer & De Corte 2000: ix). Wording of a WP is supposed to be brief and unambiguous. Nevertheless, studies in different countries that have looked at word problems in traditional textbooks have revealed that, especially in the early grades of elementary school, most WPs are phrased as semantically impoverished, stereotyped verbal vignettes (cf. Verschaffel et al. 2010). The length of a WP (number of words, number of sentences) and/or the presence of "potentially difficult" vocabulary (terminology, compound words, words of foreign origin) have been an often researched and tested factor proven as influencing the solvers' success, cf. Abedi & Lord 2001; Plath & Leiss 2018; Bergqvist, Theens & Österholm 2018. At the same time, any WP text usually comprises all the necessary pieces of

information, often in the order corresponding to the order of mathematical operations required for a successful solution.

1.3 In general, the most obvious reason of WPs' trickiness is the cognitive load which any WP puts on the pupils. In order to solve the task presented by a WP, it is necessary to process the complex of meanings encoded by natural language: Verbal description of situations or actions relying on the knowledge of conceptual meanings of lexical units and on the intuitive knowledge of relations among them (these two constituents being modified by pupils' extra-linguistic experience) must be mentally processed (comprehended). The solver needs to create his/her own concept of the problem (step 1, text comprehension). In step 2, the solver has to identify which pieces of information within the text represent the elements of the mathematics problem, i.e., a new meaning must be extracted from a text. The irrelevant information must be filtered off and the relevant mathematical information must be elicited. In step 3, the new (mathematical) model is built. In order to create the new model, the relevant items to be operated with must be determined (coded into math concepts/entities). What must be established is the relation between the verbal description of a real-life action or an array of data and an abstract mathematical operation. (Towards the concept of operation in relation to mathematical concepts, cf. Piaget 1967, pp. 33, 119–155.). This new model is processed (solved arithmetically) in step 4. The final step 5 is represented by a verbal answer to the question/task description. (Cf. Reusser's model construction, 1985; 1992; 1995, pp. 81–82, 90–95, 255). The level of cognitive load in various versions / wordings / formulations of WPs was the subject of the research project this paper is related to, see 2.1 below.

1.4 There are two crucial distinctions between mathematical concepts and the concepts of natural language:

a) there is no empirical referential value related to mathematical concepts;
b) there is no set of "elementary concepts'' in natural language. Such a set, though, does exist in mathematics. (Cf. Materna, Pala, & Zlatuška 1989; Materna 2000, p. 90).

Concepts of natural language are objective abstract (ideal) entities which can be represented by natural language expressions; nevertheless, there can be more language expressions corresponding to one concept/notion and vice versa; cf. (Materna 2000, p. 17). A regular (default) comprehension of a language expression is based on a presupposed identification of the represented concept as well as on the identification of the potential referent (an extra-linguistic entity) to which the expression can be related when used in an utterance-token (one individual instance of a sentence use occuring in a particular situational context). As for WPs, it can be supposed that it is the primary orientation of the solvers towards the empirical reference of language expressions (emerging from individual experience of each solver) which causes delays or obstacles in the required transfer from language concepts to mathematical ones. The point is that the potential referential value of a language expression regarding a WP's contextual frame can vary for each solver (despite the constant notional content of the expression). It can be influenced by a solver's individual extra-linguistic experience plus his/her communicative competence. Solvers' experience concerning the relations of extra-linguistic objects/entities named in a WP's text (and pragmatic inferences launched by the wording) can correlate with the required mathematizing (creating of a mathematical model); then they facilitate the correct/successful solution. Nonetheless, in numerous cases the adequate relations of WP constituents need not be obvious at first sight. Before they are expressed in mathematical units, those relations must be found and comprehended. To the problems concerning the WPs semantic interpretation, cf., e.g., Bassok 2001; Martin & Bassok 2005. As for modelling problems, cf. Reusser 1985, 1992, 1995; Cummins et al. 1988.

2. Background of the paper

2.1 The present paper is related to the project "Word problems as a key to the application and understanding of mathematical concepts''

(*Slovní úlohy jako klíč k aplikaci a porozumění matematickým pojmům,* reg. no. 16-06134S, Czech Science Foundation; 2016–2018). The project included a text analysis of WPs both in Czech textbooks and in some international testing materials (such as TIMSS and PISA problems). The analysis was aimed at the identification of key features of the task texts (mathematical/structural, psychological, and linguistic ones). Further, among the most prominent features of WPs those potentially problematic (difficulty influencing, possibly failure causing) were distinguished. Linguistic features chosen for further testing were the following:

a) extra-linguistic (experiential) context (i.e., a text based in a context supposedly familiar vs. unfamiliar to the pupils);
b) the presence of a superfluous information (incl. redundant numerical data; longer vs. shorter wording of texts);
c) the influence of a non-verbal constituent (a picture, a diagram) in the text of a WP;
d) linguistic expliciteness of the WP text (the presence of stereotypical formulaic constructions which can be semantically overloaded or underdetemined; as a result, such texts are potentially ambiguous or they can launch false implications);
e) order of information in the WP text;
f) the presence of a distractor (an expression leading to an operation opposite to the required one), cf., e.g., Nesher & Teubal 1975. (In the treatises dealing with the didactics of mathematics, similar phenomena are sometimes termed "inconsistent language'', cf. Lewis & Mayer 1987; Pape 2003).

2.2 In the next phase of the research, the goal was to determine how the influence of these features changes with the age and experience of pupils, beginning with Grade 3 and ending with Grade 9 of primary schools. The complete description of the project including testing methodology, test samples, questionnaires distributed to pupils and statistic as well as didactic analyses of the elicited data can be found in full in a monograph authored by a group of scholars. (Vondrová et al. 2019).

2.3 In this paper, we do not tackle the structural/mathematical features nor the psychological ones. Nevertheless, it is obvious that among the features listed in 2.1, e.g., the a) feature (the supposed un/familiarity of the context frame) involves both linguistic and psychological properties and, analogically, features e) and f) can be considered an interface of linguistic and structural/mathematical properties. The order of information provided to a solver suggests the required order of operations; the choice of a distractor is a part of the WP formulation strategy. (The way how the problem is communicated to a solver tests the solver's ability to set the correct algorithm.) Feature d) appears to be the most complex one since the phenomena describable as adding to the text expliciteness and/or ambiguity often overlap each other.

2.4 In the paragraphs below we will pursue one of the linguistic aspects pertaining to feature d/, namely, the use of causal conjunctions concerning the task assignments in WP texts. At the same time, we will deal with the issue of the communicative nature of WPs: we will render a WP as a specific type of text/genre of (educational) communication. We proceed from the assumption that the process of a WP solution includes, among other, certain preliminary information concerning WP as a text type, see paragraph 6.2 below. What we wish to present here is the pragmalinguistic perspective of WPs solution process: We analyze a WP text as a type of communication and aim for showing some of the linguistic facts related to this specific area of educational communication.

3. WP as a text type/genre

3.1 As a type of text (genre of communication, cf. Bakhtin 1986; Ferguson 1994; Engel 1996, Hirschová 2017), a WP can be defined by its major function: to set a task for the addressee (the solver) testing his/her ability to find a mathematical structure "covered" in the text. The way in which the pieces of information are presented to the solver is an indispensable component of a WP wording nature. As a text type

(a communication/a message), a WP involves three basic functional parts:

A. The initial part is usually invisible for solvers. It includes the producer´s planning and form-choosing mental activities aimed at assigning the task to an addressee – a solver with a presupposed level of both language and mathematics competence. (In textbooks for young children, the role of the text producer can be personified in a character of a guide/story-teller or a "pupils' companion" – a toy, an animal, etc.) The core of the task/problem is a structure built on a set of data which are expected to be processed by a combination of mathematical operations aiming at a certain result (a solution). What the producer seeks at this stage is an "emballage" (to be shown in part B) for a structure pertaining to a particular topic in the mathematics curriculum. From the viewpoint of a producer, the A part is dynamic while the B) and C) parts of a WP text type can be seen as relatively stable general constituents of a scenario to be re-performed.

B. Here the task/problem itself is presented in a description of some situation, in a form of a short story or in an array of data in a discourse unit related to a common context (sports, travelling, shopping, etc.). At the same time, there is an additional (implicit) information conveyed to a solver, namely, "this is a WP".

C. In this part, the job required of the addressee is specified by a question (*how many? – kolik?/ how long? – jak dlouho?* etc.); sometimes the producer addresses the solver by an imperative (*find out – zjisti / form – utvoř / calculate – vypočítej*). It is typically in the C part where the conditional conjunction (in Czech *jestliže/jestli/pokud/ /když*) occurs suggesting the relation of principial data (in B) to be operated with, e.g., *how long will it take ... if both vehicles move ... – jak dloho potrvá ... jestliže se obě vozidla pohybují ...*; *how many rooms have to be booked if the number of participants is ... – kolik pokojů je třeba rezervovat, jestliže počet účastníků je ...,* etc. (In further text, we use "*if*-clause" as a general label for clauses with the mentioned type of conjunction.)

3.2 From the viewpoint of a solver, the relation of the B and the C parts is crucial: The task can be successfully completed only if the data presented in B) are identified, mathematized and arranged into an adequate model/structure (see 1.3 above). To a great extent, completion of C depends on B, on its linguistic comprehensibility and lucidity. The very formulation of B, e.g., the order of individual pieces of numeric information, can suggest the steps of the solution process.

4. Conditional clauses in conversation

4.1. Primarily, the use of *if/ jestliže* in complex sentences indicates an implicative relation, namely "if *a,* then *b*", cf. [1]:

[1] Jestliže teplota stoupne nad bod mrazu, sníh začne tát.
If the temperature raises above the freezing point, the snow starts to melt.

It means that the raise of the temperature (a protasis/antecedent) is a condition upon meeting of which the snow melts (an apodosis/consequent), i.e., melting of the snow invariably follows the raise of the temperature. At the same time, such a necessary and/or sufficient condition *a* can be considered a cause of *b*; cf. Davidson 1967, p. 701; van Dijk 1973, p. 62. (In this paper, though, we do not mean to explore the area of problems concerning teaching/explaining the topics related to formal (material and/or strict) implication, cf., e.g., McCarthy 1987; Durand-Guerrier 2003; i.e., we do not study students' understanding of logical implication.) There is a difference between the strictly logical (and philosophical) function of *if* (or *iff*) / *jestliže* and a wide range of relations between phenomena *a* and *b* represented by clauses in a complex sentence with a conjunction (not a connector) *if*. In everyday use of conditional conjunctions, sentences like [2] – [5] can be considered customary:

[2] Jestliže budeš jíst hodně zeleniny, zhubneš.
If you eat a lot of vegetables, you will lose weight.

[3] Jestli se bude pořádně učit, určitě zkoušku udělá.
If he studies hard, he will certainly pass the exam.

[4] Když umyješ nádobí, já zaliju zahradu.
If you do the dishes, I will water the garden.

[5] Pokud si můžu vybrat, radši bych zůstala doma.
If you let me make a choice, I would rather stay at home.

It is not in the capacity of this paper to cover all kinds of conditional sentences (open condition, hypothetical condition, negative condition, etc.); any grammar of Czech or English as well as grammars of other languages deal with the topic extensively; cf., e. g., Leech & Svartvik 1975; Karlík 1995; Engel 1996; Karlík 2017. We have chosen examples [2] – [5] because they show a noticable contrast to [1]. In [2] and [3], the *if*-clause (formally *a*) does not state a condition which, when met, can be sufficient for the content of the other clause (formally *b*) to come true: It is not sure that a person eating a lot of vegetables will lose weight; even a hard working student can fail in an exam, etc. There is an assumed connection between the contents of propositions in sentences *a* and *b* in [2] and [3]. The *if*-clauses (*a*) describe activities we expect/believe (based on our experience or observation) to be usual reasons of a state of affairs described in *b* or usually having such an outcome; cf. Toulmin 2003, pp. 92–93. The use of *if*-clauses in [4] and [5] is different from [2] and [3], see 4.1.1 below. In all of them, though, the occurence of *if*-clauses can be seen and grasped as motivated by empiric factors describable in the framework of pragmatic linguistics.

4.1.1 In everyday communication, complex sentences like [2] – [5] most probably are interpreted by interlocutors not primarily on the basis of *a* and *b* propositional truth values, i.e., as logical implications but, besides the shared "usual experience", also on the basis of pragmatic factors, especially the conversational implicature (cf. Grice 1975). (Under normal circumstances, we suppose that in our examples, the speaker/producer follows the cooperative principle, cf. Grice 1975, p. 45). The speaker of both [2] and [3] apparently assumes that the addressee is interested in a positive outcome of *a*, i.e., on the content of *b* clause. But, at the same time, the speaker is reluctant to commit him/herself to the truthfulness/realization/validity of a "strong"

statement (*zhubneš - you will lose weight; udělá zkoušku - he will pass the exam*) which, as a matter of fact, need not come true. So his/her use of an *if*-clause in [2] and [3] is motivated by the speaker's attempt not to violate one of the submaxims of quality ("Do not say that for which you lack adequate evidence"; Grice 1975, p. 46). As for [4] and [5], they back up Grice's claim that the conversational maxims apply not only when observed but also when opted out, flouted or violated (Grice 1975, p. 49–50). In [4], the speaker's use of the *if*-clause flouts Grice's second submaxim of quantity ("Do not make your contribution more informative than is necessary"; Grice 1975, p. 45), and the second and the third submaxims of manner ("Avoid ambiguity; Be brief (avoid unnecessary prolixity)"); Grice 1975, p. 46). In [4], the speaker's obvious goal is to make the addressee to do the dishes, so, in order to support this goal, s/he says more than is necessary (indirectly promises his/her own watering of the garden). At the same time, s/he avoids the unambiguous form of the utterance – the imperative *do the dishes,* because such a direct request might not be favourably accepted. (One of the addressee's possible reactions in such a dialogue might be *Ne, ty umyj nádobí, a já zaliju zahradu - No, you do the dishes and I will water the garden,* i.e., the addressee could reject the speaker's implicit illocutionary point.) In [5], the use of the *if*-clause flouts the maxim of relevance – it is not clear why the addressee should not let the speaker make a choice between staying at home or not (why the speaker mentions it). In both [4] and [5], the functioning of Grice's maxims can be completed by Leech's (1983, pp. 79–103) contribution to the theory of speech behaviour, namely by his politeness principle and its subordinate maxims. In [4], using the *if*-clause, the speaker applies the maxim of generosity (meaning "we will split the domestic chores"), in [5], the speaker of *if*-clause applies the maxim of agreement (meaning "I do not want you to be offended by directly rejecting your suggestion to go out"). The *if*-clause in [5] is a polite hedge the function of which is to moderate the rejection.

4.1.2 The difference between [1] and [2] – [5] can be supported by one more property pertaining to such everyday use of *if*-clauses. As

has been shown (Geis & Zwicky 1971), in connection with sentences like [1], where the implicative-causal relation of *a* and *b* holds, many speakers of a natural language expect that this relation applies also under negation, i.e., *if* non-*a, then* non-*b* (Geis & Zwicky 1971, p. 562):

[1a] Jestliže teplota nestoupne nad bod mrazu, sníh nezačne tát.
If the temperature does not raise above the freezing point, the snow does not start to melt.

This relation may hold in the example above (which is not a negated implication, Geis and Zwicky call it Conditional Perfection; Geis & Zwicky 1971, p. 563) but not in other cases. E. g., *Jestli se Jan bude vyklánět z okna, spadne – If John leans out of that window any further, he will fall* certainly does not imply that if John does not lean out, he will not fall. (John can fall for some other reason.) Geis and Zwicky call the relation of *a* and *b* clauses in such sentences Invited (suggested) Inference (Geis & Zwicky 1971, p. 563). Similar situation can be seen in our examples [2] – [5].

4.1.3 Our brief sketch of [2] – [5] as tokens of speech behaviour in everyday conversations has shown that the *if*-clauses in them obviously cannot be considered straightforward counterparts of those expressing an implicative-causal relation like the *if*-clause in [1]. Their interpretation is based on empirical expectations and on inferences launched by conversational implicatures.

5. The *if*-clauses in WP texts

5.1 Many WPs are, regarding their wording and the nature of data presented in B, analogical to [1]. The *if*-clause in them does represent a sufficient condition for the correct solution since there is an implicative-causal relation between an array of data in B and the answer to the question in C:

[6] Do knihkupectví doručili 6 krabic nových učebnic. Kolik vážily všechny krabice dohromady, jestliže jedna vážila 10 kg?

6 boxes of new textbooks were delivered to a bookstore. How many kilograms did the delivered boxes weigh altogether, if each of them weighed 10 kilograms?

[7] Marie právě začala rekreačně běhat, běhá 11 km týdně. Anna je lepší běžkyně. Kolik kilometrů týdně Anna uběhne, když běhá třikrát víc než Marie?

Marie has just started jogging, she runs 11 kilometres per week. Anna is a better trained jogger. How many kilometres does she run per week if she runs three times more than Marie?

In [6] and [7], the implicative-causal relation is a way of expressing multiplication; it is based on an unambiguous input into that numeric operation: **if** each of the boxes weighs 10 kilograms, **then** 6 of them weigh 10 • 6. **If** one person runs 3 times more km per week than a another person who runs 11 km per week, **then** the person runs 3 • 11. Functioning of *if*-clauses in [6] and [7] can be considered proper and, considering a WP text type, unmarked/basic because they correspond to the required structural model.

5.2 In some other WPs, though, the relations of the assignment data to the required solution seem less clear:

[8] Petr má 400,- Kč. Kolik obědů ve školní jídelně si může zaplatit, jestliže ví, že 1 oběd stojí 37,- Kč?

Peter has 400,- CZK. How many lunches in his school cafeteria can he buy right now, if he knows that one lunch is 37 CZK?

In [8], the implicative-causal relation exists between the total sum of money available and the number of lunches which can be purchased for it (**if** 1 lunch costs 37 CZK, **then** the number of lunches for 400 CZk is 400 : 37; there will be 30 CZK left). Nevertheless, in [8], the literal wording of the task does not present the relation of the two numbers. It is Peter's awareness of the price of a single lunch which is put forward as a condition sufficient for his ability to purchase a certain number of lunches. To inform the solver about being aware of something or knowing something is a piece of information concerning a cognitive attitude ascribed to Peter but it is not a fact (a prerequisite/condition) necessary for determining the number of lunches to be

bought. The number of lunches calculated in [8] would be the same even without the clause *if he knows that*. The information about Peter's knowledge formulated as an antecedent is not in an implicative-causal relation to the number asked for, the consequent. It means that in this case, the WP wording cannot be considered precise. The *if*-clause represents a redundant/superfluous information which has to be filtered off by the solver. More importantly, the wording of [8] does not state that Peter is obliged to spend all the money for his school lunches – following the wording, he can buy 6 or 7 of them and spend the rest of cash some other way. The expected calculation is based on a mere conventional assumption connected to a type of WP (division with remainder). In the next example the calculation is based on an assumption, too:

[9] Marie má 42 jablka. Když je rozdělí do 6 košíků, kolik jablek bude v každém košíku?

Marie has 42 apples. If she divides them into 6 baskets, how many apples will be in each basket?

The assignment in [9] assumes that there will be the same number of apples in each basket – which, of course, is not true since it has not been explicitly stated. Examples like [9] count on pupils' experience with similar WPs, see par. 6.2 below.

Example [10] represents a slightly different case, nevertheless, it relies on a conventional assumption, too:

[10] Marie a Anna sbíraly na prázdninové brigádě jahody. Výdělek byl 10 Kč za 1 kg. Na konci směny dostaly dohromady 1180,- kč. Kolik korun dostala Marie, když Anna nasbírala o 25% víc?

As a one-day summer job, Marie and Anna picked strawberries at a farm. The pay was 10 CZK per kilo. At the end of their shift, they were paid 1180,- CZK altogether. How much did Marie get if Anna picked 25% strawberries more than Marie?

In [10], the task to be solved builds on an conventional assumption that the proportional share of the money received by each of the girls is identical with the picked amount of strawberries, i.e., **if** Anna picked

25% more strawberries, **then** she gets 25% more money than Marie. It is a situation analogous with examples [2] – [3] above. The data description in the assignment does not allow an unambiguous calculation: Anna might have picked 25% more strawberries than Marie but her strawberries might get damaged before turning them in so in fact the amount of her strawberries might have been smaller. The task formulation in (10) relies on conventional expectation, not on an unambiguous assignment formulation (e.g., *Anna odevzdala o 25% víc – Anna turned in 25% strawberries more* than Marie). A comparable text arrangement occurs in [11]:

[11] Zoo chová 6 žiraf. Krmivo pro ně stojí 49 980 Kč týdně. Pokud se zoo rozhodne koupit další dvě žirafy, kolik bude týdně stát krmivo pro všechny?

A zoo keeps 6 giraffes. Their feed costs make 49 980 CZK per week. If the zoo decides to acquire 2 more giraffes, how much will make their weekly feed costs altogether?

The recquired calculation in [11] tacitly relies on assumptions that each of the kept animals consumes the same amount of feed and that the zoo orders the same amount of feed every week.

5.3 On the other hand, authors/producers of some WPs take the importance of the input data nature into account:

[12] Jana chystá hamburgery na odpolední grilování. Má tři balíčky mletého masa, které váží 625 g, 545 g a 824 g. Jestli každý připravený hamburger bude vážit asi 125 g, kolik celých hamburgerů může Jana udělat?

Jana is making hamburgers for a barbecue. She has bought three packages of ground beef with masses of 0.652 kg, 0.545 kg, and 0.824 kg. If each hamburger she makes has a mass of about 0.125 kg, how many whole hamburgers can she make?[1]

Wording of [12] seems to be using the same formulaic stereotype as [10] and [11]. The producer, though, is aware of the fact that in this case, simple division (2.021 kg : 0.125) cannot bring a realistic result,

[1] This WP has been adopted from http://k12resources.nelson.com/math/9780176813659/student/attachments/a_student_text/nm8sb062.pdf [retrieved 22. 7. 2019].

hence the use of *asi – mass of about* and *celých - whole hamburgers*, i.e., an additional (this time important) piece of information telling the solver that the remainder in the division calculation can be neglected. In some other WPs, the formulaic stereotype relying on a conventional assumption is intentionally disclaimed and the producer openly points at the fact that the input data represent a mere assumption:

[13] Dvě nákladní auta vozí materiál na stavbu. Menší uveze 4 tuny, větší 6 tun. Dohromady bylo přivezeno 44 tun materiálu. Větší auto přivezlo o jeden náklad méně než menší. Kolikrát každé auto jelo, když předpokládáme, že pokaždé byla obě plně naložena?

Two lorries bring material to a construction site. The smaller of them has the carrying capacity of 4 tons, the bigger one can carry 6 tons. Altogether they have brought 44 tons of material. The bigger truck brought one load less than the smaller one. How many drives each of them made, if we suppose that every time both of them were fully loaded?[2]

Here, the producer ascribes the mentioned cognitive attitude (supposition) to him/herself, or, more accurately, in the form of an inclusive plural *předpokládáme/we assume* both to him/herself and the solver. (In the WPs, the use of *we* with the meaning of the inclusive plural can be interpreted as a manifestation of shared activity, i.e., a factor supporting the successful communication towards the solver and, in that way, enhancing a chance for a successful solution.) Such an assignment variation does not influence the model of calculation but it observes a requirement of expliciteness/precision of the formulation.

5.4 Examples [8] – [11], unlike [6] and [7], or the use of *if*-clauses in them, represent a kind of a formulaic stereotype occuring in many WPs: Since a WP calculation should be based on an implicative-causal relation and since most of WPs with an *if*-clause reflect such a real implicative-causal relation, every use of an *if*-clause in a WP is expected to be interpreted that way even though it corresponds rather to the lay use of if. The stories in [8] – [11] seem to neglect the extra-

[2] The source of this WP is Kotyra & Sivošová, 1997, p. 14.

linguistic reality, namely the referential potential of the lexical items naming the "data carriers'' (purchase of lunches, strawberries picking, animal feed) in their assignments; they also neglect pupils' possible experience with such real world phenomena. The assumptions those WPs are based on are connected to WPs textual stereotypy.

6. Are the lay *if*-clauses inappropriate?

6.1 We have already seen that in many complex sentences using *if*-clauses, the relations of clauses cannot be considered proper implicative-causal ones. Does it mean that such a lay use of *if*-clauses should be corrected, at least in mathematics teaching? Or, regarding the required successful solution of a WP, can such an imprecise use of of a connection be a problematic (difficulty causing) factor? In 4.1 we have seen that in ordinary conversation, an assumed (expected or observed) connection between two phenomena is generally understood as an equivalent of an implicative-causal relation, i.e., the *if*-clauses in [2] and [3] are not seen as inappropriate. The "pragmatic propriety'' holds with [4] and [5], too, even though the use of *if*-clauses in them is chiefly motivated by the goals of the speaker's speech behaviour (his/her goals in the act of communication) and that functionally, they may reflect various aspects of the speaker-addressee mutual relations, not always rational. In WPs like [8] – [11], the *if*-clauses were apparently motivated not by conversational implicature but by the producer's following the pattern shown in examples [6] and [7] which are structurally/mathematically appropriate. The *if*-clauses in WPs similar to those in [8] – [11[could be considered inappropriate if they were proven as infavourably influencing the rate of successful solutions of WPs with them. In the project we referred to in 2.1, the presence of *if*-clauses not expressing the implicative-causal relation was tested together with other features (mostly with structural ones, but also together with the order of information and the presence of a distractor word). In none of the tests, the improper *if*-clauses did turn out to be a major factor causing incorrect solutions (cf. Vondrová et al. 2019, pp. 171–172).

6.1.1 The WPs with *if*-clauses were tested in grades 3, 4, 6, and 8. In the testing materials, the pupils were presented with pairs of WPs; one of the variations included a complex sentence with an *if*-clause, the other one used two independent sentences. The first was structurally identical with [6] above, its variation is [14]:

[14] Do knihkupectví doručili 6 krabic nových učebnic. Jedna krabice váží 10 kg. Kolik váží všechny krabice dohromady?

6 boxes of new textbooks were delivered to a bookstore. Each of them weighed 10 kilograms. How many kilograms did the delivered boxes weigh altogether?

In general, the testing found out that the influence of in/explicit wording was lower than initially expected.

6.1.2 The implicative-causal relation occuring between parts B and C defines the principal function of a WP, it is always present, even though it is not expressed explicitly or properly. Let us assess example [15] which can be considered a variation of [7] above (both represent the same structural/mathematical model:

[15] Marie běhá 11 km týdně. Je to 1/3 toho, co běhá týdně Anna. Kolik kilometrů běhá Anna za 4 týdny?

Marie runs 11 kilometres a week. It is 1/3 of what Anna runs a week. How many kilometres runs Anna in 4 weeks?

There is no *if*-clause in [15] but the relation of data in the assignment [7] and in [15] (just as in [6] and [14]) above) and the operations to be done can have a single result; they do not allow additional circumstances to be taken into consideration: If 11 (km) is 1/3 of Anna's weekly track, then Anna runs 6 • 3 miles a week. In 4 weeks, Anna runs (6 • 3) • 4. The other difference between [7] and [15] is the presentation order of data and, as a result, their level of difficulty: since [15] requires a hierarchical rearrangement of the input data, it is more difficult. As for the influence of the order of data, it has been shown that the so-called mixed order of data (not corresponding to the sequence of operations needed to solve a WP) makes the problem more difficult. Cf. Hembree 1992; Vicente et al. 2008). But the solutions of WPs

where the implicative-causal relation was not exactly and precisely formulated (similarly to [8] – [11]) were not adversely affected. It is apparent that it is the WP model which suggests the presence of a "regular" relation of data in B and the task in C, even though the formulation of a WP as a whole may be underdetermined (imprecise).

6.2 It was presumed that versions such as [14] or [15], i. e., the versions splitting the complex sentence in two will be easier than the ones with the *if*-clause (the shorter sentences are easier to grasp). It is beyond the scope of this paper to present the methodology of our research in its complexity, the reader is referred to Vondrová at al. 2019. In the following, we provide only a sample.

6.2.1 Each variant was solved by 60 to 120 pupils; it was assigned to equally competent groups (determined with the use of techniques of the Item Response Theory – IRT). In order to identify the differences of the pupils' achievement between the variants, an independent two-sample t-test was used. The results (described in detail Vondrová et al. 2019, pp. 189–208) showed that the assumed influence of the *if*-clause presence is not as straightforward as expected. Its influence on the WP difficulty was detected only when combined with another complicating variable. In the grade 8 test papers, the following pair (Vondrová et al. 2019, p. 404) was used:

[16] Ševčíkovi byli na dovolené v hotelu. Na každých 21 eur za jídlo pro dospělé rodiče zaplatili 30 eur za jídlo pro děti. Když za jídla pro dospělé zaplatili 630 eur, kolik utratili za jídlo celkem?

The Sevcik family took their summer holiday in a hotel. To each 21 EUR for the meals for adults, the parents paid 30 EUR for meals for children. If they spent 630 EUR for the meals for adults, how much did they spend for food altogether?

[16a] Ševčíkovi byli na dovolené v hotelu. Na každých 21 eur za jídlo pro děti rodiče zaplatili 30 eur za jídlo pro dospělé. Za jídlo pro dospělé zaplatili 630 eur. Kolik utratili za jídlo celkem?

The Sevcik family took their summer holiday in a hotel. To each 21 EUR for meals for children, the parents paid 30 EUR for the meals for adults. They spent 630 EUR for the meals for adults. How much did they spend for food altogether?

Both [16] and [16a] rely on an assumed shared experience that the price of the meals for children is usually lower than that for adults. The variation [16] where this assumption (Martin and Bassok, 2005, speak about "semantic cues") was purposefully violated (the food for children was more expensive) turned out to be more difficult even though it includes a proper *if*-clause. Table 1 presents numerical results for the pair [16] – [16a]. The difference in difficulty was statistically significant.

Table 1. Parameters for the Sevcik Family problem (Grade 8) and results of a test: N – number of pupils, a – discrimination, i. e., potential of a problem to differentiate between pupils as for a lower/higher level of competence); b – difficulty; s. e. – standard error). For the detailed analysis, see Vondrová et al. 2019, pp. 41–56.

	N	Success rates	a	s. e. (a)	b	s. e. (b)
if-clause	118	42 %	1.21	0.26	0.38	0.18
no if-clause	120	56 %	1.90	0.37	–0.08	0.13

Both [16] – [16a] can be considered tokens of an implicative-causal relation; the use of an *if*-clause was not "improper". It was the violated assumption concerning the food expenses in [16] and [16a] which apparently claimed solvers' attention first; that moves this WP towards those based on an invited inference (i.e., those with the "improper" use of *if*).

7. Conclusions

7.1 In 3.1 above, we have shown a general text-pattern of a WP. There can be certain variations in the formulation of a B part (such as in the order of data, unfamiliar context, etc.) but the presence of an *if*-clause (or, more exactly, the presence of an implicative-causal relation) in a C part seems to be one of the properties conventionally (in the sense of a communicative convention) establishing a WP. (Another frequent convention, the expectation always to divide a number/amount into identical parts, see [9], [10], and [11], was mentioned in 5.2.) In mathematics teaching, pupils encounter WPs since their early age (1st grade) so they are well acquainted with the WP model and the basic requirements of its solution. The implicative-causal relation introduced in part B/ and formulated in C/ apparently defines the principal function of a WP.

7.2 In the logical theory of reasoning/argumentation concerning the relations "*if* a – *then* b" (plus *if non*-a *then non*-b; b, *because* a), cf. Toulmin 2003: 96-100, the scheme of argumentation is extensive and rather complex. In treatises dealing with ways of reasoning in natural language (van Dijk 1973; Karlík 1995; Levinson 2000), it has been stated that complex sentences conveying a statement that *if* a, *then* b mostly rely on interlocutors' knowledge/experience of a standard/habitual connection between phenomena (together with other information pertaining to speech behaviour). In our opinion, these findings can account for the general use of the formula "if *a*, then how many *b*s'', etc. in WPs. Since the presence of an implicative-causal relation is a constitutive property of any WP token as a representative of a text type, the solving procedure leading to a required solution can build on the underdetermined formulaic stereotypes (leading to customary inferences) as sufficient for the necessary operation. In other words, it is the awareness of the WP text type (the previous experience with it) that gives the impetus to the modelling process. For that reason, pupils' WP solving activity can be considered meta-communicative – the presence of the implicative-causal relation is itself implicated by a WP text type (it is expected to be present in it). At this stage of their schooling, pupils' recognition of a text as a WP becomes an important part of their educational communicative competence within the process of their math learning and, in a broader sense, of their education in general.

References

A b e d i, J. & L o r d, C., 2001, The language factor in mathematics tests. *Applied Measurement in Education*, 14(3), 219–234.

B a k h t i n, M. M., 1986, *Speech genres and other late essays.* C. Emerson & M. Holquist (eds.), transl. V. W. McGee. Austin: University of Texas Press.
B a s s o k, M., 2001, Semantic alignments in mathematical word problems. In Centner, D., Holyoak, K. J. & Kokinov, B. N. (eds.), *The analogical mind: Perspectives from cognitive science*, 401–433. Cambridge, Massachusetts: The MIT Press.
B e r g q v i s t, E., T h e e n s, F. & Ö s t e r h o l m, M., 2018, The role of linguistic features when reading and solving mathematics tasks in different languages. *Journal of Mathematical Behavior,* 51, 41–55.
B o v e n m y e r L e w i s, A. & Mayer, R. E., 1987, Students' miscomprehension of relational statements in arithmetic word problems. *Journal of Educational Psychology,* 79/4, 363–371.
C u m m i n s, D. et al., 1988, The role of understanding in solving word problems. *Cognitive Psychology*, 20(4), 405–438.
D a r o c z y, G., W o l s k a, M., M e u r e r s, W. D. & N u e r k, H. C., 2015, Word problems: a review of linguistic and numerical factors contributing to their difficulty. *Frontiers in Psychology*, 6(348), 22–34.
D a v i d s o n, D., 1967, Causal Relations. *The Journal of Philosophy* 64/21, 691–703.
E n g e l, U., 1996, *Deutsche Grammatik.* Heidelberg: Julius Groos.
D i j k, T. A. van, 1973, Text grammar and text logic. In: J.S. Petöfi and H. Rieser (eds.), *Studies in text grammar.* Dordrecht: Reidel, 17–78.
D u r a n d - G u e r r I e r, V., 2003, Which notion of implication is the right one? From logical considerations to a didactic perspective. *Educational Studies in Mathematics*, 53(1), 5–34.
F e r g u s o n, Ch. A., 1994, Dialect, register, and genre. Working assumptions about conventialization. In: D. Biber & E. Finegan (eds.), *Sociolinguistic perspectives on genre.* Oxford: OUP, 15–30.
G e i s, M. L. & Zwicky, A. M., 1971, On invited inferences. *Linguistic Inquiry* 2, 561–566.
G r i c e, H. P., 1975, Logic and conversation. In P. Cole – J.L. Morgan (eds.), *Syntax and Semantics* 3. New York: Academic Press, 41–58.
H e m b r e e, R., 1992, Experiments and relational studies in problem solving: A meta-analysis. *Journal for Research in Mathematics Education*, 23(3), 242–273.
H i r s c h o v á, M., Matematická slovní úloha jako komunikát. *Český jazyk a literatura,* 68(2), 69–75.
K a r l í k, P., 1995, *Studie o českém souvětí.* Brno: Masarykova Univerzita.
K a r l í k, P., 2016, Příslovečné určení podmínky. In: P. Karlík, M. Nekula, J. Pleskalová (eds.), *Nový encyklopedický slovník češtiny*. Praha: NLN, 1495–1499.
Kotyra, D. & Sivošová, A., 1997, *Jak se naučím řešit slovní úlohy z matematiky?* Praha: Educo.
L e e c h, G., 1983, *Principles of Pragmatics.* London and New York: Longman.
L e e c h, G. & S v a r t v i k, J., 1975, *A Communicative Grammar of English.* Burnt Mill, Harlow: Longman.

L e v i n s o n, S. C., 2000, *Presumptive meanigs: The theory of generalized conversational implicature.* Cambridge, Mass.: The MIT Press.
L e w i s, A. B. & M a y e r, R. E., 1987, Students' Miscomprehension of Relational Statements in Arithmetic Word Problems. *Journal of Educational Psychology* 79 (4), 363–371.
M a r t i n, S. A. & B a s s o k, M., 2005, Effects of semantic cues on mathematical modeling: Evidence from word-problem solving and equation construction tasks. *Memory & Cognition*, 33(3), 471–478.
M a t e r n a, P., 2000, *Svět pojmů a logika*. Praha: Filosofia, Nakladatelství Filozofického ústavu AV ČR.
M a t e r n a, P., P a l a, K. & Z l a t u š k a, J., 1989, *Logická analýza přirozeného jazyka*. Praha: Academia
M c C a r t h y, T., 1987, Modality, invariance, and logical truth. *Journal of Philosophical Logic,* 16, 423–443.
N e s h e r, P., & T e u b a l, E., 1975, Verbal cues as an interfering factor in verbal problem solving. *Educational Studies in Mathematics*, *6*, 41–51.
P a p e, S. J., 2003, Compare word problems: Consistency hypothesis revisited. *Contemporary Educational Psychology* 28, 396–421.
P i a g e t, J., 1967, *The psychology of intelligence*. London: Routledge.
P l a t h, J. & L e i s s, D., 2018, The impact of linguistic complexity on the solution of mathematical modelling tasks. *ZDM Mathematics Education*, 50 (1–2), 159–171.
R e u s s e r, K., 1985, *From situation to equation. On formulation, understanding and solving „situation problems"*. Technical Report no. 143. Institute of Cognitive Science, University of Colorado.
R e u s s e r, K., 1992, Kognitive Modellierung von text-, situations- und mathematischem Verständnis beim Lösen von Textaufgaben. In K. Reiss, M. Reiss & H. Spandl (Hrsg.), *Maschinelles Lernen. Modellierung von Lernen mit Maschinen,* 225–249. Berlin: Springer Verlag.
R e u s s e r, K., 1995, *Vom Text zur Situation zur Gleichung. Kognitive Simulation von Sprachverständnis und Mathematisierung beim Lösen von Textaufgaben* [Habilitationschrift 1989]. Zürich: Universität Zürich, Pädagogisches Institut.
T o u l m i n, S. E., 1958/2003, *The Uses of Argument.* Cambridge – New York – Melbourne: CUP.
V e r s c h a f f e l, L., G r e e r, B., & D e C o r t e, E., 2000, *Making sense of word problems*. Lisse: Swets & Zeitlinger Publishers.
V e r s c h a f f e l, L. et al., 2010, Reconceptualising Word Problems as Exercises in Mathematical Modelling. *Journal für Mathematik-Didaktik, 31*(1), 9–29.
V i c e n t e, S., et al., 2008, Influence of situational and mathematical information on situationally difficult word problems. Studia Psychologica, 50(4), 337–356.
V o n d r o v á, N. et al., 2019, *Matematická slovní úloha: mezi matematikou, jazykem a psychologií*. Praha: Karolinum.