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EWS-GARCH: New Regime Switching Approach to Forecast Value-at-Risk

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Abstract: In the study, the two-step EWS-GARCH models to forecast Value-at-Risk is presented. The EWS-GARCH allows different distributions of returns or Value-at-Risk forecasting models to be used in Value-at-Risk forecasting depending on a forecasted state of the financial time series. In the study EWS-GARCH with GARCH(1,1) and GARCH(1,1), with the amendment to the empirical distribution of random errors as a Value-at-Risk model in a state of tranquillity and empirical tail, exponential or Pareto distributions used to forecast Value-at-Risk in a state of turbulence were considered. The evaluation of Value-at-Risk forecasts was based on the Value-at-Risk forecasts and the analysis of loss functions. Obtained results indicate that EWS-GARCH models may improve the quality of Value-at-Risk forecasts generated using the benchmark models. However, the choice of best assumptions for the EWS-GARCH model should depend on the goals of the Value-at-Risk forecasting model. The final selection may depend on an expected level of adequacy, conservatism and costs of the model.

Keywords: value-at-risk, state of turbulence, GARCH, tail distributions, market risk.

JEL Codes: C53, C58, G17

1 Introduction

Market risk is regarded as one of the three main risks in banks. The obligation to manage market risk in banks is imposed by the international regulations established by the Basel Committee on Banking Supervision. As a part of the risk management, a very important task is its measurement. A basic requirement for an internal model is that the measurement has to be based on Value-at-Risk (and Stressed Value-at-Risk, which is derived as the Value-at-Risk for the worst year for a particular portfolio). The Basel Committee is considering switch to Expected Shortfall as a market risk measure, but as Expected Shortfall is defined as an expected value for losses below Value-at-Risk, Value-at-Risk will be still of high interest after the change.

According to the results obtained by researchers, it is not possible to determine one best method of measuring the Value-at-Risk, which would allow in every situation to achieve the best forecasts. Therefore, the analysis of the quality of Value-at-Risk forecasts generated on the basis of different models is a topic widely discussed in the literature − among others, in Engle (2001) and Engle (2004); Alexander and Lazar (2006); Angelidis *et al.* (2006); Engle and Manganelli (2001); McAleer *et al.* (2013); Marcucci (2005); Ozun *et al.* (2010); Dimitrakopoulos *et al.* (2010), Brownlees *et al.* (2011), Degiannakis *et al.* (2012) and Abad *et al.* (2014).

Even though there is no best model to forecast Value-at-Risk, a lot of researchers are trying to find a model that provides the best Value-at-Risk forecasts. In most cases, the choice depends largely on the specificity of an analysed portfolio (the specifics of assets and the market), some of the researchers indicate that the preferred models should include distributions with lighter tails (e.g., Engle 2001), but most of the researchers show that distributions with fatter tails should be preferred (e.g., Barone-Adesi and Giannopulos 2001; Engle, 2004; Gençay and Selçuk 2004; Dimitrakopoulos *et al.* 2010; Nozari et al 2010; Ozun *et al.* 2010).

Very interesting conclusions can be drawn from McAleer *et al.* (2013) and Degiannakis *et al.* (2012) who showed that for periods of tranquillity (pre-crisis) distributions with relatively thinner tails and for turbulent periods (the period of the financial crisis), models that consider the distributions with fatter tails should be preferred. These results indicate that the selection of a Value-at-Risk model should also depend on the current state of the portfolio.

The fact that the portfolio might be in different states is considered by researchers developing regime switching models (including Hamilton and Susmel 1994; Cai 1994; Grey 1996; Alexander and Lazar 2006; Chan and McAleer 2002). Specificity of the proposed models is, that in all states, losses come from the same distribution but with different parameters. This property stays in contradiction with the findings stated in McAleer *et al.* (2013) and Degiannakis *et al.* (2012), where models with different distributions were found to be the best in different states.

The aim of the study is to present the EWS-GARCH model that allows different distributions to be considered in different states of a portfolio. In these models, the Value-at-Risk forecasts are calculated in two steps. First, the state of the portfolio is forecasted (the state of tranquillity or the state of turbulence − EWS states for an Early Warning System as proposed first step models are inspired by them) and then, depending on the state forecasted, a different model is used to forecast Value-at-Risk. The EWS-GARCH models give the opportunity to use models to forecast Value-at-Risk in the state of tranquillity assuming a distribution of returns with relatively thinner tails, and in the state of turbulence models with fatter tails.

The construction of EWS-GARCH models should, on the one hand, enable an effective protection against market risk by including highly fat tailed nature of the distribution of returns in the state of turbulence, but on the other hand, in the state of tranquillity, it does not force maintaining excessive levels of capital, which should be its advantage over models that take into account strong fat tail nature of the distribution of returns also in the tranquillity state (i.e., EVT models).

In a study, to assess the quality of the Value-at-Risk forecasts, different EWS-GARCH models were compared to each other and with benchmark models (GARCH(1,1), GARCH(1,1) with the correction due to empirical distribution of random error, EGARCH(1,1,1) and GARCH-t (1,1) − model was parametrised assuming unit variance and the number of degrees of freedom greater than 2). The evaluation of the quality of Value-at-Risk forecasts was based on the Value-at-Risk forecasts adequacy (the excess ratio, the Kupiec test, the Christoffersen test, the asymptotic test of unconditional coverage and the backtesting criteria defined by the Basel Committee – both for Value-at-Risk and Stressed Value-at-Risk) and the analysis of loss functions (the Lopez quadratic loss function, the Abad & Benito absolute loss function, the 3rd Caporin loss function and the function of excessive cost, which is proposed in the paper).

The paper is organized as follows: in the first section, methods of forecasting Value-at-Risk are briefly discussed; in the second section, a concept of EWS-GARCH models is presented; and in the third section, an empirical verification of Value-at-Risk forecasts obtained from EWS-GARCH models is analysed.

2 Value-at-Risk as a measure of market risk

Value-at-Risk ($VaR_\alpha(t)$) is defined as a value that a loss would not excess with a certain probability α within a specified period of time in normal market situation. Value-at-Risk can be defined as follows (Engle and Manganelli 1999):

$$P(r_t < VaR_\alpha(t) \mid \Omega_{t-1}) = \alpha \quad (1)$$

where r_t is a return at time t, $VaR_\alpha(t)$ is Value-at-Risk at time t and Ω_{t-1} is a set of information available at time t-1.

In the literature, many different methods of Value-at-Risk measurement have been developed. In essence, most of them differ in the method of estimating distribution of returns. All methods can be divided into three basic groups: nonparametric, parametric and semi-parametric methods. In the nonparametric methods, Value-at-Risk is calculated directly based on empirical data; in the parametric methods, Value-at-Risk is calculated through models that use only estimated parameters describing the distribution of returns of the analysed portfolio. The semi-parametric methods combine the two previous approaches and partly use the estimated parameters and partly use information obtained directly from the empirical distribution of returns (see detailed discussion of methods in Abad *et al.* 2014).

In the group of parametric models, the most popular methods are EWMA models (including the RiskMetrics™ model) and ARCH/GARCH models. Parametric methods are often extended by nonparametric analysis. Semi-parametric models are characterized in a way that they contain parametric part, but simultaneously part of the model is determined based on non-parametric results (i.e., using empirical analysis or expert judgement). The semi-parametric approach is used, for example, in Monte Carlo simulation models, models with amendment to the empirical distribution of random errors, Filtered Historical Simulation models, Extreme Value Theory models or Conditional Autoregressive Value-at-Risk models.

The aforementioned approaches are the most popular approaches used to forecast Value-at-Risk in the literature. Each of them has a lot of options that can significantly affect Value-at-Risk forecasts. In example for the ARCH/GARCH class of models, Bollerslev (2008) described over 100 possible versions.

Having so many options, it is almost impossible to find one best model for every case. Even though, a lot of researchers are trying to find a model that provides the best Value-at-Risk forecasts in every situation. Important findings showing that such an approach is almost impossible to achieve may be found in papers analysing the quality of Value-at-Risk predictions for different models before, during and after the crisis, namely in McAleer *et al.* (2013) and Degiannakis *et al.* (2012). In both cases, the authors showed that GARCH models assuming the normality of the distribution of random error provides high-quality Value-at-Risk forecasts in pre-crisis 2007−2009 period, but the quality significantly decreases during and after the crisis. The study of McAleer *et al.* (2013) found that during the crisis, the best model was RiskMetrics™, and after the crisis EGARCH-t model. In the study of Degiannakis *et al.* (2012), during the crisis, the best model was APARCH with skewed Student's t distribution. These results show that in a state of tranquillity, best models are less conservative (with thinner tails), but during the crisis, their superiority is shown by models that consider the distributions of returns with fatter tails. Degiannakis *et al.* (2012) state that these claims stay valid both for developed countries, as well as, for developing countries. The presented results show not only that there is no one model that would always be the best, but also that a choice of the best model to forecast Value-at-Risk depends on the analysed period, which could be an evidence of the existence of a states in financial time series data.

Despite the conclusions drawn from the aforementioned articles, the use of regime switching models in the Value-at-Risk forecast has a rather niche character. Moreover, the results obtained by the researchers analysing such models in terms of forecasting Value-at-Risk are inconclusive.

Alexander and Lazar (2006) showed that models that take into account more than one state better reflects the nature of the observed foreign exchange time series than models with only one state. They also showed that it is appropriate to define only two states. The inclusion of the third state does not produce tangible benefits and only makes that estimates of the period of turbulence are highly unstable, which may lead to a significant decrease of quality of such a models. The authors also compare the models in terms of the quality of their Value-at-Risk forecasts. The results show the superiority of the regime switching models in comparison to one-state (classic) GARCH class models, but they are not unequivocal. In most cases, the regime switching model indeed provide better Value-at-Risk forecasts than the classic models, but there are also exceptions − for example, for the exchange rate of EUR/USD, it turned out that GARCH with skewed Student's t distribution is the best model.

Similar conclusions can be found in the Marcucci (2005). Author indicates that, in principle, the regime switching models are better suited to financial time series as they have a higher predictive power. But this supremacy is not held with respect to the quality of Value-at-Risk forecasts; however, it is worth noting that the two states models are also better in terms of Value-at-Risk forecasts than its one state counterparts (i.e., GARCH(1,1) with two states in comparison to classical GARCH(1,1) with only one state).

All the regime switching models compared above are built on assumption that the returns are under the same process for the period of tranquillity and turbulence. Differences are in the values of estimated parameters. This assumption stays in contradiction to findings stated by McAleer *et al*. (2013) and Degiannakis *et al*. (2012), where different models turned out to be the best in different states. In the next chapter, EWS-GARCH models are presented. The models have been developed to take into account a stylized fact of the existence of states and high efficiency of different models in different states.

3 EWS-GARCH models

The concept of EWS-GARCH models is based on three basic assumptions. The first assumption is that a time series of financial data has two states (the state of tranquillity and the state of turbulence), which may vary considerably in terms of their nature. This assumption means that Value-at-Risk forecasts would be provided from a different model in the state of tranquillity and different model in the state of turbulence. The second assumption is that the conditional volatility in financial data has a tendency to cluster and that other stylized facts about the characteristics of financial markets may be relevant, which makes use of the GARCH class models reasonably. The third assumption is that tail returns may be better described by a different distribution than all returns together (the Extreme Value Theory is built on this assumption).

A Value-at-Risk forecasting procedure based on EWS-GARCH models consists of two steps. In the first step, the state of time series for the next day is forecasted; then, in the second step, Value-at-Risk for the next day is forecasted. The Value-at-Risk forecast is provided from an appropriate model regarding the state forecasted in the first step. The general concept of Value-at-Risk forecasting using EWS-GARCH model is shown in Fig. 1.

In the EWS-GARCH models, it is proposed that the prediction of the state should be carried out by a model for binary dependent variable: a logit, a probit or a cloglog. Each of these models can be defined in a similar manner that differs only with regards to a random error distribution. The logit model assumes a logistic distribution, the probit model, a normal distribution and the clog log the Gompertz distribution of random errors. These models can be defined as follows:

$$y_t^* = \boldsymbol{\beta X}_t + \varepsilon_t \tag{2}$$

$$y_t = \begin{cases} 1 & y_t^* > 0 \\ 0 & y_t^* \leq 0 \end{cases} \tag{3}$$

where y^* is a latent dependent variable, β is a vector of parameters describing the relationship between independent variables and the unobserved dependent variable, X_t is a vector of the observations of independent variables that have an impact on an unobservable dependent variable, ε_t is a random error coming from the relevant distribution, and y_t is observable result of the modelled phenomenon.

In the process of forecasting the state of turbulence, the y_t is equal to 1 for a certain percentage of the lowest observed returns (5% or 10%). Here, the state of turbulence is defined as a state of the highest losses and not the highest volatility; such a definition is consistent with the goal of the model – to forecast more conservative VaR when the losses are expected to be high and less conservative otherwise (it is assumed that a bank is holding only a long position, the model can be extended assuming that the state of turbulence is defined in both distribution tail. However, this definition is also related to extreme losses and not extreme volatility and is consistent with the whole concept of the EWS-GARCH). Independent variables in the model describe a current

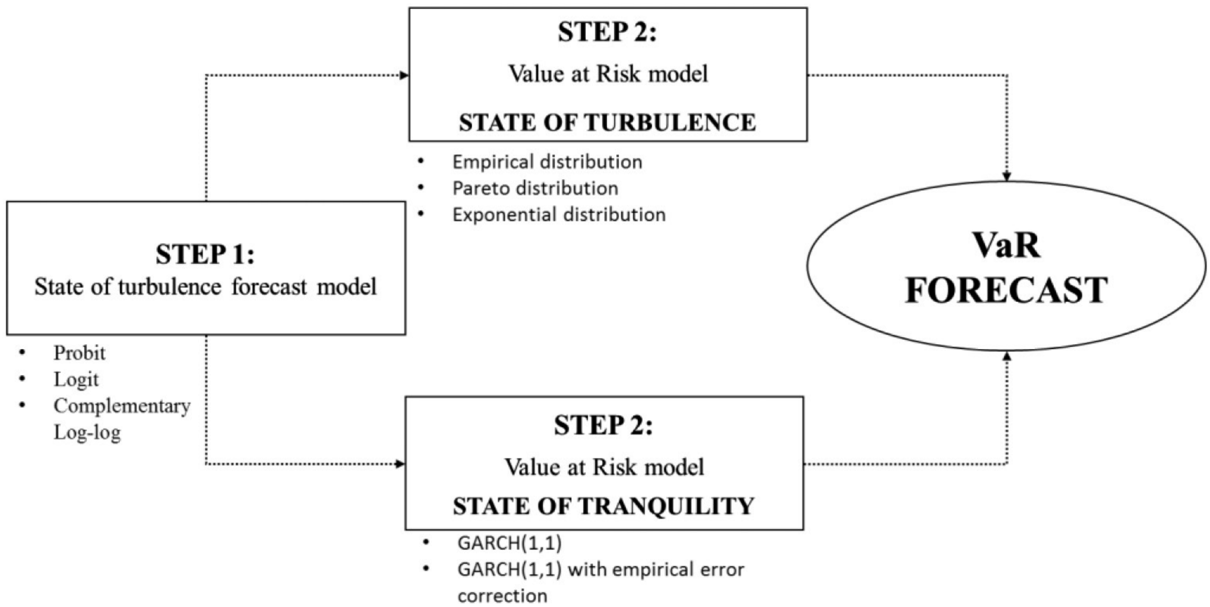

Fig. 1. The concept of Value-at-Risk forecasting using an EWS-GARCH model

Source: Author's own elaboration.

situation on the stock, exchange rates and short-term interest rates markets (current returns on of the stock index; current returns on exchange rates for major currencies and current short-term interest rates). Moreover, to achieve the best possible forecast quality, a selection of an optimal cut-off point for the event forecast is considered (it is set up to 5% and 10% for the 5% and 10% definitions of y_t relevantly). The choice of a link function for a binary variable model, the definition of the observable dependent variable, the choice of independent variables and the optimal cut-off threshold have been established in accordance with the results obtained in the study of Chlebus (2016). In the aforementioned article, the quality of the EWS models were also discussed. Additionally, it is also worth considering the two methods of independent variables' set selection. In the first approach, the forecasts of a state would be based on the whole set of independent variables. In the second approach, a set of independent variables will be limited only to variables statistically significant at the 5% significance level selected by a stepwise selection method.

The model to predict a state gives the opportunity to distinguish two states (the state of tranquillity and the state of turbulence) in a time series, which can vary considerably in their nature (with respect to expected returns, volatility, etc.). In order to take into account different specificities of these two states, in each state, different models to forecast Value-at-Risk should be used. According to the definition of the state of turbulence (5% or 10% of worst returns), the state of turbulence should be connected with the tail of the returns distribution.

In EWS-GARCH models GARCH models are considered as Value-at-Risk forecasting models (GARCH(1,1) and GARCH(1,1) with amendment to the empirical distribution of random errors) in the state of tranquillity. It is worth noting that the design of EWS-GARCH models allows to include other models to forecast Value-at-Risk. The choice of the aforementioned models stems from the lessons learned from the literature, and according to it, these models perform well in predicting the Value-at-Risk, especially at a time of relative tranquillity.

In the GARCH models, it is assumed that the return r comes from the i.i.d. distribution with parameters (μ, σ^2). In the model, conditional expected value μ_t (assumed that this value is equal to 0) and conditional variance σ_t^2 is estimated. The GARCH/ARCH class models mainly differ from each other according to the assumption how the conditional variance equation is defined. However, they may also differ in other assumptions, such as the distribution of returns or the conditional expected value equation definition. The GARCH(1,1) model can be written as:

$$r_t = \mu_t + \varepsilon_t = \mu_t + \sigma_t \xi_t \tag{4}$$

where r_t is a return on assets analysed at time t, ε_t is a random error in time t and ε_t can be expressed as the product of the conditional standard deviation σ_t and standardized random error ξ_t at time t, which satisfies the assumption $\xi_t \sim$ i.i.d.(0,1). The equation of conditional variance in the GARCH(1,1) can be written as:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{5}$$

where ω is a constant that satisfies the assumption $\omega > 0$, α_1 and β_1 are parameters that satisfy the assumptions $\alpha_1 \geq 0$ and $\beta_1 \geq 0$.

For the GARCH(1,1), the Value-at-Risk for the long position is estimated based on the following formula:

$$VaR_\alpha(t) = \widehat{\mu_t} + k_\alpha * \sqrt{\widehat{\sigma}_t^2} \tag{6}$$

where $VaR_\alpha(t)$ is a forecast of Value-at-Risk on α tolerance level at time t, $\widehat{\mu_t}$ is a forecast of conditional mean at time t, k_α is a value of quantile α from assumed random error distribution and $\widehat{\sigma}_t^2$ is a forecast of conditional variance at time t.

The Basel Committee requirements state that Value-at-Risk should be estimated with a 99% confidence (the α is assumed to be equal to 1%). Value-at-Risk forecasts from GARCH(1,1) with the amendment to the empirical distribution of random errors (Engle and Manganelli 2001) is obtained in a similar manner as in GARCH(1,1), the difference lies in the use of a quantile from the empirical distribution of returns instead of quantile from the normal distribution. The aim of this change is to include fatter tails than in the normal distribution. Such an amendment is possible because of the property of the GARCH model that the MLE estimator is consistent, even when a random error does not come from the normal distribution (Bollerslev and Wooldridge 1992). This property allows to use the GARCH processes to standardize residuals from the model even if a different distribution than normal is assumed for the standardised errors.

In the state of turbulence, as it is associated with the period of expected major losses, the tail distributions are considered, namely the empirical tail distribution and two parametric tail distributions Pareto and exponential distributions. The use of these distributions in Value-at-Risk forecasting is a practice met in an operational risk measurement (see Panjer 2006). The proposed parametric distributions are special cases of a generalized Pareto distribution used in the EVT models.

The Pareto distribution is characterized by a very thick tail and is used when there is a relatively high probability of very negative realization of returns. The Pareto distribution may be defined with one or two parameters. A cumulative distribution function for the version with two parameters may be described by the following formula:

$$F_{PAR}(x) = 1 - \left(\frac{\theta}{\theta + x}\right)^\alpha \tag{7}$$

where θ is a scale parameter and α is a shape parameter.

The second tail distribution considered is the exponential distribution, which is defined with only one parameter. Its cumulative distribution function can be written as:

$$F_{EXP}(x) = 1 - e^{-x/\theta} \tag{8}$$

where θ is a scale parameter.

For the tail models, the Value-at-Risk is forecast simply as a value of the quantile of distribution. A problem in this case is a determination that the quantile of the distribution provide the confidence equal to 99%. Two quantiles may be considered: the conservative and liberal assumption. The conservative assumption is that the Value-at-Risk forecast for the state of turbulence is equal to the 99th percentile of the tail returns distribution. Such an approach should not raise doubts about the satisfaction at least 99% level of confidence. However, the liberal assumption may be considered as well. The liberal assumption takes into account the fact that the analysis of turbulence refers to a specified percentage of the worst cases. Then, the confidence level should be obtained as the product of a definition of the state of turbulence and a quantile of tail distribution. According to the liberal approach, 99% confidence level for 10% definition of state of turbulence is obtained for the 90th percentile of the tail distribution and accordingly, for 5% definition of state of turbulence, the 80th percentile of the tail distribution should be used. In the study, the analysis of validity of the conservative and liberal assumption will be verified empirically.

EWS-GARCH models were designed in such a way as to enable an effective measurement of market risk not only at the time of relative tranquillity and turbulence, but also not to force keeping excessive levels of the regulatory capital during tranquillity. The first element gives an advantage over one state models. The second element should give the advantage over the models that take into account a fat tail nature of the distribution also during relative tranquillity (i.e., models with Student's t distribution or EVT models).

4 Estimation procedure

In the EWS-GARCH model, Value-at-Risk forecast is obtained on the basis of a two-step approach. Appropriate models on each of these steps are estimated separately. In the first step, the state forecasting model is estimated and in the second step, two different models are estimated each for both states. The advantage of this approach is its simplicity, because there is no need to develop a new complex estimation procedure. The disadvantage of this approach is the fact that at each stage, an estimation error is committed, which could cause that the estimation error is greater if the estimation of the entire model would be estimated at once.

The two-stage nature of the EWS-GARCH model results in two elements being forecasted: the state of turbulence, and Value-at-Risk. Forecasts of the state and the Value-at-Risk at time t+1 are based on the data available at time t. A data set to forecast states is prepared using the recursive window approach. Data set for Value-at-Risk forecasting is prepared using the rolling window approach (the window width was set to 1004 observations, which corresponds to about 4 years of one day returns). During the study, it is assumed that the dependent variable in GARCH models is a continuous one-day rate of return, which may be expressed as $r_t = 100\% * (\ln(p_t) - \ln(p_{t-1}))$.

Additionally, for GARCH models, significance of a constant in the conditional expected value equation was performed; in cases when the constant was statistically insignificant, no constant in the GARCH model was incorporated.

5 Testing framework

Performing a thorough analysis of the quality of EWS-GARCH models requires the development of multi-dimensional testing process. In order to confirm the quality of Value-at-Risk forecasts and comparisons of the models in terms of their quality, it was decided to carry out tests of the adequacy of Value-at-Risk forecasts and loss functions analysis. As a part of Value-at-Risk forecasts adequacy, analyses of the following were performed: the excess ratio comparison, the Kupiec test, the Christoffersen test, the asymptotic test of unconditional coverage and the backtesting criterion specified by the Basel Committee. The excess ratio and the backtesting criterion was analysed for Value-at-Risk and Stressed Value-at-Risk.

The excess ratio may be calculated as:

$$ER = \frac{\sum_{t=1}^{N} 1_{r_t < VaR_t}}{N} \tag{9}$$

where N is a number of the Value-at-Risk forecasts and $1_{r_t<VaR_t}$ is a number of cases when a realized rate of return is smaller than a forecasted Value-at-Risk.

Using the excess ratio, each of the Value-at-Risk models can be assigned to one of the Basel backtesting criterion zones – green, yellow or red. The Basel Committee requires comparing the quality of the models based on the Value-at-Risk forecasts results; however, it is also worthwhile to consider the quality of the models with regards to the Stressed Value-at-Risk. For this purpose, the worst excess ratio (from the set of 250 consecutive days with the highest excess ratio) from the out-of-sample was calculated. The result shows how the model works in the worst possible conditions observed. Analogously to the Value-at-Risk forecasts, in this case, the excess ratio can be attributed as well to one of the backtesting zones defined by the Basel Committee.

The Kupiec test (also called the unconditional coverage test) (Kupiec 1995) can be written as:

$$\begin{aligned} LR_{UC} = 2[&ln(\hat{\alpha}^X(1-\hat{\alpha})^{N-X}) \\ &- ln(\alpha^X(1-\alpha)^{N-X})] \sim \chi^2_\alpha(1) \end{aligned} \tag{10}$$

where α is an expected excess ratio (according to the Basel Committee requirements, it should be 1%), $\hat{\alpha}$ is an observed excess ratio, X is an observed number of Value-at-Risk exceedances and N is a number of Value-at-Risk forecasts. In the null hypothesis, it is assumed that the expected and observed excess ratio is equal to each other. In contrast to the backtesting criterion, the Kupiec test identifies models that both underestimate and overestimate Value-at-Risk; however, there is no straightforward method to assess whether the analysed model tends to overestimate or underestimate the Value-at-Risk forecasts. Such an analysis is possible based on a backtesting criterion statistics, also called an asymptotic test of unconditional coverage (see Abad *et al.* 2014). The backtesting criterion statistics come from the asymptotic standard normal distribution. This test is two-tailed. Strongly negative values of the test statistics indicate overestimation of the Value-at-Risk forecasts, while strongly positive, underestimation of these forecasts. The test statistic can be calculated according to the following formula:

$$z_{BT} = \frac{(N\hat{\alpha} - N\alpha)}{\sqrt{N\alpha(1-\alpha)}} \sim N(0,1) \tag{11}$$

where α is an expected excess ratio, $\hat{\alpha}$ is an observed excess ratio and N is a number of Value-at-Risk forecasts.

The Christoffersen test (the conditional coverage test) proposed by Christoffersen (1998) is an extension of the Kupiec test. This test extends the Kupiec test by inclusion of an independency of Value-at-Risk exceedances testing. The test statistic comes from the asymptotic χ^2 distribution with 2 degrees of freedom and can be formulated as:

$$LR_{CC} = LR_{UC} + LR_{IND} \sim \chi^2_{\alpha}(2) \tag{12}$$

where LR_{UC} is the Kupiec test statistics and LR_{IND} is an independency of exceedances statistics. The LR_{IND} is equal to

$$2[\ln((1-\pi_{01})^{N_{00}}\pi_{01}{}^{N_{01}}(1-\pi_{11})^{N_{10}}\pi_{11}{}^{N_{11}})$$

$$-\ln((1-\hat{\alpha})^{N_{00}+N_{10}}\hat{\alpha}^{N_{01}+N_{11}})],$$

where $\hat{\alpha}$ is an observed excess ratio, N_{ij} is the number of observation for which a state j (exceedance or not exceedance) is observed under condition that a state i (exceedance or not exceedance) was observed in the previous period, π_{01} is a probability of observing Value-at-Risk exceedances conditional on not observing them in the previous period and π_{11} is a probability of observing Value-at-Risk exceedances conditional on observing them in the previous period.

The tests presented above allow to evaluate Value-at-Risk models based on the adequacy of its forecasts. Additionally, an analysis of the cost (loss) compares on the one hand the losses resulting from exceeding the Value-at-Risk, and on the other hand, accuracy and cost efficiency of the used models. The cost (loss) functions analysis are not formal tests; during the analysis, the score is calculated, which allows to compare the Value-at-Risk models with each other.

The first cost (loss) function considered is the quadratic Lopez function (see Lopez 1999), which may be defined as:

$$CL_t = \begin{cases} 1 + (r_t - VaR_\alpha(t))^2 \; for & r_t < VaR_\alpha(t) \\ 0 \; for & r_t \geq VaR_\alpha(t) \end{cases} \tag{13}$$

where r_t is a realised rate of return at the moment t and VaR_t is a Value-at-Risk forecast for the same moment t. The score is calculated as

$$\sum_{t=1}^{N} CL_t$$

(where N is a number of Value-at-Risk forecasts). The Lopez function considers the two aspects of Value-at-Risk forecasts: a number and a severity of exceedances. Each exceedance increases a score by at least 1, where the excess over 1 is calculated with respect to its severity and is calculated as $(r_t - VaR_t)^2$. The main disadvantage of the Lopez quadratic function is that it does not give an easy interpretation. The solution may be a function proposed by Abad and Benito (2013), which can be written as:

$$CA_t = \begin{cases} |r_t - VaR_\alpha(t)| \; for & r_t < VaR_\alpha(t) \\ 0 \; for & r_t \geq VaR_\alpha(t) \end{cases} \tag{14}$$

In this case, a score is calculated as an average of severity of exceedances with respect to a number of Value-at-Risk forecasts considered, which can be calculated as

$$\sum_{t=1}^{N} CA_t / N.$$

This loss function differs from the previous one in two basic dimensions. Firstly, an average is minimized instead of the sum; therefore, the number of exceedances is not taken into account. This may cause models with a larger number of exceedances to be preferred. Secondly, the absolute deviation is analysed, which makes the interpretation easier.

Both aforementioned functions consider non-zero values only in the case of exceedance. From a perspective of use of Value-at-Risk models in a financial institutions, it is also reasonable to consider the cost (loss) functions that take into account the costs associated with both exceedances and lack of exceedances (opportunity costs). First considered function of this type is a function presented by Caporin (2008). In his study, he proposed three different cost functions, which assume that a cost of deviations of a forecasted Value-at-Risk from a realized rate of return is equally important regardless of whether the exceedance was observed or not. In the study the following cost function is considered:

$$CC_t = \begin{cases} |r_t - VaR_\alpha(t)| \; for \; r_t < VaR_\alpha(t) \\ |r_t - VaR_\alpha(t)| \; for \; r_t \geq VaR_\alpha(t) \end{cases} \quad (15)$$

Caporin proposes that in order to compare the Value-at-Risk forecasts, a sum of all CC_t should be used; however, in the study, the average of these values is considered. Both analyses lead to similar conclusions, but the average can be interpreted as the average absolute error of the Value-at-Risk forecasts.

Additionally, an absolute excessive cost function was analysed. The absolute excessive cost function, like the Caporin loss function, includes costs either in the case of the Value-at-Risk exceedance or lack of exceedance. The difference is that the analysis is focused rather on the excessive cost of use of the model than the precision of the forecast. Therefore, the process of assigning point values is divided into three cases and focuses precisely on the costs made by the model:

$$CAE_t = \begin{cases} |VaR_\alpha(t)| & for \; r_t \geq VaR_\alpha(t) \; and \; r_t \geq 0 \\ |VaR_\alpha(t) - r_t| & for \; r_t \geq VaR_\alpha(t) \; and \; r_t < 0 \\ |r_t| & for \qquad r_t < VaR_\alpha(t) \end{cases} \quad (16)$$

Value-at-Risk models should be compared in terms of mean value of excessive cost function for the analysed number of forecasts

$$\overline{CAE} = \frac{\sum_{t=1}^{N} CAE_t}{N} .$$

The $\overline{CAE}$ may be interpreted as a measure of excessive model conservatism. The higher the $\overline{CAE}$ is, the more conservative the model is, which means that the model predicts on average more conservative Value-at-Risk than needed to cover losses arising from changes in a value of analysed assets.

In the study, except for the Value-at-Risk quality analysis, also the accuracy of assumptions, for which the EWS-GARCH models were built were considered, the following tests were performed: the verification of ARCH process existence and its reduction (by LM and Q tests), the verification of the assumption about stationarity of returns distributions (Philips-Perron and KPSS tests), as well as the occurrence of an autocorrelation of random errors (Durbin-Watson test).

6 Empirical results

6.1 Data

The quality of Value-at-Risk forecasts obtained from EWS-GARCH models was analysed for 79 time series of returns for companies listed on the Warsaw Stock Exchange (a detailed list of companies is presented in Appendix in Tab. A1). Single assets time series may be considered as a portfolio with one asset. Due to the fact that in a single asset portfolio, risk diversification is not possible; it is expected to (having 79 different times series) observe extreme portfolios, which give an opportunity to asses quality of Value-at-Risk for extreme cases. Assets were selected randomly in order to avoid selection bias (selected assets were good representation of the whole stock, with respect to size, capitalisation and branches).

The empirical study was performed for the series of returns from 1 January 2006 to 31 January 2012. The period from the beginning of 2006 to the end of 2009 constituted the in-sample period and from the beginning of 2010, the forecast sample starts and ends at the end of the whole sample, thereby giving 525 predictions of Value-at-Risk for each asset. All calculations and estimations were performed in SAS 9.4.

All the considered models used to forecast Value-at-Risk have been developed in such a way as to meet the requirements set by the Basel Committee to internal models for market risk measurement. The measure of market risk is based on the one-day Value-at-Risk predictions satisfying 99% confidence level. For the quality of Value-at-Risk forecasts, only one-day predictions are required and sufficient. The assessment was carried out for 525 observations, which is about two years, which is more than expected in the Basel regulations of the minimum equal to 250 observations. The model takes into account the risk factors that may affect the level of market risk. In the state forecasting models, three key risk factors are directly addressed: the situation on the stock, the situation on the exchange rates and the situation on the interest rates market. The considered Value-at-Risk models assume that the impact of all relevant risk factors are reflected in the price of assets analysed.

6.2 Results

In the study, analogous to the practice used in the literature, EWS-GARCH models are evaluated and compared on the basis of the Value-at-Risk forecasts quality, so the quality of states forecasts are not discussed in details.

At the beginning of the Value-at-Risk quality analysis for the EWS-GARCH models, it is worth noting that the two benchmark models (GARCH(1,1) and GARCH(1,1) with the amendment to the empirical distribution of random errors) are included in EWS-GARCH models as models used to forecast Value-at-Risk in the state of tranquillity. So, the EWS-GARCH models (with the assumed tranquillity state model) can be considered as models that extends one state (benchmark) models by taking into account the state of turbulence. Therefore, the results obtained for EWS-GARCH models can be regarded as the evaluation of how incorporation of the state of turbulence can improve, according to certain criteria, the results obtained by the benchmark models.

The discussion of the results for the EWS-GARCH model was divided into two parts. In the first part, results for the EWS-GARCH model with GARCH(1,1) and in the second part, the GARCH(1,1) with the amendment to the empirical distribution of random errors as a model in a state of tranquillity were discussed. In order to maintain transparency of the results, a crossover comparison between models of different EWS-GARCH groups (with different state of tranquillity models) was omitted.

In this paper, the results for EWS-GARCH models with the Pareto distribution in the state of turbulence are not presented. This is due to the fact that Value-at-Risk forecasted in the state of turbulence based on such models always significantly exceeded the level of 100% (the total value of the portfolio). These models apparently fulfil the requirements of the Basel Committee, but do not bring any added value over the assumption that the level of Value-at-Risk should be equal to the value of the entire portfolio, which is an unacceptable assumption from the risk managing perspective. The most likely cause of such a high forecast of Value-at-Risk is the fact that the Pareto distribution is characterized by a very thick tail. For models using the exponential distribution in the state of turbulence, similar problems were not identified.

The last thing worth analysing before the discussion of results for EWS-GARCH models is the definition of improvement of Value-at-Risk forecasts. The improvement of the quality of Value-at-Risk forecasts may be defined in two ways: the first definition can be called absolute (conservative criterion), while the other relative (adequacy criterion):

- According to the conservative criterion, the improvement of the quality of Value-at-Risk forecasts should be connected with reduction of the number of exceedances. According to the definition, the more conservatively the model predicts Value-at-Risk (smaller excess ratio, more frequent assignment to the green zone according to the Basel Committee approach, smaller cost associated with exceedances), the better the model is acknowledged.
- According to the adequacy criterion, the closer the excess ratio to the expected 1% is, the better the selected model is to forecast Value-at-Risk (according to the standard confidence level assumed to be equal to 99%).

Evaluation of the Value-at-Risk quality was carried out due to both mentioned criteria definitions.

Value-at-Risk forecasts quality – the EWS-GARCH(1,1) models

The evaluation of the Value-at-Risk forecasts quality for the EWS-GARCH models began with EWS-GARCH(1,1) models. The results of the analysis of the Value-at-Risk exceedances and the cost functions for these models are presented in Tab. 1. Analysis for the EWS-GARCH(1,1) models was divided into two parts. In the first part, the models were compared with regard to the Value-at-Risk exceedances and the cost functions; in the other part, the models were compared with regard to results of the coverage tests (the same division was made for the EWS-GARCH(1,1) with the amendment to empirical error distribution models).

From the results, it can be seen that almost every version of the EWS-GARCH(1,1) model lower the excess ratio and the average value of the Lopez cost function in comparison to the GARCH(1,1) model. The only exception is the model that assumes that Value-at-Risk is defined as the 80th percentile of the empirical distribution at the 5% definition of the state of turbulence and the probit model used to forecasts the state of turbulence. Also, the Abad & Benito cost function for most of the EWS-GARCH(1,1) versions, on average, is smaller than for the GARCH (1,1). Exceptions are models assuming that Value-at-Risk is defined, as 90th and 80th percentile

Tab. 1. The results of the analysis of the quality of Value-at-Risk forecasts obtained from the EWS-GARCH(1,1) models

SFM	TSVM	TUSVM	VALUE-AT-RISK (WHOLE OUT-OF-SAMPLE)									STRESSED VALUE-AT-RISK (THE WORST 250 DAYS)				
			EN	ER	ABAD	LOPEZ	CAPORIN	EXCOST	GREEN	AT LEAST YELLOW	RED	EN	ER	GREEN	AT LEAST YELLOW	RED
NONE	GARCH-t	NONE	1.25	0.24%	6.3%	2.46	12.5%	11.6%	98.7%	98.7%	1.3%	1.03	0.4%	97.5%	98.7%	1.3%
PROBIT SEL	GARCH	EX9_10	4.39	0.84%	8.4%	4.51	11.1%	10.3%	93.7%	98.7%	1.3%	3.56	1.4%	72.2%	93.7%	6.3%
LOGIT SEL	GARCH	EX9_10	4.44	0.85%	8.6%	4.56	11.0%	10.1%	93.7%	98.7%	1.3%	3.61	1.4%	68.4%	93.7%	6.3%
CLOGLOG SEL	GARCH	EX9_10	4.53	0.86%	8.7%	4.59	10.9%	10.0%	93.7%	98.7%	1.3%	3.66	1.5%	68.4%	93.7%	6.3%
PROBIT	GARCH	EX9_10	4.56	0.87%	8.5%	4.62	11.0%	10.2%	93.7%	97.5%	2.5%	3.63	1.5%	70.9%	93.7%	6.3%
PROBIT SEL	GARCH	EM9_10	4.58	0.87%	8.8%	4.71	8.7%	7.8%	92.4%	97.5%	2.5%	3.71	1.5%	68.4%	92.4%	7.6%
CLOGLOG	GARCH	EX9_10	4.59	0.88%	8.6%	4.66	10.8%	10.0%	93.7%	97.5%	2.5%	3.67	1.5%	73.4%	93.7%	6.3%
NONE	GARCH EMP	NONE	4.61	0.88%	9.2%	4.67	7.2%	6.4%	94.9%	97.5%	2.5%	3.73	1.5%	68.4%	96.2%	3.8%
LOGIT	GARCH	EX9_10	4.62	0.88%	8.6%	4.68	10.9%	10.0%	92.4%	97.5%	2.5%	3.67	1.5%	70.9%	93.7%	6.3%
LOGIT SEL	GARCH	EM9_10	4.63	0.88%	9.0%	4.76	8.6%	7.8%	92.4%	98.7%	1.3%	3.78	1.5%	64.6%	93.7%	6.3%
PROBIT SEL	GARCH	EX0_10	4.67	0.89%	9.0%	4.80	7.9%	7.1%	92.4%	96.2%	3.8%	3.80	1.5%	65.8%	91.1%	8.9%
CLOGLOG SEL	GARCH	EM9_10	4.71	0.90%	9.1%	4.77	8.5%	7.7%	92.4%	98.7%	1.3%	3.82	1.5%	64.6%	93.7%	6.3%
LOGIT SEL	GARCH	EX0_10	4.75	0.90%	9.1%	4.81	7.8%	7.0%	92.4%	97.5%	2.5%	3.90	1.6%	59.5%	92.4%	7.6%
CLOGLOG SEL	GARCH	EX9_5	4.76	0.91%	9.0%	4.82	17.1%	16.3%	93.7%	98.7%	1.3%	3.81	1.5%	69.6%	93.7%	6.3%
PROBIT SEL	GARCH	EX9_5	4.77	0.91%	8.9%	4.84	21.6%	20.8%	93.7%	98.7%	1.3%	3.84	1.5%	69.6%	93.7%	6.3%
PROBIT	GARCH	EM9_10	4.77	0.91%	8.9%	4.84	8.6%	7.8%	91.1%	97.5%	2.5%	3.80	1.5%	68.4%	93.7%	6.3%
LOGIT SEL	GARCH	EX9_5	4.80	0.91%	9.0%	4.86	17.1%	16.3%	93.7%	98.7%	1.3%	3.87	1.5%	69.6%	92.4%	7.6%
CLOGLOG SEL	GARCH	EX0_10	4.80	0.91%	9.2%	4.80	7.8%	7.0%	92.4%	97.5%	2.5%	3.91	1.6%	59.5%	92.4%	7.6%
CLOGLOG	GARCH	EM9_10	4.81	0.92%	9.0%	4.88	8.6%	7.7%	91.1%	97.5%	2.5%	3.84	1.5%	69.6%	92.4%	7.6%
LOGIT	GARCH	EX9_5	4.84	0.92%	8.8%	4.90	16.8%	16.0%	92.4%	98.7%	1.3%	3.95	1.6%	64.6%	92.4%	7.6%
PROBIT	GARCH	EX9_5	4.84	0.92%	8.8%	4.90	16.9%	16.1%	92.4%	98.7%	1.3%	3.95	1.6%	64.6%	92.4%	7.6%
LOGIT	GARCH	EM9_10	4.84	0.92%	9.0%	4.90	8.6%	7.7%	91.1%	97.5%	2.5%	3.85	1.5%	68.4%	93.7%	6.3%
CLOGLOG	GARCH	EX9_5	4.86	0.93%	8.8%	4.93	16.8%	16.0%	92.4%	98.7%	1.3%	3.99	1.6%	64.6%	92.4%	7.6%
PROBIT	GARCH	EX0_10	4.86	0.93%	9.1%	4.86	7.9%	7.1%	89.9%	97.5%	2.5%	3.87	1.5%	65.8%	92.4%	7.6%
CLOGLOG SEL	GARCH	EM9_5	4.87	0.93%	9.3%	4.94	8.6%	7.8%	92.4%	98.7%	1.3%	3.92	1.6%	64.6%	93.7%	6.3%
PROBIT SEL	GARCH	EM9_5	4.87	0.93%	9.1%	4.94	8.6%	7.8%	92.4%	98.7%	1.3%	3.94	1.6%	67.1%	92.4%	7.6%
LOGIT SEL	GARCH	EM9_5	4.91	0.94%	9.2%	4.98	8.6%	7.8%	92.4%	98.7%	1.3%	3.99	1.6%	64.6%	91.1%	8.9%
CLOGLOG	GARCH	EX0_10	4.91	0.94%	9.2%	4.92	7.8%	7.0%	89.9%	96.2%	3.8%	3.92	1.6%	65.8%	88.6%	11.4%
LOGIT	GARCH	EX0_10	4.92	0.94%	9.2%	4.93	7.8%	7.0%	89.9%	94.9%	5.1%	3.92	1.6%	65.8%	89.9%	10.1%
LOGIT	GARCH	EM9_5	4.95	0.94%	9.1%	5.02	8.5%	7.6%	91.1%	98.7%	1.3%	4.04	1.6%	64.6%	92.4%	7.6%
PROBIT	GARCH	EM9_5	4.95	0.94%	9.1%	5.02	8.5%	7.7%	91.1%	98.7%	1.3%	4.06	1.6%	63.3%	92.4%	7.6%
CLOGLOG	GARCH	EM9_5	4.97	0.95%	9.1%	5.04	8.5%	7.7%	91.1%	98.7%	1.3%	4.08	1.6%	64.6%	92.4%	7.6%

Tab. 1. The results of the analysis of the quality of Value-at-Risk forecasts obtained from the EWS-GARCH(1,1) models (continue)

SFM	TSVM	TUSVM	VALUE-AT-RISK (WHOLE OUT-OF-SAMPLE)									STRESSED VALUE-AT-RISK (THE WORST 250 DAYS)				
			EN	ER	ABAD	LOPEZ	CAPORIN	EXCOST	GREEN	AT LEAST YELLOW	RED	EN	ER	GREEN	AT LEAST YELLOW	RED
CLOGLOG SEL	GARCH	EX8_5	5.23	1.00%	10.1%	5.23	7.3%	6.5%	91.1%	96.2%	3.8%	4.23	1.7%	58.2%	89.9%	10.1%
PROBIT SEL	GARCH	EX8_5	5.25	1.00%	10.0%	5.26	7.3%	6.5%	92.4%	96.2%	3.8%	4.25	1.7%	60.8%	89.9%	10.1%
LOGIT	GARCH	EX8_5	5.27	1.00%	9.9%	5.27	7.3%	6.5%	89.9%	97.5%	2.5%	4.32	1.7%	55.7%	89.9%	10.1%
LOGIT SEL	GARCH	EX8_5	5.28	1.01%	10.1%	5.28	7.3%	6.5%	92.4%	96.2%	3.8%	4.28	1.7%	59.5%	87.3%	12.7%
PROBIT	GARCH	EX8_5	5.29	1.01%	9.9%	5.30	7.3%	6.5%	89.9%	97.5%	2.5%	4.33	1.7%	54.4%	89.9%	10.1%
CLOGLOG	GARCH	EX8_5	5.29	1.01%	9.9%	5.30	7.3%	6.5%	88.6%	97.5%	2.5%	4.35	1.7%	55.7%	89.9%	10.1%
CLOGLOG SEL	GARCH	EM0_10	6.01	1.15%	12.6%	6.02	6.8%	6.0%	82.3%	92.4%	7.6%	4.99	2.0%	50.6%	79.7%	20.3%
PROBIT SEL	GARCH	EM0_10	6.03	1.15%	12.4%	6.03	6.8%	6.1%	82.3%	93.7%	6.3%	4.99	2.0%	51.9%	79.7%	20.3%
LOGIT SEL	GARCH	EM0_10	6.04	1.15%	12.7%	6.04	6.8%	6.0%	83.5%	92.4%	7.6%	5.01	2.0%	48.1%	79.7%	20.3%
PROBIT	GARCH	EM0_10	6.19	1.18%	12.7%	6.20	6.8%	6.0%	81.0%	92.4%	7.6%	5.08	2.0%	46.8%	81.0%	19.0%
CLOGLOG	GARCH	EM0_10	6.20	1.18%	12.8%	6.21	6.8%	6.0%	79.7%	92.4%	7.6%	5.05	2.0%	49.4%	79.7%	20.3%
LOGIT	GARCH	EM0_10	6.23	1.19%	12.8%	6.23	6.8%	6.0%	81.0%	92.4%	7.6%	5.09	2.0%	48.1%	79.7%	20.3%
CLOGLOG SEL	GARCH	EM8_5	6.29	1.20%	13.2%	6.30	6.7%	5.9%	81.0%	91.1%	8.9%	5.15	2.1%	43.0%	75.9%	24.1%
PROBIT SEL	GARCH	EM8_5	6.37	1.21%	13.1%	6.37	6.7%	5.9%	81.0%	92.4%	7.6%	5.22	2.1%	43.0%	77.2%	22.8%
LOGIT SEL	GARCH	EM8_5	6.37	1.21%	13.2%	6.37	6.7%	5.9%	79.7%	89.9%	10.1%	5.24	2.1%	43.0%	77.2%	22.8%
LOGIT	GARCH	EM8_5	6.39	1.22%	12.9%	6.40	6.7%	5.9%	79.7%	92.4%	7.6%	5.29	2.1%	36.7%	79.7%	20.3%
CLOGLOG	GARCH	EM8_5	6.39	1.22%	12.9%	6.40	6.7%	5.9%	78.5%	92.4%	7.6%	5.32	2.1%	38.0%	78.5%	21.5%
NONE	GARCH	NONE	6.42	1.22%	12.5%	6.42	6.6%	5.8%	78.5%	93.7%	6.3%	5.18	2.1%	39.2%	78.5%	21.5%
PROBIT	GARCH	EM8_5	6.43	1.22%	13.1%	6.44	6.7%	5.9%	79.7%	91.1%	8.9%	5.34	2.1%	35.4%	78.5%	21.5%
NONE	EGARCH	NONE	6.53	1.24%	12.5%	6.54	6.7%	5.9%	78.5%	92.4%	7.6%	5.19	2.1%	40.5%	81.0%	19.0%

Notes: In the table, white fields refer to the EWS-GARCH models, while grey fields to the benchmark models.

In the table, the following abbreviations are used: SFM – the state forecasting model, TSVM – the Value-at-Risk forecasting model in the state of tranquillity, TUSVM − the Value-at-Risk forecasting model in a state of turbulence, EN − the average number of exceedances, ER – the average excess ratio, ABAD − the average value of the Abad & Benito cost function, LOPEZ − the average value of the Lopez cost function, CAPORIN − the average value of the Caporin cost function, EXCOST − the average value of the excessive cost function, GREEN – the average frequency of a model being in the green zone, AT LEAST YELLOW – the average frequency of a model being at least in the yellow zone, RED – the average frequency of a model being in the red zone. In the states forecasting model abbreviation SEL means that stepwise selection process was used.

Short names of Value-at-Risk models in the state of turbulence are in the form DRQ_CP, where the DR defines a distribution of returns, Q defines the quantile for which Value-at-Risk was forecasted and CP defines the cut-off point that was used to forecast the state of turbulence in the states forecasting model. For the distributions in the state of turbulence following abbreviations are used: EX − exponential distribution, EM − empirical distribution; Q equal to 9 represents the 99th percentile, 0 represents the 90th percentile, and 8 represents the 80th percentile; 5% cut-off is denoted by 5 and the cut-off point equal to 10% by 10.

Source: Author's own calculations.

Tab. 2. The results of the analysis of the quality of Value-at-Risk forecasts obtained from the EWS-GARCH(1,1) models – coverage tests results

SFM	TSVM	TUSVM	LR_{UC}	LR_{IND}	LR_{CC}	Z_{UC}	Z^{D}_{UC}	Z^{G}_{UC}
LOGIT	GARCH	EX8_5	5.06%	6.33%	5.06%	12.66%	2.53%	10.13%
PROBIT	GARCH	EX8_5	5.06%	6.33%	5.06%	12.66%	2.53%	10.13%
CLOGLOG	GARCH	EX8_5	5.06%	6.33%	5.06%	13.92%	2.53%	11.39%
PROBIT SEL	GARCH	EX8_5	6.33%	12.66%	6.33%	10.13%	2.53%	7.59%
CLOGLOG	GARCH	EX9_5	6.33%	10.13%	6.33%	12.66%	5.06%	7.59%
LOGIT	GARCH	EX9_5	6.33%	10.13%	6.33%	12.66%	5.06%	7.59%
PROBIT	GARCH	EX9_5	6.33%	8.86%	6.33%	12.66%	5.06%	7.59%
CLOGLOG	GARCH	EM9_5	6.33%	8.86%	6.33%	13.92%	5.06%	8.86%
LOGIT	GARCH	EM9_5	6.33%	8.86%	6.33%	13.92%	5.06%	8.86%
PROBIT	GARCH	EM9_5	6.33%	7.59%	6.33%	13.92%	5.06%	8.86%
LOGIT SEL	GARCH	EX9_5	6.33%	12.66%	8.86%	11.39%	5.06%	6.33%
LOGIT SEL	GARCH	EM9_5	6.33%	12.66%	8.86%	12.66%	5.06%	7.59%
NONE	GARCH EMP	NONE	7.59%	8.86%	5.06%	10.13%	5.06%	5.06%
LOGIT SEL	GARCH	EX8_5	7.59%	11.39%	6.33%	11.39%	3.80%	7.59%
CLOGLOG SEL	GARCH	EX8_5	7.59%	11.39%	6.33%	12.66%	3.80%	8.86%
CLOGLOG SEL	GARCH	EX9_5	7.59%	12.66%	8.86%	12.66%	6.33%	6.33%
CLOGLOG SEL	GARCH	EM9_5	7.59%	12.66%	8.86%	13.92%	6.33%	7.59%
PROBIT SEL	GARCH	EM9_5	7.59%	12.66%	8.86%	13.92%	6.33%	7.59%
PROBIT SEL	GARCH	EM0_10	7.59%	12.66%	8.86%	18.99%	1.27%	17.72%
NONE	GARCH	NONE	8.86%	8.86%	7.59%	24.05%	2.53%	21.52%
PROBIT SEL	GARCH	EX9_5	8.86%	12.66%	8.86%	13.92%	7.59%	6.33%
LOGIT SEL	GARCH	EM0_10	8.86%	13.92%	11.39%	17.72%	1.27%	16.46%
CLOGLOG SEL	GARCH	EM0_10	8.86%	15.19%	11.39%	18.99%	1.27%	17.72%
LOGIT	GARCH	EM0_10	8.86%	12.66%	11.39%	20.25%	1.27%	18.99%
PROBIT	GARCH	EM0_10	8.86%	12.66%	11.39%	20.25%	1.27%	18.99%
PROBIT SEL	GARCH	EM8_5	8.86%	11.39%	11.39%	20.25%	1.27%	18.99%
CLOGLOG	GARCH	EM0_10	8.86%	15.19%	12.66%	21.52%	1.27%	20.25%
LOGIT	GARCH	EM8_5	8.86%	11.39%	12.66%	21.52%	1.27%	20.25%
CLOGLOG	GARCH	EM8_5	8.86%	11.39%	12.66%	22.78%	1.27%	21.52%
CLOGLOG SEL	GARCH	EX0_10	10.13%	11.39%	7.59%	15.19%	7.59%	7.59%
LOGIT	GARCH	EX9_10	10.13%	12.66%	7.59%	15.19%	7.59%	7.59%
LOGIT	GARCH	EM9_10	10.13%	12.66%	7.59%	16.46%	7.59%	8.86%
LOGIT SEL	GARCH	EX0_10	10.13%	8.86%	8.86%	15.19%	7.59%	7.59%
CLOGLOG SEL	GARCH	EM9_10	10.13%	11.39%	8.86%	16.46%	8.86%	7.59%
NONE	EGARCH	NONE	10.13%	5.06%	8.86%	24.05%	2.53%	21.52%
PROBIT	GARCH	EM8_5	10.13%	11.39%	13.92%	21.52%	1.27%	20.25%
LOGIT	GARCH	EX0_10	11.39%	12.66%	6.33%	16.46%	6.33%	10.13%
CLOGLOG	GARCH	EX0_10	11.39%	11.39%	6.33%	17.72%	7.59%	10.13%
PROBIT	GARCH	EX0_10	11.39%	12.66%	6.33%	18.99%	8.86%	10.13%

Tab. 2. The results of the analysis of the quality of Value-at-Risk forecasts obtained from the EWS-GARCH(1,1) models – coverage tests results (continue)

SFM	TSVM	TUSVM	LR_{UC}	LR_{IND}	LR_{CC}	Z_{UC}	Z^{D}_{UC}	Z^{G}_{UC}
CLOGLOG	GARCH	EX9_10	11.39%	11.39%	7.59%	15.19%	8.86%	6.33%
CLOGLOG	GARCH	EM9_10	11.39%	11.39%	7.59%	17.72%	8.86%	8.86%
CLOGLOG SEL	GARCH	EX9_10	11.39%	11.39%	8.86%	16.46%	10.13%	6.33%
PROBIT SEL	GARCH	EX9_10	11.39%	8.86%	11.39%	16.46%	10.13%	6.33%
LOGIT SEL	GARCH	EM9_10	11.39%	8.86%	11.39%	17.72%	10.13%	7.59%
CLOGLOG SEL	GARCH	EM8_5	11.39%	11.39%	11.39%	21.52%	2.53%	18.99%
LOGIT SEL	GARCH	EM8_5	11.39%	11.39%	12.66%	21.52%	1.27%	20.25%
PROBIT	GARCH	EX9_10	12.66%	12.66%	7.59%	16.46%	10.13%	6.33%
PROBIT	GARCH	EM9_10	12.66%	12.66%	7.59%	18.99%	10.13%	8.86%
PROBIT SEL	GARCH	EX0_10	12.66%	8.86%	10.13%	16.46%	8.86%	7.59%
LOGIT SEL	GARCH	EX9_10	12.66%	8.86%	11.39%	17.72%	11.39%	6.33%
PROBIT SEL	GARCH	EM9_10	12.66%	8.86%	11.39%	17.72%	10.13%	7.59%
NONE	GARCH-t	NONE	77,22%	2,53%	51,90%	77,22%	75,95%	1,27%

Notes: In the table, white fields refer to the EWS-GARCH models, while grey fields to benchmark the models.

In the table, the following abbreviations are used: SFM – the state forecasting model, TSVM – the Value-at-Risk forecasting model in a state of tranquillity, TUSVM − the Value-at-Risk forecasting model in a state of turbulence, LR_{UC} − the ratio of cases in which the null hypothesis was rejected in the Kupiec test, LR_{IND} − the ratio of cases in which the null hypothesis was rejected in the LR_{IND} part of the Christofferssen test, LR_{CC} − the ratio of cases in which the null hypothesis was rejected in the Christofferssen test, Z_{UC} − the ratio of cases in which the null hypothesis was rejected in the asymptotic test of unconditional coverage, Z^{D}_{UC} − the ratio of cases in which the null hypothesis was rejected in the asymptotic test of unconditional coverage in favour of alternative hypothesis that the actual excess ratio is significantly lower than expected, Z^{G}_{UC} − the ratio of cases in which the null hypothesis was rejected in the asymptotic test of unconditional coverage in favour of an alternative hypothesis that the actual excess ratio is significantly higher than expected. All tests were performed for the 5% significance level, except the asymptotic test of unconditional coverage, where level of significance was set up to 10% (5% for each tail).

Short names of the Value-at-Risk models in the state of turbulence are in the form DRQ_CP, where the DR defines a distribution of returns, Q defines the quantile for which Value-at-Risk was forecasted and CP defines the cut-off point that was used to forecast the state of turbulence in the states forecasting model. For the distributions in the state of turbulence following abbreviations are used: EX − exponential distribution, EM − empirical distribution; Q equal to 9 represents the 99th percentile, 0 represents the 90th percentile, and 8 represents the 80th percentile; 5% cut-off is denoted by 5 and the cut-off point equal to 10% by 10.

Source: Author's own calculations.

of the empirical distribution, with the 10% and 5% definition of the state of turbulence, respectively.

Improvement in the excess ratio and the costs associated with the occurrence of exceeding (expressed by the Lopez and Abad & Benito cost functions), is associated with an increase in the costs of the use of the model (expressed by the Caporin and excess costs functions). The increase in the cost of use of models is growing steadily along with the decrease of the excess ratio. Exceptions are models in which Value-at-Risk is calculated as the 99th percentile of the exponential distribution at the 5% definition of the state of turbulence, in which case the increase of the cost of the model is significant.

In principle, all models characterized by the lower excess ratio, on the average, are more likely to be assigned to the green zone in the Basel Committee backtesting approach. Frequency of being assigned at least to the yellow zone is smaller than for GARCH(1,1) for models assuming the empirical distribution of returns and the liberal way of estimating Value-at-Risk in the state of turbulence for both definitions of the state of turbulence.

Improvement in results for the Stressed Value-at-Risk (reduction of the excess ratio and more frequent qualification to the green zone and at least the yellow zone) may be seen for all EWS-GARCH models except models that assume that Value-at-Risk is defined as the 80th percentile of the empirical distribution with 5% state of turbulence definition.

The results for the EWS-GARCH(1,1) models with different assumptions about the state forecasting models may be summed up as following. The choice of the model to predict states does not cause a significant change in the quality of Value-at-Risk forecasts. However, using a stepwise selection process reduces the excess ratio and the costs associated with the Value-at-Risk exceedance, while increasing the costs of using the model. Although the results for different states forecasting models do not differ significantly, some tendency in model preference may be noticed. For the models that assumed 10% definition of a state of turbulence in 3 of 4 cases, the probit model with the stepwise selection turned out to be the best. For the models assuming 5% definition of the state of turbulence, the best was always the cloglog with stepwise selection.

Selecting the Value-at-Risk forecasting model in the state of turbulence is of crucial importance for the quality of Value-at-Risk forecasts. It can be stated that the more conservative Value-at-Risk model assumptions in the state of turbulence are (appropriate percentile definition and assumed distribution), the lower excess ratio, the lower costs associated with the Value-at-Risk exceedance and higher costs associated with the usage of the model.

The results of coverage tests for EWS-GARCH(1,1) models are shown in Tab. 2. On the basis of the results analysed high quality EWS-GARCH models may be divided into two groups:

1. Models worse than the benchmark and conservative – this group includes models for which, based on the coverage test results, the null hypothesis is rejected more frequently than for the benchmark model, but based on the results of the asymptotic unconditional coverage test, it can be assumed that this is due to the frequent rejection of this hypothesis in favour of the hypothesis that the observed number of exceedance is smaller than expected. Distinguishing this group of models is apparent from the fact that these models are better than benchmark models from the perspective of the conservative criterion (however, they are worse from the adequacy criterion perspective).
2. Models better than the benchmark − this group includes models for which the coverage tests rejected the null hypothesis less frequent or equally as for the benchmark models.

The models assuming the conservative definition of appropriate percentile to forecast Value-at-Risk in the state of turbulence and the 10% definition of this state or the model assuming the 90th percentile from the exponential distribution as the Value-at-Risk forecast may be assigned to the worse and conservative group of models. For these models, it can be seen that worse results are caused by more frequent incidences of significantly smaller excess ratio observed than expected. It should be noted that these models are characterized by, on average, the lowest excess ratio.

Better models than the benchmark include the models assuming a conservative approach to defining appropriate percentile to forecast the Value-at-Risk and the 5% definition of the turbulence state or the model assuming the 80th percentile of the exponential distribution as Value-at-Risk in the state of turbulence. It turned out that the EWS-GARCH(1,1) model with the 5% state of turbulence definition provides more conservative Value-at-Risk forecasts than the GARCH (1,1) model, but also closer to 1% excess ratio.

Based on the described results, it can be stated that the EWS-GARCH(1,1) models are characterized by lower excess ratio than the GARCH(1,1). In addition, for all of them (with the best assumptions regarding the state of turbulence forecasting) observed excess ratio is closer to the expected (in absolute value) than for GARCH(1,1). The most conservative EWS- GARCH(1,1) model assuming the 10% definition of the state of turbulence and Value-at-Risk calculated as the 99th percentile of empirical or exponential distribution. The aforementioned models are even more conservative than the GARCH (1,1) with the amendment to the empirical distribution of random errors. None of the models come close to the level of conservatism specific to the GARCH-t(1,1). However, in terms of adequacy (if the goal is to provide Value-at-Risk forecasts with the observed excess ratio as close as possible to 1%), the best models are models assuming that Value-at-Risk in the state of turbulence should be calculated as the 80th percentile of the exponential distribution.

Value-at-Risk forecasts quality – the EWS-GARCH(1,1) with the amendment to the empirical distribution of random errors models

After the results for the EWS-GARCH(1,1), results for the EWS-GARCH(1,1) with the amendment to the empirical errors distribution models may be discussed. The results with respect to exceedances and the cost functions are shown in Tab. 3.

For the EWS-GARCH(1,1) with the amendment to the empirical error distribution, only the results of models that improve (reduce) the excess ratio (the conservative criterion analysis) will be discussed. The GARCH(1,1) with the amendment to the empirical error distri-

Tab. 3. The results of the analysis of the quality of Value-at-Risk forecasts obtained from the EWS-GARCH(1,1) models with the amendment to the empirical distribution of random errors

SFM	TSVM	TUSVM	VALUE-AT-RISK (WHOLE OUT-OF-SAMPLE)									STRESSED VALUE-AT-RISK (THE WORST 250 DAYS)				
			EN	ER	ABAD	LOPEZ	CAPORIN	EXCOST	GREEN	AT LEAST YELLOW	RED	EN	ER	GREEN	AT LEAST YELLOW	RED
NONE	GARCH-t	NONE	1.25	0.24%	6.3%	2.46	12.5%	11.6%	98.7%	98.7%	1.3%	1.03	0.4%	97.5%	98.7%	1.3%
PROBIT SEL	GARCH EMP	EX9_10	3.06	0.58%	6.4%	3.27	11.6%	10.7%	100.0%	100.0%	0.0%	2.56	1.0%	88.6%	100.0%	0.0%
LOGIT SEL	GARCH EMP	EX9_10	3.09	0.59%	6.4%	3.26	11.4%	10.6%	100.0%	100.0%	0.0%	2.58	1.0%	89.9%	98.7%	1.3%
CLOGLOG SEL	GARCH EMP	EX9_10	3.16	0.60%	6.6%	3.34	11.3%	10.5%	100.0%	100.0%	0.0%	2.62	1.0%	91.1%	98.7%	1.3%
CLOGLOG	GARCH EMP	EX9_10	3.18	0.61%	6.3%	3.31	11.3%	10.5%	100.0%	100.0%	0.0%	2.67	1.1%	88.6%	100.0%	0.0%
PROBIT	GARCH EMP	EX9_10	3.18	0.61%	6.3%	3.31	11.5%	10.7%	100.0%	100.0%	0.0%	2.66	1.1%	87.3%	100.0%	0.0%
LOGIT	GARCH EMP	EX9_10	3.20	0.61%	6.3%	3.33	11.3%	10.5%	100.0%	100.0%	0.0%	2.68	1.1%	88.6%	100.0%	0.0%
PROBIT SEL	GARCH EMP	EM9_10	3.25	0.62%	6.8%	3.48	9.1%	8.3%	100.0%	100.0%	0.0%	2.72	1.1%	86.1%	100.0%	0.0%
LOGIT SEL	GARCH EMP	EM9_10	3.28	0.62%	6.8%	3.46	9.1%	8.3%	100.0%	100.0%	0.0%	2.76	1.1%	87.3%	98.7%	1.3%
CLOGLOG SEL	GARCH EMP	EM9_10	3.34	0.64%	7.0%	3.52	9.0%	8.2%	100.0%	100.0%	0.0%	2.77	1.1%	88.6%	98.7%	1.3%
PROBIT SEL	GARCH EMP	EX0_10	3.34	0.64%	6.9%	3.52	8.4%	7.5%	100.0%	100.0%	0.0%	2.80	1.1%	86.1%	100.0%	0.0%
CLOGLOG	GARCH EMP	EM9_10	3.39	0.65%	6.8%	3.53	9.0%	8.2%	100.0%	100.0%	0.0%	2.81	1.1%	84.8%	100.0%	0.0%
LOGIT SEL	GARCH EMP	EX0_10	3.39	0.65%	7.0%	3.53	8.3%	7.5%	100.0%	100.0%	0.0%	2.86	1.1%	84.8%	98.7%	1.3%
PROBIT	GARCH EMP	EM9_10	3.39	0.65%	6.7%	3.53	9.1%	8.3%	100.0%	100.0%	0.0%	2.80	1.1%	83.5%	100.0%	0.0%
PROBIT SEL	GARCH EMP	EX9_5	3.39	0.65%	6.5%	3.48	22.1%	21.3%	100.0%	100.0%	0.0%	2.81	1.1%	87.3%	100.0%	0.0%
LOGIT	GARCH EMP	EM9_10	3.42	0.65%	6.8%	3.56	9.0%	8.2%	100.0%	100.0%	0.0%	2.82	1.1%	84.8%	100.0%	0.0%
CLOGLOG SEL	GARCH EMP	EX0_10	3.43	0.65%	7.1%	3.57	8.3%	7.5%	100.0%	100.0%	0.0%	2.85	1.1%	86.1%	98.7%	1.3%
CLOGLOG SEL	GARCH EMP	EX9_5	3.43	0.65%	6.7%	3.52	17.6%	16.8%	100.0%	100.0%	0.0%	2.81	1.1%	87.3%	100.0%	0.0%
LOGIT SEL	GARCH EMP	EX9_5	3.47	0.66%	6.6%	3.56	17.6%	16.8%	100.0%	100.0%	0.0%	2.87	1.1%	83.5%	100.0%	0.0%
LOGIT	GARCH EMP	EX9_5	3.48	0.66%	6.3%	3.57	17.3%	16.5%	100.0%	100.0%	0.0%	2.92	1.2%	87.3%	100.0%	0.0%
PROBIT	GARCH EMP	EX0_10	3.48	0.66%	6.9%	3.57	8.3%	7.5%	100.0%	100.0%	0.0%	2.87	1.1%	82.3%	98.7%	1.3%
CLOGLOG	GARCH EMP	EX0_10	3.49	0.67%	7.0%	3.59	8.3%	7.5%	100.0%	100.0%	0.0%	2.90	1.2%	82.3%	98.7%	1.3%
CLOGLOG	GARCH EMP	EX9_5	3.49	0.67%	6.3%	3.59	17.3%	16.5%	100.0%	100.0%	0.0%	2.95	1.2%	86.1%	100.0%	0.0%
PROBIT	GARCH EMP	EX9_5	3.49	0.67%	6.4%	3.59	17.4%	16.6%	100.0%	100.0%	0.0%	2.92	1.2%	87.3%	100.0%	0.0%
PROBIT SEL	GARCH EMP	EM9_5	3.49	0.67%	6.7%	3.59	9.1%	8.3%	100.0%	100.0%	0.0%	2.89	1.2%	87.3%	100.0%	0.0%
LOGIT	GARCH EMP	EX0_10	3.51	0.67%	7.0%	3.60	8.3%	7.5%	100.0%	100.0%	0.0%	2.90	1.2%	82.3%	98.7%	1.3%

Tab. 3. The results of the analysis of the quality of Value-at-Risk forecasts obtained from the EWS-GARCH(1,1) models with the amendment to the empirical distribution of random errors (continue)

SFM	TSVM	TUSVM	VALUE-AT-RISK (WHOLE OUT-OF-SAMPLE)									STRESSED VALUE-AT-RISK (THE WORST 250 DAYS)				
			EN	ER	ABAD	LOPEZ	CAPORIN	EXCOST	GREEN	AT LEAST YELLOW	RED	EN	ER	GREEN	AT LEAST YELLOW	RED
CLOGLOG SEL	GARCH EMP	EM9_5	3.54	0.68%	6.8%	3.59	9.1%	8.3%	100.0%	100.0%	0.0%	2.90	1.2%	86.1%	100.0%	0.0%
LOGIT SEL	GARCH EMP	EM9_5	3.58	0.68%	6.7%	3.63	9.1%	8.3%	100.0%	100.0%	0.0%	2.96	1.2%	83.5%	100.0%	0.0%
LOGIT	GARCH EMP	EM9_5	3.59	0.68%	6.6%	3.64	9.0%	8.2%	100.0%	100.0%	0.0%	3.01	1.2%	87.3%	100.0%	0.0%
CLOGLOG	GARCH EMP	EM9_5	3.61	0.69%	6.6%	3.66	9.0%	8.2%	100.0%	100.0%	0.0%	3.04	1.2%	86.1%	100.0%	0.0%
PROBIT	GARCH EMP	EM9_5	3.61	0.69%	6.6%	3.66	9.0%	8.2%	100.0%	100.0%	0.0%	3.03	1.2%	87.3%	100.0%	0.0%
PROBIT SEL	GARCH EMP	EX8_5	3.87	0.74%	7.6%	3.93	7.8%	7.0%	100.0%	100.0%	0.0%	3.19	1.3%	78.5%	100.0%	0.0%
CLOGLOG SEL	GARCH EMP	EX8_5	3.90	0.74%	7.7%	3.90	7.8%	7.0%	100.0%	100.0%	0.0%	3.19	1.3%	79.7%	98.7%	1.3%
LOGIT	GARCH EMP	EX8_5	3.91	0.75%	7.4%	3.91	7.8%	7.0%	100.0%	100.0%	0.0%	3.29	1.3%	82.3%	98.7%	1.3%
CLOGLOG	GARCH EMP	EX8_5	3.92	0.75%	7.4%	3.93	7.8%	7.0%	100.0%	100.0%	0.0%	3.32	1.3%	81.0%	98.7%	1.3%
LOGIT SEL	GARCH EMP	EX8_5	3.95	0.75%	7.6%	3.95	7.8%	7.0%	100.0%	100.0%	0.0%	3.24	1.3%	77.2%	98.7%	1.3%
PROBIT	GARCH EMP	EX8_5	3.95	0.75%	7.5%	3.95	7.8%	7.0%	100.0%	100.0%	0.0%	3.30	1.3%	79.7%	98.7%	1.3%
NONE	GARCH EMP	NONE	4.61	0.88%	9.16%	4.67	7.23%	6.43%	94.9%	97.5%	2.5%	3.73	1.49%	68.4%	96.2%	3.8%
NONE	EGARCH_EMP	NONE	4.84	0.92%	9.46%	4.84	7.29%	6.49%	93.7%	96.2%	3.8%	4.01	1.61%	74.7%	94.9%	5.1%
NONE	GARCH	NONE	6.42	1.22%	12.48%	6.42	6.58%	5.79%	78.5%	93.7%	6.3%	5.18	2.07%	39.2%	78.5%	21.5%
NONE	EGARCH	NONE	6.53	1.24%	12.48%	6.54	6.68%	5.90%	78.5%	92.4%	7.6%	5.19	2.08%	40.5%	81.0%	19.0%

Notes: In the table, white fields refer to the EWS-GARCH models, while grey fields to benchmark the models.

In the table the following abbreviations are used: SFM – the state forecasting model, TSVM – the Value-at-Risk forecasting model in the state of tranquillity, TUSVM − the Value-at-Risk forecasting model in a state of turbulence, EN − the average number of exceedances, ER – the average excess ratio, ABAD − the average value of the Abad & Benito cost function, LOPEZ − the average value of the Lopez cost function, CAPORIN − the average value of the Caporin cost function, EXCOST − the average value of the excessive cost function, GREEN – the average frequency of a model being in the green zone, AT LEAST YELLOW – the average frequency of a model being at least in the yellow zone, RED – the average frequency of a model being in the red zone. In the states forecasting model abbreviation SEL means that stepwise selection process was used.

Short names of the Value-at-Risk models in the state of turbulence are in the form DRQ_CP, where the DR defines a distribution of returns, Q defines the quantile for which Value-at-Risk was forecasted and CP defines the cut-off point that was used to forecast the state of turbulence in the states forecasting model. For the distributions in the state of turbulence following abbreviations are used: EX − exponential distribution, EM − empirical distribution; Q equal to 9 represents the 99th percentile, 0 represents the 90th percentile, and 8 represents the 80th percentile; 5% cut-off is denoted by 5 and the cut-off point equal to 10% by 10.

Source: Author's own calculations.

Tab. 4. The results of the analysis of the quality of Value-at-Risk forecasts obtained from the EWS-GARCH(1,1) models with the amendment to the empirical distribution of random errors – coverage tests results

SFM	TSVM	TUSVM	LR_{UC}	LR_{IND}	LR_{CC}	Z_{UC}	Z^{D}_{UC}	Z^{G}_{UC}
NONE	GARCH EMP	NONE	7.59%	8.86%	5.06%	10.13%	5.06%	5.06%
CLOGLOG	GARCH EMP	EX8_5	8.86%	5.06%	1.27%	8.86%	8.86%	0.00%
LOGIT	GARCH EMP	EX8_5	8.86%	5.06%	1.27%	8.86%	8.86%	0.00%
PROBIT	GARCH EMP	EX8_5	8.86%	5.06%	1.27%	8.86%	8.86%	0.00%
NONE	GARCH	NONE	8.86%	8.86%	7.59%	24.05%	2.53%	21.52%
LOGIT SEL	GARCH EMP	EX8_5	10.13%	7.59%	2.53%	10.13%	10.13%	0.00%
PROBIT SEL	GARCH EMP	EX8_5	10.13%	8.86%	3.80%	10.13%	10.13%	0.00%
NONE	EGARCH	NONE	10.13%	5.06%	8.86%	24.05%	2.53%	21.52%
CLOGLOG SEL	GARCH EMP	EX8_5	11.39%	7.59%	2.53%	11.39%	11.39%	0.00%
CLOGLOG	GARCH EMP	EM9_5	11.39%	6.33%	3.80%	11.39%	11.39%	0.00%
LOGIT	GARCH EMP	EM9_5	11.39%	6.33%	3.80%	11.39%	11.39%	0.00%
PROBIT	GARCH EMP	EM9_5	11.39%	5.06%	3.80%	11.39%	11.39%	0.00%
CLOGLOG	GARCH EMP	EX9_5	11.39%	6.33%	5.06%	11.39%	11.39%	0.00%
LOGIT	GARCH EMP	EX9_5	11.39%	6.33%	5.06%	11.39%	11.39%	0.00%
PROBIT	GARCH EMP	EX9_5	11.39%	6.33%	5.06%	11.39%	11.39%	0.00%
LOGIT SEL	GARCH EMP	EM9_5	13.92%	8.86%	6.33%	13.92%	13.92%	0.00%
LOGIT SEL	GARCH EMP	EX9_5	13.92%	8.86%	7.59%	13.92%	13.92%	0.00%
LOGIT	GARCH EMP	EX0_10	15.19%	8.86%	5.06%	15.19%	15.19%	0.00%
CLOGLOG SEL	GARCH EMP	EM9_5	15.19%	8.86%	6.33%	15.19%	15.19%	0.00%
CLOGLOG SEL	GARCH EMP	EX0_10	15.19%	8.86%	7.59%	15.19%	15.19%	0.00%
CLOGLOG SEL	GARCH EMP	EX9_5	15.19%	8.86%	7.59%	15.19%	15.19%	0.00%
CLOGLOG	GARCH EMP	EX0_10	16.46%	7.59%	5.06%	16.46%	16.46%	0.00%
PROBIT	GARCH EMP	EX0_10	16.46%	8.86%	5.06%	16.46%	16.46%	0.00%
LOGIT	GARCH EMP	EM9_10	16.46%	8.86%	6.33%	16.46%	16.46%	0.00%
PROBIT SEL	GARCH EMP	EM9_5	16.46%	8.86%	7.59%	16.46%	16.46%	0.00%
LOGIT	GARCH EMP	EX9_10	17.72%	8.86%	6.33%	17.72%	17.72%	0.00%
PROBIT	GARCH EMP	EM9_10	17.72%	8.86%	6.33%	17.72%	17.72%	0.00%
CLOGLOG	GARCH EMP	EM9_10	17.72%	7.59%	7.59%	17.72%	17.72%	0.00%
LOGIT SEL	GARCH EMP	EX0_10	17.72%	7.59%	7.59%	17.72%	17.72%	0.00%
PROBIT SEL	GARCH EMP	EX9_5	17.72%	8.86%	7.59%	17.72%	17.72%	0.00%
CLOGLOG SEL	GARCH EMP	EM9_10	17.72%	8.86%	10.13%	17.72%	17.72%	0.00%
PROBIT	GARCH EMP	EX9_10	18.99%	8.86%	6.33%	18.99%	18.99%	0.00%
CLOGLOG	GARCH EMP	EX9_10	18.99%	7.59%	7.59%	18.99%	18.99%	0.00%
PROBIT SEL	GARCH EMP	EX0_10	20.25%	6.33%	8.86%	20.25%	20.25%	0.00%
CLOGLOG SEL	GARCH EMP	EX9_10	20.25%	7.59%	10.13%	20.25%	20.25%	0.00%
LOGIT SEL	GARCH EMP	EM9_10	21.52%	7.59%	11.39%	21.52%	21.52%	0.00%
PROBIT SEL	GARCH EMP	EM9_10	21.52%	6.33%	12.66%	21.52%	21.52%	0.00%

Tab. 4. The results of the analysis of the quality of Value-at-Risk forecasts obtained from the EWS-GARCH(1,1) models with the amendment to the empirical distribution of random errors – coverage tests results (continue)

SFM	TSVM	TUSVM	LR_{UC}	LR_{IND}	LR_{CC}	Z_{UC}	Z^{D}_{UC}	Z^{G}_{UC}
PROBIT SEL	GARCH EMP	EX9_10	22.78%	6.33%	12.66%	22.78%	22.78%	0.00%
LOGIT SEL	GARCH EMP	EX9_10	24.05%	6.33%	11.39%	24.05%	24.05%	0.00%
NONE	GARCH-t	NONE	77.22%	2.53%	51.90%	77.22%	75.95%	1.27%

Notes: In the table, white fields refer to the EWS-GARCH models, while grey fields to benchmark the models.

In the table, the following abbreviations are used: SFM – the state forecasting model, TSVM – the Value-at-Risk forecasting model in the state of tranquillity, TUSVM − the Value-at-Risk forecasting model in a state of turbulence, LR_{UC} − the ratio of cases in which the null hypothesis was rejected in the Kupiec test, LR_{IND} − the ratio of cases in which the null hypothesis was rejected in the LR_{IND} part of the Christofferssen test, LR_{CC} − the ratio of cases in which the null hypothesis was rejected in the Christofferssen test, Z_{UC} − the ratio of cases in which the null hypothesis was rejected in the asymptotic test of unconditional coverage, Z^{D}_{UC} − the ratio of cases in which the null hypothesis was rejected in the asymptotic test of unconditional coverage in favour of alternative hypothesis that the actual excess ratio is significantly lower than expected, Z^{G}_{UC} − the ratio of cases in which the null hypothesis was rejected in the asymptotic test of unconditional coverage in favour of an alternative hypothesis that the actual excess ratio is significantly higher than expected. All tests were performed for the 5% significance level, except the asymptotic test of unconditional coverage, where level of significance was set up to 10% (5% for each tail).

Short names of the Value-at-Risk models in the state of turbulence are in the form DRQ_CP, where the DR defines a distribution of returns, Q defines the quantile for which Value-at-Risk was forecasted and CP defines the cut-off point that was used to forecast the state of turbulence in the states forecasting model. For the distributions in the state of turbulence following abbreviations are used: EX - exponential distribution, EM - empirical distribution; Q equal to 9 represents the 99th percentile, 0 represents the 90th percentile, and 8 represents the 80th percentile; 5% cut-off is denoted by 5 and the cut-off point equal to 10% by 10.

Source: Author's own calculations.

bution is a conservative model itself – the excess ratio on average is smaller than the expected 1%. According to that, seeking for EWS-GARCH(1,1) with the amendment to empirical error distribution models that provide excess ratios closer to 1% than the GARCH(1,1) with the amendment to the empirical error distribution model would lead to a choice of models debilitating conservatism of GARCH(1,1) with the amendment to empirical error distribution in the state of turbulence model, which is not a purpose of the EWS-GARCH models' development and will not be discussed.

As noted above, the GARCH(1,1) with the amendment to the empirical error distribution is on average conservative. The average excess ratio is equal to 0.88%. Therefore, reducing the excess ratio requires a relatively conservative approach to use in the state of turbulences. It is possible for all models assuming Value-at-Risk is equal to the 99th percentile of a distribution in the state of turbulence. Additionally, the reduction of excess ratio is possible also by the models which assume the liberal approach to forecast Value-at-Risk using the exponential distribution in the state of turbulence.

Use of any of the EWS-GARCH models presented in Tab. 3 reduce costs associated with Value-at-Risk exceedances (both based on the Lopez and Abad & Benito cost functions), the EWS-GARCH models are also more often assaying to the green zone and to at least the yellow zone due to the backtesting criterion in comparison to the GARCH(1,1) with the amendment to the empirical error distribution model. It is worth mentioning that all the EWS-GARCH(1,1) with the amendment to the empirical error distribution models are qualified in 100% of cases to the green zone, which is even more frequently than the much more conservative GARCH-t (1,1) model. A similar decrease in the excess ratio and the frequency to the green zone qualification can also be observed for the Stressed Value-at-Risk.

The improvement in all the discussed measures, as in previous cases, is associated with an increase of excess costs of using the model. Again, the excess cost grows steadily with the reduction of the excess ratio (except models in which the excessive cost is inappropriately high assuming that Value-at-Risk forecast is calculated as the 99th percentile of the exponential distribution with the 5% definition of the turbulent state).

According to the choice of the best assumptions for the EWS-GARCH(1,1) with the amendment to the empirical error distribution, again, the stepwise selection increase the quality of Value-at-Risk forecasts (the probit model as a states forecasting model is the best for all combinations of the rest assumptions).

Results for the coverage tests are presented in a Tab. 4. It can be seen that all the models with a smaller excess ratio than the GARCH(1,1) with the amendment to the empirical error distribution belong to the worse and conservative group. This means that for the analysed EWS-GARCH models, the null hypothesis in the Kupiec test is rejected more often than for the GARCH(1,1) with the amendment to the empirical error distribution, but according to the asymptotic unconditional coverage test, this is only due to the fact that for these models the excess ratios are lower than expected. Moreover, according to the same tests, it may be noted that for the EWS-GARCH models analysed, the excess ratio is never higher than expected.

7 Conclusions

To sum up all the results, it can be stated that EWS-GARCH models provide Value-at-Risk forecast with sufficient quality and can be used as Value-at-Risk forecasting models. In addition, when appropriate assumptions would be chosen, the quality of Value-at-Risk forecasts may be improved due to both the conservative and adequacy criterion. The obtained results may be generalized and summarized in the following points:

1. Consideration of two states (the state of tranquillity and turbulence) can lead to the quality Value-at-Risk forecasts improvement (for appropriately chosen assumptions concerning the state forecasting model and the Value-at-Risk model in states of tranquillity and turbulence), both in accordance to the conservative and adequacy criterion.
2. Models that take distributions with fatter tails into account improve Value-at-Risk forecasts quality (especially the EWS-GARCH and GARCH with the amendment to the empirical distribution of random errors models).
3. More conservative Value-at-Risk forecasts are provided by the EWS-GARCH models taking into account the exponential than the empirical distribution in the state of turbulence.
4. The GARCH model with Student's t distribution is very conservative and leads to very low excess ratio.
5. EWS-GARCH models taking into account the Pareto distribution in the state of turbulence are extremely conservative and leads to far too large Value-at-Risk forecasts.
6. Among all the models analysed, the most appropriate Value-at-Risk forecasts were provided by the EWS-GARCH model assuming the 5% definition of the state of turbulence and a conservative approach in calculating the Value-at-Risk forecasts in the state of turbulence for both the exponential and the empirical distribution or a liberal approach to calculate the Value-at-Risk for the exponential distribution.

It is worth comparing the results discussed with the results obtained by other researchers.

The positive impact of the state of turbulence in Value-at-Risk forecasts model was also found by Alexander and Lazar (2006). Proposed by them, the NM-GARCH models in most cases provided better forecasts of Value-at-Risk than one-state models. Slightly different conclusions were presented by Marcucci (2005). According to his results, the models that involve more than one state (MRS-GARCH) should be considered as better than one-state models only due to the predictions' quality criteria. According to the criteria for assessing the quality of Value-at-Risk forecasts, better results are achieved by one-state models.

Indirectly, similar conclusions can be drawn from the McAleer *et al.* (2013) and Degiannakis *et al.* (2012). In their research, it is indicated that different models are the best for Value-at-Risk forecasting in the state of tranquillity and different during the state of turbulence. This duality of choice shows that it is worthwhile to consider models that allow the inclusion of two states.

The high quality of the Value-at-Risk forecasts based on models that take into account fatter tails distributions is indicated by many researchers. Such results were obtained, among others, by Angelidis *et al.* (2006), Ozun *et al.* (2010), Dimitrakopoulos *et al.* (2010) and most of the researchers and the results are described in Abad *et al.* (2014).

High quality of the Value-at-Risk forecasts based on models that take a tail distribution or the empirical distribution into account leads to a conclusion that comes from some new research. This is one of the main conclusions that were formulated by Abad *et al.* (2014), similar conclusions also arrive from Angelidis *et al.* (2006), Ozun *et al.* (2010) and Dimitrakopoulos *et al.* (2010).

In summary, EWS-GARCH models can be a valuable tool for forecasting Value-at-Risk, which satisfies the Basel Committee expectations. The obtained results indicate that EWS-GARCH models can improve the quality of Value-at-Risk forecasts in comparison to the benchmark models. The choice of optimal assumptions for EWS-GARCH models should depend on the goals set towards the Value-at-Risk forecasting model. The final selection may be due to adequacy, conservatism and costs of the approach. The use of the EWS-GARCH models can increase conservatism for each of the one-state equivalent, while not excessively increasing the cost of the model usage. For the EWS-GARCH (1,1), it is also possible to build a model that generates Value-at-Risk forecasts characterized by the closest excess ratio to the expected, equal to 1%.

Even though the EWS-GARCH models provide Value-at-Risk of good quality and may be used to measure market risk, there is still some room for improvements. Firstly, the states forecasting models may be extended by considering the use of additional variables or incorporating an autoregressive process into the model. Secondly, the different Value-at-Risk models in both states may be considered (other GARCH models for the state of tranquillity or distributions such as lognormal, gamma, Weibull or GDP for the state of turbulence). Another improvement may be preparing a one-step estimation process. The aforementioned extension are worth considering in the future; however, the EWS-GARCH models give promising results in the way that they were defined in the study.

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Appendix

Tab. A1. Industries and capitalizations in the end of the year 2011 of companies considered in the modelling

COMPANY NAME	ORIGIN	INDUSTRY	CAPITALIZATION (MLN €)
AMPLI S.A.	PL	WHOLESALE TRADE	€ 1
ASSECO POLAND S.A.	PL	IT INDUSTRY	€ 852
ATLANTA S.A.	PL	WHOLESALE TRADE	€ 15
ATLANTIS S.A.	PL	FINANCE - OTHER	€ 13
ATM GRUPA S.A.	PL	MEDIA	€ 21
AWBUD S.A.	PL	CONSTRUCTION	€ 32
BBI ZENERIS NFI S.A.	PL	FINANCE - OTHER	€ 12
BETACOM S.A.	PL	IT INDUSTRY	€ 3
BIOTON S.A.	PL	PHARMACEUTICAL	€ 90
BRE BANK S.A.	PL	BANKS	€ 2345
CENTROZAP S.A.	PL	METAL INDUSTRY	€ 12
CERAMIKA NOWA GALA S.A.	PL	BUILDING MATERIALS	€ 27
CEZ A.S.	FOREIGN	ENERGETICS	€ 18,043
COGNOR S.A.	PL	WHOLESALE TRADE	€ 49
DM IDM S.A.	PL	CAPITAL MARKET	€ 64
DOM DEVELOPMENT S.A.	PL	DEVELOPERS	€ 164
DUDA S.A.	PL	FOOD INDUSTRY	€ 40
ECHO INVESTMENT S.A.	PL	DEVELOPERS	€ 313
EFEKT S.A.	PL	WHOLESALE TRADE	€ 3
ELEKTRO BUDOWA S.A.	PL	CONSTRUCTION	€ 104
ELZAB S.A.	PL	IT INDUSTRY	€ 5
ENERGOMONTAŻ-POŁUDNIE S.A.	PL	CONSTRUCTION	€ 30
ENERGOPOL-POŁUDNIE S.A.	PL	CONSTRUCTION	€ 18
EUROCASH S.A.	PL	RETAIL	€ 885
FAM GK S.A.	PL	METAL INDUSTRY	€ 9
FAMUR S.A.	PL	ELECTROMECHANICAL INDUSTRY	€ 313
FARMACOL S.A.	PL	WHOLESALE TRADE	€ 121
FERRUM S.A.	PL	METAL INDUSTRY	€ 46
FORTE S.A.	PL	PULP AND PAPER INDUSTRY	€ 51
GLOBE TRADE CENTRE S.A.	PL	DEVELOPERS	€ 462
HYDROTOR S.A.	PL	ELECTROMECHANICAL INDUSTRY	€ 11
IMPEXMETAL S.A.	PL	METAL INDUSTRY	€ 158
INSTAL KRAKÓW S.A.	PL	CONSTRUCTION	€ 20
INTER GROCLIN AUTO S.A.	PL	AUTOMOTIVE	€ 14
IZOLACJA JAROCIN S.A.	PL	BUILDING MATERIALS	€ 2
KCI S.A.	PL	DEVELOPERS	€ 4
KGHM S.A.	PL	RAW MATERIALS	€ 5,008
KOGENERACJA S.A.	PL	ENERGETICS	€ 234
LPP S.A.	PL	RETAIL	€ 811
MCLOGIC S.A.	PL	IT INDUSTRY	€ 16
MENNICA POLSKA S.A.	PL	METAL INDUSTRY	€ 147
MOSTOSTAL WARSZAWA S.A.	PL	CONSTRUCTION	€ 72
MOSTOSTAL- EXPORT S.A.	PL	CONSTRUCTION	€ 7
MOSTOSTAL PŁOCK S.A.	PL	CONSTRUCTION	€ 7
MOSTOSTAL ZABRZE - HOLDING S.A.	PL	CONSTRUCTION	€ 43

Tab. A1. Industries and capitalizations in the end of the year 2011 of companies considered in the modelling (continue)

COMPANY NAME	ORIGIN	INDUSTRY	CAPITALIZATION (MLN €)
MUZA S.A.	PL	MEDIA	€ 3
NORDEA BP S.A.	PL	BANKS	€ 489
NOVITA S.A.	PL	LIGHT INDUSTRY	€ 11
OPAKOWANIA PLAST-BOX S.A.	PL	PLASTICS INDUSTRY	€ 23
ORCO PROPERTY GROER S.A.	FOREIGN	DEVELOPERS	€ 61
PBS FINANSE S.A.	PL	FOOD INDUSTRY	€ 13
PEPEES S.A.	PL	FOOD INDUSTRY	€ 16
PKO BP S.A.	PL	BANKS	€ 9090
POLCOLORIT S.A.	PL	BUILDING MATERIALS	€ 5
POLICE S.A.	PL	CHEMICAL INDUSTRY	€ 169
POLNORD S.A.	PL	DEVELOPERS	€ 73
PRÓCHNIK S.A.	PL	LIGHT INDUSTRY	€ 8
PROJPRZEM S.A.	PL	CONSTRUCTION	€ 9
PULAWY S.A.	PL	CHEMICAL INDUSTRY	€ 348
REDAN S.A.	PL	RETAIL	€ 16
SOPHARMA AD	FOREIGN	PHARMACEUTICAL	€ 211
STALEXPORT AUTOSTRADY S.A.	PL	SERVICES - OTHER	€ 68
STOMIL SANOK S.A.	PL	AUTOMOTIVE	€ 71
SUWARY S.A.	PL	PLASTICS INDUSTRY	€ 15
SWISSMED CENTRUM ZDROWIA S.A.	PL	SERVICES - OTHER	€ 8
SYGNITY S.A.	PL	IT INDUSTRY	€ 48
TELECOMMUNICATION POLSKA S.A.	PL	TELECOMMUNICATION	€ 5,210
TELL S.A.	PL	RETAIL	€ 15
TRAVELPLANET.PL S.A.	PL	SERVICES - OTHER	€ 5
TRION S.A.	PL	BUILDING MATERIALS	€ 12
ULMA S.A.	PL	CONSTRUCTION	€ 77
VISTULA GROER S.A.	PL	RETAIL	€ 20
WASKO S.A.	PL	IT INDUSTRY	€ 44
WILBO S.A.	PL	FOOD INDUSTRY	€ 2
WISTIL S.A.	PL	LIGHT INDUSTRY	€ 1
ZELMER S.A.	PL	ELECTROMECHANICAL INDUSTRY	€ 92
ZETKAMA S.A.	PL	METAL INDUSTRY	€ 27
ZO BYTOM S.A.	PL	LIGHT INDUSTRY	€ 10
ŻYWIEC S.A.	PL	FOOD INDUSTRY	€ 1,198