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Strategic option pricing[1]

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Abstract: In this paper an extension of the well-known binomial approach to option pricing is presented. The classical question is: What is the price of an option on the risky asset? The traditional answer is obtained with the help of a replicating portfolio by ruling out arbitrage. Instead a two-person game from the Nash equilibrium of which the option price can be derived is formulated. Consequently both the underlying asset's price at expiration and the price of the option on this asset are endogenously determined. The option price derived this way turns out, however, to be identical to the classical no-arbitrage option price of the binomial model if the expiration-date prices of the underlying asset and the corresponding risk-neutral probability are properly adjusted according to the Nash equilibrium data of the game.

Keywords: option pricing, game theory, Nash equilibrium.

JEL codes: G12, G13, C72.

Introduction

Traditional option pricing is based on the assumption that risk management is a single person decision game. Alternatively it could be said that it is a game against nature. The idea behind this concept is that price movements are assumed to be governed by exogenously given event uncertainties. The price movements are independent of all actions of other decision makers. The consequences are clear. Because the outcome is not affected by actions of the agents it is a matter of stochastical and statistical techniques to calculate the probabilities of potential

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outcomes. That is what Markowitz (1952) based stochastic risk management is all about. In this note the no-arbitrage equilibrium is replaced by a Nash equilibrium.[5]

Game theory is a major field of economic theory. It deals with decision-making of rational agents. A game is an abstract which is defined as a formal description of a strategic (interactive) situation that encompasses players, strategies and payoffs. Games are everywhere. The benefit of using game theory is at least twofold. Firstly, game theory is a method for better understanding the rules of a game and for finding the optimal strategy in a given market setting. Secondly, game theory can help to structure and shape interactions with a view to steer and influence the behaviour of other players towards a desirable outcome. The strategic view is essential in decision making. A Nash equilibrium is a stable situation where all players are making the best choice they can, given the choices of their rivals. It is important to find whether a best strategy for a given game exists. Therefore knowledge in game theory ensures more insight into the true nature of the real driving forces of the financial dynamic. Finding the right strategies and making the right decisions improve decision-making sustainability. One basic intention of the presented note is to separate the problem of the valuation of payoffs from the analysis of strategic interaction.

The study formulates and solves an extension of the well-known binomial approach to option pricing for the single-period case (see, e.g., Black & Scholes, 1973; Cox, Ross, & Rubinstein, 1979). Hence a two-date, two-asset, two-state economy as the starting point is taken. There is one risky asset and one risk-free asset whose initial price and rate, respectively, is exogenously given. The state of the economy at the second date is chosen according to a binomial distribution the parameter of which is also given exogenously. The final or end-of-period price of the risky asset is state-dependent. It is customary to express this price in terms of the initial price. These end-of-period prices are considered as a date of the economy. They are not affected by actions of market participants. The classical option pricing question in this framework is: What is the time-zero price of an option on the risky asset? The traditional answer is obtained with the help of a replicating portfolio by ruling out arbitrage.

Instead it is assumed that, unlike the traditional approach, there is indeed an impact on the end-of-period prices by interdependent actions of market participants. Hence a two-person game from the Nash equilibrium is formulated from which the option price can be derived. Consequently both the underlying asset's price at expiration and the price of the option on this asset are endogenously determined. The game-theoretic option price derived this way turns out, however, to be identical to the classical no-arbitrage option price of the single-period binomial model if the expiration-date prices of the underlying asset and the corresponding risk-neutral probability are properly adjusted

[5] For using game theory to model option pricing from a strategic point of view see Rubinstein (1991), Thakor (1991), Ziegler (2010) and Bieta, Broll and Siebe (2014).

according to the Nash-equilibrium data of the game. In this sense the model of the paper can be considered as an extension of the classical binomial framework opening the latter for interpersonal decision making.

Before beginning the mathematical part of the paper two possible applications of the research are briefly discussed. Firstly, a "pension fund, life insurance"—scenario is referred to (i). After that, a "shareholder; bondholder conflict"—scenario (ii) is outlined.

(i) Consider a classical life insurance contract with a one-off payment. Suppose the contract is a with-profits policy. That is to say the policyholder participates in the return generated by the insurer's investment portfolio, often up to 80%. In addition the insurance company or pension fund usually guarantees the insured a minimum rate of return on his or her capital investment. Hence the insurance company or the company's shareholders respectively, completely bear the risk of the rate of return on their investment portfolio falling short of the guaranteed rate.

From an options angle the following picture unfolds: The buyer of the insurance policy gets a share in the company's investment portfolio and an option on the portfolio's rate of return, the exercise price of which is the guaranteed rate of return; i.e., in case the portfolio's rate of return falls short of the guaranteed rate on the date of maturity the policyholder's implicit or embedded option will automatically be exercised by transferring the guaranteed amount stipulated in the contract to the policyholder's account. In other words the insurance buyer makes a one-off investment and hedges this investment with a put option sold by the insurance company.

The insurance company and the insured have divergent interests. The insurance buyer prefers a high-risk investment portfolio in contrast to the preferences of the insurance company and their shareholders who prefer a rather low-risk portfolio. The reason for this divergence of preferences can be found in the asymmetric participation of the company and the policyholder in rates of return both above and below the guaranteed rate, with the company only participating in the surplus with 20% and fully participating in rates of return below the guaranteed rate.

There are two aspects that relate this situation to the paper's highly stylized model. First, the insurance company is able to pursue their interests by independently choosing the investment strategy. This way the insurance company as seller of the (implicit) put option affects the underlying value on the maturity date; i.e., the rate of return on the investment portfolio. Moreover the insurer has to avoid the peril of mispricing the insurance contract by paying too little attention to this implicit option. The insurance premium has to take the price of the embedded put option into account. Thus the insurance company will be interested in the maximum price they can charge for this implicit option, bearing in mind that a prospective insurance buyer will not accept any surcharge. The paper's general framework can be adapted to capture this situation, eventually resulting in an interactively computed maximum surcharge.

(ii) Consider a partly leveraged company. Of course the company's debt is not risk-free: In case of default the ownership of the company would pass from the shareholders to the bondholders due to shareholders' limited liability.

From an options angle the following picture now unfolds: The bondholders implicitly have sold a put option on the company's value to the shareholders, with the face value of the debt plus necessary interest as the exercise price. However the shareholders solely decide on the investment strategy. Hence in this case it is the buyer of the option who is able to affect the underlying value on the expiration date; i.e., the company's value on the bonds' date of maturity. There are again divergent interests: Shareholders are willing to assume high-risk projects, thereby aiming at enhancing the value of equity, whereas the bondholders generally prefer a less risky investment policy. The lenders to the company should take care of a surcharge on the bond's coupon payments or should think of an additional lump-sum charge in order to account for the put option which they are implicitly selling to the shareholders. Of course the shareholders will not accept any surcharge. Therefore the lenders will be interested in the maximum price they can charge for this implicit option. The paper's general framework, when properly adapted to this situation, will eventually produce an interactively computed maximum surcharge.

The rest of this paper is organized as follows. Section 1 delineates the standard approach of option pricing for the binomial case, i.e., the no-arbitrage equilibrium. Section 2 develops the concept of strategic option pricing. The main result is derived. In Section 3 we further shed light on the properties of the solution. The final section concludes.

1. The no-arbitrage equilibrium

A call option is a right to buy one unit of the underlying asset at the strike price at the expiration date. In economics a right is characterized by a positive price. What is the market price of the option considered? In this section the standard approach of option pricing.[6] briefly reviewed There are two basic components to be taken into consideration: the concept of perfect arbitrage and the no arbitrage principle. This model is used to determine the current option price C. All prices of the underlying stock are assumed to be publicly known, i.e. the current stock price S and the future stock price S_1.

[6] For more information, see Froot, Scharfstein and Stein (1993), Wong, Filbeck and Baker (2015), Broll and Wong (2017).

A simple stochastic process is the binomial process. In the binomial world the stock price moves up and down over time, but the stock price can take only two outcomes at $t = 1$:

$$S_1 = \begin{cases} S^+ = (1+u)S \\ S^- = (1+d)S \end{cases} \tag{1}$$

where u = percentage increase (up) of the stock price in $t = 1$ with $u > 0$ and d percentage decrease (down) of the stock price in $t = 1$ with $-1 \leq d < 0$. It is assumed that the states are known to all participants in the market place.

Comparable to S_1 the expiration price of the call option can take two outcomes only:

$$C_1 = \begin{cases} C^+ \; if \; S_1 = S^+ \\ C^- \; if \; S_1 = S^- \end{cases} \tag{2}$$

The expiration price (2) is the larger of zero and the difference between the stock price at the expirations date and the strike price X:

$$C_1 = \max\{S_1 - X, 0\} \tag{3}$$

The basic idea of perfect arbitrage portfolios is as follows: It is possible to compose a portfolio consisting of the underlying stock and a risk free bond that perfectly matches the future cash flows of the option. As a result in a no arbitrage situation the option must have the same current price as the arbitrage portfolio. In other words a perfect arbitrage portfolio is a combination of securities that perfectly replicates the future cash flows of the derivative security.

The next step is to calculate the optimal combination of the underlying stock and the risk free bond in the perfect arbitrage portfolio at $t = 0$. Assuming a bond price of one, the volume of the bond investment is F and the risk free rate is r, with $u > r > 0 > d$. The volume of the stock investment is the product of the number of stocks Y times the stock price S. The result is SY.

In the simple binomial world a no arbitrage equilibrium holds if the following conditions are satisfied at $t = 1$:

$$S^+Y + (1 + r)F = C^+ \tag{4a}$$

$$S^-Y + (1 + r)F = C^- \tag{4b}$$

From (4a) and (4b) we get:

$$Y = \frac{C^+ - C^-}{(u-d)S} \tag{5}$$

$$F = \frac{1}{(1+r)} \frac{(1+d)C^{+} - (1+u)C^{-}}{(u-d)} \tag{6}$$

The present value of (4a) and (4b) is given by:

$$SY + F = C^{*} \tag{7}$$

From (5) and (6) we calculate the current option price:

$$C^{*} = \frac{1}{(1+r)} \left(q^{*} C^{+} + (1-q^{*}) C^{-} \right) \tag{8}$$

where:

$$q^{*} = \frac{r-d}{u-d} \tag{9}$$

and:

$$1 - q^{*} = \frac{u-r}{u-d} \tag{10}$$

The probabilities (9) and (10) are called quasi-probabilities or risk neutral probabilities. For the remainder of the discussion without a loss of generality $S = 1$ is assumed.

In contrast to Cox and others (1979) (in the following CRR), in this model (Section 2) the two possible expiration date prices are not exogenously given. The seller sets the expiration prices endogenously according to his strategic goals. In other words, the seller is representing the market thus giving up the assumption of exogenously given market prices. On the opposite side of the market, the buyer is making strategic investment decisions in both the underlying and the financial option. In order to determine the outcome of the strategic interaction of the players the Nash equilibrium is adopted. The Nash equilibrium is the generally accepted concept when it comes to strategic decision making (Rubinstein, 1991).

2. The non-cooperative equilibrium

This section develops the concept of strategic option pricing. The case of a call option for a single period horizon as analysed in Section 1 is still considered. However the option seller is now setting the prices of the underlying in both states along the lines of strategic reasoning. In addition probabilities to the states up and down are assigned. The state u occurs with the probability q and the state d occurs with the probability $1 - q$. The risk free rate is r. As an additional constraint we assume $u > r > 0 > d > -1$.

An individual investor (the buyer) who can choose from two alternatives at $t = 0$ denoted by (i) B: to buy the stock at the rate r and (ii) NB: not to buy the stock is considered.

Assuming the outcome in the state u is equal to $(1 + u)$ and in the state d the outcome is equal to $(1 + d)$ it is found that buying the stock means choosing the lottery $L_B = [(u - r), (d - r); q, (1 - q)]$. Alternatively not buying results in choosing a degenerated lottery with a sure payoff of zero. It is assumed that the buyer is risk neutral and, consequently, the buyer is planning to maximize the expected payoff.

Now the seller enters the stage and offers the buyer to decide on buying or not buying the stock when the financial market state will be disclosed at $t = 1$. This comes to offering the buyer a call option with the strike price $X = 1 + r$. For the buyer holding this option means being in the situation of the lottery L^* given by:

$$L^* = [(u - r), 0; q, (1 - q)]. \tag{11}$$

L^* is labelled the (call) option lottery. What is the maximum premium the buyer is willing to pay for this option?

First, the prices of the underlying are no more predetermined as the seller is able to set these prices. In other words the seller decides what prices the underlying will take at the expiration date $t = 1$. It is assumed that the set of strategies of the seller, A, is given by:

$$A = \{a : \max\{r - u, |d| - 1\} \leq a \leq r - d\} \tag{12}$$

If the seller chooses any $a \in A$, then S_1, the value of the underlying at the end of the period, takes on one of the two possible values:

$$S_1 = \begin{cases} S^+(a) = 1 + (u + a) \\ S^-(a) = 1 + (d + a) \end{cases} \tag{13}$$

To keep the model still simple only consider a 1-dimensional price strategy for the seller is considered. A price change specific to each state ($u(+)$ or $d(-)$) does not come under consideration. More formally in this model the seller only chooses $a = a^+ = a^-$ instead of $a^+ \neq a^-$.

There are three types of possible strategies: The choice of $a = 0$ means choosing inactivity, which is the roulette situation. Now the market exclusively determines the prices of the underlying in both states. Alternatively $a > 0$ means the choice of a price increasing strategy for the underlying. The price of the underlying will be raised in both states of the market. Finally, $a < 0$ means the choice of a price reducing strategy. The price of the underlying will be cut in both states. Condition (12) prevents negative prices occurring.

By assumption the buyer cannot observe the seller's choice. Both the buyer and the seller are assumed to be risk neutral. The seller's expected payoff depends on which alternative the buyer will choose at $t = 0$ in the absence of the call option offer. The payoff to the seller is assumed to be the difference of the expected value of the call option lottery and the buyer's expected payoff given alternative actions. The implicit assumption is that the seller is a premium maximizer.

Hence, the expected payoff to the buyer playing his/her (pure) strategy B (or NB) is given by:

$$A_2(a, B) = q((u + a) - r) + (1 - q)((d + a) - r) \tag{14}$$

$$A_2(a, NB) = 0 \tag{15}$$

Referring to the buyer's choice at $t = 0$ and noting that E^* is the fair value of L^* what means that the seller does not offer L^* for a price smaller than E^* the expected payoff to the seller is given by:

$$A_1(a, B) = E^* - A_2(a, B) \tag{16}$$

$$A_1(a, NB) = E^* \tag{17}$$

Let the expected payoff of the option lottery L^* be denoted by E^*, i.e.:

$$E^* = q((u + a) - r) \tag{18}$$

$$A_1(a, B) = (1 - q)(r - (d + a)) \text{ is obtained} \tag{19}$$

and

$$A_1(a, NB) = q((u + a) - r) \tag{20}$$

The preceding setting is referred to as the one period hypothetical option game with risk neutral agents. Next the question is asked: What choice of $a \in A$ will be made by the seller when aiming at a maximum premium for the call option?

First, the seller will take into consideration that his/her choice of increasing or reducing the price of the underlying will be reflected by the action the buyer will decide upon at $t = 0$ given the choice a. Second it can be observed that in equilibrium the buyer will randomize between his/her two actions if:

$$0 < q < \frac{1+r}{u-d} \tag{21}$$

holds true for the exogenously given probability q of the state u.

The intuition is as follows: Suppose to the contrary that the buyer in equilibrium chooses a pure strategy, that is to say, his/her equilibrium choice were either B or NB: (i) say, it were B. In equilibrium the choice of the seller must be a best reply to the choice of the buyer. It can be concluded from (19) that the seller's choice must be $a = |d| - 1$ or $a = r - u$, (ii) now say, the buyer's equilibrium choice were NB. Then, (20) demonstrates that the seller's best response is $a = r - d$ and (iii) however, in either case it is easy to see that the respective choice of the buyer is not an optimal answer to the seller's choice of the value of the underlying. This, however, must be the case in equilibrium: If, for instance, the equilibrium choice of the seller was $a = |d| - 1$ or $a = r - u$, then the buyer's best response is, according to (14) and (15), given by NB.

Note that in this case according to (14), the buyer's payoff is given by $A_2(|d| - 1, B) = q(u - d) - (1 + r)$ or $A_2(r - u, B) = (1 - q)(d - u)$. We conclude that $A_2(|d| - 1, B) < A_2(|d| - 1, NB)$ must hold true (see (21)). Thus, neither $(|d| - 1, B)$ nor $(r - u, B)$ can be an equilibrium. If, on the other hand, the equilibrium choice of the seller was $a = r - d$, then the buyer's best response is, according to (14) and (15), given by B: The payoff to the buyer at NB equals $A_2(r - d, NB)$. It is concluded that $A_2(r - d, NB) < A_2(r - d, B)$ (see (21)) or $A_2(r - u, B) < A_2(r - u, NB)$ must hold true. Thus $(r - d, NB)$ cannot be an equilibrium.

As a result it is known that in equilibrium the buyer will employ a mixed strategy. Each of his/her alternatives must generate the same payoff to him/her in equilibrium, for otherwise he/she could do better by not randomizing and choosing a pure strategy instead, that is to say, by choosing either B or NB. Therefore in equilibrium it must hold true that:[7]

$$A_2(a^*, B) = A_2(a^*, NB) \tag{22}$$

with a^* denoting the seller's equilibrium strategy. From this we obtain:

$$a^* = r - (qu + (1 - q)d) \tag{23}$$

If the seller employs his/her equilibrium strategy, then the value of the underlying at the end of the period, S_1, takes on one of the following two values (see (13)):

$$S_1 = \begin{cases} S^+(a^*) = (1+r) + (1-q)(u-d) \\ S^-(a^*) = (1+r) + q(d-u) \end{cases} \tag{24}$$

[7] See (i) Section 3 for an argument that a^* is indeed the best reply of the seller to the mixed equilibrium strategy of the buyer.

Furthermore, from (22) we conclude that:

$$A_1(a^*, B) = A_1(a^*, NB) \tag{25}$$

must hold true. In particular:

$$A_1(a^*, B) = E^* - A_2(a^*, NB), \tag{26}$$

which gives:

$$A_1(a^*, B) = q(1-q)(u-d) \tag{27}$$

Discounting (27) gives the strategically determined option premium:

$$C_s = \frac{1}{1+r} q(1-q)(u-d) \tag{28}$$

Now, recall that the quasi-probability in the one period option pricing formula q^*, is given by (see (9)):

$$q^* = \frac{r-d}{u-d} \tag{29}$$

The CRR quasi-probability of the single period case, q^*, satisfies condition (21) if the borderline case $d = -1$ is excluded. In particular, if the exogenously given probability for the state u, q, equals q^*, we conclude from (28) that:

$$C_s = \frac{1}{1+r}\left(\frac{r-d}{u-d}(u-r)\right) \tag{30}$$

must hold true. Furthermore, the no-arbitrage pricing formula derived in CRR entails for the single period case the option premium:

$$C^* = \frac{1}{1+r} q^*(u-r) \tag{31}$$

Drawing a comparison with (8) it can be seen that $C^+ = u - r$ and $C^- = 0$ must hold in addition. Thus the main result is obtained.

Proposition. The strategic option premium of the one period option game with risk neutral agents equals the single period no-arbitrage premium if, and only if, the exogenously given probability for the state u equals the quasi-probability in the one period option pricing formula, i.e. $C_s = C^*$, if and only if $q = q^*$.

3. Properties of the solution

In this section further details of the solution are given. Some of the features are helpful in understanding the model´s rationale.

(i) Recall that the buyer in equilibrium chooses a mixed strategy denoted by $Y^* = (y^*, 1 - y^*)$. As the buyer chooses B with the probability y^* $y^* = q$ (32) is obtained. According to $\max\{r - u, |d| - 1\} < a^* < r - d$ the seller's equilibrium choice a^* is an interior solution. $A_1(a, Y^*)$ denotes the expected payoff of the seller given the mixed strategy of the buyer. Then observe that $\frac{\partial}{\partial a} A_1(a, Y^*)$ is constant and conclude from $\frac{\partial}{\partial a} A_1(a, Y^*)$ that (32) holds.

(ii) In case $1 > q > (r + 1)/(u - d)$ and $\max\{r - u, |d| - 1\} = |d| - 1$ hold true, $(|d| - 1, B)$ constitutes a Nash equilibrium. With (19) it can be seen that the seller's best response to B is $a = |d| - 1$. Then (14) and (15) illustrate that B is the buyer's best response to $|d| - 1$ if, and only if, $q[(u + |d| - 1) - r] + (1 - q)[(d + |d| - 1) - r] > 0$ holds true. The latter is the case if, and only if, $q > (r + 1)/(u - d)$ holds true. The result is $q > q^*$ since $d - 1$; the quasi-probability in the one period option pricing formula by CRR allows of the representation $q^* = (r - d)/(u - d)$. However, plausible values of r, u and d imply $(r + 1)/(u - d) > 1$. Hence a necessary condition for $(|d| - 1, B)$ being a Nash equilibrium of the one period hypothetical option game is not satisfied in general. Thus condition (21) is not very restrictive.

(iii) In case $1 > q > (r + 1)/(u - d)$ holds but $\max\{r - u, |d| - 1\}$ equals $r - u$ (instead of $|d| - 1$), then neither $(r - u, B)$ nor $(r - d, NB)$ can be a Nash equilibrium. Again, the equilibrium is given by (a^*, Y^*) (see (23), (32)).

(iv) Given that q fulfils (21) it can be seen that $C_s = C^*$ holds if and only if q equals the risk neutral probability q^* (assuming $|d| < 1$). The distinguish the cases (a) $q^* > 0.5$ and (b) $q^* < 0.5$ are distinguished. In case (a) it can be observed that $C_s > C^*$ holds if, and only if, $1 - q^* < q < q^*$ is valid. In case (b) we obtain $C_s > C^*$ if, and only if, $q^* < q < 1 - q^*$ is true. If $q^* = 0.5$ holds then the result is $C_s < C^*$ for all q q^*.

(v) Again q is satisfying (21) is assumed. Given $q = q^*$ the result $a^* = 0$ is obtained. The result can be described as follows: If the exogenously given expectation of state u is identical with the CRR quasi-probability of state u it is found that the seller is completely passive. He/she is accepting the exogenously given expiration date prices without considering any change.

(vi) As above q fulfils (21) is assumed. The expiration date prices of the underlying are $(1 + u + a^*)$ and $(1 + d + a^*)$ in states u and d respectively. Calculating the CRR quasi-probability, $q^{\sim *}$ $q^{\sim *} = (r - d - a^*)/(u - d)$ is found. As a result the CRR premium $C^{\sim *} = q(1 - q)(u - d)/(1 + r)$ is determined. Alternatively it can be said that the strategic option premium, C_s, is identical with the CRR premium taking care of the adjusted data situation.

Conclusions

In this study a one period option pricing game is developed and presented. If the buyer of a call option bets on what will happen and the seller of a call option decides what happens, then, without a doubt, insight into the true nature of openness (risk) is required. Given the strategic behaviour of both the seller and the buyer of a call option two results are presented. Firstly it has been shown how the seller's maximum premium is calculated. Secondly it is we demonstrated that in a specific strategic setting and in the non-strategic Cox and others (1979) setting, the results are the same, if the objective probabilities for the states of the world and the so-called CRR risk neutral probabilities for the states of the world are the same.

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