sciendo

Economics and Business Review, Vol. 8 (22), No. 2, 2022: 80-91
DOI: 10.18559/ebr.2022.2.5

Market risk, value-at-risk and exponential weighting[1]

***Udo Broll*[2], *Andreas Förster*[3]**

Abstract: Banks and financial intermediaries are exposed to market risk. The aim of the paper is to explore the implications of legal requirements on market risk valuation. The focus is on the calculation of the permissible weighting factor of the concept of value-at-risk (VaR). When measuring market risk, banks and financial intermediaries may deviate from equally weighting historical data in their value-at-risk calculation and instead use an exponential time series weighting. The use of exponential weighting in the value-at-risk calculation is very popular because it takes into account changes in market volatility (immediately) and can therefore quickly adapt to VaR. In less volatile market phases this leads to a reduction in VaR and thus to lower own funds' requirements for banks and financial intermediaries. However, in the exponential weighting a high volatility in the past is quickly forgotten and the VaR can be underestimated. To prevent this banks and financial intermediaries are not completely free to choose a weighting (decay) factor. The exchange rate between Polish zloty and euro is used to estimate the value-at-risk as an example and exceptions to the general legal requirements are also discussed.

Keywords: banks, financial intermediaries, risk management, market risk, exponentially weighted moving average, weighting scheme, value-at-risk.

JEL codes: C22, E 58, G18, G28.

[1] Article received 24 February 2022, accepted 25 June 2022. We would like to thank the editor of this journal and our referees for helpful comments and suggestions. This paper is an updated and extended version of a previous working paper (WP), 04/2020, Market risk: exponential weighting in the value-at-risk calculation. CEPIE (Center of Public and International Economics), Department of Business and Economics, TU Dresden.

[2] Center of International Studies (ZIS), Faculty of Business and Economics, Technische Universität Dresden, Helmholtzstr. 10, 01062 Dresden, Germany, corresponding author: Udo.Broll@tu-dresden.de, ORCID: https://orcid.org/0000-0002-3036-2622.

[3] Faculty of Business and Economics, Technische Universität Dresden, Helmholtzstr. 10, 01062 Dresden, Germany, ORCID: https://orcid.org/0000-0002-9439-9598.

Introduction

Financial risk and returns are at the core of financial intermediation. Banks and financial intermediaries (FI) taking deposits and holding portfolios need to manage market risk, credit risk, liquidity risk and operational risk (see Baker & Filbeck, 2015; Broll, Guo, & Welzel, 2017). In the banking industry value-at-risk is a widely used instrument to measure and control risk (see e.g., Leippold, 2015). Furthermore regulators favour value-at-risk (VaR) measure. The aim of this research is to describe two legally permissible approaches to market risk valuation (equal weighting vs. exponential weighting) and their impact on banks and financial intermediaries. It is demonstrated why the weighting method plays a crucial role in determining own funds' requirements.

Calculating the VaR requires historical data such as the last 500 daily returns of a risk factor (e.g. Hull, 2015, 2021). This data can be incorporated into the calculation with equal weighting or exponentially weighting, so-called EWMA-models for Exponentially Weighted Moving Average and supported by RiskMetrics (Alexander, 2008). When using the exponential weighting method changes in the present are accorded greater weight than changes that are further in the past. This has the advantage that the VaR can adapt more quickly to volatile market phases in order to adequately capture risks (e.g., Leippold, 2015). If on the other hand the volatile phase is far in the past it is taken into account less and the VaR mainly reflects the less volatile market phase. The VaR begins to "forget" the volatile time. This raises the question of which of the two approaches (equal weighting vs. exponential weighting) should be chosen to calculate the market risk or how the weighting factor should be selected.

The rest of the paper is organized as follows. Section 1 examines how the legal requirement affects the weighting factor in market risk measurement. Section 2 delineates the concept of value-at-risk with the exchange rate between the Polish zloty and euro. Exponential weighting in the value-at-risk calculation is demonstrated in Section 3. Weighting of trading days in value -at-risk calculation is presented in Section 4. The main point of reference of this study is European financial market regulation. Exemptions for a specific German alternative investment fund institution are discussed in Section 5. The final section concludes.

1. Market risk measurement and legal requirement

Banks and financial intermediaries (FI) are not completely free to choose the weighting factor in market risk measurement. Article 365 Regulation (EU) No 575/2013 requires for market risk measurement: "1. The calculation of the value-at-risk number referred to in Article 364 [Regulation (EU) No 575/2013] shall be subject to the following requirements: […] (d) an effective historical obser-

vation period of at least one year except where a shorter observation period is justified by a significant upsurge in price volatility". The European Banking Authority (EBA) explains what is meant by this (EBA, 2015, p. 72) in Article 38 (Observation period):

> Where competent authorities verify that the VaR numbers are computed using an effective historical observation period of at least one year, in accordance with point (d) of Article 365(1) of Regulation (EU) No 575/2013, competent authorities shall verify that a minimum of 250 business days is used. Where institutions use a weighting scheme in calculating their VaR, competent authorities shall verify that the weighted average time lag of the individual observations is not less than 125 business days.

The EBA equally requires an effective historical observation period of at least one year or alternatively 250 working days (equal to 250 observations). Of course banks and FI remain free to use a longer observation period. The requirement also allows the use of different weighting schemes. In the event of a deviation from the equilibrium the weighted average must not fall below the period of six months (Balance Point, BP). This is achieved if no weighting is applied at all: in this case all historical observations are included in the calculation in the same way and with full weight. Therefore, the "balance point" which divides the weighted observations into two equal parts is at the time of six months. The same requirement applies when using a weighting scheme. In practice weighting schemes of a more current date are used. Recent observations have a higher impact on the VaR than observations in the past. Accordingly, the higher weightings for the more recent observations must be compensated by a greater number of more recent observations to ensure the six-month balance point (see also ECB, 2019 guide to internal models—Risk-type-specific chapters).

In summary this leads to the following conclusions: a historical observation period of at least 250 days is required; within the context of exponential weighting, the balance point may not fall below the duration of six months (or 125 trading days) meaning that the most recent trading days must at least cover the last six months. If the weighting factor is set too low, the balance point shifts too much in the direction of recent history and eventually falls below the legal required length of at least 125 trading days.

2. Example: value-at-risk of exchange rate Polish euro/zloty

In Figure 1 the daily log return rates of the exchange rate euro and Polish zloty is shown. From this dataand by using the standard deviation the value-at-risk was calculated for various historical time horizons (125 days, 250 days, 500 days and 1,000 days) with a level of significance of 99% each. The longer the histori-

cal time horizon chosen , the lower the variation of the value-at-risk. A shorter historical time horizon (125 days) leads to stronger fluctuations of the VaR. In Figure 1, the historical data was weighted equally. The legal requirements for the VaR calculation of banks are set out in Article 365 Regulation (EU) No 575/2013 (CRR, Capital Requirements Regulation):

1. The calculation of the value-at-risk number referred to in Article 364 shall be subject to the following requirements:
   (a) daily calculation of the value-at-risk number;
   (b) a 99th percentile, one-tailed confidence interval;
   (c) a 10-day holding period;
   (d) an effective historical observation period of at least one year except where a shorter observation period is justified by a significant upsurge in price volatility;
   (e) at least monthly data set updates.

   The institution may use value-at-risk numbers calculated according to shorter holding periods than ten days scaled up to ten days by an appropriate methodology that is reviewed periodically.

The confidence level (99%) is thus fixed by law and cannot be varied (99th percentile). This guarantees a level playing field for banks with regard to the VaR calculation and the associated capital requirements (Basel Committee, 2019; Wahl & Broll, 2005; Broll, Sobiech, & Wahl, 2012). The VaR calcula-

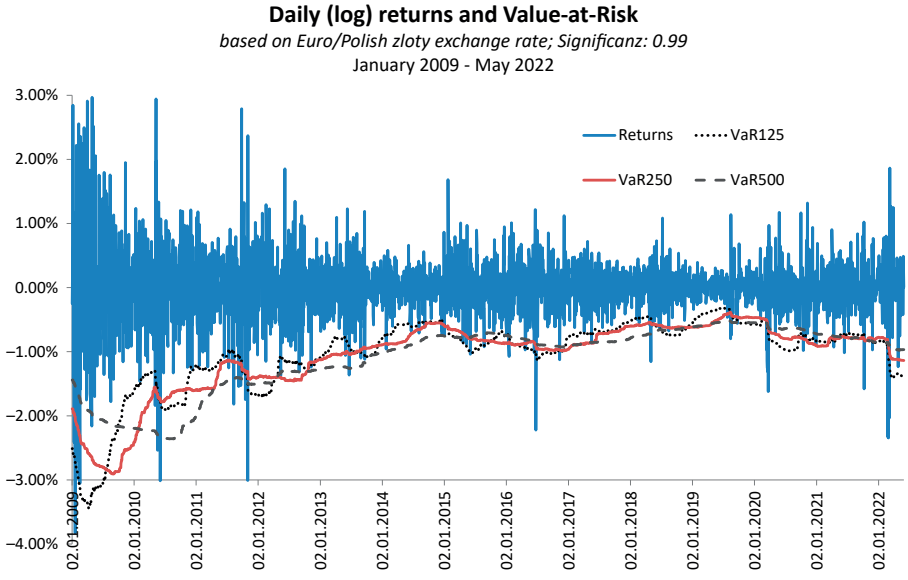

Figure 1. Daily log-return for exchange rate euro/zloty and value-at-risk (at 99% level of significance), from January 2009 to May 2022
Source: Own calculations based on (ECB, 2022).

tion is relevant for the "Own Funds Requirements for Market Risk" (Title IV (EU) No. 575/2013) and especially Article 364 Regulation (EU) No 575/2013. A higher VaR leads to higher own funds' requirements so banks want to minimize own funds' requirements and thus the VaR. However, the VaR cannot be calculated arbitrarily low.

One basic idea is illustrated in Figure 1 and demonstrated in Table 1. A wrongly chosen VaR leads to many outliersand log-return under the VaR (in Figure 1). Too many regulatory back-testing outliers lead to "multiplication factors" for the VaR according to Article 366. This is calculated and shown in Table 1 that is the outliers create a higher VaR and thus higher capital requirements. When calibrating the market risk model banks face a trade-off between reducing the VaR (low equity requirements, but also keeping the number of outliers low (e.g. by a high VaR) to avoid a (higher) multiplication factor for the VaR calculation. At a 99% level of significance 2.5 outliers are expected in 250 trading days.

Table 1. Number of outliers and multiplication factors for VaR

Number of outliers	Multiplication factor for VAR
Fewer than 5	0.00
5	0.40
6	0.50
7	0.65
8	0.75
9	0.85
10 or more	1

Source: Own calculation.

Banks and Financial Intermediaries (FI) can freely choose the underlying historical observation period. However, it has to cover at least the last year (see Article 365). A shorter period on the other hand would be more flexible in responding to shocks (see Figure 1 on the left-hand side). A long observation period leads to a very inflexible VaR but to stable forecasts regarding own funds' requirements.

For banks and FI weighting the historical time series as part of market risk measurement is an adjustment parameter (in Germany, e.g., KAGB, 2013). It is possible to deviate from an equal weighting of the historical data and instead use of an exponential weighting scheme (in Germany, e.g., Verordnung, 2013; BaFin, 2013). This weighting means that the present fluctuation will have a higher impact on calculating volatility than past fluctuations. If the exchange rate fluctuates more strongly this is immediately reflected in the market risk model

with an exponential time series weighting. The impact of weighting scheme in the exchange rate can be seen directly in Figure 2.

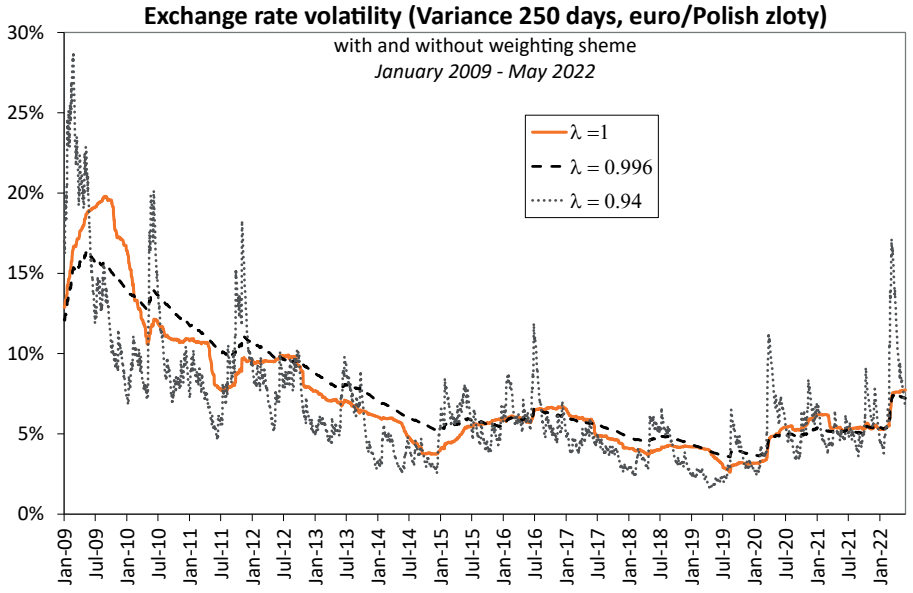

Figure 2. Exchange rate volatility (variance) based on the exchange rate euro/zloty from January 2009 to May 2022 (with and without a weighting scheme)
Source: Own calculations based on (ECB, 2022).

A risk increase (higher volatility) increases the value-at-risk and banks' capital requirements rise. In contrast, the outliers can be reduced by using an exponential weighting with an increase in volatility if necessary. Heightened risks can easily be taken into account when calculating the VaR in volatile market phases. Volatile market phases with an exponential time series weighting also become negligible. The risk can also be estimated to be lower than with an equal weighting of the time series, see Figure 2 for details.

3. Calculating the balance point

For exponential weighting a weighting factor (decay factor) λ is determined. This factor λ is less than one ($\lambda = 1$ means equal weighting). Assuming a historical observation period of 501 trading days (hence 500 rates of change) the most recent rate of change will be indexed with zero and the oldest with 499 so that 500 observations will be included in the calculation. For the weighting factor of a given day t applies t^{λ} thus the latest rate of change is included one hundred percent into the risk calculation: $0^{\lambda} = 1$. The smaller the decay fac-

tor chosen the more the changes in the past are "discounted" and the less they are included in the risk calculation. Figure 3 illustrates this with different decay factors. With a decay factor of 0.94 historical trading days which are more than four months (80 days) in the past become virtually irrelevant for the VaR calculation. To prevent this from happening the legislator has issued the above requirements.

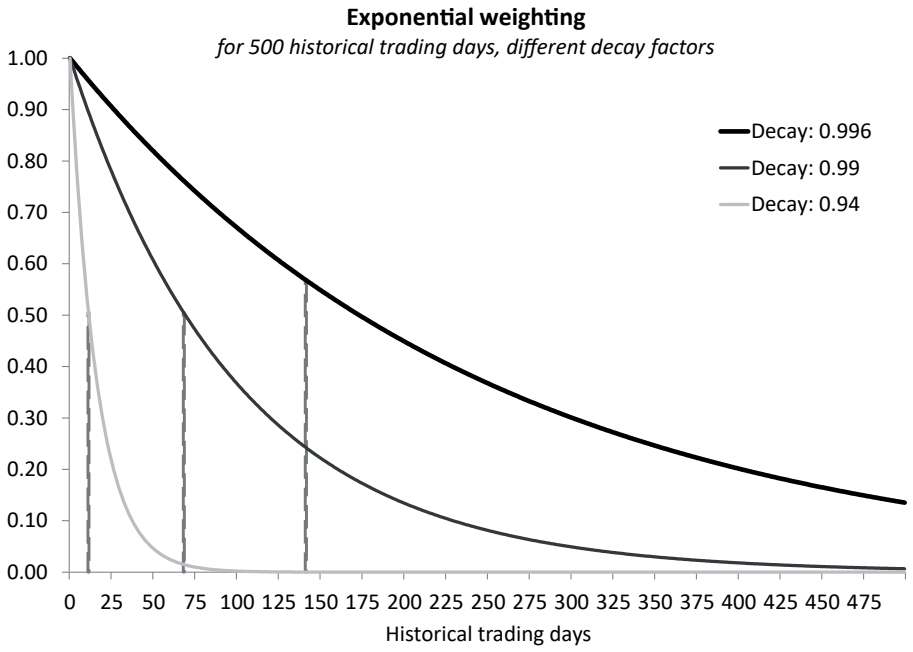

Figure 3. Exponential weighting of historical trading days for different weighting factors
Source: Own calculation.

The balance point (BP) is the day on which the weighted observations are separated into two parts whereas the more recent observations (e.g., trading days 0 to 141) equals the older observations (trading days 142 to 499 shown in Figure 3) for a total of 500 historical data points. The impact of different weighting factors in case of exponential weighting is illustrated in Figure 3.

Since the younger observations have a higher impact on VaR than the older ones more trading days in the distant past must be included in the calculation. Expressed mathematically the definite integral of data point 0 up to the balance point (BP) must correspond to the definite integral of trading days from the balance point (BP) to the last observation data point (499). To calculate the definite integral of the total period:

$$\int_0^{499} e^{-(1-)\cdot t}\,dt$$

applies where T represents the trading day and λ the decay factor. The result is divided by 2 and the area is obtained from day 0 to the balance point (or from BP to the oldest observation). If e.g. a decay factor of $\lambda = 0.996$ is used, this results in a value of approximately 108 (or a total surface area of approximately 216). Now the day of the balance point (BP) can be calculated:

$$108 = \int_0^{BP} e^{-0{,}004\cdot t}\,dt$$

The balance point (BP) is at a decay factor of 0.996 on trading day $t = 141$ (and with 500 historical trading days). Since the recent past is more than a half year (125 trading days) the choice of this decay factor is permissible. With $\lambda = 0.99$, the balance point is reached after 68 trading days; this decay factor would therefore be inadmissible. The same applies to the decay factor of 0.94 (as proposed by RiskMetrics, see J.P.Morgan/Reuters, 1996) where the balance point would be on the eleventh trading day and would therefore be completely unacceptable.

According to the legal requirements, an exponential weighting for a historical observation period of 250 days is not allowed for the VaR calculation since the balance point would always be less than 125 days. Longer (than 500 days) historical observation periods allow only a slightly lower decay factor than 0.996. In addition, it would not make sense to outweigh a smaller decay factor with a longer time series.

4. Weighting of trading days

In addition to the previous approach weighting or—for the calculation of the balance point—cumulative weighting can be considered for each trading day. If a decay factor $\lambda = 0.996$ is chosen for 500 historical trading days, the initial weight α will be $\alpha = (1 - \lambda)/(1 - \lambda)^{500}) = 0.46\%$, which is well above 0.2% and would result in equal weight. If all data are equally weighted each trading day is weighted at 0.2% for a historical observation period of 500 days. Figure 4 helps explain this—a straight line is given for the presentation of the cumulative equal weighted trading days.

With a weighting of $\lambda = 0.996$, the most recent day has a weight of 0.46% ($\alpha \cdot \lambda^0 = 0{,}46\% \cdot 0{,}996^0 = 0.46\%$), while the oldest observation weighs only $\alpha \cdot \lambda^{499} = 0.0046 \cdot 0.996^{499} = 0.06\%$ assigned. The sum of all daily weights (days 0 to 499) advises. At the balance point the younger and the older past cumu-

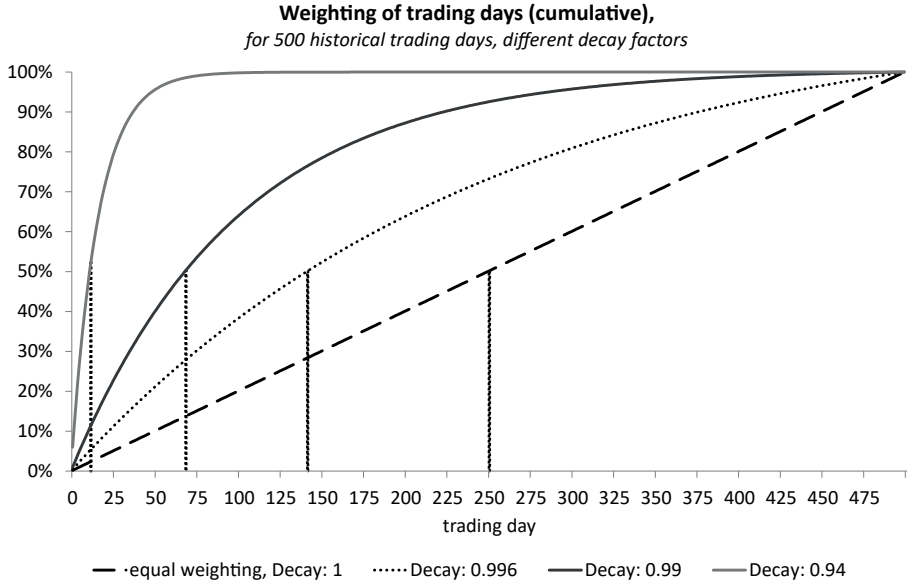

Figure 4. Cumulative weighting of historical trading days with different weighting factors
Source: Own calculation.

latively each weigh about 50%. Due to the integerity problem (here a full day) the recent past is slightly larger than 50% up to the balance point.

With a decay factor of 0.996 the balance point would be on day 141 of the time series: on this day the cumulative weight of trading days 0 to 141 (about 50.16%) corresponds to the weight of the trading days 142 to 499 (almost 50%). If the decay factor were chosen to be smaller the weight of the recent past is higher ($\lambda = 0.99$ or $\lambda = 0.94$) and the balance point in Figure 4 would continue to shift to the left. Thus, the recent past would be overweighted and the requirement of the CRR (balance point after 125 trading days) would be violated.

5. Exemptions

Due to the fact that capital and financial markets in the European Union (EU) are not fully integrated some national financial institutions and country specific financial regulations exist. In general banking regulation theory distinguishes between the regulation of structure and the regulation of conduct (see Freixas and Rochet, 2008).

First there is one exemption in the banking sector within the EU. No. 2 Article 38 EBA (2015, p. 72) explains:

> Where according to point (d) of Article 365 (1) of Regulation (EU) No 575/2013 the calculation of the VaR is subject to an effective historical observation period of less than one year competent authorities shall verify that the institution has in place procedures to ensure that the application of a shorter period results in daily VaR numbers greater than daily VaR numbers computed using an effective historical observation period of at least one year.

It is questionable whether banks will voluntarily make use of this exemption in stressed market phases: a stronger weighting of a recent volatile past will lead to a higher VaR and thus to increased own funds requirements (Article 364 CRR Regulation No. 575/2013). For this reason there is no incentive to recognize the risk for banks in stressful market phases at this point. The exemption should not result in the weighted VaR being lower than the unweighted VaR (EBA, 2015, p. 72).

Banks and financial intermediaries calculate the market risk potential under stress conditions as a second reference point in accordance with Article 365 (2) CRR Regulation No. 575/2013—but here without the inclusion of weighted time series data (EBA, 2012). However, there is no hybrid approach as this would lead to leaps in the capital requirement. In addition to the increased calculation effort for a VaR with an exponential time series weighting, the VaR is "biased" as it does not simply serve as a risk indicator but is used in the banking sector for the capital requirements: a volatile VaR is not always desirable from this point of view. A more flexible VaR can also help to reduce the outliers in back-testing the VaR and thus reduce the capital requirements.

Second there is another exemption. One well known institution in Germany, i.e., the Federal Financial Supervisory Authority, called BaFin is discussed. The German Derivatives Regulation provides for an exemption for Alternative Investment Fund Manager (AIFM): it is possible to derogate from the weighting scheme due to exceptional market conditions and the prior approval of BaFin, 2013. This would allow for an effective balance point of less than six months under certain circumstances when using exponential weighting. Such exponential weighting which results in a lower balance point may be allowed in the context of extreme market conditions characterized by exceptionally volatile markets. As recent observations become more influential the volatility estimate will increase more rapidly in increasingly volatile markets. On the other hand such a weighting also leads to a greater drop in the volatility estimate in calming markets. In this respect, a precise balance must be made here and the balance point at least be not too short. In order to prevent the exponential weighting from being used to widen the market risk limit the BaFin (2013) requires prior approval. The suitability of the model must be substantiated in the corresponding application. The BaFin can combine the approval with conditions.

The requirements of the German Derivatives Regulation and the CRR are intended to ensure that the past is not forgotten too quickly in the VaR calcu-

lations. A hybrid approach would also be interesting for banks and FI, with the VaR calculated using equally weighted and exponentially weighted data. This means that two risk ratios would have to be determined which would involve enormous computational effort. However, the calculation would be helpful for investment management companies to obtain a second reference point for the risk assessment; i.e., a risk assessment for a longer history (equal weighting) and an estimate for the most recent past (weighted time series).

Conclusions

In banking and finance the most important risk measure is the value-at-risk. This study sheds light on market risk and value-at-risk for banks and financial intermediaries. The focus is specifically on this risk measure because of its practical relevance in the banking industry. Furthermore, the European financial regulation favour value-at-risk as a risk measure. In the process of measuring financial market risk banks and other financial intermediaries may deviate from equally weighting historical financial data in their value at risk calculation. For the banking industry the use of weighting historical time series can be seen as an adjustment parameter. In case of exponential weighting when financial markets are less volatile this leads to a reduction in value-at-risk and thus lowers own funds' requirements. In order to prevent this observation banks and other financial intermediaries are regulated and controlled when they choose a weighting factor (Admati & Hellwig, 2013; Kakati & Roy, 2021). One further research question would be to derive an endogenous factor indicating which weighting might be optimal for banks and financial intermediaries.

This study described the impact of the EU und German legal framework on the calculation of the value-at-risk for financial portfolios. The analysis has shown that in order to satisfy the regulatory requirements for market risk measurement (EWMA-models) the decay factor should not be too small; at 500 historical rates of change it must be around 0.996. This fulfills the legal requirement of a balance point after 125 business days. A small weighting factor of about 0.94 is not permitted by law. An exponential weighting of the historical time series over an observation period of 250 days is also inadmissible. Even a weighted time series will not be able to replicate an increase in volatility due to behavioural risks in the risk model within seconds however risks following a shock can be processed more quickly and if necessary more adequately in the market risk model. An advantage of the risk assessment seems to be calculating the value-at-risk with a weighted and an unweighted historical time series as there are two reference points for the risk assessment.

References

Admati, A., & Hellwig, M. (2013). *The bankers' new clothes*. Princeton: Princeton University Press.

Alexander, C. (2008). *Market risk analysis*. Vol. 4: *Value at risk models*. New York: Wiley.

BaFin. (2013). Erläuterungen zur Derivateverordnung in der Fassung vom 16. Juli 2013, Bonn/Frankfurt. Retrieved from: https://www.bafin.de/dok/7863988

Baker, H. K., & Filbeck, G. (2015). *Investment risk management*. Oxford: Oxford University Press.

Basel Committee on Bank Supervision. (2019). Minimum capital requirements for market risk. Basel.

Broll, U., Guo, X., Welzel, P. (2017). Risk sharing markets and hedging a loan portfolio: A note. *Economics and Business Review*, *3*(17), 47–54.

Broll, U., Sobiech, A., & Wahl, J. E. (2012). Banking firm, equity and value at risk. *Contemporary Economics*, *6*, 50–53.

EBA—European Bank Authority. (2012). *Guidelines on stressed value at risk (stressed VaR)*. EBA/GL/2012/2. Paris.

EBA—European Bank Authority. (2015). *Consultation Paper EBA/CP/2015/27*. Paris.

ECB—European Central Bank. (2019). *ECB guide to internal models*. Frankfurt.

ECB—European Central Bank. (2022). *Eurosystem, statistical data*. Frankfurt.

European Union. (2013). Regulation (EU) No 575/2013 of the European Parliament and of the Council of 26 June 2013 on prudential requirements for credit institutions and investment firms and amending Regulation (EU) No 648/2012 (Document 32013R0575). Official Journal of the European Union.

Freixas, X., & Rochet, J. C. (2008). *Microeconomics of banking* (2nd ed.). Cambridge, MA: MIT Press.

Hull, J. C. (2015). *Risk management and financial institutions* (4th ed.). New York: Wiley.

Hull, J. C. (2021). Options, futures, and other derivatives (11th ed.). New York: Wiley.

J.P. Morgan/Reuters. (1996). *RiskMetrics—Technical Document* (4th ed.). New York.

Kakati, S., & Roy, A. (2021). Financial sustainability: An annotated bibliography. *Economic and Business Review*, *7*(3), 35–60.

Kapitalanlagegesetzbuch (KAGB). (2013). Retrieved from: https://www.gesetze-im-internet.de/kagb/BJNR198110013.html

Leippold, M. (2015). Value-at-risk and other risk measures. In: H. K. Baker & G. Filbeck (Eds.), *Investment risk management* (Chapter 15). Oxford: Oxford University Press.

Verordnung über Risikomanagement und Risikomessung beim Einsatz von Derivaten, Wertpapier-Darlehen und Pensionsgeschäften in Investmentvermögen nach dem Kapitalanlagegesetzbuch (Derivateverordnung—DerivateV). 2013. Retrieved from: https://www.gesetze-im-internet.de/derivatev_2013/BJNR246300013.html

Wahl, J. E., & Broll, U. (2005). Value at risk, bank equity and credit risk. In: M. Frenkel, U. Hommel & M. Rudolf (Eds.), *Risk management* (pp. 159–168). Berlin, Heidelberg: Springer.