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Change-point detection in CO_2 emission-energy consumption nexus using a recursive Bayesian estimation approach

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ABSTRACT

This article focuses on the synthesis of conditional dependence structure of recursive Bayesian estimation of dynamic state space models with time-varying parameters using a newly modified recursive Bayesian algorithm. The results of empirical applications to climate data from Nigeria reveals that the relationship between energy consumption and carbon dioxide emission in Nigeria reached the lowest peak in the late 1980s and the highest peak in early 2000. For South Africa, the slope trajectory of the model descended to the lowest in the mid-1990s and attained the highest peak in early 2000. These change-points can be attributed to the economic growth, regime changes, anthropogenic activities, vehicular emissions, population growth and industrial revolution in these countries. These results have implications on climate change prediction and global warming in both countries, and also shows that recursive Bayesian dynamic model with time-varying parameters is suitable for statistical inference in climate change and policy analysis.

Key words: dynamic model, Bayesian inference, CO_2, climate change, energy.

1. Introduction

A major reason for the burgeoning popularity of the recursive Bayesian estimation approach where new estimates are required each time a new measurement arrives in empirical science is the increasing prominence of numerical simulations by computational algorithms, which relies heavily on the Markov chain Monte Carlo methods (Ng and Young, 1990). Significant breakthroughs in the application of recursive Bayesian models with time-varying parameters in econometrics has been recorded by the works of authors like Pollock (2003), Chow et al. (2011), Del Negro and Otrok (2008) and Young (2011).

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Recursive Bayesian algorithms are mainly based on statistical dependence of random variables. Two random variables, say x and y, are statistically independent if and only if their joint distribution is equal to the product of their marginal distribution.

$$p(x, y) = p(x)p(y) \tag{1}$$

intuitively,

$$p(x, y) = p(x)p(y|x) = p(y)p(x|y) \tag{2}$$

represent the fact that the conditional distribution of one random variable, given the other, is not a function of what it is being conditioned on. So,

$$p(x|y) = p(x) \tag{3}$$

and

$$p(y|x) = p(y) \tag{4}$$

This definition can easily be extended to more than two random variables. The joint distribution of a collection of independent random variables is the product of their marginal distributions (Petris et al., 2009; Hillebrand and Koopman, 2016). Basically, all conditional distributions are independent of the random variables they are conditioned on. In statistical dependence, knowledge of x tells us something about y. Using Bayes' rule, we can easily show that the reverse is also true, i.e. knowledge of y also tells us something about x. Another important definition to consider is that of conditional independence. Two random variables x and y are conditionally independent given another random variable z if and only if:

$$p(x, y|z) = p(x|z)p(y|z) \tag{5}$$

or equivalently,

$$p(x|y, z) = p(x|z) \tag{6}$$

$$p(y|x, z) = p(y|z) \tag{7}$$

In Bayesian analysis, the simplest dependence structure is conditional independence. It can be assumed, in many applications, that the random observations $y_1, \dots, y_n$ are conditionally independent and identically distributed given the parameter θ. Mathematically,

$$f(y_1, \dots, y_n|\theta) = \prod_{t=1}^{n} (y_t|\theta) \tag{8}$$

In particular,

$$p(y_1, y_2) = \int p(y_1, y_2|\theta)p(\theta)d\theta$$
$$= \int p(y_1|\theta)p(y_2|\theta)p(\theta)d\theta$$

Suppose that the observations $y_1, \dots, y_n$ provide us information about the unknown parameter θ, and through θ we can also obtain information about the next observation $y_{n+1,}$ then y_{n+1}depends on the past observations $y_1, \dots, y_n$ in a probabilistic sense (Petris et al., 2009). Furthermore, the predictive densities for the case above can be computed as

$$(y_{n+1}|y_1, \dots, y_n) = \int f(y_{n+1}, \theta|y_1, \dots, y_n)d(\theta) \tag{9}$$

$$= \int f(y_{n+1}|\theta)\pi(\theta|y_1, \dots, y_n)d(\theta) \tag{10}$$

where $\pi(\theta|y_1, \dots, y_n)$ is the posterior density of θ, conditional on the data $(y_1, \dots, y_n)$. The posterior density can be computed by Bayes' formula as noted earlier.

$$\pi(\theta|y_1, \dots, y_n) = \frac{f(y_1, \dots, y_n|\theta)\pi(\theta)}{p(y_1, \dots, y_n)} \; \alpha \prod_{t=1}^{n} (y_t|\theta)\, \pi(\theta) \tag{11}$$

where the marginal density $p(y_1, \dots, y_n)$ is playing the role of a normalizing constant and hence does not depend on θ. More so, with the assumption of conditional independence, the posterior distribution can be computed recursively. This implies that we do not need all the previous data to be kept in storage and processed every time a new measurement is taken.

At times (n – 1), the information available about the parameter θ can be described by the conditional density

$$\pi(\theta|y_1, \dots, y_{n-1}) \; \alpha \; \prod_{t=1}^{n-1} (y_t|\theta)\,\pi(\theta) \tag{12}$$

The density (12) above can then play the role of prior at time n. Once the new observation y_n becomes available, we would just compute the likelihood, which is given as

$$f(y_n|\theta, y_1, \dots, y_{n-1}) = f(y_n|\theta) \tag{13}$$

by the assumption of conditional independence and then update the prior $\pi(\theta|y_1, \dots, y_{n-1})$ by Bayes' rule to obtain

$$\pi(\theta|y_1, \dots, y_{n-1}, y_n) \; \alpha \; \pi(\theta|y_1, \dots, y_{n-1}) f(y_n|\theta) \; \alpha \; \prod_{t=1}^{n-1} f(y_t|\theta)\,\pi(\theta) f(y_n|\theta) \tag{14}$$

which is equivalent to equation (12). This recursive structure of the posterior distribution is crucial in the study of dynamic linear models with time-varying parameters, which is used to analyze climate variables in this present work. The treatment of recursive regression has a Bayesian flavour and relies on the calculus of conditional expectations, whose essentials have been provided in equation (1) – (7). Essentially, this article presents how the nexus of energy consumption and carbon dioxide emission can be studied and predicted using the recursive Bayesian estimation approach.

Essentially, the main objective of this study is to contribute to global warming and climate change research by investigating the change points in CO_2 emission-energy consumption nexus over time in two major African countries using a new modified recursive Bayesian estimation approach of (Awe and Adepoju, 2018), which has been found to be computationally less intensive. To achieve this objective, we apply a model where the observational variance in the Bayesian dynamic linear model of West and Harrison (1997) is constant and the evolution variance, which is time-varying, is represented as a fraction of the filtering variance. This present work is a continuation and application of our previous work.

2. State space representation of the Bayesian Dynamic Linear Model (DLM)

The Bayesian state space model has been defined by Petris et al. (2009), Awe et al. (2015) as one which consists of an R^p-valued time series $\beta_t: t = 1,2,\dots,T$ and an R^k-valued time series $y_t: t = 1,2,\dots,T$, which satisfies the following assumptions:

- β_t is a Markov chain
- Conditional on β_t, the $y_t's$ are independent and depend on β_t only.

State space models (dynamic linear models, in particular) are useful for modelling time-varying scenarios (Doh and Connolly, 2013), whose applications exist heavily in environmental science, economics and engineering. An archetypical dynamic linear (state space) model takes the following general form:

$$y_t = F_t'\beta_t + v_t \qquad v_t \sim N(0, V_t) \tag{15}$$

$$\beta_t = G_t\beta_{t-1} + w_t \qquad w_t \sim N_P(0, W_t) \tag{16}$$

$$\beta_0 \sim N_P(m_0, C_0) \tag{17}$$

where y_t is a vector of observed time series of dimension $m \times 1$.

Equation (15) is known as the observation equation, while equation (16) is a first order Markov process called the evolution equation. G_t and F_t are known matrices of order $p \times p$ and $m \times p$ respectively, which determine how the observation and state equation evolve in time (Lee, 2012). B_t are Markov-modulated time-varying parameters, which are to be estimated and whose structural behaviour we want to study. It also contains time-varying intercepts $\propto_t$, which were estimated. The matrices $F_t, V_t, G_t\, and\, W_t$ are known as the system matrices and contain non-random elements. If they do not depend deterministically on t, the state space system is time invariant, otherwise they are time varying. The initial state distribution is assumed to be normally distributed with parameters $m_0\, and C_0$ as shown in (17), where E$(v_t\beta_t') =$ 0, E$(w_t\beta_t') = 0$ for $t = 1,2,\dots,T$. It is often convenient to study the properties of a process when the model is in the state space form because of the Markovian property of the model which assumes that the knowledge of the present state is relevant to the predictions about the future of the system, although additional information about the past state is irrelevant.

3. Recursive estimation of model parameters

Assuming normality, we estimated βt and f_t by using the Kalman filter algorithm (Kalman, 1960). V_t is assumed to be fixed and distributed inverse-gamma a priori and is estimated using a Gibbs sampler. We propose the use of discount factors to estimate w_t in the spirit of Awe et al. (2015).

Basically, the estimation of the Bayesian state space model involves three important stages: prediction, filtering and smoothing. Prediction has to do with forecasting future values of the time-varying state parameters. Filtering makes the best estimate of the current values of the state from the record of observations including the current observation. Smoothing involves making the best estimate of past values of the states given the record of observations. Suppose data $d_t$obtained till time t is represented as $D_t = (D_{t-1}, y_t)$ meaning combination of data until time t – 1 and observations at time t. Using Bayes' formula, and supposing the parameter of interest is β_t, we have

$$\pi(\beta_t)|D_{t-1}, y_t) = \frac{\pi(y_t|\beta_t, D_{t-1})\pi(\beta_t|D_{t-1})}{\pi(y_t|D_{t-1})}$$

$$\pi(y_t|\beta_t|D_{t-1}) = \pi(y_t|\beta_t) \tag{18}$$

and

$$\pi(\beta_t|D_{t-1}, y_t)\ \alpha\ \pi(y_t|\beta_t)\pi(\beta_t|D_{t-1})$$

where $$\beta_t|D_t \sim N(m_t, C_t).$$

The prior distribution for β_t is

$$\beta_t|D_{t-1} \sim N(a_t, R_t),$$

The likelihood is

$$y_t|\beta_t \sim N(F_t\beta_t, V_t).$$

The posterior for β_t is

$$\beta_t|D_t \sim N(m_t, C_t).$$

m_t and C_t are iteratively computed for $t = 1, \dots, n$ from the following Kalman filtering equations:

$$a_t = G_t m_{t-1}$$

$$f_t = F_t a_t \tag{19}$$

$$R_t = G_t C_{t-1} G_t' + w_t \tag{20}$$

$$Q_t = F_t' R_t F_t + V_t \tag{21}$$

$$e_t = y_t - f_t \tag{22}$$

$$A_t = R_t F_t Q_t^{-1} \tag{23}$$

$$m_t = a_t + A_t e_t \tag{24}$$

$$C_t = R_t - A_t Q_t A_t' \tag{25}$$

(26)

From the algorithm, f_t is the predicted value of observation at time t+1. e_t represents the prediction error. In order to predict from the posterior estimate of the state parameter β_t, a one-step-ahead prediction is made as follows:

$$E(y_{t+1}|D_t) = E(F_{t+1}\beta_{t+1} + v_{t+1}|D_t) \tag{27}$$

$$= F_{t+1}E(\beta_{t+1}|D_t)$$

hence,

$$F_{t+1}' a_{t+1} = f_{t+1} \tag{28}$$

also,

$$V[y_{t+1}|D_t] = V[F_{t+1}\beta_{t+1} + V_{t+1}|D_t] \tag{29}$$

$$= F_{t+1}V[\beta_{t+1}|D_t]F_{t+1} + V_{t+1}) \tag{30}$$

$$= F_{t+1}R_{t+1}F_{t+1}+V_{t+1} \tag{31}$$

$$= Q_{t+1}$$

For the smoothing aspect, we used the Recursive Forward Filtering Backward Smoothing algorithm of Carter and Kohn (1994) and Awe et al. (2015), where the best estimates of past values of the states given the record of observations are obtained using the rule of total probability:

$$\pi(\beta_{t-1}|D_t) = \int \pi(\beta_{t-1}|\beta_t, D_t)\,\pi(\beta_t|D_t)d\beta_t \tag{32}$$

and

$$\pi(\beta_{t-1}|\beta_t, D_t) = \pi(\beta_{t-1}|\beta_t, D_{t-1}) \tag{33}$$

and

$$\pi(\beta_{t-1}|\beta_t, D_{t-1}) = \frac{\pi(\beta_t|\beta_{t-1}, D_{t-1})\pi(\beta_{t-1}|D_{t-1})}{\pi(\beta_t|D_{t-1})} \tag{34}$$

given that,

$$\beta_{t-1}|D_t \sim N(a_{t-1}, R_{t-1})$$

where

$$a_{t-1} = m_{t-1} + \beta_{t-1}(m_t - a_t) \tag{35}$$

$$R_{t-1} = C_{t-1} - \beta_{t-1}(R_t - C_t)\,B'_{t-1} \tag{36}$$

$$B_{t-1} = C_{t-1}G'_tR_t^{-1} \tag{37}$$

We denote $p(\theta_0, \dots, \theta_T|D_T = \prod_{t=0}^{T} p(\theta_t|\theta_{t+1}, \dots, \theta_T, D_T)$, then sample form $p(\theta_T|D_T)$ using the filtering density above.

By the Markov property,

$$p(\theta_t|\theta_{t+1}, \dots, \theta_T, D_T) = p(\theta_t|\theta_{t+1}, D_T)$$

And we proceed recursively until we have a complete sample from $p(\theta_0\,, \dots, \theta_T|D_T)$

A sample from the posterior state parameter was generated using the algorithm documented in Awe and Adepoju (2018).

4. Change point detection in carbon dioxide (CO_2) emission-energy consumption nexus

Our empirical application involves the estimation of time-varying parameters of the dynamic state space model with Markov-modulated structure presented in equations (15-17) above for analyzing the nexus of annual carbon dioxide emission and energy consumption in Nigeria and South Africa over time. The data were obtained from the World Development Index (WDI) database. Carbon dioxide (CO_2) series is the endogenous variable (y_t) while energy consumption is the exogenous variable. For the estimated Bayesian dynamic model, the techniques in the previous section were used and time-varying slope parameters (β_t) were estimated for both Nigeria and South Africa (two of the richest economies in Africa in the period before the fall of the global oil price (1970-2010). It is necessary to study past trends to aid future projections.

This study is important because energy consumption has been known to be an increasing function of CO_2 emission. In fact, CO_2 emission significantly depends on energy consumption and economic growth (Sulaiman and Abdul-Rahim, 2017). This implies that CO_2 emission increases with an increase in both energy consumption and economic growth. Carbon dioxide is also known as a greenhouse gas (GHG), a gas that absorbs and emits thermal radiation, thereby creating the 'greenhouse effect'. A limited number of countries are largely responsible for the African CO_2 emissions from fossil fuels. South Africa accounts for 38% of the continental total carbon dioxide emission, while 46% comes from Nigeria, Algeria, Egypt, Morocco and Libya combined. For every tonne of coal burned (energy consumption), approximately 2.5 tonnes of CO_2 are produced into the atmosphere (Aye and Edoja, 2017). The contribution of each of these sources has changed significantly through time, and still shows large differences by regions and countries. Rapid industrial development and growth of cities in major African countries have raised the quest for increasing understanding of the correlated relationship between pollution in the form of carbon dioxide emission, and energy consumption (burning of fossil fuels).

The time plot of the variables for the two countries shown in Figures 1 and 2 reveals a random walk nature during the period under study. The African continent is likely to be severely affected by climate change and global warming effects. Major catastrophes from climate change would affect the natural resources, energy consumption and economies of African nations (Aye and Edoja, 2017). Hence, there is a need to know the change points and dates in which the effect of energy consumption affects climate change in order to make adequate projections for proper future planning and policy recommendations based on past trends in the respective countries. There are only few studies in the literature that monitor the year-to-year effect of energy consumption on CO_2 emission in Africa. Subjective Bayesian methods were proposed for use in climate modelling as early as 1997 (West and Harrison, 1997).

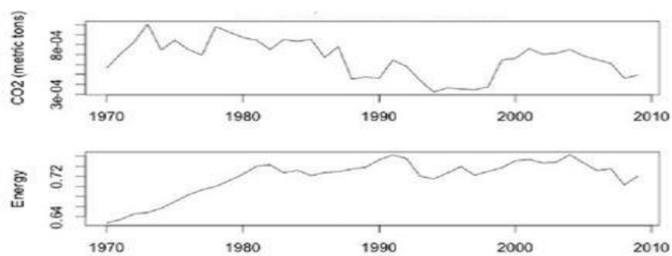

Figure 1. Time Plot of CO_2 Emission and Energy Consumption (Nigeria)

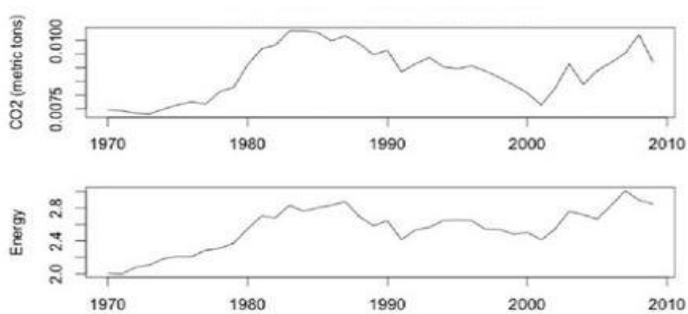

Figure 2. Time Plots of CO_2 Emission and Energy Consumption (South Africa)

The Bayesian model used in this study is therefore a suitable model which can help influence policy because many of the research statements made on climate change over the past few years took on an increasingly Bayesian flavour (Fienberg, 2011, IPCC, 2014). More so, the tradition of elicitation of expert judgments, which is what Bayesian model portends, is perfectly in line with climate policy making as suggested in the works of Morgan and Keith (1995), and Zickfeld et al. (2007). Knowing about the past trend of climate change with respect to change points and periods of intervention can aid future policies. It has been well argued that a good predictive climate model is one that is able to adequately capture the past (Parker, 2011). Therefore, this study provides a historical time-varying perspective of how the relationship between CO_2 emissions and energy consumption have evolved over time. The probable causes of seasonal variation in concentration of atmospheric carbon dioxide due to energy consumption (burning of fossil fuels) are also discussed in the present study. A key advantage of the Bayesian method used in this work is that it allows us to combine estimates from study data with relevant past information (a prior probability distribution), to derive a posterior distribution. The prior probability distribution is capable of reflecting all information available to date on the model parameters. This is only possible and realistic via the Bayesian method adopted in this study.

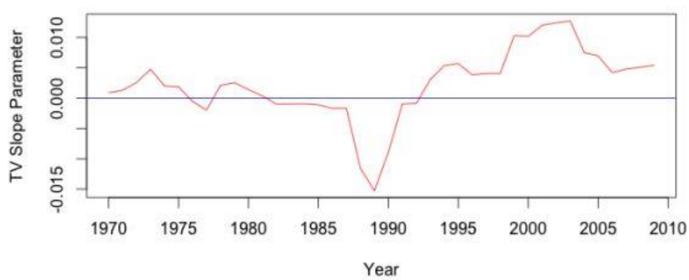

Figure 3. Time-Varying Slope (CO_2 Emission vs. Energy Consumption) for Nigeria

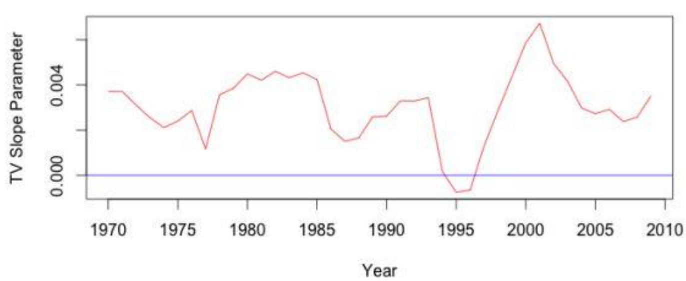

Figure 4. Time-Varying Slopes (CO_2 Emission vs. Energy Consumption) for South Africa

4.1. Result and discussion

The trend observed in Figures 3 and 4 is approximately a true reflection of the cross-sectional relationship of the observed climate variables over time across the two major countries considered. However, the trends differ for individual countries as time evolved. If these trends are viewed over the timeline from 1990 onwards, we can see that there are large variations in the evolution of carbon intensities in both countries. Figure 3 shows that the relationship between energy consumption and CO_2 emission in Nigeria experienced the lowest peak in the late 1980s while that of South Africa (Figure 4) experienced the lowest peak in the mid-1990s. It, however, experienced the lowest peak in the mid-1990s around the period of independence in South Africa. Also, from early 2000, it can be shown that South Africa and Nigeria had a positive and significant impact on environmental degradation through the continuous release of carbon dioxide (CO_2) emission as revealed by the positive slopes. This increase (peak) within the studied period can be attributed to economic growth, anthropogenic activities, vehicular emission as a result of rural-urban migration, population growth, proliferation of industries and provision of social amenities with respect to new carbon emitting technologies. It also shows that the slopes exhibit a seasonal and random walk pattern from 1970 in both countries. These results agree with the use of CO_2 emissions as a pollution indicator, GDP and the production of nuclear electricity as economic indicators as reported by Baek and Pride (2014), who used the vector autoregressive co-integrated model and Johnsen cointegration during a study of countries in the major nuclear production within the period 1990–2011.

5. Concluding remarks

In this study, we have proposed a Bayesian time-varying parameter dynamic state space regression model for change-point detection in dynamic environmental and climatic processes. The model was estimated via a recursive Bayesian estimation approach for obtaining time-varying parameter shifts and change-point detection. It was observed that the contribution of energy consumption to CO_2 emission in two of the richest economies in Africa have changed significantly and erratically through time.

The relationship between energy consumption and CO_2 emission in Nigeria (Africa's largest economy) experienced the lowest peak in the late 1980s (after the Structural Adjustment Programme (SAP)), and the highest peak in mid-2000s (during a major regime change and industrial revolution) while the slope in South Africa (14^{th} highest emitter of CO_2 in the world and second largest economy in Africa (nominal GDP)) experienced the lowest peak in the mid-1990s (around the time of their independence) and the highest peak in early 2000 (during the time of their economic boom). Recursive Bayesian estimation of Bayesian state space models with time-varying parameters is useful for statistical inference on volatility of parameter estimates and change-point detection in climate change. The empirical applications and findings in this study reveal that the increase and decrease in the relationship between carbon dioxide and energy consumption within this studied period can be attributed to economic growth, regime change, anthropogenic activities, vehicular emission as a result of rural-urban migration, population growth, proliferation of industries and provision of social amenities with respect to new carbon emitting technologies.

The need for low carbon technologies which are capable of plummeting carbon emissions and enhancing sustainable economic growth in South Africa and Nigeria is hereby recommended and emphasized. This may include policies to enhance efficiency of energy through modification from non-renewable energy to renewable energy, thereby reducing the impending greenhouse effect.

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REFERENCES

AWE, O. O., ADEPOJU, A. A., (2018). Modified Recursive Bayesian Algorithm for Estimating Time-Varying Parameters in Dynamic Linear Models. Statistics in Transition, 19(2), pp. 239–258.

AWE, O. O., CRANDELL, I., ADEPOJU, A. A., (2015). A Time-Varying Parameter State-Space Model for Analyzing Money Supply-Economic Growth Nexus, Journal of Statistical and Econometric Methods, 4(1), pp. 73–95.

AYE, G. C., EDOJA, P. E., (2017). Effect of Economic Growth on CO_2 Emission in Developing Countries: Evidence from a Dynamic Panel Threshold Model, Cogent Economics & Finance, 5(1), 1379239.

BEAK, J., PRIDE, D., (2014). On the Income–Nuclear Energy–CO_2 Emissions Nexus Revisited, Energy Economics, 43, pp. 6–10. http://dx.doi.org/10.1016/j.eneco.2014.01.015.

CARTER, C. K., KOHN, R., (1994). On Gibbs Sampling for State Space Models, Biometrika, 81(3), pp. 541–553.

CHOW, S-M, ZU, J, SHIFREN, K., ZHANG, G., (2011). Dynamic Factor Analysis Models with Time-Varying Parameters, Multivariate Behavioral Research 46(2), 303-339. DOI: 10.1080/00273171.2011.563697.

DEL NEGRO, M., OTROK, C., (2008). Dynamic Factor Analysis Models with Time-Varying Parameters, FRB of New York Staff Report 326. DOI: 10.2139/ssrn.1136163.

DOH, T., CONNOLLY, M., (2013). The State Space Representation and Estimation of Time-Varying Parameter VAR with Stochastic Volatility, Springer, 2013.

FIENBERG, S. E., (2011). Bayesian Models and Methods in Public Policy and Government Settings, Statistical Science, 26(2), pp. 212–226.

HILLEBRAND, E, KOOPMAN, S. J., (2016). Dynamic Factor Models, ISBN: 978–1785603532.

IPCC, (2014). Climate Change 2014: Synthesis Report. Contribution of Working Groups I, II and III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change [Core Writing Team, R.K. Pachauri and L.A. Meyer (eds.)], IPCC, Geneva, Switzerland, 151 pp. Available online.

KALMAN, R. E., (1960). A New Approach to Linear Filtering and Prediction Problems, Journal of Fluids Engineering, 82(1), pp. 35–45.

LEE, J., (2012). Measuring Business Cycle Co-Movement in Europe: Evidence from a Dynamic Factor Model with Time-Varying Parameters. Economic Letters, 115(3), pp. 438–440. DOI: 10.1016/j.econlet.2011.12.125.

MORGAN, M. G., KEITH, D. W., (1995). Subjective Judgments by Climate Experts, Environmental Science & Technology, 29(10), pp. 468A–476A.

NG, C. N., YOUNG, P. C., (1990). Recursive Estimation and Forecasting of Non-Stationary Time Series. Journal of Forecasting, 9(2), pp.173–204.

PARKER, W. S., (2011). When Climate Models Agree: The Significance of Robust Model Predictions, Philosophy of Science, 78(4), pp. 579–600.

PETRIS, G., PETRONE, S., CAMPNAGOLI, P., (2009). Dynamic Linear Models with R. Springer, 2009.

POLLOCK, D. S. G., (2003). Recursive Estimation in Econometrics, Computational Statistics & Data Analysis, 44(1), pp. 37–75.

SULAIMAN, C., ABDUL-RAHIM, A. S. (2017). The Relationship between CO_2 Emission, Energy Consumption and Economic Growth in Malaysia: a Three-Way Linkage Approach, Environmental Science and Pollution Research, 24(32), pp. 25204–25220.

WEST, M., HARRISON, P. J., (1997). Bayesian Forecasting and Dynamic Models. Springer-Verlag, New York, 2nd Edition.

YOUNG, P. C. (2011). Recursive Estimation and Time Series Analysis: An Introduction for the Student and Practitioner, Springer.

ZICKFIELD, K., LEVERMANN, A., MORGAN, M. G., KUHLBRODT, T., RAHMSTORF, S., & KEITH, D. W., (2007). Expert Judgements on the Response of the Atlantic Meridional overturning Circulation to Climate Change, Climatic Change, 82(3-4), pp. 235–265.