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Bankruptcy prediction of small- and medium-sized enterprises in Poland based on the LDA and SVM methods

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ABSTRACT

The impact the last financial crisis had on the small- and medium-sized enterprises (SMEs) sector varied across countries, affecting them on different levels and to a different extent. The economic situation in Poland during and after the financial crisis was quite stable compared to other EU member states. SMEs represent one of the most important segments of the economy of every country. Therefore, it is crucial to develop a prediction model which easily adapts to the characteristics of SMEs.
Since the Altman Z-Score model was devised, numerous studies on bankruptcy prediction have been written. Most of them involve the application of traditional methods, including linear discriminant analysis (LDA), logistic regression and probit analysis. However, most recent studies in the area of bankruptcy prediction focus on more advanced methods, such as case-based reasoning, genetic algorithms and neural networks. In this paper, the effectiveness of LDA and SVM predictions were compared. A sample of SMEs was used in the empirical analysis, financial ratios were utilised and non-financial factors were taken account of. The hypothesis assuming that multidimensional discrimination was more effective was verified on the basis of the obtained results.

Key words: discriminant analysis, support vector machines, bankruptcy prediction, SMEs.

1. Introduction

In Poland, the economic situation during the financial crisis 2009-2010 and after was quite stable compared to other European countries. Poland was a "green island" on the map of Europe.

The last financial crisis affected the SMEs sector in different countries at different levels and strength. Even among EU some economies suffered less compared to other ones. SMEs are the most important sector of the economy of every country. Therefore, we need a prediction model that easily adapts to SMEs characteristics.

Poland was the leader of growth among all OECD countries (Raport… 2012, p.9). Faster than ever we decreased the distance to Western Europe. We still kept high

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financial credibility, avoided the recession and dramatic currency crises or debt crises. Those phenomena deeply concerned other European countries including our central and eastern European neighbours. One of the pillars of this success was effectively operating Polish companies. Corporations were able to flexibly adjust the operations to the crises environment. Adjustment abilities among small companies were of course obvious but their expectations about finishing the recession were more visible. This behaviour is not the most effective from the company's point of view but is a visible sign of Polish transformation success. Our companies are doing as well as their highly capitalized competitors from abroad. They have the past history full of hard conditions of survival and now they are more resistant to the crisis. SMEs are mostly flexible and adjustable (due to low employment and expenses limitations) to changing economic conditions. Resistibility to crises depends also on low export level. SMEs do not use the external funding and are more conservative to expansion and in consequence are not involved in risky financial operations and big investment projects. Majority of Polish SMEs are involved in services sector which suffer the least from the crises. SMEs react on crises decreasing the employment. Orłowski et al. (2010) conducted the project to give the answer to the questions: is SME sector able to overcome this crisis by its own? Or any external help is needed? Can this crisis be helpful in improvement of enterprises functioning? Results show that for some SMEs this crisis is more a "medial" fact than real. They are more pessimistic about the influence of this crisis on the situation in the country and the sector than on an individual enterprise. Opinions are formulated not only by the enterprises but also by media and social opinions (pessimism). Only medium enterprises and exporters declare that they have to cope with real problems. Enterprises do cope with less number of orders but they do not feel lower profitability and delays in payments, which are more characteristic for the recession. Crisis is not the main threat for enterprises' functioning. More dangerous are typical difficulties like taxes, competition. Reaction among SMEs is more often passive than active like restructuring or new markets.

Since the Altman's Z-Score model (Altman 1968), many bankruptcy prediction studies have been written. Most of them use the traditional methods, like discriminant analysis, logistic regression (Back et al. 1996), probit analysis (Zmijewski 1984). Recent studies in this area are focused on more advanced methods, like case-based reasoning CBR (Bryant 1997; Yip 2006; Sartori, Mazzucchelli, Di Gregorio 2016), genetic algorithms GA (Back et al. 1996) and neural networks NN (Desai et al, 1996, Abdou et al. 2008, Derelioğlu and Gürgen, 2011, Blanco et al. 2013) or support vector machines SVM (Huang et al, 2004, Kim et al. 2010).

In their publication Sartori, Mazzucchelli, Di Gregorio (2016) used the case-based reasoning (CBR) method to forecast the bankruptcy and compared the results with the Z-Score model. The authors found that the CBR approach could be useful for clustering enterprises according to similarity metrics. Another method used for bankruptcy prediction was Genetic algorithms (GAs). Gordini (2014) compared the genetic algorithms with two other methods, namely logistic regression (LR) and support vector machine (SVM). The results suggest that GAs is a quite effective and promising compared with LR and SVM, especially in small misclassification rate type II. In this paper the size of companies and the geographical area were analysed. Both characteristics seem to influence the accuracy of the models. The author built separate models for separate geographical areas. GAs prediction accuracy in each area was higher than that of the other models separately. Sohn, Kim and Yoon (2016) applied fuzzy logistic regression for the default prediction models. According to the authors the proposed approach outperforms the logistic regression in terms of discriminatory power. Similarly, Chaudhuri and De (2011) also used the fuzzy model but using support vector machines. Those models outperformed traditional bankruptcy prediction models. According to Psillaki, Tsolas, Margaritis (2010) non-financial indicators are also useful. They proved that management is an important indicator of enterprises' risk, mostly financial risk. More efficient firms and firms with more liquid assets are much less risky. Analogous to Kalak and Hudson (2016), who emphasized the differences between small and micro firms, also Gupta et al. (2015) investigated how the SMEs size can affect bankruptcy risk. Their research results suggest that separate models for micro firms are desired. In case of small and medium companies, there is no such a need as the determinants present a similar level of hazard. A two-stage genetic programming (2SGP) model was proposed by Huang et al. (2006). This approach achieves better results. Also, Berg (2007) used accounting-based models for bankruptcy prediction. The generalized additive models are more effective compared to models: linear discriminant analysis, neural networks and generalized linear models. Modina and Pietrovito (2014) identified that capital structure and interest expenses of SMEs play more important role than economic characteristics while specifying the determinants of company's default probability. The approach presented by Andreeva et al. (2014) combines the use of Generalized Extreme Value (GEV) regression. Additionally authors compared two different ways of treating the missing values, namely multiple imputation and the Weights of Evidence approach. According to the results obtained, the BGEVA approach outperforms the logistic regression, where in the case of missing values WoE showed better results. In order to identify defaulted SMEs, Calabrese et al. (2015) investigated a binary regression accounting-based model. Results obtained suggest that their approach outperformed the classical logistic regression model for different default horizons considered.

The literature of bankruptcy prediction in Poland is very reach. When first bankruptcies were registered in 1990 (economic transition) the researchers started to be interested in this subject. It is not possible to mention all the literature from this area.

Only some of the articles can be cited. One of the first authors who apply the Altman model for Polish enterprises was Mączyńska (1994). Researchers used financial ratios at that time and built national models for the bankruptcy prediction (Wędzki 2000; Stępień, Strąk 2003; Prusak 2005, Pogodzińska, Sojak 1995; Gajdka, Stos 1996; Hadasik 1998; Wierzba 2000). They used multivariate discriminant functions based on small samples of data. Next, analysis were expanded to more frequent samples and more advanced statistical models, logit models (Hołda 2001; Sojak and Stawicki 2000; Gruszczyński 2003; Michaluk 2003; Mączyńska 2004; Appenzeller, Szarzec 2004; Korol 2004; Hamrol et al. 2004; Wędzki 2004; Stępień, Strąk 2004; Prusak, Więckowska 2007; Jagiełło 2013; Pociecha et al. 2014; Karbownik 2017). Only recently more advanced data mining and survival models have been applied (Pociecha et al. 2014; Ptak-Chmielewska 2016, Korol 2010b; Gąska 2016; Zięba et al. 2016). Many models for different sectors and sizes of enterprises have been estimated (Jagiełło 2013; Brożyna et al. 2016; Balina, Bąk 2016; Karbownik 2017). Model that used macroeconomic variables and non-financial variables have been much rarer (Korol 2010a; Ptak-Chmielewska and Matuszyk 2017). Only one of the papers included the economic cycle and suggested that enterprise bankruptcy prediction models should include measures reflecting changes in economic conditions (Pociecha and Pawełek 2011). Prusak (2018) proposed a very original method of research using a literature review as the database for rating model. His database covered a long period (Q4 2016–Q3 2017) mostly from Google Scholar and ResearchGate and wide space of Central and Eastern European countries. He collected material including countries like: Poland, Latvia, Lithuania, Estonia, Russia, Ukraine, Hungary, Slovakia, Czech Republic, Bulgaria, Romania, Belarus. Based on this literature review, he proposed the ratings (Prusak 2018, p. 17) from Rating 0 — no studies in enterprise bankruptcy prediction in a given country, up to Rating 4 — most advanced methods are utilised in enterprise bankruptcy prediction in this country and researchers proposed new solutions that affect discipline development. According to this assessment, Poland got the highest grade equal to 4.0 (Prusak 2018, p.17). Only Czech Republic got a grade as high as Poland (4.0).

The main goal of this paper is to check and verify the effectiveness of multidimensional discrimination like support vector machines in comparison with traditional linear multivariate discrimination. Empirical verification was based on the sample of data for Polish enterprises. Discriminant analysis and support vector machines methods were applied as a research tool. Multivariate discriminant analysis is based on linear discrimination. Support vector machines are based on hyperplane discrimination and can be considered as more advanced and flexible discrimination compared do linear discriminant analysis. High accuracy of prediction bankruptcy in the small and medium enterprises (SME) sector is always in scope of interest not only researchers but also policy makers and business. This paper contributes to research area on this topic.

2. Materials, Methods and Results

The database used in this paper covered 806 enterprises. The sample was quite balanced consisting of 311 bankruptcies and 495 firms in a good condition. Firms were mostly small and medium enterprises (SME) from the Polish market. The balanced sample enables to estimate the robust misclassification rates. The information about financial ratios came from financial statements covering the homogenous period of 3 years 2008-2010. The bankruptcy events were recorded for the period 2009-2012. Only standard 12-month period of observation was taken for analysis. The data sample (anonymous) was delivered by one of the consulting firm form Poland. Only selected 16 financial ratios were considered (see Table 1).

Table 1. Financial ratios utilised in the models (calculated at the date of FS)

Ratio	Name	Formula
X_1	current liquidity	$\frac{\text{current assets}}{\text{short term liabilities}}$
X_2	quick ratio	$\frac{\text{current assets} - \text{inventory} - \text{prepayments}}{\text{short term liabilities}}$
X_3	cash liquidity	$\frac{\text{cash}}{\text{short term liabilities}}$
X_4	capital share in assets	$\frac{\text{current assets} - \text{short term liabilities}}{\text{total assets}}$
X_5	gross margin	$\frac{\text{gross profit/loss on sale}}{\text{operating expenses}}$
X_6	operating profitability of the sales	$\frac{\text{profit/loss on operating activities}}{\text{total revenues}}$
X_7	operating profitability of the assets	$\frac{\text{profit/loss on operating activities}}{\text{total assets}}$
X_8	net profitability of the equity	$\frac{\text{net profit/loss}}{\text{equity}}$
X_9	assets turnover	$\frac{\text{total revenues}}{\text{total assets}}$
X_10	current assets turnover	$\frac{\text{total revenues}}{\text{current assets}}$

Table 1. Financial ratios utilised in the models (calculated at the date of FS) (cont.)

Ratio	Name	Formula
X_11	receivables turnover	$\frac{\text{total revenues}}{\text{receivables}}$
X_12	inventory turnover	$\frac{\text{total revenues}}{\text{inventory}}$
X_13	capital ratio	$\frac{\text{equity}}{\text{total liabilities}}$
X_14	coverage of the short-term liabilities by the equity	$\frac{\text{equity}}{\text{short term liabilities}}$
X_15	coverage of the fixed assets by the equity	$\frac{\text{equity}}{\text{fixed assets}}$
X_16	share of the net financial surplus in the total liabilities	$\frac{\text{net profit/loss} + \text{amortisation} + \text{interests}}{\text{total liabilities}}$

Financial ratios were only static for one period, not including dynamic ratios. Among non-financial variables we considered only five of them due to limited information. Non-financial variables are presented in Table 2. In our opinion the selection of variables is very important in analysis of bankruptcy. In most cases only financial ratios are considered (Du Jardin 2009), but very often it is not sufficient to achieve a very good prediction accuracy of the model. For our purpose the sample was partitioned in the proportion 70%:30%.

Univariate analysis was based on t-test for interval variables and chi-sqr test for binary variables (significance level 0.1). Additionally, the correlation analysis eliminated correlated ratios (r>0.7). For the model only 9 variables were selected. Financial ratios used in the models estimation were the following: current liquidity, gross margin, operating profitability of sales, assets turnover, current assets turnover, capital ratio, coverage of short-term liabilities by equity. Non-financial factors (binary) used in estimation were the following: region_low_risk and legal_form_group2.

Table 2. Non-financial factors utilized in the models

Name	Attributes/categories
Sector of activity	Equal proportion of companies from sectors: Production, Trade and Services. This variable was dichotomized and reference category was set to Services (lowest risk of bankruptcy).
Cluster of regions	16 regions grouped into 3 clusters according to bankruptcy rate (“low risk”, “average risk”, “high risk”) and dichotomized. Reference category was set to “high risk” group. Clusters grouped by k-means clustering method based on bankruptcy rate.

Table 2. Non-financial factors utilized in the models (cont.)

Name	Attributes/categories
Legal form	group1: limited liability company and group2: joint stock company, limited partnership company, other (cooperative, association, etc.). Reference category was set to group1.
Age of the company	Interval variable (age in completed years at the start of the observation period).
Number of employees	Interval variable (number of employed workers on the date of FS).

One of the most frequently used methods in bankruptcy prediction is discriminant analysis. The main idea of this method is to classify all cases (individuals) into two (or more) classes. In this case into two classes: bankrupted and non-bankrupted enterprises. Discriminant (linear) function is used to classify using the training sample. To construct this function the explanatory variables are used. Those variables must be carefully selected, must follow the normality distribution and must not be correlated to each other. The classification and the function construction is based on the criterion of maximization the distance between groups. A very important assumption in this method is the assumption of equality of variances between subsamples. This assumption must be positively verified before drawing conclusions on the results received from analysis.

The estimation of misclassification errors is not biased when the sample is close to balanced (equal proportion of observations in both groups). In the classic approach the discriminant function is linear – the Fisher discriminant function (Ptak-Chmielewska 2012):

$$Z = a_0 + a_1X_1 + a_2X_2 + \dots + a_nX_n,$$

where:

Z – dependent variable,

a_0 – intercept,

a_i, $i = 1, 2, \dots, n$ – discriminative coefficients (weights),

$X_1, X_2, \dots, X_n$ – explanatory variables (financial ratios).

After estimating the value of this discriminant function the cut-off point value must be set up to classify all possible cases (firms) into a potentially bankrupted or non-bankrupted firm. The most frequently used method for cut-off calculation is the half of the distance between averages of the values in two groups. If the value of this function for a particular enterprise is below this cut-off point than the enterprise is classified as potentially bankrupted and in the case where it is below this cut-off value the enterprise is classified as potentially non-bankrupted one. Discriminant model applies simultaneously many variables using weights. It transforms multivariable space into one dimension. It is possible to estimate and interpret the impact of each variable on the dependent one. A very important advantage of this model is that it can be applied on a small sample and by this it is useful in bankruptcy prediction. This method is available in all popular statistical packages (Ptak-Chmielewska 2012). There are also some disadvantages of this method. The possibility of using qualitative variables is very limited. The assumption of normality distribution is very often violated. This assumption is not critical. More critical is the assumption of equality of variances between groups. Sometimes this assumption is also violated. If the sample is not balanced we must remember about the decision matrix to be specified (cost of misclassification). Sometimes the linear separation is not optimal, and non-linear solution is more effective. The probability of bankruptcy is not estimated directly like in logistic regression (Ptak-Chmielewska 2012). Despite such a limitation this method is still popular in prediction of bankruptcy. The final estimated models with linear discriminant functions are presented in Table 3.

Table 3. Discriminant analysis – results

Discriminant linear function	0	1
Intercept	-0.56368	-0.60428
X_1 - current liquidity	0.00674	0.00919
X_5 – gross margin	0.00457	0.05883
X_6 – operating profitability of the sales	0.31414	-1.97663
X_9 – assets turnover	0.01844	-0.02436
X_10 – current assets turnover	0.07558	0.16014
X_13 – capital ratio	0.19445	0.08945
X_14 – coverage of the short-term liabilities by the equity	-0.00100	0.00662
region_low_risk (binary)	1.24260	0.39534
legal_form2 (binary)	1.65670	0.95146

All included explanatory variables were significant at least at the significance level 0.1. A higher value of gross margin classifies to bankrupted enterprises. Higher values of operating profitability of sales ratio and assets turnover ratio classify to non-bankrupted enterprises. But higher values of current assets turnover ratio classify to the bankrupted enterprises group. High capital ratio classifies to the good enterprises group. Activity situated in low risk region classifies to the non-bankrupted enterprises group.

The accuracy of the model was presented in Table 4. Overall accuracy on the training sample amounted to 68.5%. Among bankrupted enterprises the accuracy was comparable to non-bankrupted and amounted to 68.2% and 68.7% respectively.

Table 4. Classification table for train sample – linear discrimination

	Model=0	Model=1	Total
Level=0	237	108	345
Level=1	69	148	217
Total	306	256	562

Support Vector Machines (SVM) is a model based on the decision planes that define decision boundaries. A decision plane separates a set of objects into different classes. Most classification tasks are not as simple as linear classification, and often more complex structures are necessary to get the optimal separation. Optimal separation is the correct classification of new objects (test cases) based on the available train cases. This classification tasks which are drawing separating lines to classify objects of different class memberships are called hyperplane classifiers. Support Vector Machines can handle such complex tasks. The objects are mapped using mathematical functions (kernels). This process of classifying the objects is known as mapping.

Support Vector Machine (SVM) is a method for classifying that performs tasks by constructing hyperplanes in a multidimensional space. This method separates cases from different classes. SVM can be both regression and classification. It utilises multiple continuous and categorical variables (categorical variables are transformed into dummies). To find an optimal hyperplane, SVM uses an iterative algorithm to minimize an error function. SVM models can be classified into four groups (error function) two for classification: C-SVM, nu-SVM and two for regression: epsilon-SVM, nu-SVM regression.

SVM for classification Type 1 (C-SVM), with the error function to be maximized:

$$\boldsymbol{\frac{1}{2} w^T w + C \sum_{i=1}^{N} \xi_i}$$

with the constraints:

$$y_i(w^T \phi(x_i) + b) \geq 1 - \xi_i \text{ and } \xi_i \geq 0, i = 1, \dots, N$$

where:

C - the capacity constant,

w - the vector of coefficients,

b - constant,

$\boldsymbol{\phi_i}$ - parameters to handle nonseparable data (inputs).

Additionally, y∈±1 represents class labels, x_i represents the independent variables. The kernel $\boldsymbol{\phi}$ is used for data transformation from the input to the feature space. Larger C, more penalize the error, and should be chosen carefully to avoid over-fitting.

SVM for classification Type 2 (nu-SVM), with the error function to be maximized:

$$\frac{1}{2} w^T w - \nu p + \frac{1}{N} \sum_{i=1}^{N} \xi_i$$

with the constraints:

$$y_i(w^T\phi(x_i)+b) \geq \rho - \xi_i \text{ and } \xi_i \geq 0, i=1,\dots,N \text{ and } \rho \geq 0.$$

The most popular kernels used in Support Vector Machines models are: linear, polynomial, radial function (RBF) or sigmoid:

Kernel Functions

$$K(X_i,X_j) = \begin{cases} X_i \cdot X_j & \text{Linear} \\ (\gamma X_i \cdot X_j + C)^d & \text{Polynomial} \\ \exp\left(-\gamma |X_i - X_j|^2\right) & \text{RBF} \\ \tanh(\gamma X_i \cdot X_j + C) & \text{Sigmoid} \end{cases}$$

where $K(X_i,X_j)=\phi(X_i)\cdot\phi(X_j)$ is the kernel function with Gamma as an adjustable parameter. The most popular choice is RBF to be used in Support Vector Machines.

As the final SVM – Support Vector Machines model was estimated with the method of optimization using the Interior Point with polynomial function (3 degree) for scaling. The C method for penalization was used with the parameter equal to 1. Maximum iterations set to 50 with tolerance 1e-08.

Table 5. Results of SVM train

Internal weights (product)	39.4414518
Burden	-22.224232
Violation of restrictions	337.700692
Longest vector	14.1651387
Number of support vectors	383
Number of support vectors on margin	373
Maximum value of F	3.36246618
Minimum value of F	-21.635810
Number of included effects	9
Data matrix (number of columns)	9
Kernel matrix (number of columns)	**220**

The first type error for the train sample (wrongly classified defaults) was equal to 0.49. The second type error (wrongly classified good firms) was to 0.07 (see Table 6).

Table 6. Classification table for train sample – SVM

	Model=0	Model=1	Total
Level=0	338	7	345
Level=1	145	72	217
Total	483	79	562

3. Models comparison

The comparison of models' accuracy was based on a test sample (see Table 7 and 8) and the overall accuracy as well as bankruptcy prediction accuracy was compared. Also bankrupted enterprises prediction accuracy (sensitivity) was compared. Despite higher overall accuracy for SVM the specificity was higher for the DISCRIM model.

Generally, in the literature only the overall prediction accuracy is counted. The input from this analysis comparing the DISCRIM and the SVM model shows that specificity is more important in bankruptcy prediction. The cost of wrong classification for enterprises in good condition is not as high as the cost of wrong classification for a bankrupted enterprise.

Applying machine learning techniques including SVM is not always the best solution in bankruptcy prediction. Simple models, understandable and clear interpretation bring more value in understanding bankruptcy risk drivers.

Table 7. Classification table for test sample – discriminant analysis and SVM

Discriminant analysis			
	Model=0	Model=1	Total
Level=0	92	57	149
Level=1	32	63	95
Total	124	120	244
SVM			
	Model=0	Model=1	Total
Level=0	142	7	149
Level=1	61	34	95
Total	203	41	244

Table 8. Overall accuracy and prediction of bankruptcy – comparison

Model	Overall accuracy	Bankruptcy prediction accuracy	Good SMEs prediction accuracy
DISCRIM	63.5%	66.3%	61.7%
SVM	72.1%	35.8%	95.3%

4. Discussion and Conclusions

In this paper the accuracy of two different methods was assessed. The first method was popular discriminant function, called the Fisher discrimination. The second was Support Vector Machines, recently developed and applied for classification. According to the results, the overall accuracy of SVM was higher compared to linear discrimination but the accuracy of bankrupted enterprises prediction (sensitivity) was higher in the case of simple linear discrimination (almost double).

False alarms for enterprises in good standing are much less costly compared to wrong classification of bankrupted ones. Such models should first of all classify bankrupted enterprises. Only with higher accuracy of bankrupted enterprises (sensitivity) make such models useful for an early warning process. Implication for policy is a possible application of such a model (based on the DISCRIM model) in an early warning signal process. It is quite an easy tool for business to assess the credibility of partners and customers (let us say suppliers).

Comparing the effectiveness of different models based only on AUC (Area Under the ROC Curve) or AR (Accuracy Ratio) does not give the full picture of the real accuracy (Zięba et al. 2016). We should always take a look on the classification of both sides: bankrupted and non-bankrupted enterprises. In this field this research brings added value to the bankruptcy prediction research area. Typically, the literature and applications focus on general accuracy measures, not being cautious about bankrupted enterprises misclassifications.

Results highlighted the importance of financial ratios like current liquidity, gross margin, operating profitability of the sales, assets turnover and the current assets turnover, capital ratio, coverage of short-term liabilities by the equity. Also non-financial information was important like the region of activity, legal form of the enterprise. This confirms the importance of different non-financial, macroeconomic aspects, etc.

Future research should focus on specificity of SME's financial analysis. In comparison with corporations, the information about SMEs is always limited. Not only financial ratios are important, non-financial information plays also an important role. Analysis cannot be based only on information on financial ratios.

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References

ABDOU, H., POINTON, J., MASRY, A. E., (2008). Neural Nets Versus Conventional Techniques in Credit Scoring in Egyptian Banking. Expert Systems with Applications, 35(2), pp. 1275–1292.

ALTMAN, E. I., (1968). Financial ratios, Discriminant analysis and the prediction of corporate bankruptcy. Journal of Finance, 23(4), pp. 589–609.

ANDREEVA, G., CALABRESE, R., OSMETTI, S. A., (2014). A comparative analysis of the UK and Italian small businesses using Generalised Extreme Value models. https://arxiv.org/pdf/1412.5351.pdf.

APPENZELLER, D., SZARZEC, K., (2004). Forecasting the bankruptcy risk of Polish public companies. Rynek Terminowy, 1, pp. 120–128.

BACK, B., LAITINEN, T., SERE, K., VAN WEZEL, M., (1996). Choosing bankruptcy predictors using discriminant analysis, logit analysis, and genetic algorithms, Technical Report, Turku Centre for Computer Science.

BALINA, R., BĄK, M. J., (2016). Discriminant Analysis as a Prediction Method for Corporate Bankruptcy with the Industrial Aspects. Waleńczów: Wydawnictwo Naukowe Intellect.

BERG, D., (2007). Bankruptcy prediction by generalized additive models. Applied Stochastic Models in Business and Industry, 23(2), pp. 129–143.

BLANCO, A., PINO-MEJÍAS, R., LARA, J., (2013). Credit scoring models for the microfinance industry using neural networks: Evidence from Peru, Expert Systems with Applications, 40(1), pp. 356–364.

BROŻYNA, J., MENTEL, G., PISULA, T., (2016). Statistical methods of the bankruptcy prediction in the logistics sector in Poland and Slovakia. Transformations in Business & Economics, 15, pp. 80–96.

BRYANT, S. M., (1997). A case-based reasoning approach to bankruptcy prediction modelling. Intelligent Systems in Accounting, Finance and Management, 6(3), pp. 195–214.

CALABRESE, R., MARRA, G., OSMETTI, S. A., (2015). Bankruptcy prediction of small and medium enterprises using a flexible binary generalized extreme value model. Journal of the Operational Research Society, 67(4).

CHAUDHURI, A., DE, K., (2011). Fuzzy support vector machine for bankruptcy prediction. Applied Soft Computing, 11(2), pp. 2472–2486.

DERELIOĞLU, G., GÜRGEN F., (2011). Knowledge discovery using neural approach for SME's credit risk analysis problem in Turkey. Expert Systems with Applications, 38(8), pp. 9313–9318.

DESAI, V. S., CROOK, J. N., OVERSTREET, G. A., (1996). A Comparison of Neural Networks and Linear Scoring Models in the Credit Union Environment. European Journal of Operational Research, 95(1), pp. 24–47.

DU JARDIN, P., (2009). Bankruptcy prediction models: How to choose the most relevant variables?. Bankers, Markets & Investors, 98, pp. 39–46.

GAJDKA, J., STOS, D., (1996). Wykorzystanie analizy dyskryminacyjnej do badania podatności przedsiębiorstwa na bankructwo. In: J. Duraj ed. Przedsiębiorstwo na rynku kapitałowym, Wydawnictwo Uniwersytetu Łódzkiego, Łódź.

GĄSKA, D., (2016). Predicting Bankruptcy of Enterprises with the use of Learning Methods. Ph.D. dissertation, Wrocław University of Economics.

GORDINI, N., (2014). A genetic algorithm approach for SMEs bankruptcy prediction: Empirical evidence from Italy, Expert Systems with Applications, 41(14), pp. 6067–6536.

GRUSZCZYŃSKI, M., (2003). Models of microeconometrics in the analysis and forecasting of the financial risk of enterprises. Zeszyty Polskiej Akademii Nauk, 23.

GUPTA, J., GREGORIOU, A., HEALY, J., (2015). Forecasting bankruptcy for SMEs using hazard function. A review of quantitative finance and accounting, 45 (4), pp. 845–869.

HADASIK, D., (1998). The Bankruptcy of Enterprises in Poland and Methods of its Forecasting. Wydawnictwo Akademii Ekonomicznej w Poznaniu, 153.

HAMROL, M., CZAJKA, B., PIECHOCKI, M., (2004). Enterprise bankruptcy–discriminant analysis model. Przegląd Organizacji, 6, pp. 35–39.

HOŁDA, A., (2001). Forecasting the bankruptcy of an enterprise in the conditions of the Polish economy using the discriminatory function ZH. Rachunkowość, 5, pp. 306–10.

HUANG, Z., CHEN, H., HSU, C. J., CHEN, W. H., WU, S., (2004). Credit Rating Analysis with Support Vector Machines and Neural Networks: A Market Comparative Study. Decision Support System, 37(4), pp. 543–558.

HUANG, J. J., TZENG, J. H., ONG, C. S., (2006). Two-stage genetic programming (2sgp) for the credit scoring model. Applied Mathematics and Computation, 174, pp. 1039–1053.

JAGIEŁŁO, R., (2013). Discriminant and Logistic Analysis in the Process of Assessing the Creditworthiness of Enterprises. Materiały i Studia, 286. Warszawa: NBP.

KALAK I. E., HUDSON, R., (2016). The effect of size on the failure probabilities of SMEs: An empirical study on the US market using discrete hazard model. International Review of Financial Analysis, 43, pp. 135–145.

KARBOWNIK, L., (2017). Methods for Assessing the Financial Risk of Enterprises in the TSI Sector in Poland. Łódź: Wydawnictwo Uniwersytetu Łódzkiego.

KIM, H. S., SOHN, S. Y., (2010). Support Vector Machines for Default Prediction of SMEs Based on Technology Credit. European Journal of Operational Research, 201(3), pp. 838–846.

KOROL, T., PRUSAK, B., (2009). Upadłość przedsiębiorstwa a wykorzystanie sztucznej inteligencji, Warszawa: CeDeWu.

KOROL, T., (2004). Assessment of the Accuracy of the Application of Discriminatory Methods and Artificial Neural Networks for the Identification of Enterprises Threatened with Bankruptcy. Gdańsk: Doctoral dissertation.

KOROL, T., (2010a). Early Warning Systems of Enterprises to the Risk of Bankruptcy. Warszawa: Wolters Kluwer.

KOROL, T., (2010b). Forecasting bankruptcies of companies using soft computing techniques. Finansowy Kwartalnik Internetowy "e-Finanse", 6, pp. 1–14.

MĄCZYŃSKA, E., (1994). Assessment of the condition of the enterprise. Simplified methods. Życie Gospodarcze, 38, pp. 42–45.

MĄCZYŃSKA, E., (2004). Early warning systems. Nowe Życie Gospodarcze, 12, pp. 4–9.

MICHALUK, K., (2003). Effectiveness of corporate bankruptcy models in Polish economic conditions. In: L. Pawłowicz, R.Wierzba ed. Corporate Finance in the Face of Globalization Processes. Warszawa: Wydawnictwo Gdańskiej Akademii Bankowej.

MODINA, M., PIETROVITO, F., (2014). A default prediction model for Italian SMEs: the relevance of the capital structure. Applied Financial Economics, 24(23), pp. 1537–1554.

ORŁOWSKI, W., PASTERNAK, R., FLAHT, K., SZUBERT, D., (2010). Procesy inwestycyjne i strategie przedsiębiorstw w czasach kryzysu, Raport PARP Warszawa.

POCIECHA, J., PAWEŁEK, B., (2011). Bankruptcy Prediction and Business Cycle, Contemporary Problems of Transformation Process in the Central and East European Countries. Paper presented at 17th Ukrainian-Polish-Slovak Scientific

Seminar, Lviv, Ukraine, September 22–24; Lviv: The Lviv Academy of Commerce, pp. 9–24.

POCIECHA, J., PAWEŁEK, B., BARYŁA, M., AUGUSTYN, S., (2014). Statistical Methods of Forecasting Bankruptcy in the Changing Economic Situation. Kraków: Fundacja Uniwersytetu Ekonomicznego w Krakowie.

POGODZIŃSKA, M., SOJAK, S., (1995). The Use of Discriminant Analysis in Predicting Bankruptcy of Enterprises. Ekonomia XXV, 299.

PRUSAK, B., (2018). Review of Research into Enterprise Bankruptcy Prediction in Selected Central and Eastern European Countries. International Journal of Financial Studies, 6, 60.

PRUSAK, B., WIĘCKOWSKA, A., (2007). Multidimensional models of discriminant analysis in the study of the bankruptcy risk of Polish companies listed on the WSE. In: B. Prusak ed. Economic and Legal Aspects of Corporate Bankruptcy. Warszawa: Difin.

PRUSAK, B., (2005). Modern Methods of Forecasting Financial Risk of Enterprises. Warszawa: Difin.

PSILLAKI, M., TSOLAS, I. E., MARGARITIS, D., (2010). Evaluation of credit risk based on firm performance. European Journal of Operational Research, 201 (3), pp. 873–881.

PTAK-CHMIELEWSKA, A., MATUSZYK, A., (2017). The importance of financial and non-financial ratios in SMEs bankruptcy prediction. Bank i Kredyt, 49(1), pp. 45–62.

PTAK-CHMIELEWSKA, A., (2016). Statistical Models for Corporate Credit Risk Assessment–Rating Models. Acta Universitatis Lodziensis Folia Oeconomica, 3, pp. 98–111.

PTAK-CHMIELEWSKA, A., (2012). Wykorzystanie modeli przeżycia i analizy dyskryminacyjnej do oceny ryzyka upadłości przedsiębiorstw. Ekonometria, 4 (38), pp. 157–172.

POLSKA AGENCJA ROZWOJU PRZEDSIĘBIORCZOŚCI, (2012). Raport o stanie sektora małych i średnich przedsiębiorstw w Polsce w latach 2010–2011, Warsaw.

SARTORI, F., MAZZUCCHELLI, A., DI GREGORIO, A., (2016). Bankruptcy forecasting using case-based reasoning: the CRePERIE approach. Expert Systems with Applications, 64, pp. 400–411.

SOHN, S. Y., KIM, D. H., YOON, J. H., (2016). Technology credit scoring model with fuzzy logistic regression. Applied Soft Computing, 43, pp. 150−158.

SOJAK, S., STAWICKI, J., (2001). Wykorzystanie metod taksonomicznych do oceny kondycji ekonomicznej przedsiębiorstw. Zeszyty Teoretyczne Rachunkowości, 3(59), pp.45−52.

STĘPIEŃ, P., STRĄK, T., (2004). Multidimensional logit models for assessing the risk of bankruptcy of Polish enterprises. In: D.Zarzecki ed. Time for Money, t. I., Szczecin: Wydawnictwo Uniwersytetu Szczecińskiego.

WĘDZKI, D., (2000). The problem of using the ratio analysis to predict the bankruptcy of Polish enterprises–Case study. Bank i Kredyt, 5, pp. 54–61.

WĘDZKI, D., (2004). Logit model of bankruptcy for the Polish economy–Conclusions from the study. In: D.Zarzecki ed. Time for Money. Corporate finance. Financing enterprises in the EU. Szczecin: Wydawnictwo Uniwersytetu Szczecińskiego.

WIERZBA, D., (2000). Early Detection of Enterprises Threatened with Bankruptcy Based on the Analysis of Financial Ratios–Theory and Empirical Research. Zeszyty Naukowe nr 9. Warszawa: Wydawnictwo Wyższej Szkoły Ekonomiczno-Informatycznej w Warszawie.

YIP, A. Y. N., (2006). Business failure prediction: a case-based reasoning approach. Review of Pacific Basin Financial Markets and Policies, 09, pp. 491−508.

ZIĘBA, M., TOMCZAK, S. K., TOMCZAK, J. M., (2016). Ensemble boosted trees with synthetic features generation in application to bankruptcy prediction. Expert Systems With Applications, 58, pp. 93−101.

ZMIJEWSKI, M. E., (1984). Methodological issues related to the estimation of financial distress prediction models. Journal of Accounting Research, 22, pp. 59−82.