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Developing calibration estimators for population mean using robust measures of dispersion under stratified random sampling

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ABSTRACT

In this paper, two modified, design-based calibration ratio-type estimators are presented. The suggested estimators were developed under stratified random sampling using information on an auxiliary variable in the form of robust statistical measures, including Gini's mean difference, Downton's method and probability weighted moments. The properties (biases and MSEs) of the proposed estimators are studied up to the terms of first-order approximation by means of Taylor's Series approximation. The theoretical results were supported by a simulation study conducted on four bivariate populations and generated using normal, chi-square, exponential and gamma populations. The results of the study indicate that the proposed calibration scheme is more precise than any of the others considered in this paper.

Key words: calibration, outliers, percentage relative efficiency (PRE), stratified sampling.

1. Introduction

In sampling survey, calibration is a commonly used technique to produce estimation weights. These calibrations weights in turn satisfy calibration equation that incorporates auxiliary information. The calibration approach consists of (a) computation of new weights that incorporate specified auxiliary information and are restrained by calibration equations (b) the use of these weights to compute linearly weighted estimate of mean, totals and other finite population parameters satisfying an objective of obtaining nearly unbiased estimate. This technique has been used to develop cosmetic estimators (estimators interpretable both as design-based and as prediction-based estimators) (see Sarndal and Wright (1984), Brewer (1995, 1999), etc.). The calibration technique has also been utilized to develop design-based estimator

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under different sampling schemes like stratified random sampling, stratified random double sampling, two-stage sampling, etc. In this direction many authors like Deville and Sarndal (1992), Singh and Mohl (1996), Estevao and Sarndal (2000), Estevao and Sarndal (2002), Singh (2003),Tracy et al. (2003), Kim et al. (2007), Barktus and Pumputis (2010), Sud et al. (2014), Clement and Enang (2016), Rao et al. (2016) and Subzar et al. (2018) have proposed estimators and studied their properties for estimating population mean under different calibration constraints in stratified random sampling. Tracy et al. (2003) obtained calibration weights for population mean by using first and second order moments of auxiliary variable in stratified random sampling. Nidhi et al. (2017) considered estimation of population mean using calibration approach in stratified and stratified double sampling schemes. Kim et al. (2007) utilized calibration approach in defining estimators for population variance in stratified random sampling. Other authors like Horvitz and Thompson (1952), Estevao and Sárndal (2006), Aditya et al. (2016), Salinas et al. (2019) considered estimation of population mean under two stage sampling scheme using the calibration approach.

In this paper, we have suggested two calibrated schemes in stratified random sampling by utilizing auxiliary information on certain robust statistical measures like Gini's mean difference, Downton's method and Probability weighted moments, all of which are insensitive against the presence of outliers in the population and are less susceptible to fluctuations in sampling whenever extreme observations are present as alternatives to Rao et al. (2016) calibration estimators.

2. Some existing estimators in literature

Let $\Theta_N = \left\{\Theta_{N_h}, h = 1, 2, ..., K \right\}$ be a stratified non-overlapping heterogeneous population with K strata of size $N = \sum_{h=1}^{K} N_h$ with units $y_{hi}, i = 1, 2, ..., N_{hi}$ and $x_{hi}, i = 1, 2, ..., N_{hi}$ for study variable y and auxiliary variable x respectively. $\bar{Y}_h = N_h^{-1} \sum_{i=1}^{N_h} y_{hi}$ and $\bar{X}_h = N_h^{-1} \sum_{i=1}^{N_h} x_{hi}$ are means of study and auxiliary variables respectively. A random sample of size $n = \sum_{h=1}^{K} n_h$ is selected from the population using SRSWOR. The conventional unbiased estimator of the population mean and its variance is given in Eq. (2.1) and Eq. (2.2), respectively.

$$\bar{y}_{st} = \sum_{h=1}^{K} \Delta_h \bar{y}_h \tag{2.1}$$

$$Var\left(\bar{y}_{st}\right) = \sum_{h=1}^{K} \Delta_h^2 \left(n_h^{-1} - N_h^{-1}\right) S_{yh}^2 \tag{2.2}$$

Where,

$$\Delta_h = N_h / N,\ \bar{y}_h = n_h^{-1}\sum_{i=1}^{n_h} y_{hi}, S_{yh}^2 = \left(N_h - 1\right)^{-1}\sum_{i=1}^{N_h}\left(y_{hi} - \bar{Y}_h\right)^2, \bar{Y} = \sum_{h=1}^{K}\Delta_h \bar{Y}_h$$

Singh (2003) suggested a design-based calibration estimator with two constraints for estimating population mean in stratified sampling. The suggested calibration estimator is given in Eq. (2.3).

$$\bar{y}_S = \sum_{h=1}^{K}\Delta_h^S \bar{y}_h \tag{2.3}$$

where Δ_h^S is the new calibration weight of stratum K^{th} to be obtained by solving (2.4).

$$\left.\begin{array}{ll} \min & Z_S = \sum_{h=1}^{K}\left(\Delta_h^S - \Delta_h\right)^2 / \Delta_h \phi_h \\ s.t & \sum_{h=1}^{K}\Delta_h^S \bar{x}_h = \sum_{h=1}^{K}\Delta_h \bar{X}_h, \quad \sum_{h=1}^{K}\Delta_h^S = \sum_{h=1}^{K}\Delta_h \end{array}\right\} \tag{2.4}$$

where ϕ_h are suitably chosen positive scale factors, which decide the form of the estimator.

Eq.(2.4) yields a calibration weight in Eq. (2.5) and the estimator $\bar{y}_S$ was obtained as in Eq. (2.6).

$$\Delta_h^S = \Delta_h + \frac{\phi_h \Delta_h \bar{x}_h \sum_{h=1}^{K}\Delta_h \phi_h - \Delta_h \phi_h \sum_{h=}^{K}\Delta_h \phi_h \bar{x}_h}{\sum_{h=}^{K}\Delta_h \phi_h \sum_{h=}^{K}\Delta_h \phi_h \bar{x}_h^2 - \left(\sum_{h=}^{K}\Delta_h \phi_h \bar{x}_h\right)^2}\left(\bar{X} - \sum_{h=}^{K}\Delta_h \bar{x}_h\right) \tag{2.5}$$

$$\bar{y}_S = \sum_{h=1}^{K}\Delta_h \bar{y}_h + \frac{\sum_{h=1}^{K}\phi_h \Delta_h \sum_{h=1}^{K}\phi_h \Delta_h \bar{x}_h \bar{y}_h - \sum_{h=1}^{K}\phi_h \Delta_h \bar{x}_h \sum_{h=1}^{K}\phi_h \Delta_h \bar{y}_h}{\sum_{h=1}^{K}\phi_h \Delta_h \sum_{h=1}^{K}\phi_h \Delta_h \bar{x}_h^2 - \left(\sum_{h=1}^{K}\phi_h \Delta_h \bar{x}_h\right)^2}\left(\bar{X} - \sum_{h=1}^{K}\Delta_h \bar{x}_h\right) \tag{2.6}$$

Clement and Enang (2016) suggested a design-based calibration estimator for the combined ratio estimator in stratified random sampling. The suggested estimators with the associated calibration constraint are given in Eq. (2.7) and Eq. (2.8).

$$\bar{y}_{CE} = \sum_{h=1}^{K}\Delta_h^{CE}\hat{R}\bar{X} \tag{2.7}$$

$$\left.\begin{array}{ll} \min & Z_{CE} = \sum_{h=1}^{K}\left(\Delta_h^{CE} - \Delta_h\right)^2 / \Delta_h \phi_h \\ s.t & \sum_{h=1}^{K}\Delta_h^{CE}\bar{x}_h = \bar{X} \end{array}\right\} \tag{2.8}$$

where $\hat{R}_h = \bar{y}_h / \bar{x}_h$ Δ_h^{CE} is the proposed calibration weight of K^{th} stratum.

The calibration weight Δ_h^*, estimator $\bar{y}_{CE}$ and $\text{var}(\bar{y}_{CE})$ were obtained as given in Eq. (2.9), Eq. (2.10) and Eq. (2.11) respectively.

$$\Delta_h^{CE} = \Delta_h + \frac{\Delta_h \phi_h \bar{x}_h}{\sum_{h=1}^{K} \Delta_h \phi_h \bar{x}_h^2} \left(\bar{X} - \sum_{h=1}^{K} \Delta_h \bar{x}_h \right) \tag{2.9}$$

$$\bar{y}_{CE} = \bar{X} \sum_{h=1}^{K} \Delta_h \bar{y}_h / \sum_{h=1}^{K} \Delta_h \bar{x}_h \tag{2.10}$$

$$Bias(\bar{y}_{CE}) = \bar{X}^{-1} \sum_{h=1}^{K} \Delta_h \left(n_h^{-1} - N_h^{-1} \right) \left(RS_{xh}^2 - S_{yxh} \right) \tag{2.11}$$

$$Var(\bar{y}_{CE}) = \left(\bar{X}^2 Var(\bar{x}_{st}) \sum_{h=1}^{K} \Delta_h^2 \left(n_h^{-1} - N_h^{-1} \right) S_h^{*2} \right) / \hat{\bar{X}}^2 Var(\bar{x}_{st}^*) \tag{2.12}$$

where

$\hat{\bar{X}} = \sum_{h=1}^{K} \Delta_h \bar{x}_h, Var(\bar{x}_{st}^*) = \sum_{h=1}^{K} \Delta_h S_{xh}^2, S_h^{*2} = S_{yh}^2 + R^2 S_{xh}^2 - 2RS_{yxh}, R = \bar{Y} / \bar{X}$,
$S_{yxh} = (N_h - 1)^{-1} \sum_{i=1}^{N_h} \left(y_{hi} - \bar{Y}_h \right) \left(x_{hi} - \bar{X}_h \right)$

Rao et al. (2016) proposed two new design-based calibration schemes by incorporating coefficient of variation in the constraint to the chi-square distance function for the new calibration weight defined to improve the precision of the sample mean estimator in stratified random sampling. The first scheme proposed is given in Eq. (2.13).

$$\bar{y}_{RTK} = \sum_{h=1}^{K} \Delta_h^* \bar{y}_h \tag{2.13}$$

where Δ_h^* is the new calibration weight such that the chi-square function Z_* is defined as

$$\left.\begin{aligned} \min \quad & Z_* = \sum_{h=1}^{K} \frac{\left(\Delta_h^* - \Delta_h \right)^2}{\Delta_h \phi_h} \\ s.t \; & \sum_{h=1}^{K} \Delta_h^* \left(\bar{x}_h + c_{xh} \right) = \sum_{h=1}^{K} \Delta_h \left(\bar{X}_h + C_{Xh} \right) \end{aligned}\right\} \tag{2.14}$$

where

$c_{xh}=s_{xh}/\bar{x}_h, C_{Xh}=S_{Xh}/\bar{X}_h, s_{xh}^2=(n_h-1)^{-1}\sum_{i=1}^{n_h}(x_{hi}-\bar{x}_h)^2, \bar{x}_h=n_h^{-1}\sum_{i=1}^{n_h}x_{hi}$

$S_{Xh}^2=(N_h-1)^{-1}\sum_{i=1}^{N_h}(x_{hi}-\bar{X}_h)^2$

Solving Eq. (2.14) and let $\phi_h=(\bar{x}_h+c_{xh})^{-1}$, the calibration weight Δ_{h1}^* and the estimator $\bar{y}_{RTK}$ are given by Eq. (2.15) and Eq. (2.16) respectively.

$$\Delta_{h1}^*=\Delta_h+\Delta_h\left(\sum_{h=1}^{K}\Delta_h(\bar{X}_h+C_{Xh})-\sum_{h=1}^{K}\Delta_h(\bar{x}_h+c_{xh})\right)\left(\sum_{h=1}^{K}\Delta_h(\bar{x}_h+c_{xh})\right)^{-1} \quad (2.15)$$

$$\bar{y}_{RTK1}=\sum_{h=1}^{K}\Delta_h\bar{y}_h\sum_{h=1}^{K}\Delta_h(\bar{X}_h+C_{Xh})\left(\sum_{h=1}^{K}\Delta_h(\bar{x}_h+c_{xh})\right)^{-1} \quad (2.16)$$

Similarly, function Z_* is also subjected to another constraint defined in Eq. (2.17),

$$\sum_{h=1}^{K}\Delta_h^*(1+\bar{x}_h+c_{xh})=\sum_{h=1}^{K}\Delta_h(1+\bar{X}_h+C_{Xh}) \quad (2.17)$$

which lead to another estimator given as

$$\Delta_{h2}^*=\Delta_h+\Delta_h\left(\sum_{h=1}^{K}\Delta_h(1+\bar{X}_h+C_{Xh})-\sum_{h=1}^{K}\Delta_h(1+\bar{x}_h+c_{xh})\right)\left(\sum_{h=1}^{K}\Delta_h(1+\bar{x}_h+c_{xh})\right)^{-1} \quad (2.18)$$

$$\bar{y}_{RTK2}=\sum_{h=1}^{K}\Delta_h\bar{y}_h\sum_{h=1}^{K}\Delta_h(1+\bar{X}_h+C_{Xh})\left(\sum_{h=1}^{K}\Delta_h(1+\bar{x}_h+c_{xh})\right)^{-1} \quad (2.19)$$

However, estimators $\bar{y}_{RTK1}$ and $\bar{y}_{RTK2}$ are functions of coefficients of variation which are easily affected outliers or extreme values.

3. Suggested calibration estimators

Motivated by Clement and Enang (2016) and Rao et al. (2016), we proposed two classes of design-based calibration estimators in stratified random sampling using robust measures such as Gini's mean difference G_{MD}, Downton's method D_M and probability weighted moments P_{WM} of the auxiliary information, which are insensitive to the presence of outliers or extreme values in the data.

Let $z\in\Re^+$ with units $z_i, i=1,2,...,N$, then:

$$G_{MD}(z) = 2N^{-1}(N-1)^{-1}\sum_{i=1}^{N}(2i-N-1)z_i \tag{3.1}$$

$$D_M(z) = 2\sqrt{\pi}N^{-1}(N-1)^{-1}\sum_{i=1}^{N}(i-(N+1)/2)z_i \tag{3.2}$$

$$P_{WM}(z) = \sqrt{\pi}N^{-2}\sum_{i=1}^{N}(2i-(N+1))z_i \tag{3.3}$$

3.1. First calibration scheme proposed

Consider an estimator defined in Eq. (3.4) under stratified sampling having distance function as given in Eq. (3.5),

$$\bar{y}_{AR} = \sum_{h=1}^{K}\Omega_{hi}^{*}\bar{y}_h,\ i = 1,2,3. \tag{3.4}$$

where Ω_{hi}^{*} is the new calibration weights such that the chi-square function Z^{*} is defined as

$$\left.\begin{aligned} &\min \quad Z^{*} = \sum_{h=1}^{K}\frac{\left(\Omega_{hi}^{*}-\Delta_h\right)^2}{\Delta_h\phi_h} \\ &s.t \sum_{h=1}^{K}\Omega_{hi}^{*}\left(\bar{x}_h+\lambda_{hi}(x)\right) = \sum_{h=1}^{K}\Delta_h\left(\bar{X}_h+\lambda_{hi}(x)\right), i=1,2,3 \end{aligned}\right\} \tag{3.5}$$

where $\lambda_{h1}(x) = G_{MDh}(x), \lambda_{h2}(x) = D_{Mh}(x),\ \lambda_{h3}(x) = P_{WMh}(x)$

To compute the new calibrated weights Ω_{hi}^{*}, we use the Lagrange multipliers function of the form given by Eq. (3.6),

$$\Phi = \sum_{h=1}^{K}\frac{\left(\Omega_{hi}^{*}-\Delta_h\right)^2}{\Delta_h\phi_h} - 2\eta\left(\sum_{h=1}^{K}\Omega_{hi}^{*}\left(\bar{x}_h+\lambda_{hi}(x)\right)-\sum_{h=1}^{K}\Delta_h\left(\bar{X}_h+\lambda_{hi}(x)\right)\right) \tag{3.6}$$

Partially differentiating Eq. (3.6) with respect to Ω_h^{*} and η and equating to zero, we have

$$\Omega_{hi}^{*} = \Delta_h + \eta\Delta_h\phi_h\left(\bar{x}_h+\lambda_{hi}(x)\right) \tag{3.7}$$

$$\sum_{h=1}^{K}\Omega_{hi}^{*}\left(\bar{x}_h+\lambda_{hi}(x)\right)-\sum_{h=1}^{K}\Delta_h\left(\bar{X}_h+\lambda_{hi}(x)\right) = 0 \tag{3.8}$$

Substituting Eq. (3.7) in Eq. (3.8) to get η and then substituting the expression obtained into Eq. (3.7). By putting $\phi_h = \left(\bar{x}_h+\lambda_{hi}(x)\right)^{-1}$, the new calibration weight Ω_{hi}^{*} is obtained as

$$\Omega_{hi}^{*}=\Delta_h+\Delta_h\left(\sum_{h=1}^{K}\Delta_h\left(\bar{X}_h+\lambda_{hi}(x)\right)-\sum_{h=1}^{K}\Delta_h\left(\bar{x}_h+\lambda_{hi}(x)\right)\right)\left(\sum_{h=1}^{K}\Delta_h\left(\bar{x}_h+\lambda_{hi}(x)\right)\right)^{-1} \quad (3.9)$$

Now, substituting Eq. (3.9) in Eq. (3.4) and letting $\lambda_{hi}(x)$ be either $G_{MDh}(x)$ or $D_{Mh}(x)$ or $P_{WMh}(x)$, the new estimators are obtained as,

$$\left.\begin{aligned}
\bar{y}_{AR1}&=\sum_{h=1}^{K}\Delta_h\bar{y}_h\sum_{h=1}^{K}\Delta_h\left(\bar{X}_h+G_{MDh}(x)\right)\left(\sum_{h=1}^{K}\Delta_h\left(\bar{x}_h+G_{MDh}(x)\right)\right)^{-1}\\
\bar{y}_{AR2}&=\sum_{h=1}^{K}\Delta_h\bar{y}_h\sum_{h=1}^{K}\Delta_h\left(\bar{X}_h+D_{Mh}(x)\right)\left(\sum_{h=1}^{K}\Delta_h\left(\bar{x}_h+D_{Mh}(x)\right)\right)^{-1}\\
\bar{y}_{AR3}&=\sum_{h=1}^{K}\Delta_h\bar{y}_h\sum_{h=1}^{K}\Delta_h\left(\bar{X}_h+P_{WMh}(x)\right)\left(\sum_{h=1}^{K}\Delta_h\left(\bar{x}_h+P_{WMh}(x)\right)\right)^{-1}
\end{aligned}\right\} \quad (3.10)$$

3.2. Second calibration scheme proposed

To obtain the second class of the proposed estimators, we let

$$\bar{y}_{AS}=\sum_{h=1}^{K}\Pi_{hi}^{*}\bar{y}_h,\ i=1,2,3. \quad (3.11)$$

where Π_{hi}^{*} is the new calibration weight such that the chi-square function U^{*} is defined as

$$\left.\begin{aligned}
&\min\quad U^{*}=\sum_{h=1}^{K}\frac{\left(\Pi_{hi}^{*}-\Delta_h\right)^2}{\Delta_h\phi_h}\\
&s.t\ \sum_{h=1}^{K}\Pi_{hi}^{*}\left(1+\bar{x}_h+\lambda_{hi}(x)\right)=\sum_{h=1}^{K}\Delta_h\left(1+\bar{X}_h+\lambda_{hi}(x)\right),\ i=1,2,3
\end{aligned}\right\} \quad (3.12)$$

Solving for Π_h^{*} using the Lagrange multipliers technique and putting $\phi_h=\left(1+\bar{X}_h+\lambda_{hi}(x)\right)^{-1}$, we have the new calibrated weight given by Eq. (2.14).

$$\Pi_{hi}^{*}=\Delta_h+\Delta_h\left(\sum_{h=1}^{K}\Delta_h\left(1+\bar{X}_h+\lambda_{hi}(x)\right)-\sum_{h=1}^{K}\Delta_h\left(1+\bar{x}_h+\lambda_{hi}(x)\right)\right)\left(\sum_{h=1}^{K}\Delta_h\left(1+\bar{x}_h+\lambda_{hi}(x)\right)\right)^{-1} \quad (3.13)$$

By putting Eq. (3.13) in Eq. (3.11) and letting $\lambda_{hi}(x)$ be either $G_{MDh}(x)$ or $D_{Mh}(x)$ or $P_{WMh}(x)$, the new estimators are obtained as

$$\left.\begin{aligned}
\bar{y}_{AS1} &= \sum_{h=1}^{K}\Delta_h \bar{y}_h \sum_{h=1}^{K}\Delta_h\left(1+\bar{X}_h+G_{MDh}(x)\right)\left(\sum_{h=1}^{K}\Delta_h\left(1+\bar{x}_h+G_{MDh}(x)\right)\right)^{-1}\\
\bar{y}_{AS2} &= \sum_{h=1}^{K}\Delta_h \bar{y}_h \sum_{h=1}^{K}\Delta_h\left(1+\bar{X}_h+D_{Mh}(x)\right)\left(\sum_{h=1}^{K}\Delta_h\left(1+\bar{x}_h+D_{Mh}(x)\right)\right)^{-1}\\
\bar{y}_{AS3} &= \sum_{h=1}^{K}\Delta_h \bar{y}_h \sum_{h=1}^{K}\Delta_h\left(1+\bar{X}_h+P_{WMh}(x)\right)\left(\sum_{h=1}^{K}\Delta_h\left(1+\bar{x}_h+P_{WMh}(x)\right)\right)^{-1}
\end{aligned}\right\} \tag{3.14}$$

3.3. Properties (bias and MSE) of the proposed estimators

To obtain bias and MSE of the suggested estimators $\bar{y}_{ARi}$, $\bar{y}_{ASi}$, the following error terms are defined: $e_0=\left(\bar{y}_{st}-\bar{Y}\right)/\bar{Y}$, $e_1=\left(\bar{x}_{st}-\bar{X}\right)/\bar{X}$ with expected values defined in Eq. (3.15)

$$\left.\begin{aligned}
&E\left(e_0\right)=E\left(e_1\right)=0, E\left(e_0^2\right)=Var\left(\bar{y}_{st}\right)/\bar{Y}^2,\\
&E\left(e_1^2\right)=Var\left(\bar{x}_{st}\right)/\bar{X}^2, E\left(e_0e_1\right)=Cov\left(\bar{y}_{st}\bar{x}_{st}\right)/\overline{YX}
\end{aligned}\right\} \tag{3.15}$$

where

$$Var\left(\bar{x}_{st}\right)=\sum_{h=1}^{K}\Delta_h^2\left(n_h^{-1}-N_h^{-1}\right)S_{xh}^2, Cov\left(\bar{y}_{st}\right)=\sum_{h=1}^{K}\Delta_h^2\left(n_h^{-1}-N_h^{-1}\right)S_{yxh}.$$

Expressing Eq. (3.10) and Eq. (3.14) in terms of $e_i, i=0,1$ and simplifying up to the second degree approximation, we obtained Eq. (3.16) and Eq. (3.17) respectively as

$$\bar{y}_{ARi}=\bar{Y}\left(1+e_0\right)\sum_{h=1}^{K}\Delta_h\left(\bar{X}_h+\lambda_{ih}(x)\right)/\left(\bar{X}e_1+\sum_{h=1}^{K}\Delta_h\left(\bar{X}_h+\lambda_{ih}(x)\right)\right) \tag{3.16}$$

$$\bar{y}_{ASi}=\bar{Y}\left(1+e_0\right)\sum_{h=1}^{K}\Delta_h\left(1+\bar{X}_h+\lambda_{ih}(x)\right)/\left(\bar{X}e_1+\sum_{h=1}^{K}\Delta_h\left(1+\bar{X}_h+\lambda_{ih}(x)\right)\right) \tag{3.17}$$

Simplifying Eq. (3.16) and Eq. (3.17), we get Eq. (3.18) and Eq. (3.19)

$$\bar{y}_{ARi}=\bar{Y}\left(1+e_0\right)\left(1+\varpi_i e_1\right)^{-1} \tag{3.18}$$

$$\bar{y}_{ASi} = \bar{Y}\left(1+e_0\right)\left(1+\vartheta_i e_1\right)^{-1} \tag{3.19}$$

where

$$\varpi_i = \sum_{h=1}^{K}\Delta_h \bar{X}_h \Big/ \sum_{h=1}^{K}\Delta_h\left(\bar{X}_h+\lambda_{ih}(x)\right),\quad \vartheta_i = \sum_{h=1}^{K}\Delta_h \bar{X}_h \Big/ \sum_{h=1}^{K}\Delta_h\left(1+\bar{X}_h+\lambda_{ih}(x)\right).$$

Simplifying Eq. (3.18) and Eq. (3.19) up to the first order approximation, we obtained

$$\bar{y}_{ARi}-\bar{Y} = \bar{Y}\left(e_0-\varpi_i e_1+\varpi_i^2 e_1^2-\varpi_i e_0 e_1\right) \tag{3.20}$$

$$\bar{y}_{ASi}-\bar{Y} = \bar{Y}\left(e_0-\vartheta_i e_1+\vartheta_i^2 e_1^2-\vartheta_i e_0 e_1\right) \tag{3.21}$$

Take expectation of Eq. (3.20), Eq. (3.21) and using the results obtained in Eq. (3.15), we obtained the $Bias\left(\bar{y}_{ARi}\right)$ and $Bias\left(\bar{y}_{ASi}\right)$ as

$$Bias\left(\bar{y}_{ARi}\right) = R\bar{X}^{-1}\varpi_i^2 Var\left(\bar{x}_{st}\right)-\bar{X}^{-1}\varpi_i C\text{ov}\left(\bar{y}_{st}\bar{x}_{st}\right) \tag{3.22}$$

$$Bias\left(\bar{y}_{ASi}\right) = R\bar{X}^{-1}\vartheta_i^2 Var\left(\bar{x}_{st}\right)-\bar{X}^{-1}\vartheta_i C\text{ov}\left(\bar{y}_{st}\bar{x}_{st}\right) \tag{3.23}$$

where $R=\bar{Y}/\bar{X}$.

Squaring Eq. (3.20) and Eq. (3.21), and taking expectations and substituting the results of Eq. (3.15), we obtained the $MSE\left(\bar{y}_{ARi}\right)$ and $MSE\left(\bar{y}_{ASi}\right)$ as given in Eq. (3.24) and Eq. (3.25) respectively.

$$MSE\left(\bar{y}_{ARi}\right) = Var\left(\bar{y}_{st}\right)+R^2\varpi_i^2 Var\left(\bar{x}_{st}\right)-2R\varpi_i C\text{ov}\left(\bar{y}_{st}\bar{x}_{st}\right),\ i=1,2,3 \tag{3.24}$$

$$MSE\left(\bar{y}_{ASi}\right) = Var\left(\bar{y}_{st}\right)+R^2\vartheta_i^2 Var\left(\bar{x}_{st}\right)-2R\vartheta_i C\text{ov}\left(\bar{y}_{st}\bar{x}_{st}\right),\ i=1,2,3 \tag{3.25}$$

3.4. Properties of the New Weights Ω_{hi}^* and $\Pi_{hi}^*, i=1,2,3$

Theorem 1: The proposed weights Ω_{hi}^* and $\Pi_{hi}^*, i=1,2,3$ are consistent.

Proof: As the sample size in each stratum tends to the stratum size, i.e. as $n_n \to N_h$, the stratum sample mean converges to the stratum population mean, i.e. $\bar{x}_h \to \bar{X}_h$. Then, the expression $\sum_{h=1}^{K}\Delta_h\left(\bar{X}_h+\lambda_{hi}(x)\right)-\sum_{h=1}^{K}\Delta_h\left(\bar{x}_h+\lambda_{hi}(x)\right)$ in

$\Omega^*_{hi}, i = 1,2,3$ and expression $\sum_{h=1}^{K} \Delta_h \left(1 + \bar{X}_h + \lambda_{hi}(x)\right) - \sum_{h=1}^{K} \Delta_h \left(1 + \bar{x}_h + \lambda_{hi}(x)\right)$ in $\Pi^*_{hi}, i = 1,2,3$ tend to zeros. So,

$$\lim_{n_h \to N_h} \frac{\Omega^*_{hi}}{\Delta_h} \to 1 \tag{3.26}$$

$$\lim_{n_h \to N_h} \frac{\Pi^*_{hi}}{\Delta_h} \to 1 \tag{3.27}$$

Theorem 2: The sum of the proposed weights Ω^*_{hi} and $\Pi^*_{hi}, i = 1,2,3$ converged to unity.

Proof: Taking the summation of Ω^*_{hi} and $\Pi^*_{hi}, i = 1,2,3$ over K, we obtained

$$\sum_{h=1}^{K} \Omega^*_{hi} = 1 + K\left(\bar{X} - \bar{x}_{st}\right) / \sum_{h=1}^{K} \Delta_h \left(\bar{x}_h + \lambda_{hi}(x)\right) \tag{3.28}$$

$$\sum_{h=1}^{K} \Pi^*_{hi} = 1 + K\left(\bar{X} - \bar{x}_{st}\right) / \sum_{h=1}^{K} \Delta_h \left(1 + \bar{x}_h + \lambda_{hi}(x)\right) \tag{3.29}$$

As $n_n \to N_h$, $\bar{x}_h \to \bar{X}_h$ and $\bar{x}_{st} \to \bar{X}$, then

$$\lim_{n_h \to N_h} \sum_{h=1}^{K} \Omega^*_{hi} = \lim_{n_h \to N_h} \sum_{h=1}^{K} \Pi^*_{hi} = 1 \tag{3.30}$$

Theorem 3: The proposed weights $0 < \Omega^*_{hi} < 1$ and $0 < \Pi^*_{hi} < 1, i = 1,2,3$.

Proof: As $n_n \to N_h$, $\bar{x}_h \to \bar{X}_h$ and $\bar{x}_{st} \to \bar{X}$, then

$$\lim_{n_h \to N_h} \Omega^*_{hi} = \lim_{n_h \to N_h} \Pi^*_{hi} = \Delta_h = N_h / N \tag{3.31}$$

Since $N_h > 0, N > 0$ and $N_h < N$, then $0 < \Delta_h < 1$.

4. Empirical study

4.1. Simulation study

In this section, we perform a simulation study to examine the superiority of the proposed estimators over other estimators considered in the study. For this, we generate a bivariate random population of size N=1000 for study population stratified into 3 non-overlapping heterogeneous groups of size 200, 300 and 500 using function defined in Table 4.1. Samples of sizes 20, 30 and 50 were selected 10,000 times by the SRSWOR method from each stratum respectively. The precision (PRE) of the considered estimators was computed using Eq. (4.1).

$$Bias\left(\hat{\theta}\right)=\frac{1}{10000}\sum_{j=1}^{10000}\left(\hat{\theta}-\bar{Y}\right) \tag{4.1}$$

$$MSE\left(\hat{\theta}\right)/Var\left(\hat{\theta}\right)=\frac{1}{10000}\sum_{j=1}^{10000}\left(\hat{\theta}_j-\bar{Y}\right)^2 \tag{4.2}$$

$$PRE\left(\hat{\theta}\right)=\left(Var\left(\bar{y}_{st}\right)/Var\left(\hat{\theta}\right)\right)100 \tag{4.3}$$

where

$$\text{var}\left(\bar{y}_{st}\right)=\frac{1}{10000}\sum_{j=1}^{10000}\left(\bar{y}_{st}-\bar{Y}\right)^2, \hat{\theta}=\bar{y}_{st}, \bar{y}_{CE1}, \bar{y}_{CE2}, \bar{y}_{RTK1}, \bar{y}_{RTK2}, \bar{y}_{ARi}, \bar{y}_{ASi}$$

Table 4.1. Populations used for Empirical Study

Population	Auxiliary variable x	Study variable y
I	$x_h \sim N\left(\mu_h,\sigma_h\right), \mu_1=60, \sigma_1=50,$ $\mu_2=50, \sigma_2=70, \mu_3=30, \sigma_3=40$	$y_{hi}=\alpha x_{hi}+x_{hi}^2+\xi_{hi},$ $\alpha=0.5,1,1.5,2.0,2.5,3$ $\xi_h \sim N\left(0,1\right), h=1,2,3$
II	$x_h \sim chsq\left(\theta_h\right), \theta_1=1, \theta_2=2, \theta_3=3$	
III	$x_h \sim \exp\left(\lambda_h\right), \lambda_1=0.2, \lambda_2=0.3, \lambda_3=0.1$	
IV	$x_h \sim gamma\left(\theta_h,\eta_h\right), \theta_1=3, \eta_1=2,$ $\theta_2=3, \eta_2=1, \theta_3=3, \eta_3=3,$	

Table 4.2 shows the biases, MSEs and PREs of the traditional, Rao et al. (2016), Clement and Enang (2016) and the proposed estimators using population I defined in Table 4.1. The proposed estimators have smaller MSEs compared to other estimators. This implies that the estimates of the proposed estimators are on average closer to the true estimate than that of other estimators. The PREs of the proposed estimators are higher than that of other estimators. The proposed estimator under has PRE of 326.4 implying 200% and 100% gain in efficiency over and respectively. However, the proposed estimators are averagely more biased compared to other estimators considered in the study.

Table 4.2. PRE of the Proposed and Some Existing Estimators using Pop. BI

Estimators	Values of α								
	0.5			1.0			1.5		
	Bias	MSE	PRE	Bias	MSE	PRE	Bias	MSE	PRE
$\bar{y}_{st}$	0.1	404126.7	100	0.1	406568.4	100	0.1	409023.1	100
Rao et al. (2016)									
$\bar{y}_{RTK1}$	-0.8	176491.7	229	-0.8	176432.9	230.4	-0.9	176374.1	231.9
$\bar{y}_{RTK2}$	-2	174926	231	-2.1	174866.3	232.5	-2.1	174806.6	234
Clement and Enang (2016)									
$\bar{y}_{CE}$	8.9	192907.5	209.5	8.9	192907.5	210.8	8.9	192907.5	212
Proposed									
$\bar{y}_{AR1}$	-17.2	123802.4	326.4	-17.2	124216.1	327.3	-17.2	124636	328.2
$\bar{y}_{AS1}$	-17.2	125626.8	321.7	-17.2	126052.3	322.5	-17.2	126484.1	323.4
$\bar{y}_{AR2}$	-17.4	119408.2	338.4	-17.4	119731.9	339.6	-17.4	120061.1	340.7
$\bar{y}_{AS2}$	-17.4	121356.7	333	-17.4	121693.5	334.1	-17.4	122035.9	335.2
$\bar{y}_{AR3}$	58.2	125249.8	322.7	58.5	125596.7	323.7	58.8	125949.3	324.8
$\bar{y}_{AS3}$	57.4	127154.8	317.8	57.7	127514.5	318.8	58.0	127879.9	319.8
Estimators	**Values of α**								
	2.0			**2.5**			**3.0**		
$\bar{y}_{st}$	0.2	411490.7	100	0.2	413971.2	100	0.2	416464.8	100.0
Rao et al. (2016)									
$\bar{y}_{RTK1}$	-0.9	176315.5	233.4	-0.9	176256.9	234.9	-1.0	176198.3	236.4
$\bar{y}_{RTK2}$	-2.1	174747	235.5	-2.2	174687.5	237	-2.2	174628.1	238.5
Clement and Enang (2016)									
$\bar{y}_{CE}$	8.9	192907.5	213.3	8.9	192907.5	214.6	8.9	192907.5	215.9
Proposed									
$\bar{y}_{AR1}$	-17.2	125062	329	-17.3	125494.3	329.9	-17.3	125932.7	330.7
$\bar{y}_{AS1}$	-17.2	126922	324.2	-17.3	127366.1	325	-17.3	127816.4	325.8
$\bar{y}_{AR2}$	-17.4	120395.9	341.8	-17.5	120736.2	342.9	-17.5	121082	344
$\bar{y}_{AS2}$	-17.5	122383.8	336.2	-17.5	122737.4	337.3	-17.5	123096.5	338.3
$\bar{y}_{AR3}$	59.1	126307.7	325.8	59.5	126671.8	326.8	59.8	127041.7	327.8
$\bar{y}_{AS3}$	58.3	128251.1	320.8	58.6	128628.1	321.8	58.9	129010.9	322.8

Table 4.3 also shows the biases, MSEs and PREs of the traditional, Rao et al. (2016), Clement and Enang (2016) and the proposed estimators using population II defined in Table 4.1 The proposed estimators have smaller MSEs compared to other estimators. These results are in conformity with that of population in Table 4.2.

Table 4.3. PRE of the Proposed and Some Existing Estimators using Pop. II

Estimators	Values of α								
	0.5			1.0			1.5		
	Bias	MSE	PRE	Bias	MSE	PRE	Bias	MSE	PRE
$\bar{y}_{st}$	0.02	3.3	100	0.03	3.6	100	0.03	4.0	100
Rao et al. (2016)									
$\bar{y}_{RTK1}$	0.05	1.4	235.7	0.04	1.4	257.1	0.02	1.5	266.7
$\bar{y}_{RTK2}$	0.03	1.7	194.1	0.02	1.8	200	0.04	2.0	200
Clement and Enang (2016)									
$\bar{y}_{CE}$	-0.1.	1.0	330	-0.1	1.0	360	-0.1	1.0	400
Proposed									
$\bar{y}_{AR1}$	-0.1	0.9	366.7	-0.1	0.8	450	-0.1	0.8	500
$\bar{y}_{AS1}$	-0.1	1.2	275	-0.1	1.2	300	-0.1	1.2	333.3
$\bar{y}_{AR2}$	-0.1	0.9	366.7	-0.1	0.8	450	-0.1	0.8	500
$\bar{y}_{AS2}$	-0.1	1.2	275	-0.1	1.2	300	-0.1	1.2	333.3
$\bar{y}_{AR3}$	0.1	0.9	366.7	0.1	0.9	400	0.1	0.9	444.4
$\bar{y}_{AS3}$	0.02	1.2	275	0.01	1.2	300	0.1	1.3	307.7
Estimators	**Values of α**								
	2.0			**2.5**			**3.0**		
$\bar{y}_{st}$	0	4.4	100	0	4.8	100	0	5.2	100
Rao et al. (2016)									
$\bar{y}_{RTK1}$	0	1.5	293.3	0	1.6	300	0	1.7	305.9
$\bar{y}_{RTK2}$	0	2.1	209.5	0	2.2	218.2	0	2.3	226.1
Clement and Enang (2016)									
$\bar{y}_{CE}$	-0.1	1	440	-0.1	1	480	-0.1	1	520
Proposed									
$\bar{y}_{AR1}$	-0.1	0.8	550	-0.1	0.8	600	-0.1	0.8	650
$\bar{y}_{AS1}$	-0.1	1.3	338.5	-0.1	1.3	369.2	-0.1	1.3	400
$\bar{y}_{AR2}$	-0.1	0.8	550	-0.1	0.8	600	-0.1	0.8	650
$\bar{y}_{AS2}$	-0.1	1.3	338.5	-0.1	1.3	369.2	-0.1	1.3	400
$\bar{y}_{AR3}$	0.1	0.9	488.9	0.1	0.9	533.3	0.1	0.9	577.8
$\bar{y}_{AS3}$	0.1	1.3	338.5	0.1	1.3	369.2	0.1	1.4	371.4

Table 4.4. PRE of the Proposed and Some Existing Estimators using Pop. III

Estimators	Values of α								
	0.5			1.0			1.5		
	Bias	MSE	PRE	Bias	MSE	PRE	Bias	MSE	PRE
$\bar{y}_{st}$	-0.1	396	100	-0.1	405.7	100	-0.1	415.5	100
Rao et al. (2016)									
$\bar{y}_{RTK1}$	-1.0	199.4	198.6	-1.0	200.4	202.4	-1.0	201.3	206.4
$\bar{y}_{RTK2}$	-1.0	223.2	177.4	-1.0	225.1	180.2	-1.0	227.1	183
Clement and Enang (2016)									
$\bar{y}_{CE}$	-1.3	175.3	225.9	-1.3	175.3	231.4	-1.3	175.3	237
Proposed									
$\bar{y}_{AR1}$	-1.4	152.7	259.3	-1.4	152.1	266.7	-1.4	151.4	274.4
$\bar{y}_{AS1}$	-1.4	170.1	232.8	-1.4	170	238.6	-1.4	170	244.4
$\bar{y}_{AR2}$	-1.4	153.8	257.5	-1.4	153.2	264.8	-1.4	152.5	272.5
$\bar{y}_{AS2}$	-1.4	172.1	230.1	-1.4	172.1	235.7	-1.4	172.1	241.4
$\bar{y}_{AR3}$	-0.5	156.8	252.6	-0.5	156.2	259.7	-0.5	155.5	267.2
$\bar{y}_{AS3}$	-0.6	175.5	225.6	-0.5	175.6	231	-0.5	175.6	236.6
Estimators	Values of α								
	2.0			2.5			3.0		
$\bar{y}_{st}$	-0.1	425.4	100	-0.1	435.5	100	-0.1	445.7	100
Rao et al. (2016)									
$\bar{y}_{RTK1}$	-1.0	202.3	210.3	-1.0	203.3	214.2	-1.0	204.2	218.3
$\bar{y}_{RTK2}$	-1.0	229	185.8	-1.0	231	188.5	-1.0	233	191.3
Clement and Enang (2016)									
$\bar{y}_{CE}$	-1.3	175.3	242.7	-1.3	175.3	248.4	-1.3	175.3	254.2
Proposed									
$\bar{y}_{AR1}$	-1.4	150.8	282.1	-1.4	150.1	290.1	-1.4	149.5	298.1
$\bar{y}_{AS1}$	-1.4	170	250.2	-1.4	170	256.2	-1.4	170.1	262
$\bar{y}_{AR2}$	-1.4	151.9	280.1	-1.4	151.3	287.8	-1.4	150.7	295.8
$\bar{y}_{AS2}$	-1.4	172.2	247.0	-1.4	172.3	252.8	-1.4	172.3	258.7
$\bar{y}_{AR3}$	-0.4	154.9	274.6	-0.4	154.2	282.4	-0.4	153.6	290.2
$\bar{y}_{AS3}$	-0.5	175.7	242.1	-0.5	175.7	247.9	-0.4	175.8	253.5

Table 4.4 also shows the biases, MSEs and PREs of the traditional, Rao et al. (2016), Clement and Enang (2016) and proposed estimators using population III. The proposed estimators with the exception of $\bar{y}_{AS3}$, which performed below Clement

and Enang (2016) estimator, have smaller MSEs compared to other estimators. These results are in conformity with that of population in Table 4.2.

Table 4.5. PRE of the Proposed and Some Existing Estimators using Pop. IV

Estimators	Values of α								
	0.5			1.0			1.5		
	Bias	MSE	PRE	Bias	MSE	PRE	Bias	MSE	PRE
$\bar{y}_{st}$	0	0.66	100	0	0.74	100	0	0.83	100
Rao et al. (2016)									
$\bar{y}_{RTK1}$	0	0.32	206.2	0	0.33	224.2	0	0.34	244.1
$\bar{y}_{RTK2}$	0	0.4	165	0	0.43	172.1	0	0.46	180.4
Clement and Enang (2016)									
$\bar{y}_{CE}$	0	0.26	253.8	0	0.26	284.6	0	0.26	319.2
Proposed									
$\bar{y}_{AR1}$	0	0.22	300	0	0.22	336.4	0	0.22	377.3
$\bar{y}_{AS1}$	0	0.31	212.9	0	0.32	231.2	0	0.33	251.5
$\bar{y}_{AR2}$	0	0.22	300	0	0.22	336.4	0	0.22	377.3
$\bar{y}_{AS2}$	0	0.31	212.9	0	0.32	231.2	0	0.34	244.1
$\bar{y}_{AR3}$	0	0.23	287	0	0.23	321.7	0.1	0.23	360.9
$\bar{y}_{AS3}$	0	0.32	206.2	0	0.33	224.2	0	0.34	244.1
Estimators	**Values of α**								
	2.0			**2.5**			**3.0**		
$\bar{y}_{st}$	0	0.92	100	0	1.02	100	0	1.13	100
Rao et al. (2016)									
$\bar{y}_{RTK1}$	0	0.35	262.9	0	0.37	275.7	0	0.39	289.7
$\bar{y}_{RTK2}$	0	0.49	187.8	0	0.52	196.2	0	0.56	201.8
Clement and Enang (2016)									
$\bar{y}_{CE}$	0	0.26	353.8	0	0.26	392.3	0	0.26	434.6
Proposed									
$\bar{y}_{AR1}$	0	0.22	418.2	0	0.22	463.6	0	0.22	513.6
$\bar{y}_{AS1}$	0	0.34	270.6	0	0.36	283.3	0	0.37	305.4
$\bar{y}_{AR2}$	0	0.22	418.2	0	0.22	463.6	0	0.22	513.6
$\bar{y}_{AS2}$	0	0.35	262.9	0	0.36	283.3	0	0.38	297.4
$\bar{y}_{AR3}$	0.1	0.23	400	0.1	0.23	443.5	0.1	0.23	491.3
$\bar{y}_{AS3}$	0	0.36	255.6	0	0.37	275.7	0.1	0.39	289.7

Table 4.5 also shows the biases, MSEs and PREs of the traditional, Rao et al. (2016), Clement and Enang (2016) and proposed estimators using population III. The proposed estimators with the exception of $\overline{y}_{AR3}$ and other estimators are unbiased. The proposed estimators $\overline{y}_{AR1}, \overline{y}_{AR2}, \overline{y}_{AR3}$ performed better compared to other estimators. However, the proposed estimators $\overline{y}_{AS1}, \overline{y}_{AS2}, \overline{y}_{AS3}$, which outperformed Rao et al. (2016) estimators and usual unbiased estimator $\overline{y}_{st}$, performed below the estimator of Clement and Enang (2016).

5. Discussion

Tables 4.2, 4.3, 4.4 and 4.5 report PREs of the sample mean in stratified sampling, Rao et al. (2016), Clement and Enang (2016) and proposed calibration estimators using populations I, II, III and IV (Normal, Chi Square, exponential and gamma distributions) respectively defined in Table 4.1 for different values of $\alpha = (0.5, 1.0, 1.5, 2.0, 2.5, 3.0)$. The results of the PREs reveal that as the values of α (coefficients of linear component of response variable model) increase, the efficiency of the all the estimators increases. The results also revealed that all the proposed estimators have higher PREs compared to their counterparts considered in the study. This implies that the proposed estimators are more efficient in estimation of population mean than other related estimators considered in this study.

6. Conclusion

In this study, we utilized auxiliary information for a heterogeneous population in the form of robust statistical measures based on Gini's mean difference, Downton's method and probability weighted moments. These measures which are not unduly affected by outliers present in the data and provide more efficient estimates of population parameters in the presence of extreme values were used as alternatives for the coefficient of variation used by Rao et al. (2016). From the results of Tables 4.2, 4.3, 4.4 and 4.5, it is observed that in general the estimators proposed under both the calibration schemes are not only robust but more efficient than the usual ratio estimator in stratified sampling, Clement and Enang (2016) and Rao et al. (2016) calibration estimators making them applicable in real life situation when data is somewhat affected by the presence of extreme values. However, the proposed estimators $\overline{y}_{AS1}, \overline{y}_{AS2}, \overline{y}_{AS3}$ performed below the estimator of Clement and Enang (2016) under population IV and generally the efficiency of the proposed estimators is higher when the study variables are characterized by outliers.

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