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A modified robust confidence interval for the population mean of distribution based on deciles

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ABSTRACT

The confidence interval is an important statistical estimator of population location and dispersion parameters. The paper considers a robust modified confidence interval, which is an adjustment of the Student's t confidence interval based on the decile mean and decile standard deviation for estimating the population mean of a skewed distribution. The efficiency of the proposed interval estimator is evaluated on the basis of an extensive Monte Carlo simulation study. The coverage ratio and average width of the proposed confidence interval are compared with certain existing and widely used confidence intervals. The simulation results show that, in general, the proposed interval estimator's performance is highly effective. For illustrative purposes, three real-life data sets are analyzed, which, to a certain extent, support the findings obtained from the simulation study. Thus, we recommend that practitioners use the robust modified confidence interval for estimating the population mean when the data are generated by a normal or skewed distribution.

Key words: robust confidence interval, decile mean, decile mean standard deviation, decile mean standard error, Monte Carlo simulation

1. Introduction

The normality assumption is the basis for many developed statistical theories. One of these theories is the estimation theory for constructing the confidence interval developed by Neyman (1937). However, in real life a lot of the data do not follow normality assumption and data are not mound shaped, rather they are skewed; that is, there is a lack of symmetry of the distribution about the mean. Skewed data may harm our results. Skewness is considered either positive or negative based on the direction

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and nature of the distribution. It has long been known that when sampling from a skewed population, with small sample sizes, the usual frequentist confidence intervals for the population mean (μ) have poor coverage properties (Meeden, 1999). The positively skewed data, for example, are common in various fields of modelling such as in psychology (Cain et al., 2016), health science (Baklizi and Kibria, 2009; Banik and Kibria, 2010; Ghosh and Polansky, 2016), environmental science (Mudelsee and Alkio, 2007), biological science (McDonald, 2014), engineering science and others. A confidence interval is an interval estimator that will capture the true parameter value in repeated samples. Abu-Shawiesh et al. (2019) defined confidence interval as a range of values that provides the user with an understanding of how precise the estimates of a parameter are. In practice, it is usual to use normal theory to construct a confidence interval for making inferences about the population mean (μ). Unfortunately, the confidence interval based on this theory suffers when samples come from skewed or non-normal populations. Therefore, it is important to construct a confidence interval of a population mean (μ) that is not limited by the assumption of population normality (Miller and Penfield, 2005). Several other methods have been described in the literature, such as transformation methods and bootstrap methods, to obtain an acceptable coverage rate and small interval width with skewed distribution and small sample sizes (Meeden, 1999; Shi and Kibria, 2007; Ghosh and Polansky, 2016). There are various methods in the literature in which confidence intervals are obtained for the population mean (μ). In practice, it is often possible to work with smaller sample sizes. In such cases, Student's t confidence interval can be preferred instead of the classical confidence interval, but it requires an assumption of normality. Luh and Guo (2001) argued that "since violation of the normality assumption may be fairly common in applied research, robust and efficient alternatives to deal with the problem are needed". Therefore, it is essential to use robust estimators which are less affected by outliers or small departures from the model assumptions (Sindhumol et al., 2016). Johnson (1978) proposed a modification of the Student's t confidence interval for skewed distributions. Since Johnson (1978), many researchers have obtained confidence intervals for population mean of a skewed distribution (Chen, 1995; Meeden, 1999; Kibria, 2006; Shi and Kibria, 2007; Baklizi, 2008; Abu-Shawiesh et al., 2009; Baklizi and Kibria, 2009; Abu-Shawiesh et al., 2011; Pek et al., 2017; Abu-Shawiesh et al., 2018; Abu-Shawiesh and Saghir, 2019; Akyuz and Abu-Shawiesh, 2020; Sinsomboonthong et al., 2020).

In this paper, we compare various methods for constructing a confidence interval for the population mean (μ) when data are normally or non-normally distributed and propose a new robust confidence interval. This proposed confidence interval is an adjustment of the Student's t confidence interval based on the decile mean and the decile mean standard deviation. Since a theoretical comparison is not possible, we investigate the performance of the proposed confidence interval by using a Monte Carlo simulation study and its implementation with three real-life data sets.

2. The decile mean (DM) and the decile mean standard deviation (SD_{DM})

The sample mean ($\bar{x}$) and sample standard deviation (s) are the most popular and frequently used classical estimators of the location and scale parameters of a probability distribution. However, they are unreliable in the presence of skewed distributions. In this paper, we estimate them with well-known and simple robust estimators of location and scale. They are the decile mean (DM) for location and the decile mean standard deviation (SD_{DM}) for scale. Furthermore, the standard error of the decile mean standard deviation (SD_{DM}) is defined.

2.1. The Deciles (D_m)

The central tendency of a data set is a measure of the location or most typical value of the data set. There are various types of descriptive statistics, such as sample mean, sample median and sample trimmed mean that can be chosen as a measure of the central tendency; under a well-behaved normal distribution, they possess some desirable properties. But there is evidence that they may perform poorly and not as well as expected in the presence of skewed distributions. Rana et al. (2012) proposed a new measure of central tendency based on deciles called the decile mean (DM). This measure is fairly robust as it automatically discards extreme observations or outliers from both tails but at the same time is more informative than the sample median in every respect. Let $X_1, X_2, \dots, X_n$ be independent identically distributed (iid) observations from a given population with mean (μ) and standard deviation (σ); then the deciles, which are a measure of position, are the values (nine in number) of the variable that divide any ordered data set $X_{(1)}, X_{(2)}, \dots, X_{(n)}$ into ten equal parts so that each part represents $\frac{1}{10}$ of the sample or population, and are denoted by $D_1, D_2, \dots, D_9$. The fifth decile (D_5) is equal to the sample median (MD). The deciles determine the values for 10%, 20% … and 90% of the data set. Now assume that $x_1, x_2, \dots, x_n$ be the sample observations of the random sample $X_1, X_2, \dots, X_n$, then the deciles can be calculated as follows:

(1) Order the observations $x_1, x_2, \dots, x_n$ according to the magnitude of the values to get the ordered data set $x_{(1)}, x_{(2)}, \dots, x_{(n)}$.

(2) To find the value of the m^{th} sample decile where $m = 1, 2, \dots, 9$, the following simple formula can be used:

$$D_m = x_{\left(\left[\frac{m\,(n+1)}{10}\right]\right)} \text{ observation} \tag{1}$$

where n is the total number (sample size) of observations.

2.2. The Decile Mean (DM)

The decile mean, denoted by DM for the random sample $X_1, X_2, \dots, X_n$, can be calculated by summing all the deciles $D_1, D_2, \dots, D_9$ and dividing the sum by the number of deciles. Thus, the formula to find the decile mean (DM) from 9 deciles is as follows:

$$DM = \frac{\sum_{i=1}^{9} D_i}{9} = \frac{D_1 + D_2 + \cdots + D_9}{9} \quad (2)$$

The main advantage of the decile mean (DM) is that it is less sensitive to extreme values than any other existing measures; also, it depends on 80% of a sample. Let X distributed like $X_1, X_2, \dots, X_n$. Then D_m is a number for which $P(X < D_m) \le \frac{m}{10} \le P(X \le D_m)$. If X has absolutely continuous distribution function $F(x) = P(X \le x)$ then $F(D_m) = P(X \le D_m) = \frac{m}{10}$. It is termed a robust estimator in this regard. Rana et al. (2012) used the bootstrap method to investigate the sampling distribution of the newly proposed decile mean (DM) with three other popular and commonly used measures of location, i.e., the sample mean, median and trimmed mean, and found that the newly proposed decile mean (DM) has the following properties:

(i) The distribution of the sample decile is quite normal in shape and irrespective of the presence of outliers.
(ii) The bias and standard error of sample decile mean (DM) are very small, and among the four compared estimators, this estimator appears to be the best in every respect.
(iii) The results presented show that all four estimators are biased, but this bias is the least for the sample decile mean (DM).

Both the bootstrap and simulation study demonstrate that the sample decile mean (DM) is a more accurate measure of central tendency or location in terms of possessing smaller bias and lower standard errors in a variety of situations, and hence can be recommended to be used as an effective measure of central tendency or location.

2.3. Decile Mean Standard Deviation (SD_{DM}) and Standard Error (SE_{DM})

Decile mean standard deviation (SD_{DM}) is a robust measure of dispersion proposed by Doullah (2018) as an alternative to the sample standard deviation (S). Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a given population with mean (μ) and standard deviation (σ); then the decile mean standard deviation (SD_{DM}) can be calculated by using the following formula:

$$SD_{DM} = \sqrt{\frac{\text{The sum of the 9 deciles of } (X_i - DM)^2}{9-1}} = \sqrt{\frac{1}{8} \sum_{i=1}^{9} (D_i - D_m)^2} \quad (3)$$

Doullah (2018) also defined the standard error of the decile mean standard deviation (SD_{DM}), denoted by SE_{DM} , to be computed as follows:

$$SE_{DM} = \frac{SD_{DM}}{\sqrt{n}} \quad (4)$$

3. Methods for estimation the confidence interval for the population mean

In this section, the used methods and the proposed robust method for confidence interval of the population mean (μ) for normal and non-normal distributions are introduced. Let $X_1, X_2, \dots, X_n$ be iid random sample of size n from a population with mean (μ) and standard deviation (σ). Our purpose is to find an interval estimate for the population mean (μ) with a specific level of confidence. Several methods have been suggested in the literature to find the confidence interval for μ. These are (a) the parametric approach, (b) the modified t approach, (c) the nonparametric approach and (d) the bootstrap approach, among others. In this study, we concentrate on (a) and (b) approaches only. The (1 – α) 100% confidence intervals for the population mean (μ) by different approaches are presented below.

3.1. The Parametric t-Approach

The parametric method to construct the (1 – α) 100% confidence interval for the population mean (μ) is the most used approach because it is well understood, simple and widely used to construct such the confidence interval. Under this approach, we consider two confidence interval methods. Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a normal distribution with mean (μ) and variance (σ^2); that is, $X_1, X_2, \dots, X_n \sim N(\mu,\ \sigma^2)$. Then, the (1 – α) 100% confidence interval for the population mean (μ) given by Student (1908) and known as the Student's t confidence interval for a small sample size n ($n \leq 30$) and unknown population standard deviation (σ) can be constructed as follows:

$$C.I. = \bar{X} \pm t_{(\frac{\alpha}{2},\ n-1)} \frac{S}{\sqrt{n}} \quad (5)$$

where $\bar{X} = n^{-1} \sum_{i=1}^{n} X_i$, $S = \sqrt{(n-1)^{-1} \sum_{i=1}^{n} (X_i - \bar{X})^2}$ and $t_{(\alpha/2, n-1)}$ is the upper $\alpha/2$ percentage point of the Student's t-distribution with $(n - 1)$ degrees of freedom. Now, since the Student's t confidence interval depends on the normality assumption, it may not be the best confidence interval and may not perform as well as expected in the presence of skewed distributions. DiCicco and Efron (1996) and Boos and Hughes-Oliver (2000) stated that the Student's t confidence interval is not very robust and can be quite inaccurate in practice for non-normal data.

3.2. The Modified Parametric t-Approach

If the iid random sample $X_1, X_2, \dots, X_n$ is from a non-normal distribution, the distribution of the t-statistic is not a Student's t distribution. In particular, the skewness of a non-normal distribution has a large impact on the validity of the Student's t-distribution; see, for example, Yanagihara and Yuan (2005). Several methods for constructing the (1 – α) 100% confidence interval for the population mean (μ) have been proposed to remove the effect of skewness by modifying the t-statistic. Here, we briefly review the most important of these methods.

3.2.1. The Johnson t-Approach

Based on the first term of the inverse Cornish–Fisher expansion, Johnson (1978) proposed the following confidence interval estimator for the population mean (μ):

$$C.I. = \left[\bar{X} + \frac{\hat{\mu}_3}{6nS^2}\right] \pm t_{\left(\frac{\alpha}{2}, n-1\right)} \frac{S}{\sqrt{n}} \tag{6}$$

where $\hat{\mu}_3 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^3}{n}$ is the estimator of the third central moment of the population (μ_3). Kibria (2006) concluded it appears that the width of the Student's t and Johnson-t confidence intervals is the same.

3.2.2. The Chen t-Approach

Using the Edgeworth expansion, Chen (1995) modified the CLT approach and proposed the following confidence interval estimator for the population mean (μ):

$$C.I. = \bar{X} \pm \left[t_{\left(\frac{\alpha}{2}, n-1\right)} + \frac{\hat{\gamma}\left(1 + 2t^2_{\left(\frac{\alpha}{2}, n-1\right)}\right)}{6\sqrt{n}} + \frac{\hat{\gamma}^2\left(t_{\left(\frac{\alpha}{2}, n-1\right)} + 2t^2_{\left(\frac{\alpha}{2}, n-1\right)}\right)}{9\,n} \right] \frac{S}{\sqrt{n}} \tag{7}$$

where $\hat{\gamma} = \frac{\hat{\mu}_3}{S^3}$ is the estimate of the coefficient of skewness.

3.2.3. The Yanagihara and Yuan t-Approach

To reduce the effect of the mean bias as well as population skewness, Yanagihara and Yuan (2005) proposed the following confidence interval estimator for the population mean (μ):

$$C.I. = \left[\bar{X} + \frac{(S\,\hat{k}_3)}{\left((4n)\left(2 + \frac{15}{n}\right)\right)}\right] \pm t_{\left(\frac{\alpha}{2}, n-1\right)} \frac{S}{\sqrt{n}} \tag{8}$$

where $\hat{k}_3 = \frac{\left(\sum_{i=1}^{n}(X_i - \bar{X})^3/n\right)}{\left(\sum_{i=1}^{n}(X_i - \bar{X})^2/n\right)^{3/2}}$.

3.2.4. The Shi and Kibria Mad t-Approach

In terms of using the sample median (MD) rather than the sample mean ($\bar{X}$) for defining the sample standard deviation, Shi and Kibria (2007) proposed another confidence interval estimator for the population mean (μ), as follows:

$$C.I. = \bar{X} \pm t_{(\frac{\alpha}{2}, n-1)} \frac{\tilde{S}_2}{\sqrt{n}} \tag{9}$$

where $\tilde{S}_2 = \frac{1}{n} \sum_{i=1}^{n} |X_i - \bar{X}|$ is the sample mean absolute deviation (Mad).

3.2.5. The Abu-Shawiesh, Banik and Kibria AADM t-Approach

Abu-Shawiesh et al. (2018) proposed a modification of the Student's t confidence interval for the population mean (μ) of a skewed distribution, called AADM-t confidence interval estimator and expressed as follows:

$$C.I. = \bar{X} \pm t_{(\frac{\alpha}{2}, n-1)} \frac{AADM}{\sqrt{n}} \tag{10}$$

where $AADM = \frac{\sqrt{\pi/2}}{n} \sum_{i=1}^{n} |X_i - MD|$ is the average absolute deviation from the sample median (Gastwirth, 1982). Gastwirth (1982) stated that the AADM is an asymptotically normally distributed, consistent estimator of the population standard deviation (σ) and almost surely converges to it.

3.3. The Confidence Interval Based on Resampling Approach

Efron and Tibshirani (1993) recommended resampling approach to generate a large number of independent bootstrap samples $x^{*1}, x^{*2}, \ldots, x^{*B}$ for a random sample from an unknown distribution with population mean. A bootstrap sample $x^* = (x_1^*, x_2^*, \ldots, x_n^*)$ is obtained by randomly resampling n times with replacement from the original data sample $x_1, x_2, \ldots, x_n$. Then, the (1 – α) 100% confidence interval based on bootstrap percentile for the population mean (μ) can be constructed as follows: this approach performs resampling technique B times and let $\hat{\mu}^{*1}, \hat{\mu}^{*2}, \ldots, \hat{\mu}^{*B}$ be the estimator of parameter μ for each independent bootstrap sample $x^{*1}, x^{*2}, \ldots, x^{*B}$. If $\hat{\mu}^*$ is a random variable drawn from the normal distribution with mean $\hat{\mu}$ and variance $\hat{\sigma}^2$, then the (1 – α) 100% bootstrap percentile confidence interval estimator for the population mean (μ) can be expressed in the form of equation (11) and (12) as follows:

$$CL = 100 \cdot {}^{\alpha}/_{2}{}^{th} \ percentile\ of\ \hat{\mu}^{*\prime}s\ distribution \tag{11}$$

$$UCL = 100 \cdot (1 - {}^{\alpha}/_{2})^{th} \ percentile\ of\ \hat{\mu}^{*\prime}s\ distribution \tag{12}$$

The bootstrap confidence interval produce a good coverage ratio for interval estimation as shown in the study by DiCiccio and Efron (1996), Marinho et al. (2018), Ghosh and Polansky (2016).

3.4. The Proposed Robust DMSD$_{DM}$ t-Approach

In this section, we propose a robust modification of the Student's t confidence interval for the population mean (μ) of a skewed population. It is a simple adjustment of the Student's t confidence interval and can be obtained with the following steps:

Step 1: Select a random sample of size (n), X_1, $X_2, \dots, X_n$, from the probability distribution of the random variable X.

Step 2: Calculate the sample decile mean (DM), which is given by equation (2).

Step 3: Calculate the decile mean standard deviation (SD$_{DM}$) and the standard error of the decile mean standard deviation (SE_{DM}), which are given by equations (3) and (4).

Step 4: The lower confidence limit (LCL) and the upper confidence limit (UCL) for the $(1-\alpha)100\%$ proposed robust confidence interval estimator–DMSD$_{DM}$-t confidence interval–of the population mean (μ) for the skewed distribution can be calculated as follows:

$$LCL = DM - t_{(\frac{\alpha}{2},n-1)} \frac{SD_{DM}}{\sqrt{n}} \tag{13}$$

$$UCL = DM + t_{(\frac{\alpha}{2},n-1)} \frac{SD_{DM}}{\sqrt{n}} \tag{14}$$

where $\boldsymbol{t}_{(\frac{\alpha}{2},\boldsymbol{n-1})}$ is the upper α/2 percentage point of the Student's t-distribution with $(\boldsymbol{n-1})$ degrees of freedom.

4. The simulation study

Since a theoretical comparison among these confidence intervals is not possible, a simulation study is conducted. All the simulation results are performed by SAS programming version 9.4.

4.1. Performance Evaluation

A Monte Carlo simulation study is presented in this section to compare the performance of eight confidence interval estimators for the population mean of three distributions. We consider a set of possible useful confidence intervals and compare them with the proposed robust method, aiming to confirm that it is appropriate for estimating the population mean (μ) of a skewed distribution. To make comparisons among confidence intervals, the coverage ratio (CR) and average width (AW) of the confidence intervals are considered as the performance criteria. A smaller width indicates a better confidence interval when the coverage ratios are the same level. Further, the higher coverage ratio indicates a better confidence interval when the widths of intervals are the same level. The sample sizes of n = 10, 20, 30, 40, 50 and 100 were randomly generated 100,000 times. For each set of samples, 95% confidence intervals were constructed for the considered methods and the construction of bootstrap percentile confidence intervals for the population mean are generated resampling 1,000 times for each situation. The coverage ratio (CR) and the average width (AW) of the confidence intervals are obtained using the following two formulas:

$$CR = \frac{\#(L \leq \theta \leq U)}{100{,}000} \quad \text{and} \quad AW = \frac{\sum_{i=1}^{100{,}000} (U_i - L_i)}{100{,}000} \tag{15}$$

4.2. Probability Distributions for the Simulation Study

To study the effect of skewness and compare the performance of the eight confidence interval estimators for the population mean (μ) of the distribution, two cases for the simulation observations, namely normal and skewed distributions, are considered in this study.

Case (a): Normal Distribution

The normal distribution is symmetric and has no skewness. The probability density function (pdf) of a normal distribution with mean μ and standard deviation σ, $N(\mu, \sigma^2)$, is given as follows:

$$f(x;\ \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}}\ e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\ ;\ -\infty < x < \infty\ ,\ -\infty < \mu < \infty\ ,\ \sigma > 0 \tag{16}$$

In the simulation algorithm of this study, the population mean μ and the population standard deviation σ are set as $\mu = 20$ and $\sigma = 5, 10, 20$.

Case (b): Skewed Distributions

The skewness of a probability distribution refers to the departure of the distribution from symmetry. A distribution with longer tail on the left is negative skewed, and a distribution with longer tail on the right is positive skewed (Sharma et al., 2009). For skewed distributions, we simulate observations from two probability distributions with varying degrees of skewness as follows:

(i) The chi-square distribution, $\chi^2_{(k)}$, where k is the number of degrees of freedom with probability density function (pdf), is given as follows:

$$f(x;\ k) = \begin{cases} \frac{1}{\Gamma(k/2)\ 2^{k-1}}\ x^{(k/2)-1}\ e^{-x/2} & ,\quad x > 0 \\ 0 & ,\ otherwise \end{cases} \tag{17}$$

The mean and the variance of the chi-square distribution are given by $\mu = k$ and $\sigma^2 = 2k$. The coefficient of skewness of the distribution is $\sqrt{8/k}$. In the simulation algorithm of this study, the parameter k for the chi-square distribution is set as k = 5, 10, 50.

(ii) The triangular distribution, $Tr(a, b, c)$, involves parameters a, b and c, where a is the minimum value, b is the maximum value and c is the most likely value (mode). The triangular distribution is selected for this study as it can be used to model both positive and negative skewed distributions. The probability density function (pdf) for the triangular distribution is given as follows:

$$f(x; a, b, c) = \begin{cases} 0 & , \quad x < a \\ \frac{2(x-a)}{(b-a)(c-a)} & , \quad a \le x \le c \\ \frac{2(b-x)}{(b-a)(b-c)} & , \quad c < x \le b \\ 0 & , \quad x > b \end{cases} \tag{18}$$

The mean and variance of the triangular distribution, $Tr(a, b, c)$, are given by $\mu = \frac{a+b+c}{3}$ and $\sigma^2 = \frac{a^2+b^2+c^2-ab-ac-bc}{18}$. The skewness coefficient of the triangular distribution is given by $\frac{\sqrt{2}(a+b-2c)(2a-b-c)(a-2b+c)}{5(a^2+b^2+c^2-ab-ac-bc)^{3/2}}$. In the simulation algorithm of this study, we simulate observations from $Tr(0, 1, 0.05), Tr(0, 1, 0.5)$ and $Tr(0, 1, 0.95)$ to represent the positive, symmetric and negative cases of the triangular distribution, respectively. Table 4.1 shows the specific distributions and their skewness coefficients used in this simulation study.

Table 4.1. Coefficients of skewness for the studied simulation probability distributions

Probability Distributions	Parameters	Coefficients of Skewness
$N(\mu, \sigma^2)$	$\mu = 20, \sigma = 5, 10, 20$	0
$\chi^2_{(k)}$	$k = 5$	1.2649
	$k = 10$	0.8944
	$k = 50$	0.4000
$Tr(0, 1, c)$	$c = 0.05$	0.5607
	$c = 0.50$	0
	$c = 0.95$	-0.5607

4.3. The Simulation Study Results

The simulation results for all studied cases are shown in Tables 4.2 to 4.10. The performance of 95% confidence intervals of the population mean for the eight methods are as follows: in the case of normally distributed data as shown in Tables 4.2 to 4.4, it is observed that the coverage ratio of $DMSD_{DM}$-t confidence interval is slightly under

0.95 for all sample sizes. However, the coverage ratio of Bootstrap-percentile confidence interval is only slightly under 0.95 for a small sample size, but it close to 0.95 for the large sample sizes. In addition, the coverage ratios of five intervals–Student's t, Johnson-t, Chen-t, YY-t and AADM-t–are close to 0.95 for all sample sizes. Further, the coverage ratio of Mad-t confidence interval is more under than the nominal level when compared with the proposed interval. When the performance of confidence intervals is compared in terms of the average width, the five methods in which the coverage ratio is close to 0.95 tend to have no difference in average width for any sample size or any of the normal distributed data. Although, the coverage ratio of the proposed method is slightly lower than that of the five methods, the average width of this proposed interval is smaller than that of the five intervals for all sample sizes and it is smaller than the average width of Bootstrap-percentile for the large sample size.

Table 4.2. Coverage ratio (CR) and average width (AW) of the 95% confidence intervals for the population mean of normal distribution with $\mu = 20$ and $\sigma = 5$

n	Performance Measures	Confidence Interval Methods							
		Student-t	Johnson-t	Chen-t	YY-t	Mad-t	AADM-t	$DMSD_{DM}$-t	Bootstrap
10	CR	0.9506	0.9506	0.9477	0.9505	0.8855	0.9438	0.9260	0.9013
	AW	7.0	7.0	7.1	7.0	5.4	6.8	6.2	5.7
20	CR	0.9496	0.9495	0.9482	0.9495	0.8826	0.9459	0.9174	0.9266
	AW	4.6	4.6	4.6	4.6	3.6	4.6	4.1	4.2
30	CR	0.9494	0.9495	0.9489	0.9495	0.8819	0.9464	0.9142	0.9345
	AW	3.7	3.7	3.7	3.7	2.9	3.7	3.3	3.5
40	CR	0.9499	0.9499	0.9493	0.9499	0.8820	0.9481	0.9138	0.9382
	AW	3.2	3.2	3.2	3.2	2.5	3.2	2.8	3.0
50	CR	0.9501	0.9502	0.9499	0.9501	0.8818	0.9483	0.9129	0.9409
	AW	2.8	2.8	2.8	2.8	2.2	2.8	2.5	2.7
100	CR	0.9501	0.9501	0.9501	0.9501	0.8818	0.9490	0.9119	0.9449
	AW	2.0	2.0	2.0	2.0	1.6	2.0	1.8	1.9

Table 4.3. Coverage ratio (CR) and average width (AW) of the 95% confidence intervals for the population mean of normal distribution with $\mu = 20$ and $\sigma = 10$

n	Performance Measures	Confidence Interval Methods							
		Student-t	Johnson-t	Chen-t	YY-t	Mad-t	AADM-t	$DMSD_{DM}$-t	Bootstrap
10	CR	0.9506	0.9506	0.9477	0.9505	0.8855	0.9437	0.9260	0.9013
	AW	13.9	13.9	14.1	13.9	10.8	13.6	12.5	11.4
20	CR	0.9496	0.9495	0.9482	0.9495	0.8826	0.9458	0.9173	0.9266
	AW	9.2	9.2	9.3	9.2	7.3	9.1	8.2	8.4
30	CR	0.9494	0.9494	0.9489	0.9495	0.8820	0.9464	0.9142	0.9345
	AW	7.4	7.4	7.4	7.4	5.9	7.3	6.6	7.0
40	CR	0.9499	0.9500	0.9493	0.9499	0.8821	0.9481	0.9138	0.9381
	AW	6.4	6.4	6.4	6.4	5.0	6.3	5.6	6.1
50	CR	0.9501	0.9502	0.9499	0.9501	0.8819	0.9483	0.9129	0.9409
	AW	5.7	5.7	5.7	5.7	4.5	5.6	5.0	5.5
100	CR	0.9501	0.9501	0.9500	0.9501	0.8818	0.9490	0.9119	0.9449
	AW	4.0	4.0	4.0	4.0	3.1	3.9	3.5	3.9

Table 4.4. Coverage ratio (CR) and average width (AW) of the 95% confidence intervals for the population mean of normal distribution with $\mu = 20$ and $\sigma = 20$

n	Performance Measures	Confidence Interval Methods							
		Student-t	Johnson-t	Chen-t	Student-t	Mad-t	AADM-t	$DMSD_{DM}$-t	Bootstrap
10	CR	0.9506	0.9506	0.9477	0.9506	0.8856	0.9437	0.9260	0.9013
	AW	27.8	27.8	28.3	27.8	21.7	27.1	25.0	22.8
20	CR	0.9496	0.9495	0.9482	0.9495	0.8826	0.9458	0.9173	0.9266
	AW	18.5	18.5	18.6	18.5	14.6	18.2	16.4	16.8
30	CR	0.9494	0.9494	0.9489	0.9495	0.8819	0.9464	0.9142	0.9345
	AW	14.8	14.8	14.9	14.8	11.7	14.7	13.1	13.9
40	CR	0.9499	0.9500	0.9493	0.9499	0.8821	0.9481	0.9138	0.9382
	AW	12.7	12.7	12.7	12.7	10.1	12.6	11.3	12.1
50	CR	0.9501	0.9502	0.9499	0.9501	0.8818	0.9483	0.9129	0.9409
	AW	11.3	11.3	11.3	11.3	9.0	11.2	10.0	10.9
100	CR	0.9501	0.9502	0.9500	0.9501	0.8818	0.9490	0.9119	0.9449
	AW	7.9	7.9	7.9	7.9	6.3	7.9	7.0	7.8

In the case of data are generated from two skewed probability distributions–chi-square and triangular distributions–with varying degrees of skewness, the performance of 95% confidence intervals of the population means are shown in Tables 4.5 to 4.10. If the coefficient of skewness for chi-square distribution is equal to 1.2649 or 0.8944, then the coverage ratio of $DMSD_{DM}$-t tends to decrease when the sample size increases. However, if the coefficient of skewness for this distribution is equal to 0.4000, then the coverage ratio of $DMSD_{DM}$-t is slightly under 0.95, and it tends to be at the same level irrespective of the sample size. Moreover, the coverage ratio of five intervals – Student's t, Johnson-t, Chen-t, YY-t and AADM-t – is close to the specified confidence coefficient level, while that of the Mad-t confidence interval is more under than the nominal level for all sample sizes when it is compared with the proposed method. For both positive and negative coefficients of skewness for triangular distribution, the coverage ratio of $DMSD_{DM}$-t tends to decrease for a large sample size. Moreover, this coverage ratio of $DMSD_{DM}$-t tends to be the same level and slightly under 0.95 for each sample size when coefficient of skewness equals zero. Additionally, the coverage ratio of five intervals – Student's t, Johnson-t, Chen-t, YY-t and AADM-t – is close to 0.95 for all sample sizes and all coefficients of skewness for triangular distributions. When considering all of the distributions in this study, it is found that the coverage ratios of the proposed confidence interval are close to the nominal level and greater than this of the Bootstrap-percentile confidence interval for a small sample size, and the average width of these two methods tends to be no difference for all sample sizes.

Table 4.5. Coverage ratio (CR) and average width (AW) of the 95% confidence intervals for the population mean of Chi-square distribution with df = 5

n	Performance Measures	Confidence Interval Methods							
		Student-t	Johnson-t	Chen-t	YY-t	Mad-t	AADM-t	$DMSD_{DM}$-t	Bootstrap
10	CR	0.9299	0.9311	0.9445	0.9306	0.8670	0.9233	0.8959	0.8869
	AW	4.3	4.3	5.2	4.3	3.3	4.2	3.8	3.5
20	CR	0.9366	0.9379	0.9563	0.9375	0.8687	0.9305	0.8650	0.9159
	AW	2.9	2.9	3.4	2.9	2.2	2.8	2.4	2.6
30	CR	0.9406	0.9417	0.9601	0.9413	0.8683	0.9350	0.8481	0.9281
	AW	2.3	2.3	2.7	2.3	1.8	2.2	1.9	2.2
40	CR	0.9420	0.9429	0.9619	0.9426	0.8669	0.9358	0.8288	0.9322
	AW	2.0	2.0	2.3	2.0	1.5	1.9	1.6	1.9
50	CR	0.9433	0.9440	0.9621	0.9437	0.8693	0.9364	0.8181	0.9353
	AW	1.8	1.8	2.0	1.8	1.4	1.7	1.4	1.7
100	CR	0.9472	0.9477	0.9633	0.9477	0.8685	0.9404	0.7580	0.9429
	AW	1.2	1.2	1.4	1.2	1.0	1.2	1.0	1.2

Table 4.6. Coverage ratio (CR) and average width (AW) of the 95% confidence intervals for the population mean of Chi-square distribution with df = 10

n	Performance Measures	Confidence Interval Methods							
		Student-t	Johnson-t	Chen-t	YY-t	Mad-t	AADM-t	$DMSD_{DM}$-t	Bootstrap
10	CR	0.9391	0.9397	0.9483	0.9393	0.8749	0.9325	0.9099	0.8926
	AW	6.2	6.2	7.1	6.2	4.8	6.0	5.5	5.0
20	CR	0.9423	0.9430	0.9566	0.9428	0.8750	0.9376	0.8894	0.9200
	AW	4.1	4.1	4.6	4.1	3.2	4.0	3.5	3.7
30	CR	0.9452	0.9458	0.9599	0.9456	0.8755	0.9410	0.8801	0.9313
	AW	3.3	3.3	3.7	3.3	2.6	3.2	2.8	3.1
40	CR	0.9458	0.9461	0.9611	0.9459	0.8764	0.9424	0.8715	0.9357
	AW	2.8	2.8	3.1	2.8	2.2	2.8	2.4	2.7
50	CR	0.9460	0.9462	0.9606	0.9462	0.8741	0.9418	0.8643	0.9374
	AW	2.5	2.5	2.8	2.5	2.0	2.5	2.1	2.4
100	CR	0.9475	0.9476	0.9596	0.9475	0.8742	0.9441	0.8349	0.9434
	AW	1.8	1.8	1.9	1.8	1.4	1.7	1.5	1.7

Table 4.7. Coverage ratio (CR) and average width (AW) of the 95% confidence intervals for the population mean of Chi-square distribution with df = 50

n	Performance Measures	Confidence Interval Methods							
		Student-t	Johnson-t	Chen-t	YY-t	Mad-t	AADM-t	$DMSD_{DM}$-t	Bootstrap
10	CR	0.9469	0.9470	0.9494	0.9471	0.8831	0.9397	0.9213	0.8985
	AW	13.9	13.9	14.9	13.9	10.8	13.5	12.4	11.3
20	CR	0.9487	0.9488	0.9546	0.9488	0.8822	0.9452	0.9121	0.9257
	AW	9.2	9.2	9.8	9.2	7.3	9.1	8.1	8.4
30	CR	0.9502	0.9503	0.9569	0.9502	0.8813	0.9465	0.9087	0.9349
	AW	7.4	7.4	7.8	7.4	5.8	7.3	6.5	7.0
40	CR	0.9504	0.9504	0.9571	0.9505	0.8825	0.9476	0.9063	0.9389
	AW	6.4	6.4	6.6	6.4	5.0	6.3	5.6	6.1
50	CR	0.9485	0.9484	0.9555	0.9484	0.8806	0.9465	0.9028	0.9389
	AW	5.7	5.7	5.9	5.7	4.5	5.6	5.0	5.5
100	CR	0.9483	0.9484	0.9540	0.9484	0.8795	0.9470	0.8946	0.9431
	AW	4.0	4.0	4.1	4.0	3.1	3.9	3.5	3.9

Table 4.8. Coverage ratio (CR) and average width (AW) of the 95% confidence intervals for the population mean of triangular distribution with a = 0, b = 1 and c = 0.05

n	Performance Measures	Confidence Interval Methods							
		Student-t	Johnson-t	Chen-t	YY-t	Mad-t	AADM-t	$DMSD_{DM}$-t	Bootstrap
10	CR	0.9405	0.9420	0.9528	0.9412	0.8848	0.9361	0.9063	0.9030
	AW	0.3	0.3	0.4	0.3	0.3	0.3	0.3	0.3
20	CR	0.9450	0.9464	0.9597	0.9458	0.8898	0.9465	0.8920	0.9275
	AW	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
30	CR	0.9466	0.9477	0.9601	0.9473	0.8935	0.9505	0.8865	0.9357
	AW	0.2	0.2	0.2	0.2	0.1	0.2	0.2	0.2
40	CR	0.9489	0.9500	0.9614	0.9496	0.8964	0.9543	0.8829	0.9406
	AW	0.1	0.1	0.2	0.1	0.1	0.2	0.1	0.1
50	CR	0.9472	0.9478	0.9584	0.9475	0.8951	0.9533	0.8766	0.9401
	AW	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
100	CR	0.9493	0.9498	0.9584	0.9496	0.8978	0.9576	0.8600	0.9460
	AW	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

Table 4.9. Coverage ratio (CR) and average width (AW) of the 95% confidence intervals for the population mean of triangular distribution with a = 0, b = 1 and c = 0.50

n	Performance Measures	Confidence Interval Methods							
		Student-t	Johnson-t	Chen-t	YY-t	Mad-t	AADM-t	$DMSD_{DM}$-t	Bootstrap
10	CR	0.9474	0.9479	0.9452	0.9477	0.8880	0.9432	0.9201	0.9025
	AW	0.3	0.3	0.3	0.3	0.2	0.3	0.3	0.2
20	CR	0.9502	0.9509	0.9495	0.9507	0.8900	0.9501	0.9140	0.9294
	AW	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
30	CR	0.9491	0.9497	0.9486	0.9494	0.8881	0.9503	0.9114	0.9362
	AW	0.2	0.2	0.2	0.2	0.1	0.2	0.1	0.1
40	CR	0.9492	0.9497	0.9490	0.9496	0.8886	0.9513	0.9107	0.9393
	AW	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
50	CR	0.9500	0.9504	0.9502	0.9503	0.8895	0.9528	0.9102	0.9420
	AW	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
100	CR	0.9501	0.9503	0.9502	0.9502	0.8914	0.9541	0.9113	0.9458
	AW	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

Table 4.10. Coverage ratio (CR) and average width (AW) of the 95% confidence intervals for the population mean of triangular distribution with a = 0, b = 1 and c = 0.95

n	Performance Measures	Confidence Interval Methods							
		Student-t	Johnson-t	Chen-t	YY-t	Mad-t	AADM-t	$DMSD_{DM}$-t	Bootstrap
10	CR	0.9411	0.9429	0.9249	0.9420	0.8858	0.9371	0.9062	0.9033
	AW	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
20	CR	0.9451	0.9464	0.9280	0.9460	0.8906	0.9462	0.8923	0.9281
	AW	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
30	CR	0.9466	0.9478	0.9310	0.9474	0.8921	0.9503	0.8847	0.9358
	AW	0.2	0.2	0.2	0.2	0.1	0.2	0.2	0.2
40	CR	0.9487	0.9496	0.9344	0.9492	0.8944	0.9538	0.8816	0.9411
	AW	0.1	0.1	0.1	0.1	0.1	0.2	0.1	0.1
50	CR	0.9491	0.9501	0.9362	0.9497	0.8969	0.9553	0.8790	0.9424
	AW	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
100	CR	0.9493	0.9500	0.9403	0.9497	0.8979	0.9580	0.8609	0.9461
	AW	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

5. Real data applications

In this section, three real-life examples from normal and skewed distributions are analyzed to illustrate the applications of the proposed robust confidence interval.

5.1. Load at failure data

The first data set was obtained from Berndt (1989). The data describe the results of tensile adhesion tests (in megapascals) on 22 U-700 alloy specimens: 19.8, 10.1, 14.9, 7.5, 15.4, 15.4, 15.4, 18.5, 7.9, 12.7, 11.9, 11.4, 11.4, 14.1, 17.6, 16.7, 15.8, 10.5, 8.8, 13.6, 11.9, and 11.4. The Kolmogorov-Smirnov (K-S) goodness-of-fit test for normality for this data set has a p-value (p-value > 0.150) greater than α = 0.05. We conclude that the data are in excellent agreement with a normal distribution with skewness = 0.07, kurtosis = -0.68, mean = 13.305 and standard deviation = 3.369.

Table 5.1. The 95% confidence intervals for the population mean of load at failure

Methods	Estimated Confidence Interval Limits		Width
	Lower Limit	Upper Limit	
Student-t	11.8108	14.7983	2.9875
Johnson-t	11.8125	14.7999	2.9874
Chen-t	11.7948	14.8143	3.0195
YY-t	11.8118	14.7992	2.9874
Mad-t	12.0611	14.5480	2.4869
AADM-t	11.7461	14.8630	3.1169
$DMSD_{DM}$-t	11.9306	14.6138	2.6832
Bootstrap	12.0159	14.6750	2.6591

The 95% CI for the population mean (μ) for load specimen failure is studied. The considered confidence intervals and their corresponding width have been given in Table 5.1. From Table 5.1, the 95% estimated confidence interval for population mean (μ) of load specimen failure, which is constructed using AADM-t method, gives the largest width, whereas the 95% of Mad-t confidence interval gives the smallest width and the secondary width is constructed for the 95% confidence interval using the $DMSD_{DM}$-t and Bootstrap-percentile methods. Therefore, the results from this real-life example as shown in Table 5.1 support the simulation study in Section 4.

5.2. Psychotropic drug exposure data

To study the average use of psychotropic drugs among non-antipsychotic drug users, the number of psychotropic drug users was reported for a random sample of $n = 20$ from different categories of drugs. The following data represent the number of users (Johnson and McFarland, 1993): 43.4, 24, 1.8, 0, 0.1, 170.1, 0.4, 150, 31.5, 5.2, 35.7, 27.3, 5, 64.3, 70, 94, 61.9, 9.1, 38.8, and 14.8. The data are checked and found to be positively skewed with skewness = 1.57, kurtosis = 2.06, mean = 42.37 and standard deviation = 48.43. The considered confidence intervals and their corresponding width are given in Table 5.2.

Table 5.2. The 95% confidence intervals for the average use of psychotropic drugs

Methods	Estimated Confidence Interval Limits		Width
	Lower Limit	Upper Limit	
Student-t	19.8445	64.8955	45.0509
Johnson-t	20.3850	65.4359	45.0509
Chen-t	13.4694	71.2706	57.8013
YY-t	20.1629	65.2139	45.0509
Mad-t	25.7607	58.9793	33.2185
AADM-t	22.7839	61.9561	39.1722
$DMSD_{DM}$-t	19.3406	50.2838	30.9432
Bootstrap	23.7750	66.1800	42.4050

From Table 5.2, the 95% estimated confidence interval for the average use of psychotropic drugs, which is constructed by using the Chen-t method, gives the largest width and differs from other methods, whereas the 95% of $DMSD_{DM}$-t confidence interval gives the shortest width, followed by Mad-t and AADM-t confidence intervals. Since this data set is positively skewed, we conclude that the results in Table 5.2 support the simulation results in the case of positively skewed distribution of this study.

5.3. Long jump distance data

The following data represent the results of the final points scores reported for 40 players in long jump distance in meters (International Olympic Committee, 2019): 8.11, 8.11, 8.09, 8.08, 8.06, 8.03, 8.02, 7.99, 7.99, 7.97, 7.95, 7.92, 7.92, 7.92, 7.89, 7.87, 7.84, 7.79, 7.79, 7.77, 7.76, 7.72, 7.71, 7.66, 7.62, 7.61, 7.59, 7.55, 7.53, 7.5, 7.5, 7.42, 7.38, 7.38, 7.26, 7.25, 7.08, 6.96, 6.84, 6.55. The data are checked and found to be negatively skewed with skewness = −1.16, kurtosis = 1.20, mean = 7.6745 and standard deviation = 0.37. The considered confidence intervals and their corresponding width have been given in Table 5.3. From Table 5.3, the 95% estimated confidence interval for the population mean (μ) of the final points scores in long jump distance in meters, which is constructed by using Student's t, Johnson-t and YY-t methods, gives the same value of the largest width, whereas the 95% of $DMSD_{DM}$-t confidence interval gives the smallest width and the secondary width is constructed by using the Mad-t confidence interval. Since this data set is negatively skewed, we conclude that the results in Table 5.3 support the simulation results in the case of negatively skewed distribution of this study.

Table 5.3. The 95% confidence intervals for the population mean of the final scores for long jump distance in meters

Methods	Estimated Confidence Interval Limits		Width
	Lower Limit	Upper Limit	
Student-t	7.5528	7.7962	0.2434
Johnson-t	7.5512	7.7945	0.2434
Chen-t	7.5668	7.7822	0.2154
YY-t	7.5517	7.7951	0.2434
Mad-t	7.5793	7.7697	0.1903
AADM-t	7.5587	7.7903	0.2316
$DMSD_{DM}$-t	7.6242	7.8029	0.1787
Bootstrap	7.5553	7.7798	0.2245

6. Summary and concluding remarks

The proposed confidence interval, $DMSD_{DM}$-t, is an adjustment of the Student's t confidence interval based on the decile mean and the decile mean standard deviation. In addition, the simulation results show that in many cases the proposed confidence interval performs better than the existing estimators when observations are sampled from both normal and skewed distributions. Even though the Mad-t confidence interval tends to provide the smallest average width in the case of observations sampled from the normal distribution, the coverage ratio of this tends to be more under the nominal level when compared with the proposed confidence interval. That is, the performance of the $DMSD_{DM}$-t method is better than the Mad-t method for both coverage ratio and average width because the coverage ratio of the $DMSD_{DM}$-t confidence interval tends to be slightly below the nominal level. Although the coverage ratio of the proposed interval is slightly lower than that of the five intervals – Student's t, Johnson-t, Chen-t, YY-t and AADM-t – and the average width of this proposed interval is smaller than that of the five intervals, especially for a small sample size and observations sampled from the normal distribution. In the case of skewed distributions, such as observations sampled from chi-square distribution with a small coefficient of skewness, the average width of the proposed interval is also smaller than that of the five intervals – Student's t, Johnson-t, Chen-t, YY-t and AADM-t – even though the coverage ratio of the proposed interval is slightly lower than that of the five intervals. The bootstrap estimator for a confidence interval is reliable at least for n not very small.

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