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Variance estimation in stratified adaptive cluster sampling

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ABSTRACT

In many sampling surveys, the use of auxiliary information at either the design or estimation stage, or at both these stages is usual practice. Auxiliary information is commonly used to obtain improved designs and to achieve a high level of precision in the estimation of population density. Adaptive cluster sampling (ACS) was proposed to observe rare units with the purpose of obtaining highly precise estimations of rare and specially clustered populations in terms of least variances of the estimators. This sampling design proved to be more precise than its more conventional counterparts, including simple random sampling (SRS), stratified sampling, etc. In this paper, a generalised estimator is anticipated for a finite population variance with the use of information of an auxiliary variable under stratified adaptive cluster sampling (SACS). The bias and mean square error expressions of the recommended estimators are derived up to the first degree of approximation. A simulation study showed that the proposed estimators have the least estimated mean square error under the SACS technique in comparison to variance estimators in stratified sampling.

Key words: variance estimator, stratified sampling, stratified adaptive cluster sampling (SACS).

Significance statement

The stratified adaptive cluster sampling technique is an efficient technique used for the population of plants or animals in biological and ecological surveys, which are effective in estimating the population variance when the measurement of the variability of observations is difficult. The main task of this study is to develop a sampling technique and efficient estimators using auxiliary information for estimation of finite population variance. On the basis of a simulation study, the performance of the proposed variance estimators using the stratified adaptive cluster sampling technique is better than the competing variance estimators using stratified random sampling for clustered, hidden and patchy populations.

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1. Introduction

In the theory of survey sampling, it is well recognized at the estimation stage known information of auxiliary variables to increase the precision of the estimators of unknown population parameter(s) in different sampling techniques. Several authors have used information of auxiliary variates like population mean, population variance, kurtosis, skewness, etc., to estimate population mean and variance of the study variable. In our daily life, variation is present everywhere. According to the law of nature, no two things or individuals are the same. For instance, a manufacturer wants stable information about the level of variation in consumer's response to his product to be able to know whether to increase or reduce his price, or improve the value of his product. For these reasons, several authors have done important work in this field such as Das and Tripathi [1], Isaki [2], Shabbir and Gupta [3], Tailor et al. [4] and Subramani and Kumarapandiyan [5] and Yasmeen et al. [8].

In ecological and environmental surveys, conventional sampling techniques are not frequently applicable as the consequences obtained from such sampling designs may not be reliable due to lack of information on characteristics of the population. Different types of population are needed in different sampling procedures according to their characteristics. Many populations of plants or animals have different types of cluster tendencies, often the size or the shape of the cluster cannot be recognized before the survey. Applications of sampling methods in real environmental sciences were discussed by Cormack [6]. He mentioned special designs and these designs are needed for environmental sciences. ACS is an efficient method for population which have cluster rare tendencies suggested by Thompson [7]. He suggested an unbiased estimator for the population mean by modifying the Hansen-Hurwitz estimator.

The greatest challenge is the estimation of COVID-19 virus cases in all over the world. Many people do not like to come to laboratories for COVID-19 test or they do not have any symptoms. In such situations for the selection of a sample using simple random sampling or many other techniques for testing to estimate the actual number of COVID-19 cases in the different countries. But precise estimates are still a challenge. Chandra et al. (2021) recommended adaptive cluster sampling technique to provide precise estimates of the number of infected persons. So in real life, the adaptive cluster sampling, stratified adaptive cluster sampling and systematic adaptive cluster sampling technique might be a useful source in finding or tracing the distribution of COVID-19-affected people, for developing the mathematical models for prediction and to get the efficient estimates of the number of COVID-19-affected cases.

In the geographical survey area for the application of the stratified adaptive cluster sampling technique, the survey region can be divided into smaller but same regions according to a recognized variable like plant species, habitat or soil type. While a large region seems to be homogeneous, the stratified sampling approach based on geographical locations can be used to produce a sample gathered over the whole area. The overall variability in the estimation can be reduced when stratification leads to reduced variability within a stratum with larger variation across strata. Moreover, the most precise estimator is obtained when the units in each stratum are as similar as possible Thompson [12].

By the motivation of many authors, we anticipated variance estimator utilizing midrange, kurtosis, the median, the tri-mean, the coefficient of correlation, the coefficient of variation, the coefficient of skewness and the quartile deviation with the help of single auxiliary variate information under stratified adaptive cluster sampling approach to estimate the finite population variance respectively.

In this paper, variance estimators have been suggested under the stratified adaptive cluster sampling (SACS) design. Section 2 presents the sampling procedure to follow the SACS design. In Section 3, the proposed estimators for the estimation of population variance have been presented. The derivation of the bias and minimum mean square error (MSE) of each estimator are obtained. The simulation study of the proposed estimators and the existing ratio estimator is accomplished in Section 4. Section 5 concludes this paper.

2. Stratified adaptive cluster sampling design for variance estimation

Suppose a finite population Q of size N divided into L non-overlapping strata of size N_h. Let y_{hi} be the value of response varíate (y) and x_{hi} be the value of auxiliary varíate (x), for ith ($i=1,2,\ldots,N_h$) population unit Q in the hth stratum (h=1,2,….,L). Suppose a sample of size n_h from hth stratum is drawn by SRSWOR, where $\left(\sum_{h=1}^{L} n_h = n\right)$. When we ignore the finite population correction factor, the usual variance of $\overline{y}_{st}$ is given by $\operatorname{var}(\overline{y}_{st}) = \sum W_h^2 \frac{s_{y(h)}^2}{n_h} = s_{y(h)}^2$ where $s_{y(h)}^2$ is the variance of population for stratum h. Suppose

$$e_{yh} = \frac{\left(s_{yh}^2 - S_{yh}^2\right)}{S_{yh}^2} \text{ and } e_{xh} = \frac{\left(s_{xh}^2 - S_{xh}^2\right)}{S_{xh}^2} \text{ so that } E\left(e_{yh}^2\right) = \frac{\left(\beta_{2y(h)} - 1\right)}{n_h};$$

$$E\left(e_{xh}^2\right) = \frac{\left(\beta_{2x(h)} - 1\right)}{n_h};$$

$$E\left(e_{yh}e_{xh}\right)=\frac{\left(\vartheta_{22(h)}-1\right)}{n_h};\ E\left(e_{xh}^2\right)=\frac{\left(\beta_{2x(h)}-1\right)}{n_h};\ \text{where } \beta_{2y(h)}=\frac{u_{40(h)}}{u_{20(h)}^2},\beta_{2x(h)}=\frac{u_{04(h)}}{u_{02(h)}^2}$$

are the coefficients of kurtosis of y and x respectively, in the hth stratum and

$$\varphi_{22(h)}=\frac{u_{22(h)}}{u_{02(h)}^2 u_{20(h)}},\ \text{where } u_{ab(h)}=\sum_{i=1}^{N_h}\left(y_{hi}-\bar{y}_h\right)^a\left(x_{hi}-\bar{x}_h\right)^b.$$

For the SACS design, let N be denoted as the total number of units in the population. Associated with unit T_{hi}, the *ith* unit of stratum *h* is a variable of interest y_{hi}. For any unit T_h of the population, the neighbourhood of unit T_{hi} is defined as a collection of units which includes T_{hi} and with the property that ,if unit $T_{h'i'}$ is in the neighbourhood of unit T_{hi}s, then unit T_{hi} is in the neighbourhood of unit $T_{h'i'}$. A unit T_{hi} is said to fulfil pre-defined condition of interest if the *y* value associated with that unit is in a specified set *C*.

The first sample of units is selected from a population using stratified random sampling technique, that is within stratum *h*, a simple random sample on n_h units designated without replacement, the selection for separate strata being made independently. When a selected unit satisfies the condition, all units in its neighbourhood not already in the sample are added to the sample. Still further units may be added to the sample when any extra-added units satisfies the pre-defined condition, so that the final sample include every unit in the neighbourhood of some sample unit satisfying the condition.

For the unit T_{hi}, the new variate w_{hi} is the total of the *y* variate of the network to which belongs to T_{hi}. Weighted by the stratum sampling fraction and divided by a weighted sum of the network-stratum intersection.

$$w_{hi}=\frac{\frac{n_h}{N_h}\sum_{k=1}^{L}\xi_{khi}}{\sum_{k=1}^{L}\frac{n_k}{N_k}m_{khi}}$$, where, suppose, m_{khi} denotes the number of units in the

intersection of stratum *h* with the network which contains unit T_{hi}. ξ_{khi} is the total value of the *y*-values in the intersection of stratum *h* with the network that consist of T_{hi} unit and m_{khi} is the number of units in this intersection. The usual variance of $\bar{y}_{wst}$ is given by $\text{var}(\bar{y}_{wst})=\sum W_h^2\frac{s_{wy(h)}^2}{n_h}=s_{y(wst)}^2$ where $s_{wy(h)}^2$ is the population variance for stratum *h*.

Suppose $e_{wyh}=\frac{\left(s_{wyh}^2-S_{wyh}^2\right)}{S_{wyh}^2}$ and $e_{wxh}=\frac{\left(s_{wxh}^2-S_{wxh}^2\right)}{S_{wxh}^2}$ so, $E\left(e_{wyh}\right)=E\left(e_{wxh}\right)=0;$

$E\left(e_{wyh}^2\right)=\frac{\left(\beta_{2wy(h)}-1\right)}{n_h};\ E\left(e_{wxh}^2\right)=\frac{\left(\beta_{2wx(h)}-1\right)}{n_h};\ E\left(e_{wyh}e_{wxh}\right)=\frac{\left(\vartheta_{22(wh)}-1\right)}{n_h}$,

$\beta_{2wy(h)}=\frac{u_{40(h)}}{u_{20(h)}^2}, \beta_{2wx(h)}=\frac{u_{04(h)}}{u_{02(h)}^2}$ are the coefficients of kurtosis of y and x, respectively, in the *hth* stratum, and $\varphi_{22w(h)}=\frac{u_{22(h)}}{u_{02(h)}^2 u_{20(h)}}$, where

$$u_{abw(h)}=\frac{\sum_{i=1}^{N_h}\left(wy_{hi}-w\overline{y}_h\right)^a\left(wx_{hi}-w\overline{x}_h\right)^b}{N_h}.$$

The variance estimator for stratified sampling given in Shabbir and Gupta [3] is

$$\delta=\sum_{h=1}^{L}\frac{W_h^2}{n_h}\left(s_{y(h)}^2\frac{S_{x(h)}^2}{s_{x(h)}^2}\right).$$

3. Proposed estimator

The following proposed estimator is suggested of finite population variance utilizing the coefficient of skewness, kurtosis, tri-mean, median, quartile deviation under stratified adaptive cluster sampling given by

$$t_G=\sum_{h=1}^{L}\frac{w_h^2}{n_h}\left(s_{wy(h)}^2\frac{\gamma S_{wx(h)}^2+\chi}{\gamma s_{wx(h)}^2+\chi}\right). \tag{3.1}$$

where $\gamma=1,\rho,\beta_1,\beta_2;\chi=0,1,TM,M_d,QD$

By using the notations given in Section 2, the proposed variance estimator (3.1) can be written as

$$t_G=\sum_{h=1}^{L}\frac{w_h^2}{n_h}\left(S_{wy(h)}^2(1+\overline{e}_{wy})\left(1-\frac{\gamma S_{wx(h)}^2 e_{wx(h)}}{\gamma s_{wx(h)}^2+\chi}\right)\right). \tag{3.2}$$

After simplifications of this estimator, applying expectations on both sides of (3.3), using the notation given in Section 2, the bias of the recommended variance estimator (3.1) is given by

$$Bias(t_G)=\sum_{h=1}^{L}\frac{w_h^2}{n_h^2}\left(-\omega_i S_{wy(h)}^2\left(Q_{22(h)}-1\right)\right). \tag{3.3}$$

In order to derive mean square error of t_G, again using (3.3), ignoring the higher order power terms, we attain

$$MSE(t_G) = \sum_{h=1}^{L} \frac{w_h^4}{n_h^3} \left(S_{wy(h)}^4 \left(B_{2y(h)} - 1 \right) + \omega_{1(h)}^2 \left(B_{2x(h)} - 1 \right) - 2\omega_{1(h)} \left(Q_{22(h)} - 1 \right) \right). \tag{3.4}$$

$$\omega_{i(h)} = \frac{\gamma S_{wx(h)}^2}{\gamma s_{wx(h)}^2 + \chi_{(h)}}, \qquad \text{i=1, 2……..17}$$

where

$$\omega_{1(h)} = 1,\ \omega_{2(h)} = \frac{S_{wx(h)}^2}{S_{wx(h)}^2 + 1},\ \omega_{3(h)} = \frac{S_{wx(h)}^2}{S_{wx(h)}^2 + TM_{(h)}},\ \omega_{4(h)} = \frac{S_{wx(h)}^2}{S_{wx(h)}^2 + MD_{(h)}},$$

$$\omega_{5(h)} = \frac{S_{wx(h)}^2}{S_{wx(h)}^2 + QD_{(h)}},\ \omega_{6(h)} = \frac{\rho_{(h)} S_{wx(h)}^2}{\rho_{(h)} S_{wx(h)}^2 + 1},\ \omega_{7(h)} = \frac{\rho_{(h)} S_{wx(h)}^2}{\rho_{(h)} S_{wx(h)}^2 + TM_{(h)}},$$

$$\omega_{8(h)} = \frac{\rho_{(h)} S_{wx(h)}^2}{\rho_{(h)} S_{wx(h)}^2 + MD_{(h)}}\ \omega_{9(h)} = \frac{\rho_{(h)} S_{wx(h)}^2}{\rho_{(h)} S_{wx(h)}^2 + QD_{(h)}},\ \omega_{10(h)} = \frac{\beta_{1(h)} S_{wx(h)}^2}{\beta_{1(h)} S_{wx(h)}^2 + 1},$$

$$\omega_{11(h)} = \frac{\beta_{1(h)} S_{wx(h)}^2}{\beta_{1(h)} S_{wx(h)}^2 + TM_{(h)}},\ \omega_{12(h)} = \frac{\beta_{1(h)} S_{wx(h)}^2}{\beta_{1(h)} S_{wx(h)}^2 + MD_{(h)}},\ \omega_{13(h)} = \frac{\beta_{1(h)} S_{wx(h)}^2}{\beta_{1(h)} S_{wx(h)}^2 + QD_{(h)}},$$

$$\omega_{14(h)} = \frac{\beta_{2(h)} S_{wx(h)}^2}{\beta_{2(h)} S_{wx(h)}^2 + 1},\ \omega_{15(h)} = \frac{\beta_{2(h)} S_{wx(h)}^2}{\beta_{2(h)} S_{wx(h)}^2 + TM_{(h)}},\ \omega_{16(h)} = \frac{\beta_{2(h)} S_{wx(h)}^2}{\beta_{2(h)} S_{wx(h)}^2 + MD_{(h)}}$$

$$\omega_{17(h)} = \frac{\beta_{2(h)} S_{wx(h)}^2}{\beta_{2(h)} S_{wx(h)}^2 + QD_{(h)}}.$$

We try these estimators but in this article we discussed three estimators which are most efficient among them.

4. Simulation study

The stratified adaptive cluster sampling design is an efficient procedure and flexible technique for capturing more information for hidden and patchy clustered populations. It is a well recognized procedure and suitable a different choice to collect a sample from population. In the section of this study, we consider the performance of SACS design having single auxiliary variable with conventional and non-conventional measures for the variance estimation of population variance. Dryver and Chao [10] and Chao et al. [11] have generated the values for the study variable utilizing the models

(4.1) and (4.2) in such a procedure that there is a strong positive correlation between study variate and simulated variate at unit level.

$$y_{hi} = 4x_{hi} + \varepsilon_{hi} \;; \quad \varepsilon_j \sim N(0, x_{hi}) \tag{4.1}$$

$$y_{hi} = 4w_{xhi} + \varepsilon_{xhi} \;; \quad \varepsilon_j \sim N(0, w_{xhi}) \tag{4.2}$$

In this system the population is simulated using the model (4.1) has recognized a clearly mentioned sign for strong correlation between variable of interest and auxiliary variable at unit level as well as network level. The model (4.2) has a clear sign for a strong correlation between auxiliary variate and response variable at the network level for simulated population. The population used for simulation is taken from Thompson [8]. The study area is divided into 2 strata and the stratum size is 20*10 = 200, units formed from 20 rows and 10 columns. For each iteration, an initial sample is selected by simple random sampling without replacement in every stratum. A neighbouring unit is distinct as the spatially adjacent units (left, right, top, and bottom) of that unit. 10,000 iterations were performed for every estimator to attain an efficient estimate. The total of x values is 223 and the average value is 0.5575. The total of y values is 690 and the average value of y values is 1.725. The values of y are generated by model 5.1. The condition of interest $c=\{y : y>0\}$ for added units in the sample is used. In each stratum, initial simple random sample sizes were varied n_h = 2, 3, 4, 5, 10, 20, 25, 30, 40, 50, 100 and initial samples n = 4, 6, 8, 10, 20, 40, 50, 60, 80, 100, 200 were used for all strata. The expected final sample of size n differs from sample to sample in SACS.

Let $E(v)$ = 7.89, 11.69, 15.41, 19.04, 36.08, 65.59, 78.64, 90.84, 113.31, 133.99, 226.06 denote the final expected sample size in stratified adaptive cluster sampling.

The expected sample size is the expected number of distinct units in the final sample, is the sum of the N inclusion probabilities of all the quadrants in the population. It is generally larger than the initial sample size and increases with the increase of the initial sample size.

The comparison of the proposed variance estimator has been made with the variance estimator given in Shabbir and Gupta [3]. The expected final sample size varies from sample to sample in ACS.

The estimated variances of all the suggested estimators considered in this paper have been computed and presented in Table 5.1. and Table 5.2. for population generated by model 4.1 and 4.2 respectively. The results obtained from Table 5.1. and Table 5.2. show that the variances of the proposed estimators are very low as compared to the variance estimator given in Shabbir and Gupta [3] estimator and the results of variances of all the suggested estimators are decreasing on different primary and expected final sample size.

Ten thousand iterations were used to generate samples from population to the achievement of the efficient estimates. In this paper, the variances of the proposed estimators better by increasing the length of the sample according to the results of the simulation study, which are shown in Table 5.1. and Table 5.2. The suggested stratified variance estimators have been developed using known population co-efficient of variation, population co-efficient of correlation, population tri-mean, median, quartile deviation, coefficient of skewness and coefficient of kurtosis with the information of interest variable and auxiliary variable. It is observed that the variances of the developed estimators are smaller than the variance estimator given in Shabbir and Gupta [3] estimator under the stratified adaptive cluster sampling procedure and as Figure 5.1. designates, the developed variance estimators perform better than the variance estimator given in Shabbir and Gupta [3] when the initial sample size is greater than 2 or more.

Hence, the class of stratified variance estimators developed in this paper can be used for enhanced results and preferred for hidden and patchy populations in practical location. Moreover, simulation studies reveal that the variances for the suggested estimators are lower than the variance estimator given Shabbir and Gupta [3] in conditions of population.

5. Concluding discussion

The consequences of the study simulation are given in Table 5.1. with model 5.1 and Table (5.2.) with model 5.2 for three proposed estimators from SACS procedure. The results of the simulation study are obtained from population provides efficient form of variances from SACS technique than the stratified simple random sampling technique for the highly clumped, patchy or hidden population. In this paper, three estimators have been proposed by using a number of conventional and non-conventional measures for interest variable with single auxiliary variable. The variances have been shown in Tables 5.1. and Table 5.2. The developed estimators are found to be more efficient as compared to variance estimator given by Shabbir and Gupta [3] for estimation of finite population variance. The last conclusion obtained from the simulation study confirmed that the proposed estimators are most efficient than the estimator given by Shabbir and Gupta [3] in SACS design on population for different sizes of the sample. The results of variances have shown in Figure 5.1. indicate that the proposed estimators have been minimum variance compare than the existing estimator. The results of Table 5.2. have presented poorer performance of the estimator given in Shabbir and Gupta [3] paper using stratified simple random sampling because of the week correlation between interest variate and auxiliary variate at the network level. According to this study of simulation the 0/0 quantity is assumed to be zero because Dryver and Chao [2] assumed 0/0 as 0 for the ratio estimator so that the suggested variance estimators using SACS technique returned the values for small sample sizes in population. The amount of estimated variance on population was found to decrease as well as the sample size increases.

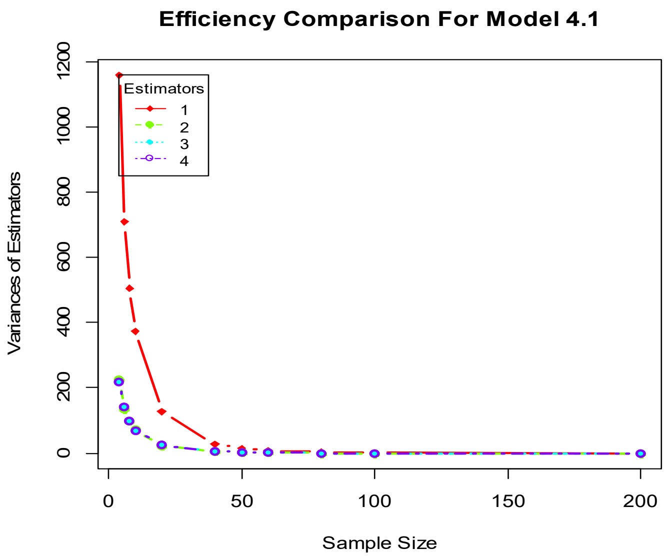
Finally, on the basis of the simulation study, the performance of the proposed variance estimators using SACS technique is better than the competing variance estimator given in Shabbir and Gupta [3] estimator using stratified random sampling for clustered, hidden and patchy populations. The results of simulation study supported that stratified adaptive cluster sampling scheme can be a cost effective and time-saving sampling scheme, and it is suitable for sampling in a spatially accumulated population.

Table 5.1. Variances for different suggested estimators under stratified adaptive cluster design under simulated model 5.1 using population

Sample size s	$E(v)$	$\mathrm{var}(\delta)$	$\mathrm{var}(t_1)$	$\mathrm{var}(t_2)$	$\mathrm{var}(t_3)$
4	7.89	1157.959	226.426	217.049	219.604
6	11.69	708.404	138.573	140.596	141.114
8	15.41	504.876	97.2011	95.733	98.784
10	19.04	373.676	70.459	70.898	69.946
20	36.08	127.508	22.929	23.543	23.844
40	65.59	28.111	5.131	5.159	5.119
50	78.64	14.823	2.681	2.672	2.643
60	90.84	8.303	1.468	1.464569	1.463
80	113.31	2.732	0.468	0.448	0.459
100	133.99	0.946	0.140	0.151	0.146
200	226.06	0.016	0.000	0.000	0.000

Table 5.2. Variances for different suggested estimators under stratified adaptive cluster design under simulated model 5.2 using population

Sample size s	$E(v)$	$\mathrm{var}(\delta)$	$\mathrm{var}(t_1)$	$\mathrm{var}(t_2)$	$\mathrm{var}(t_3)$
4	7.89	13217511	240.646	218.97	216.392
6	11.69	7393951	151.805	138.892	138.683
8	15.41	5064571	102.362	94.193	98.191
10	19.04	3959354	74.795	69.168	70.735
20	36.08	1363876	24.810	23.598	23.677
40	65.59	432186.7	5.202	5.145	5.131
50	78.64	277516.3	2.706	2.686	2.694
60	90.84	161725.8	1.463	1.465	1.454
80	113.31	65089.37	0.462	0.466	0.452
100	133.99	29852.82	0.143	0.142	0.144
200	226.06	194.552	0.000	0.000	0.000



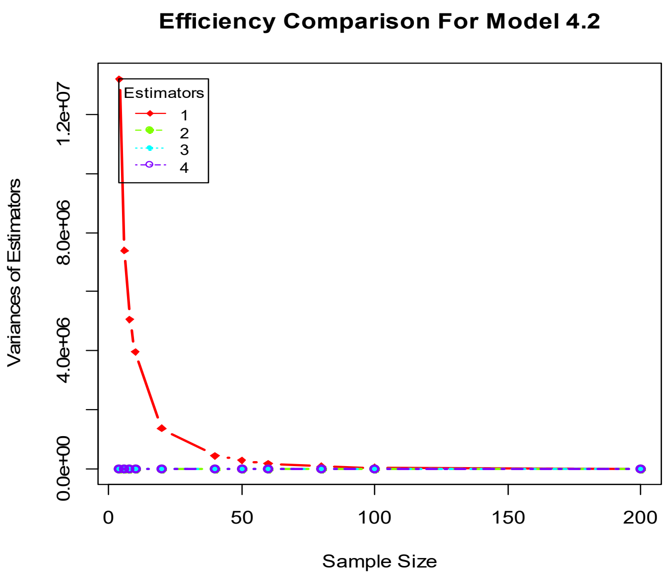


Figure 5.1. Variances of developed estimators under SACS design to the variance estimator given in Shabbir and Gupta [10] under stratified sampling with comparable sample sizes under models.

REFRENCES

Boomer, K., Werner, C., Brantley, S., (2000). CO2 emissions related to the Yellowstone volcanic System, J. Geophys. Res., 105(B5), pp. 10817-10830.

Chandra, G., Tiwari, N., & Nautiyal, R., (2021). Adaptive cluster sampling-based design for estimating COVID-19 cases with random samples, Current Science (00113891), 120(7).

Chao, C. T., Dryver, A. L., Chiang, T. C., (2011). Leveraging the Rao-Blackwell Theorem to improve ratio estimators in adaptive cluster sampling, Environmental and Ecological Statistics 18, pp. 543-568.

Cormack, R. M, (1988). Statistical challenges in the environmental sciences, A Personal view, Journal of the Royal Statistical Society: Series A, 151, pp. 201-210.

Das, A. K. , Tripathi, T. P., (1978). Use of auxiliary information in estimating the finite population variance, Sankhya, 40, pp. 139-148.

Dryver, A. L.,Chao, C. T., (2007). Ratio estimators in adaptive cluster sampling, Environmetrics, 18, pp. 607–620.

Isaki, C. T., (1983). Variance estimation using auxiliary information, Journal of the American Statistical Association, 78, pp. 117-123.

Shabbir, J., Gupta, S., (2010). Some estimators of finite population variance of stratified sample mean, Communications in Statistics–Theory and Methods, 16, pp. 3001-3008.

Subramani, J, Kumarapandiyan, G., (2013). Estimation of variance using known co-efficient of variation and median of an auxiliary variable, Journal of Modern Applied Statistical Methods, 12, pp. 58-64.

Tailor, R., Tailor, R., Parmar, R., Kumar, M., (2012). Dual to ratio cum product estimator using known parameters of auxiliary variables, Journal of Reliability and Statistical Studies, 5, pp. 65-71.

Thompson, S. K., (1990). Adaptive Cluster Sampling, Journal of the American Statistical Association, 85, pp. 1050–1059.

Thompson S., (1991b). Stratified adaptive cluster sampling, Biometrika, 78(2), pp. 389-397.

Thompson, S. K., (2002). Sampling, New York: Wiley.

Yasmeen, U., Noor ul Amin, M., Hanif M., (2018). Exponential Estimators of Finite PopulationVariance Using Transformed Auxiliary Variables, Proceedings of the National Academy of Sciences, India Section A: Physical Sciences.

APPENDIX

Table C.1. x-population

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	23	14	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	36	34	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	3	63	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	16	3	0	0	0	2	12	12	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	2	57	65	17	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	5	14	5	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0