ORIGINAL ARTICLE

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The impact of bequest taxation on savings and transfers

Abstract

Research background: The paper investigates the impact of bequest taxation on saving and transfers when parents and children make decisions consistently. It complements the predictions of Gale and Perozek's life-cycle modeling (2001) when decisions of parents and children are set independently and can be time-inconsistent.

Purpose of the article: The paper strives to answer the question of whether taxation of bequest harms savings and *inter vivos* transfers. The previous results indicated that this is possible for some bequest motives. Our results show that this is not likely for the most typical values of parameters.

Methods: The analysis involves economic modeling of four bequest motives: altruistic, paternalistic, accidental, and exchange. The method is based on the overlapping generation approach and life-cycle approach in the case of a paternalistic bequest. The results are supplemented with numerical simulations.

Findings & value added: For the altruistic motive, savings are smaller if interest rates are low relative to the tax rate and the utility of one's consumption is more valuable than the utility of the next generations. For the accidental motive, savings decrease with small interest rates, high taxation and long-life expectancy. For the paternalistic motive, savings decrease when the interest rate and the value of future utility are low. For the exchange motive, savings are higher after a tax increase, but it depends on the value of attention and life expectancy. The general conclusion is that higher bequest taxation hamper saving behavior and may disturb the intergenerational exchange process. The bequest tax should, therefore, be set low, especially for aging populations, in order to induce higher savings and force the young to provide the elderly with higher attention.

Introduction

Inheritance and gift taxation (bequest taxation) affect the saving of parents and their children and modify transfers (gifts) between these two groups. There is some evidence that transfer taxes can significantly influence the size and timing of *inter vivos* transfers (Glogowsky, 2021; Hines *et al.*, 2019; Tsoutsoura, 2015). The total effect can be positive or negative, as Gale and Perozek (2001) have theoretically shown in the life cycle approach, and depends on the assumed bequest motive. Unfortunately, in the life-cycle approach saving and transfers are modeled separately for parents and children, allowing for the inconsistency of their choices when young versus old. Particularly, the decision undertaken by the child (when young) can be inconsistent with the decision of the same person acting as the parent (when old) because the interests of agents are different at different moments of life. For example, a young individual can provide a different level of attention than required by the parent and the parent can offer the inheritance but finally not transfer it to the child. To overcome the limitation of the life cycle approach, we analyze the same problem assuming a consistent response of both groups to the tax incentives irrespective of the time frame. Our proposition is based on an overlapping generation approach (OLG) where parents' and children's utilities are optimized together. This

type of modeling approach is popular when modeling saving behavior in the presence of taxation (cf. Uchida & Ono, 2021).

In this paper, we strive to predict the expected reaction of saving and *inter vivos* transfers to the changes in bequest taxation under the assumption that people make rational economic choices, and their decisions have long-term implications. It complements the theory of bequest taxation in the life cycle approach which is focused on modeling the short-term decisions of parents and children. The analysis includes four cases related to the bequest motives: altruistic (the utility of the next generations is present in the objective function), paternalistic (the joy of giving — the utility of the act of transfer is present in the objective function), accidental (there are no transfers in the objective function), and exchange (the objective function includes attention which is received in exchange for transfers) (Kopczuk, 2009, 2013). These models allow the formulation of some expectations on the behavior of taxpayers/individuals after an increase in bequest taxation. They are new and different from the results obtained in previous literature, especially by Gale and Perozek.

Particularly, our results indicate why the impact of the bequest tax increase is sometimes positive and sometimes negative for the same bequest motive, emphasizing the importance of such factors as the valuation of children's utility, the validation of the act of giving, or the characteristics of the exchange concerning transfers and attention. We also provide some numerical illustrations of the impact of taxation on savings and transfers for the probable values of the model parameters. It gives new insight regarding a comparison between myopic (individuals behave according to their current interests) and long-term (individuals behave according to their current and future interests) decision-making processes.

This paper supplements the results of life-cycle models (we juxtaposed the results of our study and the study of Gale and Perozek in Table 1) and provides some expectations regarding savings and transfers which can be tested empirically in future research. The paper also contributes to economic policy as it predicts the effects of an increase in inheritance and gift taxes on savings and transfers. Based on this implication a government can choose a policy stimulating the creation of savings or facilitating transfers between older and younger parts of the population. This latter policy can at least partially substitute for the state provision of care and financial support to the elderly.

The paper also fills the gap in modeling the impact of inheritance taxation on saving in an overlapping generation framework and the life-cycle framework when the paternalistic motive of bequest is considered. We perceive the topic of the paper as important because a large proportion of wealth transfers in modern societies result from inheritance and intergenerational transfers (some detailed studies from the United States include: Saez and Zucman (2016) and Bricker *et al.* (2017); from Piketty (2011) and Sweden Klevmarken (2004)). After the development of inheritance theory, inheritance taxation has also gained importance, even though the tax burden imposed on bequests remains relatively small. It is probably caused by a negative social judgment of high taxation on wealth transferred between close relatives and inherited by younger individuals (Abraham *et al.*, 2018; Gross *et al.*, 2017) or the low perceived importance of wealth taxation (Bastani & Waldenström, 2021). Low inheritance taxation can be also perceived as detrimental to saving and wealth transfers. The last statement is especially challenging, as our modeling results indicate the opposite impact for the plausible set of parameters: high taxation of bequest reduces saving and transfers. The opposite statement would require special conditions to be met or a special motive of bequest (e.g., high valuation of the next generation's utility or transfers in exchange for attention).

To be comparable with previous studies, we follow the notation and some assumptions of the Gale-Perozek life cycle model with the inheritance taxation and lump-sum transfer unrelated to the amount of taxes. The life-cycle models include two periods. In the first period, a parent (donor) offers the transfer of wealth to the child (recipient) and in the second period, the child can expect the inheritance with some probability. Tax is imposed on the transfer and inheritance at the same rate, and this affects the decisions on the value of transferred wealth and savings. Taxation does not consider the individual situation of the recipient and the relationship between the donor and the recipient, so there is no difference between the estate tax (the tax imposed on the whole estate at death or the moment of transfer) and inheritance tax (the tax imposed on the share received by the recipient). The detailed solutions of Gale and Perozek involved four cases: accidental bequest, bequest as exchange, and two cases of the altruistic model, with and without commitment power. Commitment power means that the promise of inheritance is always held.

The proposed OLG model implies not only the commitment but also the symmetry of an agent's behavior. The life-cycle model of Gale and Perozek

allows for asymmetry as young and old individuals make decisions separately. Our models assume the behavior of both sides is the same and consistent, thus we omit the altruistic model with no commitment. Instead, we have completed the theory by developing the paternalistic[1] (joy of giving) model in both approaches (LC and OLG), which is a novelty.

Our results of modeling different bequest motives point out that the effect of tax on saving is negative if the parents are altruistic and the interest rate is relatively small to the tax rate, the salience of the children's utility is small, or the well-being of children is relatively high in relation to parent's utility. If a bequest is accidental, the effect of the tax increase is similar for low-interest rates and high taxation, or high life expectancy. For the paternalistic motive, the effect on saving is ambiguous because it also depends on the relative size of the transfer and savings. However, for low-interest rates and small transfers in comparison with overall savings, it is negative. Finally, for the exchange motive savings decrease if the interest rate is sufficiently small relative to the tax rate, life expectancy is high and the demand for attention is not very high which manifests in low transfers in comparison to savings. The tax effects on transfers are similar, but they are not affected by the value of attention.

In the later part of the paper, we briefly describe all four bequest models with the literature and then calculate the derivatives of savings and transfers with respect to the bequest tax rate. The formulas are provided for each type of bequest motive. Each part is summarized with a conclusion and includes a brief comparison with the life cycle model equivalent. The paper ends with a comparison of all models for different frameworks and bequest motives, a brief discussion of results and final conclusions.

Literature review on bequest motives

There are several bequest motives postulated in the literature. The altruistic motive encourages parents to take care of their utility and the utility of their children (Becker, 1974; Becker & Tomes, 1979). Since an increase in bequest taxes reduces the net transfers received by children, parents are more willing to relocate wealth between their consumption and transfers to their children to maximize their (i.e., children's) utility. In general, this

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[1] In paternalistic bequests, the utility of the recipient is not considered but the sole act of giving provides some utility to the donor.

reallocation depends on the elasticity of the marginal utility of consumption and the elasticity of the marginal utility of bequests. This model predicts that parents try to compensate for the losses incurred by children because all generations act like individuals (they agree on the economic trade-off) (Bernheim, 1989). Different results are obtained when *inter vivos* and *post mortem* transfers are taxed with different tax rates (Nordblom and Ohlsson, 2006; Niimi, 2019). The difference stimulates the avoidance behavior of parents inducing a higher or lower transfer of gifts or inheritance, whichever tax is lower.

A subtype of the altruistic model is a paternalistic model — also called "warm-glow" or "joy of giving" (Andreoni, 1990; Bevan & Stiglitz, 1979; Glomm & Ravikunar, 1992; Kotlikoff & Spivak, 1981; Kopczuk, 2013). The bequest is as consumption expenditure providing additional utility to the parents but not to children. This model can tackle the phenomenon suggested by Altonji *et al.* (1992), that there is no automatic transfer of wealth when the parent's and children's resources change, and siblings with different incomes do not receive different inheritances (higher for lower-income children and lower for higher-income children). It stems from the fact that in the "warm glow" model, there is no intentional smoothing of consumption across generations.

An accidental bequest means that individuals do not consume all their accumulated wealth during their life and there are no *inter vivos* transfers. The *post mortem* transfer is a deliberate choice or an effect of uninsurable risk (Yaari, 1965; Davies, 1981; Abel, 1985; Hurd, 1987, 1989; Andreoni, 1990). In general, the bequest behavior is not affected by tax policy (Cremer & Pestieau, 2011), even if the 100% tax rate is applied and consequently saving should stay unaffected, too. Some evidence on the anticipated character of bequest provides Spiteri and von Brockdorff (2022). The accidental model can explain a large fraction of wealth accumulation (about 35–45% according to Davies and Shorrocks, (2000)).

In the model of exchange (strategic bequest), parents pay for attention provided by their children (Bernheim *et al.*, 1985; Cox, 1987, Glazer *et al.*, 2003). Transfers in this model are positively related to the services provided by children, and an increase in the inheritance tax increases the price that parents must pay for their children's attention. Therefore, the response of parents depends on the elasticity of demand for their children's attention concerning a tax change.

For inelastic demand, parents should not reduce transfers; in fact, they should increase them. The briefly described motives of giving are still in development and remain very popular (Kopczuk, 2009; Ferrara & Missios, 2020). The motives can be developed further by combining different motives. For example, the altruistic motive of parents with a similar altruistic motive from their children (two-sided altruism) (Takaaki *et al.*, 2019). However, we would like to restrict our attention only to four of the most important bequest motives, intensively investigated in the literature on inheritance taxation.

We do not consider the optimal taxation of a bequest and refer only to the impact on savings and transfers. In the optimal theory literature (e.g., Cremer & Pestieau, 2011), there is a belief that transfers should be treated differently when they occur for different reasons. Formally, altruistic transfers should not be taxed or even can be subsidized (Boadway & Cuff, 2015), but the size of the bequest can matter (Casamatta, 2023). Conversely, accidental transfers should be taxed heavily, because such taxation does not change the behavior of agents and exchange transfers are treated as "an investment". The paternalistic bequest should be taxed heavily or leniently depending on the impact on social welfare. One of the most interesting postulates is taxation depending on the expected life longevity (Leroux & Pestieau, 2022). Irrespective of the bequest motive, there is also a view that taxation should depend on the concentration of wealth, the elasticity of the bequest, and society's preferences for wealth inequality (Piketty & Saez, 2013).

Our analysis is focused on determining what is expected, but not what should be taxed from a normative point of view. Therefore, the results are not comparable with the optimal taxation predictions, as they neither refer to the efficiency of taxation nor redistribution. Redistribution is present in our models, but only as an exogenous transfer from a government (*T*). It is because we consider the decision-making process from the perspective of an individual who perceives transferred money as not related to the previously paid tax. In the case of bequest taxation, it is very likely because the redistribution by transfers from a government is not replicating the original distribution of wealth.

The paper verifies two hypotheses. First, the general direction of tax-increase-impact on saving should be negative irrespectively of the bequest motive and a positive response is possible when the other needs fulfilled

by the bequest are very important for the utility. Second, the impact of tax changes on inheritance and gifts should work in the same direction.

Research methods and results

The altruistic model

We assume that an agent maximizes the utility U derived from consumption, c, in two periods (when she is young and old). The utility function includes also the utility of her child V to capture the altruistic orientation of the parent's behavior. The utility of a child is defined as the utility from consumption in two periods. The utility of the child has the same form as the utility of the parent but we assume that the child derives utility only from her consumption (the child is neither altruistic to the parent nor the descendants). The first derivative of the utility functions is positive and the second is negative. The rate β reflects the time preference of the parent and the rate θ measures the salience of the child's utility for the parent. The objective function can include further components of the next generation's utilities, but if we assume that the next generations behave the same as the child's generation, it can be approximated by the sufficient increase of θ value. Nevertheless, we assume that the value of θ is finite.

The parameters of the models include net transfer (I_t) granted by the parent to her child in time t when the parent is alive (gift) and savings created when an agent is young (s_t). The bequest has two forms: when the parent is alive, it is a form of investment (*inter vivos* transfer, *a* gift) and when the parent is dead, it is an inheritance. The latter is composed of the parent's savings (s_t) increased by the interest (r) and decreased by the tax on bequest (τ). A tax of the same size is also imposed on *inter vivos* transfers to avoid tax arbitrage. We assume that tax should not exceed one. The interest rate r is positive but not greater than one. There is a transfer of money from the government in the next period to the agent (T) which is exogenous (its value is not related to the sum of the collected tax). There are also exogenous incomes y_t and y_{t+1} received in subsequent periods and for simplicity, we assume that they are equal. The annuity market is complete. The subscript denotes the period and superscripts denote whether an individual is young (y) or old (o).

The value of Q determines whether the parent is dead or not with some probability of death equaling *1-q* and is lower than one. The q is the probability of being alive and will be referred further as "life expectancy". It should be emphasized that this unusual assumption about the use of the alternative of receiving or not inheritance is necessary to accurately describe the decision-making process. An individual is not certain whether they receive an inheritance or not, but has to make a decision about the saving, knowing only the probability of her parent's death. This decision is different from the decision in the state of perfect information when she knows whether she gets an inheritance or not. For example, if there is no inheritance then taxation has no effect on saving decisions. Moreover, q affects also the value of future consumption. Low life expectancy (low q) makes future consumption of little value and discourages saving.

The maximization problem of youth is to choose the savings (in period *t*) with the given *I**. The problem of an elder (in period *t+1*) is to decide *I*, taking *s** as given. The objective function takes the form:

$$max_{s_t, I_{t+1}}\{U(c_t^y) + \beta q U(c_{t+1}^o) + \theta[V(c_{t+1}^y) + \beta q V(c_{t+2}^o)]\} \qquad (1)$$

The consumption in the first period is equal to the exogenous income (y_t) less saving (s_t) and increased by a net wealth of bequest (I_t^*) (in the case of a living parent) or the net value of accumulated savings (in the case of a dead parent):

$$c_t^y = y_t - s_t + (1 - Q)I_t^* + Q(1 + r)(1 - \tau)s_{t-1}, \qquad (2)$$

where

$$Q = \begin{cases} 0 \text{ with probability } q \text{ (parent is not dead)} \\ 1 \text{ with probability } 1 - q \text{ (parent is dead)}. \end{cases}$$

The consumption in the second period depends positively on the exogenous income (y_{t+1}), transfer received from the government (T), accumulated savings $(1 + r)s_t$, and negatively on the gross value of the bequest granted to the child $\frac{I_{t+1}}{1-\tau}$:

$$c_{t+1}^o = y_{t+1} + T + (1 + r)s_t - \frac{I_{t+1}}{1-\tau} \qquad (3)$$

The consumption of a child in *t+1* and *t+2* periods can be described similarly. In the equilibrium, all *inter vivos* transfers and all savings (irrespective of the periods subscript) are equal.

To solve the problem, we need to calculate the derivatives of s and I with respect to τ. The subscripts and superscripts are omitted. After the calculation of first-order conditions for optimal s and I we can derive $\frac{\partial s}{\partial \tau}$ and $\frac{\partial I}{\partial \tau}$ using the implicit function theorem (cf. Appendix 1). The derivative of saving with respect to bequest taxation τ is equal to:

$$\frac{\partial s}{\partial \tau} = \frac{qI_{t+1}+(1-q)(1+r)(1-\tau)s_t}{(1-\tau)[r(1-\tau)-\tau]} - \frac{(1-q)(1+r)\theta V'[\beta U_o''+q(1-\tau)^2\theta V_y'']}{\beta[r(1-\tau)-\tau]U_o''[U_o''-(1+r)(1-\tau)\theta V_y'']} - \frac{qU_o'\{(1-\tau)U_o''+(1+r)[\beta U_o''-(1-q)(1-\tau)^2\theta V_y'']\}}{(1-\tau)[r(1-\tau)-\tau]U_o''[U_o''-(1+r)(1-\tau)\theta V_y'']}. \tag{4}$$

Assuming that $U_o' > 0, U_o'' < 0$ are the derivatives of the parent's utility function and $V_y' > 0; V_y'' < 0$ are derivatives of the child's utility function $0 < \tau, r, q < 1$, the first part of this formula is negative if r is not too high $\tau/(1-\tau) > r$. For $r \to \tau/(1-\tau)$ the part $r - q(1+r)(1-\tau) - (1+r)\tau \to -q$. Therefore, for $\tau/(1-\tau) > r$ the sign of the second part of equation 4 depends on the sign of $U_o'' - (1+r)(1-\tau)\theta V_y''$. If the latter expression is negative, then the second part of the derivative is negative. It is met if the change in the marginal utility of children decreases less than the change in the marginal utility of parents, and the valuation of children's consumption θ remains relatively low. It means that the situation of the children is relatively good and that parents do not value the utility of children highly.

The sign of the third part of this formula is not determined *a priori* and it depends on the utility functions and on the relationship between β and q. If $U_o'' - (1+r)(1-\tau)\theta V_y'' < 0$, β is high (close to one meaning a high valuation of future consumption) or q is high (close to one meaning a high probability of being alive), the last part of formula 4 is negative. The numerical example calculated for the logarithmic utility function confirms that this derivative is negative for some plausible values of parameters (Appendix 3, Figure 1).

Inheritance taxation reduces the possible benefits from saving and discourages their creation if the initial incentives encourage saving (high valuation of future consumption and low death probability). We can guess that there is a trade-off between egoistic behavior focused on the maximization of one's own utility and altruistic behavior focused on the maximization of

children's utility. Egoistic motivation is more important when an individual evaluates his future consumption high and expects long life, taxation is higher than gains from saving, the salience of the utility of future generations is low, and children are wealthy. In this situation, the best reply to an increase in tax is to lower saving to compensate for the drop in consumption. Conversely, if the utility of future generations is very important for an individual and children's wealth is low, saving should be increased, especially if the valuation of future consumption and life expectancy are both low.

Similarly, we can calculate the derivative of transfers with respect to the tax:

$$
\begin{aligned}
\frac{\partial I}{\partial \tau} &= \frac{[r - q(1+r)(1-\tau) - (1+r)\tau]I - (1-q)(1+r)^2(1+\tau)^2 s]}{(1-\tau)} + \\
&+ \frac{(1-q)(1+r)(1-\tau)\theta V_y'\{(1+r)\beta U_o'' - \theta(1-\tau)[r - q(1+r)(1-\tau) - (1+r)\tau]V_y''\}}{\beta U_o''[U_o'' - (1+r)(1-\tau)\theta V_y'']} + \\
&+ \frac{U_o'\{[q(1+r)(1-\tau) + (1+r)\tau - r][U_o'' - (1+r)(1-q)(1-\tau)\theta V_y''] + (1+r)^2 q\beta U_o''\}}{U_o''[U_o'' - (1+r)(1-\tau)\theta V_y'']}.
\end{aligned}
\tag{5}
$$

Considering the same assumption about signs of derivatives and limiting the parameters t, r and q to the range 0-1, we can assess the possible tax-change impact. The first term of the formula (5) is negative for $[q + \tau(1-q)]/[(1-\tau)(1-q)] > r$. To meet this requirement, it is sufficient if the probability of being alive is greater than the interest rate. The sign of the second part depends on the sign of $U_o'' - (1+r)(1-\tau)\theta V_y''$. The last formula compares the decrease of marginal utility between older and younger populations modified by the increase of consumption in the future $(1+r)(1-\tau)$ and the importance of children's utility for the parents θ. If the probability of staying alive is high, the valuation of children's utility in the objective function is low, children are relatively rich (the marginal utility of children decreases slower than the marginal utility of parents) and the net value of investment earnings is low (savings provide low after-tax gains) then the second part of the derivative is negative. The third term can be positive or negative. But if q is sufficiently higher than r and $U_o'' - (1+r)(1-\tau)\theta V_y'' < 0$ then this part of the derivative is also negative. In Appendix 3 (Figure 2) the numerical example for the logarithmic utility function is provided for the same value of parameters as for the derivative of savings. The derivative is negative so the decrease of the transfer (I) with tax rates is expected.

Transfer reacts to the tax change similarly to saving, but the factors related to the valuation of the future consumption of the donor (*β* and *q*) are unimportant. The valuation of the children's utility and their well-being determines the reaction of donors. The high valuation or bad situation of the future generation induces growth of the transfer.

In the life-cycle approach (Gale & Perozek, 2001), the corresponding model is the model of altruism with commitment power because there are altruistic transfers and there is no possibility of not committing. The result of the life-cycle model is a decrease in transfers with a tax rate increase and an ambiguous change in the parent's and child's savings. Tax increases should lower the parent's savings and increase the child's savings with an unknown total effect.

In the model proposed here, saving is always affected and there is no difference between the parent's and child's behavior. In ordinary situations (low-interest rate in relation to the tax rate and high probability of long life and relatively good situation of children), taxation decreases the gains from saving and transfers, making them lower.

Accidental bequest

The bequest is accidental when there is no transfer from parent to child during her life and all transfer occurs *post mortem*. Therefore, contrary to the altruistic model, the utility of the child is not present in the utility function of a parent and there is no transfer *I*. Then attention is constant and independent on *I*, so *a(I)=a* and it does not change with the transfer. Thus, the optimization of the objective function is only with respect to *s*.

$$\max_{s_t} U(c_t, -a) + \beta q U(c_{t+1}, a), \tag{6}$$

with

$$c_t = y_t - s_t + Q(1+r)(1-\tau)s_{t-1}, \tag{7}$$

And

$$c_{t+1} = y_{t+1} + T + s_t(1+r), \tag{8}$$

where

$$Q = \begin{cases} 0 \; with \; probability \; q \; (parent \; is \; not \; dead) \\ 1 \; with \; probability \; 1 - q \; (parent \; is \; dead). \end{cases}$$

The problem of the child is the same as the problem of the parent because they are both egoists and do not care about transfers. The derivative of saving with respect to tax rate is:

$$\frac{\partial s}{\partial t} = \frac{-(1-q)(1+r)s}{q(1+r)[1+\beta(1+r)-\tau]-r(1-\tau)+\tau}. \tag{9}$$

The sign of the formula is always negative if $r < \frac{\tau}{1-\tau}$. The formula can be positive if q and τ are both small and r is relatively high (see also Appendix 3, Figure 3). Taxation affects the value of inheritance received from the parents, so a consumer saves less to sustain the level of consumption in time t, but this also affects the savings received in the form of inheritance by the next generation. A high-interest rate can mitigate this detrimental effect of taxation because then the higher taxation can be compensated by higher savings. It also must be accompanied by a high probability of death, making inheritance more important for the consumer utility than their own saving. There is no impact of bequest tax on *inter vivos* transfer (because there is no such transfer).

In the life-cycle model of Gale and Perozek (2001), taxation does not affect the saving of parents and increases the saving of their children. The taxation in the life-cycle model decreases the future wealth of young individuals so they are induced to save more. One can notice that in the proposed model, the older and younger generations are forced to save less, while in the life-cycle model, neither parents nor children decrease saving.

Paternalistic bequest

Paternalistic transfers mean that the utility of the younger population is not present in the utility function of the parents. Parents make a transfer only to increase their utility, but they do not care about the effect they exercise on their children. This transfer contributes to a higher utility of parents by function *V(I)*. The bequest received by the child is not optimized, as it does not affect the choice of transfer, so we describe it as T_t. The transfer

from the government to the parents is now T_{t+1}. The objective function and the budget constraints are given by:

$$max_{s_t, I_{t+1}} U(c_t) + \beta q U(c_{t+1}) + \beta q V(I_{t+1}), \tag{10}$$

with

$$c_t = y_t - s_t + T_t + Q(1+r)(1-t)s_{t-1}, \tag{11}$$

and

$$c_{t+1} = y_{t+1} + T_{t+1} + s_t(1+r) - I_{t+1}/(1-\tau), \tag{12}$$

where

$$Q = \begin{cases} 0 \ with \ probability \ q \ (parent \ is \ not \ dead) \\ 1 \ with \ probability \ 1 - q \ (parent \ is \ dead). \end{cases}$$

Having applied the implicit function theorem, we can find the derivatives of saving and transfer with respect to tax rate:

$$\frac{\partial s}{\partial t} = \frac{q\beta[U'/(1-\tau)+IV'']-(1-q)s[U''+(1-\tau)^2V'']}{q\beta(1+r)(1-\tau)^2V''+[q(1-\tau)+\tau-r/(1+r)][U''+(1-\tau)^2V'']}, \tag{13}$$

Assuming that the second derivative of V is negative, the denominator of this derivative is negative if r is sufficiently small compared to q and τ ($r < \frac{q(1-t)+\tau}{(1-q)(1-\tau)}$). The nominator of (13) can be positive or negative. If $q\beta I - (1-q)(1-\tau)^2s < 0$ then the nominator is positive and together with the low-interest rate it makes the derivative negative. The sign is affected by the relative size of s and I and by the values of q, β and τ. Especially, the sufficiently low value of q ascertains the negative impact of tax on savings. The positive value of this derivative also requires a high value of the second derivative of the utility from giving (V'').

A parent decides how much to save when the tax rate goes up depending on the valuation of the transfer and current and future consumption. We can guess that if the act of giving is very valuable for the donor (which is indicated by the high I relative to s and the high value of the second derivative of V) and the valuation of future utility ($q\beta$) is high, then the transfer will be preferred over saving and the latter will be reduced. However,

this situation describes the extremely high valuation of gifts and should be rare in practice. The numerical simulation, for the logarithmic function and for similar parameters to the other models (Appendix 3, Figure 4), reveals the negative value of this derivative. Thus savings should be lower after an increase in the tax.

$$\frac{\partial I}{\partial t} = \frac{-\left\{\left[q(1+r)+\frac{\tau}{1-\tau}-r\right]I+(1-q)(1+r)^2(1-\tau)s\right\}U''}{(1+r)\left\{q\beta(1+r)(1-\tau)^2V''+\left[q(1-\tau)+\tau-\frac{r}{1+r}\right][U''+(1-\tau)^2V'']\right\}} +$$
$$+ \frac{\{q(1+r)[1+\beta(1+r)-\tau]-r(1-\tau)+\tau\}U'}{(1+r)\{q\beta(1+r)(1-\tau)^2V''+\left[q(1-\tau)+\tau-\frac{r}{1+r}\right][U''+(1-\tau)^2V'']\}}. \tag{14}$$

The derivative of transfers is negative for $r < \frac{q(1-\tau)+\tau}{(1-q)(1-\tau)}$. Besides the situation of very high r in relation to q and τ, taxation decreases *inter vivos* transfer. The intuition behind this result is as follows; higher taxation decreases the value of the transfer and consequently the utility of the donor. This effect can be compensated by a higher return on savings but this requires high interest rate. (Illustration: Appendix 3, Figure 5)

There is no solution to the life-cycle model in the work of Gale and Perozek (2001), so we have calculated the algebraic solution of the life-cycle paternalistic model in the appendix. We use the CES function to be in line with the life-cycle altruistic model. Bequest taxation negatively affects saving if γ — (the parameter of the utility function) — is lower than the tax rate t. So, it is quite similar to the OLG results, where the form of the utility function determines the final effect. The transfer is always negatively affected by the increased tax rate.

Bequest as exchange

The utility of parent and child depends on the attention (described by the function *a(It)*) provided by a young individual to the old one at the price of inter vivos transfer in time *t (It)*. This function is increasing with I but the changes of attention are decreasing $a'(I_t) > 0$ and $a''(I_t) < 0$. Thus, this function is similar to the ordinary work supply function with transfers instead of wages. The net worth of the transfers is I/(1-τ) as the inheritance

tax is imposed[2]. Attention decreases the utility of a child but increases the utility of a parent.

$$max_{s_t, I_{t+1}}\{U(c_t, -a(I_t)) + \beta q U(c_{t+1}, a(I_{t+1}))\} \tag{15}$$

with

$$c_t = y_t - s_t + (1 - Q)I_t + Q(1 + r)(1 - \tau)s_{t-1}, \tag{16}$$

where

$$Q = \begin{cases} 0 \ with \ probability \ q \ (parent \ is \ not \ dead) \\ 1 \ with \ probability \ 1 - q \ (parent \ is \ dead). \end{cases}$$

The consumption in the second period depends positively on the exogenous income (y_2), the transfer received from the government (T), accumulated savings $(1 + r)s$, and negatively on the gross value of the bequest granted to the child:

$$c_{t+1} = y_{t+1} + T + (1 + r)s_t - \frac{I_{t+1}}{1-\tau}. \tag{17}$$

In time *t* the saving is set and in time *t+1* the transfer to the next generation. In equilibrium, all savings and transfers should be equal irrespectively of the subscripts. Once again, using the implicit function theorem, the model can be solved for $\frac{\partial s}{\partial t}$ and $\frac{\partial I_a}{\partial t}$. If we assume that mixed derivatives are equal to zero (the attention expression is separable from the saving expression), we can obtain the following algebraic results:

$$\frac{\partial s}{\partial t} = \frac{q^2\beta[1+(1+r)\beta-\tau]U_1' + I_{1+t}[bq(1+r)\beta(1-\tau) - q^2\beta U_1'']}{(1-\tau)\{bc - q\beta[r(1-\tau) - \tau]U_1''\}} - \\ + \frac{s_t(1-q)(1+r)(1-\tau)[b(1-\tau)^2 + q\beta U_1'']}{(1-\tau)\{bc - q\beta[r(1-\tau) - \tau]U_1''\}}. \tag{18}$$

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[2] We do not follow the assumptions of Gale and Perozek (2001) that attention is set optimally at a level *a**(s)*, so $\left(a\ (s)\right)' = 0 = \left(a\ (s)\right)''$. This assumption means that there is no motivation to change the saving behavior (of parents) and the attention (of children) in the equilibrium.

and

$$\frac{\partial I}{\partial t} = \frac{-q\beta(\{q(1+r)[1+\beta(1+r)-\tau]-r(1-\tau)+\tau\}U_1')}{bc+q\beta[r(1-\tau)-\tau]U_1''} +$$
$$+ \frac{q\beta\{[q(1+r)+\tau/(1-\tau)-r]I_{1+t}+(1-q)(1+r)^2(1-\tau)s_t\}U_1''}{bc+q\beta[r(1-\tau)-\tau]U_1''}. \tag{19}$$

where

$b = (1-q\beta)U_2'a'' - (1+q\beta)(a')^2U_2''$ and $c = (1-\tau)^2\{q(1+r)[1+\beta(1+r)-\tau]-r(1-\tau)+\tau\}U_1'$ is the first derivative of the objective function with respect to s, and U_1'' is the second derivative of the objective function with respect to s. U_2' is the first derivative of the objective function with respect to I and U_2'' is the second derivative of the objective function with respect to I. a' is the first derivative of the attention function with respect to I, and a'' is the second derivative of the attention function with respect to I.

The b measures the relative importance of attention for the utility of a consumer. The sign of b can be positive or negative and it depends on the curvatures of utility (U) and attention functions a. For example, b is positive when $\frac{(1-q\beta)a''}{(1+q\beta)(a')^2} > \frac{U_2''}{U_2'}$. This condition means that the curvature of the utility function $\frac{-U_2''}{U_2'}$ (Arrow-Pratt absolute risk coefficient) is higher than the curvature of the attention function $\frac{-a''}{a'}$ multiplied by $\frac{(1-q\beta)}{(1+q\beta)a'}$. The latter term is lower than one. However, it should be noted, that it is sufficient for b to be positive if $q\beta \geq 1$ (the expected future consumption is valued similarly or higher than current consumption) irrespectively to the curvature of the utility function. The sign of c is positive if r is smaller than $\tau/(1-\tau)$ or $r < q$.

The derivative $\frac{\partial s}{\partial t}$ can be positive or negative. The nominator of 18 is positive for q close to 1. The denominator is negative for r smaller than $\tau/(1-\tau)$ and when the product of multiplication of b and c is small. If these conditions are met the saving decreases with an increase of tax. Therefore, for q close to 1, r smaller than $\tau/(1-\tau)$ and low importance of attention (small b) taxation decreases saving. If r is smaller than $\tau/(1-\tau)$ then the nominator of 19 is negative and the last part of the denominator is positive. Therefore, for $q\beta \geq 1$, r smaller than $\tau/(1-\tau)$, $\frac{\partial I}{\partial t}$ is negative and bequest taxation decreases transfers. $q\beta$ informs about the value of expected future consumption when an individual is alive in the next period. The numerical simula-

tion for the logarithmic function of utility and the square root function of attention is included in Appendix 3, Figures 6 and 7. The simulation shows the importance of the relationship between interest and tax rate for the signs of derivatives.

Saving can be higher after a tax increase if the higher transfers for attention are necessary for the next period. This is most likely when also the conditions for the increase of transfers are met (attention and future consumption are valued high). If transfers are low in relation to savings, it means that the cost of receiving attention is low (transfers are the price of buying attention) and the effect of lower inheritances is more important than the effect of sustaining the attention level. Therefore, like in the accidental model, taxation makes an individual save less to compensate for the drop in current consumption caused by lower inheritance.

It follows then the transfers should be lower after a tax increase as they decrease the level of consumption of the elderly. Higher transfers provide the trade-off between the utility of consumption of goods and the utility of attention received but in equilibrium, the marginal utility from consumption of goods and from attention should be the same. Transfers can be used only in one way (to buy attention) as the decision about their size is taken when a consumer is old. Conversely, savings can increase the consumption of goods in the future and allow for higher future transfers. Therefore, applications of savings are more universal but also more complicated.

In the life-cycle model presented by Gale and Perozek (2001), the effect of bequest taxation on saving is undetermined *a priori* and it depends on the elasticity of demand for attention (the results of all the presented models from sections 3-6 are compared with the life-cycle model results in Table 1.).

Discussion

In most cases, savings and transfers are affected by bequest taxation. The detailed predictions of the impact of taxation depend not only on the prevailing bequest motive but also on the modeling framework (LC or OLG). Sometimes the expected behaviors of taxpayers with the same motive (but with a different modeling framework) provide opposite predictions. For example, in the accidental model total saving in the life-cycle framework should be higher, while in the overlapping generation framework saving

should be lower if the interest rates are small enough. It is important for the evaluation of policies focused on the stimulation of saving because such policies can be ineffective. This inefficiency stems from the different ways of decision-making. In the life-cycle framework, the decisions of the young and the elderly are separated while the OLG framework considers situations in different moments of life.

In the presented models, saving is negatively affected by bequest taxation for altruistic, accidental, paternalistic and exchange motives if interest rates are not very high in relation to the tax rate (cf. Table 1). For altruistic, accidental and exchange bequests, the probability of being alive in the next period should also be high. These results refer directly to the profitability of saving and the valuation of future consumption. Bequest taxation reduces the profitability of savings. A taxpayer receives less from inheritance and transfers, which makes their current consumption lower and discourages savings. Only if interest rates are sufficiently high to cover the payment of taxes can the opposite effects be observed. These results are modified in altruistic and paternalistic models by the utility of children in the first case and the utility of the donor in the second case.

Taxation can discourage saving if the value of these additional aspects of taxpayers' behavior is low. In the altruistic model, it occurs when a donor does not evaluate children's utility or when the financial support to children is less valuable than the increase of the consumption of a donor. In the paternalistic model, it is when satisfaction from giving is relatively low or the value of future consumption is low. In the exchange model, an increase in taxation decreases saving, particularly if the value of attention is low or the cost of attention (transfers) is high. High transfers indirectly hint that attention is more important for the taxpayer than the consumption of other goods, so the taxpayer would increase transfers despite the lower consumption of other goods and the necessity of paying higher taxes.

Recall, that in all models transfers occurred in the second period of a donor's life but the decision about the saving size is taken in the first period. Therefore, the savings can be spent on the increase of future consumption or on transfers. Transfers occur when an individual is old, thus they can be used only to achieve additional purposes (the higher utility of future generations, the higher joy of giving, or higher attention). The valuation of these purposes is crucial for understanding the transfer behavior. In general, if the purposes are very important the transfers should react opposite to the changes in taxation. This result confirms the first hypothesis.

The transfers are not present in the accidental model and negatively affected in the altruistic, paternalistic and exchange models if the interest rate is relatively low compared to the tax rate. Additionally, in the altruistic model, the importance of children's utility affects the impact of taxation and is similar to the effect exerted on savings. In the exchange model, transfers respond negatively to the tax increase when the probability of being alive in the next period is high. A long-living individual prefers higher consumption of goods in the next period and lower value of more taxed transfers. This argument is not valid in accidental and altruistic motives. In the latter motive, the fact of being alive is not so important, as the welfare of future generations is present in the objective function. The numerical simulation provides the same direction of tax impact on transfers as on saving. It is consistent with the second hypothesis but it does not preclude that in specific situation transfer can change oppositely to savings. For example, it is possible when an agent needs intensive care (and attention) in the second period of her life.

Comparison of results with the life-cycle model's predictions (Table 1) directly indicates factors affecting the negative or positive sign of the derivatives. Particularly, it refers to the results of the exchange and altruistic model which are described as ambiguous in the life-cycle approach but can be identified in the proposed OLG models. The results for saving in the life-cycle accidental bequest model are opposite of those predicted here, while the results for the paternalistic life-cycle model are opposite among children and parents and do not consider the importance of the act of transfer to the utility of a donor. Although, we can expect that the behavior of a donor should be similar to the accidental model when a transfer is not valuable and altered when the act of giving is very valuable. This aspect is not captured by the life-cycle model and saving and transfers are related only to τ — the tax rate, and γ — the parameter of the CES utility function. It should be also noted that in the exchange motive attention is fixed in the life-cycle (Gale & Perozek, 2001) while we allow for the adjustment of attention relative to the cost of it.

The presented models assume no recycling of the tax revenues. For further research, it seems reasonable to explore whether recycling affects the results, as it eliminates the income effect (taxpayers are no poorer after the tax payment). Similarly, some other modifications can be added to the models. The most promising seems the extension of the altruistic model to two-sided altruism.

Conclusions

On the whole, the policy of higher bequest taxation reduces savings so the inheritance taxes should be small. This observation can explain why bequest taxation (taxation of inheritance and gifts) provides a relatively small part of government revenues (Drometer *et al.*, 2018). This conclusion is expected in the altruistic motive (as bequest taxation decreases the utility of donor and recipient), but as one can see from the obtained results also for exchange motive.

In most developed countries we observe population aging (an increase in the number of elderly and a decrease in the number of youth). It makes the exchange motive more important and stimulates higher demand for attention as not all elderly are receiving a sufficient level of attention. Higher demand for attention requires higher transfers and higher savings to finance them. Therefore, the increase in taxation seems unnecessary, as it disturbs the exchange, making it more costly for the donors. Similarly, the exchange between the young and the elderly at least partially relieves the state of offering care to the elderly, thus it should not be reduced by high taxation of the bequests.

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Acknowledgments

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Annex

Table 1. The differences between the life-cycle and overlapping generation models' responses to the bequest tax increase

Framework type	Value affected	Altruistic	Accidental	Paternalistic	Exchange
OLG	Saving	Saving decreases for small r relative to τ, high q and β, low valuation of children consumption θ or relatively high well-being of children	Saving decreases for small r and sufficiently high τ and q	Saving decreases for relatively small r, small q, τ and β or when saving is respectively higher than transfer in the equilibrium	Saving decreases for small r relative to τ, high probability of being alive q, and low importance of attention in the future.
	Inter vivos transfer	Transfer decreases for small r relative to τ, low valuation of children consumption θ or relatively high utility of children	There is no *inter vivos* transfer	Transfer decreases for small r relative to τ and q	Transfer decreases for small r relative to τ, and high valuation of expected future utility $q\beta$
LC†	Saving	Ambiguous	Child saving increases with taxation while the parent's saving is not affected	CES function (cf. Appendix 2): Parent saving decreases with τ if $\gamma < 1$. Child saving increases with τ. Total saving increases for $\tau > \gamma$	Ambiguous
	Inter vivos transfer	Negative	There is no *inter vivos* transfer	CES function (cf. Appendix 2): Parent's transfer decreases with τ	Ambiguous

Source: † results taken from (Gale and Perozek, 2001). The Paternalistic life-cycle model is calculated separately with the same set of assumptions.

Appendix 1

Let $F(s_t, I_{t+1}; \tau) = U(c_t^y) + \beta q U(c_{t+1}^o) + \theta[V(c_{t+1}^y) + \beta q V(c_{t+2}^o)]$ together with (2) and (3) be the objective function of the maximization problem (1). First-order conditions are

$$\frac{\partial F(s_t, I_{t+1}; \tau)}{\partial s_t} = F_1(s_t, I_{t+1}; \tau) = 0,$$

$$\frac{\partial F(s_t, I_{t+1}; \tau)}{\partial I_{t+1}} = F_2(s_t, I_{t+1}; \tau) = 0,$$

(A.1)

and define implicitly s_t and I_{t+1} as functions of τ. Inserting $s_t = s_t(\tau)$ and $I_{t+1} = I_{t+1}(\tau)$ into (A.1) and implicitly differentiating w.r.t. to τ yields the two equations:

$$\frac{\partial F_1}{\partial \tau} + \frac{\partial F_1}{\partial s_t}\frac{ds_t}{d\tau} + \frac{\partial F_1}{\partial I_{t+1}}\frac{dI_{t+1}}{d\tau} = 0,$$

$$\frac{\partial F_2}{\partial \tau} + \frac{\partial F_2}{\partial s_t}\frac{ds_t}{d\tau} + \frac{\partial F_2}{\partial I_{t+1}}\frac{dI_{t+1}}{d\tau} = 0.$$

(A.2)

From (A.2) we find derivatives of s_t and I_{t+1} w.r.t. τ

$$\begin{bmatrix} \dfrac{ds_t}{d\tau} \\ \dfrac{dI_{t+1}}{d\tau} \end{bmatrix} = - \begin{bmatrix} \dfrac{\partial F_1}{\partial s_t} & \dfrac{\partial F_1}{\partial I_{t+1}} \\ \dfrac{\partial F_2}{\partial s_t} & \dfrac{\partial F_2}{\partial I_{t+1}} \end{bmatrix}^{-1} \begin{bmatrix} \dfrac{\partial F_1}{\partial \tau} \\ \dfrac{\partial F_2}{\partial \tau} \end{bmatrix}$$

Appendix 2

The paternalistic model in the life-cycle framework is not derived from Gale and Perozek (2001). However, we can resolve the optimization problem starting from the altruistic model without the utility of children consumption in the utility function of the parent (old) but with the utility function V of a transfer I:

$$max U_{s,I} U(c_1) + \beta q U(c_2) + \beta q V(I).$$

(A.3)

The consumption of the parent (old) is equal to

$$c_{1o} = y_1 - s_o \qquad (A.4)$$

in the first period of the parent's life where subscript o stands for old and

$$c_{2o} = y_2 + T + s_o(1+r) - I/(1-\tau), \qquad (A.5)$$

where

$$Q = \begin{cases} 0 \ with \ probability \ q \ (parent \ is \ not \ dead) \\ 1 \ with \ probability \ 1-q \ (parent \ is \ dead) \end{cases}$$

In the second period of the parent's life, to be consistent with the approach of Gale and Perozek we use the CES function of utility in the form:

$$U = \frac{c_{1o}^{1-\gamma}}{1-\gamma} + \beta q \frac{c_{2o}^{1-\gamma}}{1-\gamma} + \beta q \frac{I^{1-\gamma}}{1-\gamma}. \qquad (A.6)$$

The derivatives of (A.4) with respect to I and s are

$$I^{-\gamma} q\beta - \frac{q\beta\left[(1+r)s_o - \frac{I}{1-\tau} + T + y_{2o}\right]^{-\gamma}}{1-\tau}, \qquad (A.7)$$

and

$$-(y_{1o} - s_o)^{-\gamma} + q(1+r)\beta\left[(1+r)s_o - \frac{I}{1-\tau} + T + y_{2o}\right]^{-\gamma}. \qquad (A.8)$$

Equalizing (A.5) and (A.6) to zero and solving for s_o and I provide the optimal saving s_o and bequest I of a parent

$$s_o = \frac{q^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}}\left[1+(1-\tau)^{\frac{1}{\gamma}}-\tau\right]\beta^{\frac{1}{\gamma}}y_1 - (1-\tau)(T+y_2)}{(1+r)(1-\tau) + q^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}}\left(1+(1-\tau)^{\frac{1}{\gamma}}-\tau\right)\beta^{\frac{1}{\gamma}}}, \qquad (A.9)$$

$$I = \frac{q^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}}(1-\tau)^{1+\frac{1}{\gamma}}\beta^{\frac{1}{\gamma}}[T+(1+r)y_1+y_2]}{(1+r)(1-\tau) + q^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}}\left(1+(1-\tau)^{\frac{1}{\gamma}}-\tau\right)\beta^{\frac{1}{\gamma}}}. \qquad (A.10)$$

The utility function of a child (young) is equal to:

$$U = \frac{c_{1y}^{1-\gamma}}{1-\gamma} + \beta q \frac{c_{2y}^{1-\gamma}}{1-\gamma}, \tag{A.11}$$

where

$$c_{1y} = y_1 - s, \tag{A.12}$$

$$c_{2y} = y_2 + I \quad + (1+r)s. \tag{A.13}$$

Calculating the derivative of (A.9) with consumptions given by (A.10) and (A.11) and equalizing it to zero one can calculate the optimal saving for young

$$s_y = \frac{ab^2 y_1 - (1+r)(1-\tau)y_2 + b\{(1-\tau)[(1+r)y_1 - y_2] - (a-1+\tau)c}{[1+r+b]\{(1+r)(1-\tau)+ba\}}, \tag{A.14}$$

where a = $1 + (1-\tau)^{\frac{1}{\gamma}} - \tau$, b = $q^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}}\beta^{\frac{1}{\gamma}}$ and c = $(1-\tau)[T + (1+r)y_1] + (2-\tau)y_2$.

The derivative of the optimal saving of old with respect to τ is then equal to

$$\frac{\partial s_o}{\partial \tau} = \frac{-q^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}}(1-\tau)^{\frac{1}{\gamma}}\beta^{\frac{1}{\gamma}}(1-\gamma)d}{\{(1+r)(1-\tau)+q^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}}\left[1+(1-\tau)^{\frac{1}{\gamma}}-\tau\right]\beta^{\frac{1}{\gamma}}\}^2\gamma}. \tag{A.15}$$

where d = $[T + (1+r)y_1 + y_2]$ This is negative if and only if $\gamma < 1$.

The derivative of the optimal saving of young with respect to τ is then equal to

$$\frac{\partial s_y}{\partial \tau} = \frac{q^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}}(1-\tau)^{\frac{1}{\gamma}}\beta^{\frac{1}{\gamma}}\{(1+r)(1-\tau)+q^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}}\beta^{\frac{1}{\gamma}}\left[1-\tau+(1-\tau)^{\frac{1}{\gamma}}\gamma\right]\}d}{\left(1+r+q^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}}\beta^{\frac{1}{\gamma}}\right)\{(1+r)(1-\tau)+q^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}}\left[1+(1-\tau)^{\frac{1}{\gamma}}-\tau\right]\beta^{\frac{1}{\gamma}}\}^2\gamma}. \tag{A.16}$$

This is always positive.

The derivative of the optimal bequest I of old with respect to τ is then equal to

$$\frac{\partial I}{\partial \tau} = \frac{-q^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}}(1-\tau)^{\frac{1}{\gamma}}\beta^{\frac{1}{\gamma}}\{(1+r)(1-\tau)+q^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}}\beta^{\frac{1}{\gamma}}\left[1+(1-\tau)^{\frac{1}{\gamma}}-\tau\right]\}d}{\{(1+r)(1-\tau)+q^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}}\left[1+(1-\tau)^{\frac{1}{\gamma}}-\tau\right]\beta^{\frac{1}{\gamma}}\}^2\gamma}. \tag{A.17}$$

This is always negative.

Finally, we can calculate the derivative of total savings for old and young:

$$\frac{\partial s}{\partial \tau} = \frac{q^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}}(1-\tau)^{\frac{1}{\gamma}}\beta^{\frac{1}{\gamma}}\{(1+r)(\gamma-\tau)+q^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}}\beta^{\frac{1}{\gamma}}\left[\gamma-t+(1-\tau)^{\frac{1}{\gamma}}\gamma\right]\}d}{\left(1+r+q^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}}\beta^{\frac{1}{\gamma}}\right)\{(1+r)(1-\tau)+q^{\frac{1}{\gamma}}(1+r)^{\frac{1}{\gamma}}\left[1+(1-\tau)^{\frac{1}{\gamma}}-\tau\right]\beta^{\frac{1}{\gamma}}\}^2\gamma}. \tag{A.18}$$

This derivative is positive if $\gamma > \tau$.


Appendix 3

The numerical examples present the impact of inheritance tax on savings $\frac{\partial s}{\partial \tau}$ and inheritances and transfers $\frac{\partial I}{\partial \tau}$, when interest rate r is changing from 0 to 0.4 and τ is changing from 0 to 0.5. We have applied logarithmic utility functions and logarithmic function of attention (in the paternalistic model). All simulations are calculated for the following set of parameters q=0.99; β=0.99; θ=2; T=0.25; y_t=1; y_{t+1}=1; y_{t+2}=1. The probability of being alive should be close to 1 as the low value of this parameter makes the saving redundant. An individual should not save if she expects to be dead in the next period. Similarly, the valuation of future utility should be high to make saving and transfer desirable. So it is set at a level close to one. There are no arguments for β to be above 1 because it would mean that future utility is more important than current. The valuation of children's utility θ is set at 2, as we would like to know what can happen when this part is important for the parent. If θ is low then the problem simplifies to the ordinary choice of savings in the two-period model. Therefore, we set the valuation of children's utility above one and, because the utility of future generations can also be taken into account by the parent, we assume that this parameter could be higher than one but finite. The transfers are set at

25% of the income, and the incomes are equal in each period and set at one. The number 25% is close to the average total tax revenue raised from households relative to GDP in OECD countries (Causa & Hermansen, 2019).
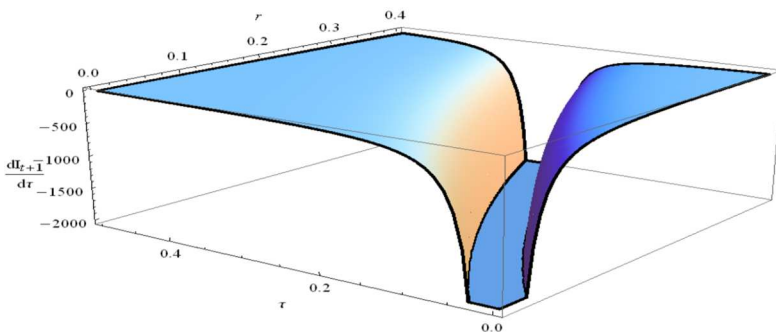
Altruistic model

The objective function: $F = \{Log(c_t^y) + \beta q Log(c_{t+1}^o) + \theta[Log(c_{t+1}^y) + \beta q Log(c_{t+2}^o)]\}$

Figure 1. The impact of τ on s $\frac{\partial s}{\partial \tau}$ in the altruistic model
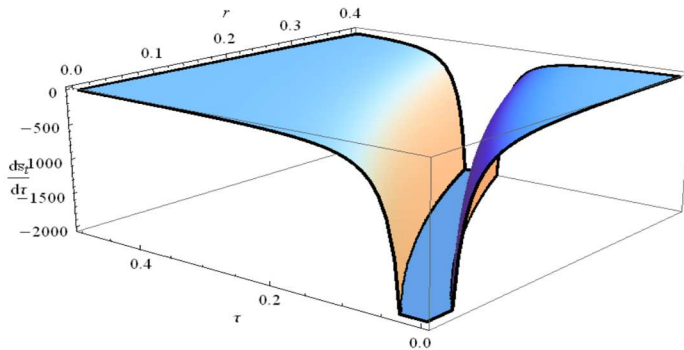
The derivative $\frac{\partial s}{\partial \tau}$ is negative. Taxation decreases savings in the altruistic model.

Figure 2. The impact of τ on I $\frac{\partial I}{\partial \tau}$ in the altruistic model

The derivative $\frac{\partial I}{\partial \tau}$ is negative. The effect of taxation is small for large τ and similar to the effect on savings. The gifts and savings are perceived as substitutes and taxation similarly decreases their values.

Accidental bequest

The objective function: $F = \{Log(c_t) + \beta q Log(c_{t+1})\}$

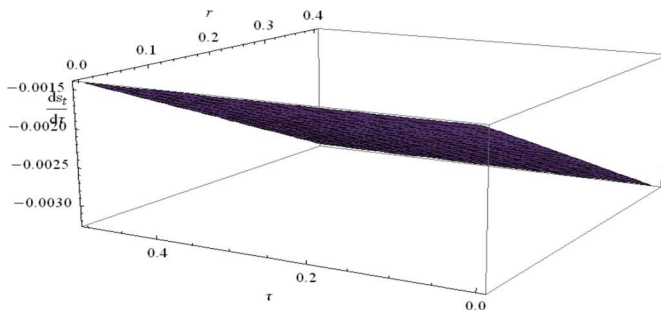**Figure 3**. The impact of τ on $s \frac{\partial s}{\partial \tau}$ in the accidental model
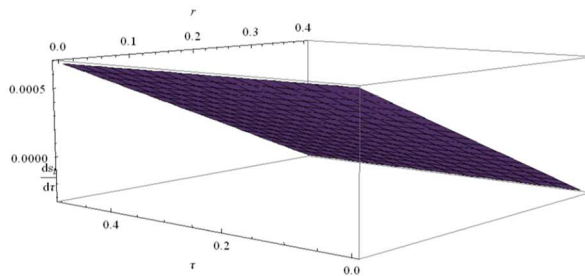
The derivative $\frac{\partial s}{\partial \tau}$ is positive. There is no *inter vivos* transfer.
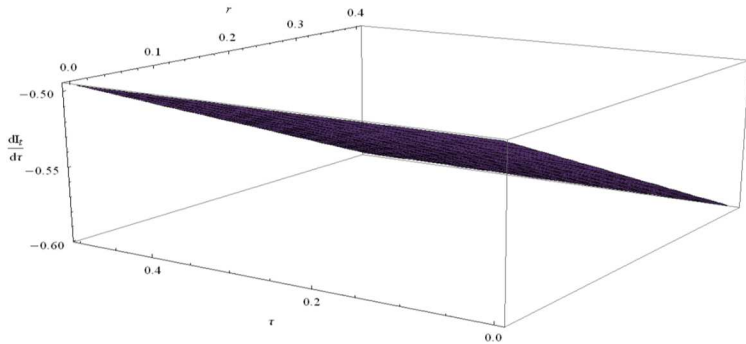
Paternalistic

The objective function: $F = \{Log(c_t) + \beta q Log(c_{t+1}) + \beta q \, 1/2 \, Log(I_t)\}$

Figure 4. The impact of τ on $s \frac{\partial s}{\partial \tau}$ in the paternalistic model

The derivative $\frac{\partial s}{\partial \tau}$ is negative.

Figure 5. The impact of τ on $I\frac{\partial I}{\partial \tau}$ in the paternalistic model
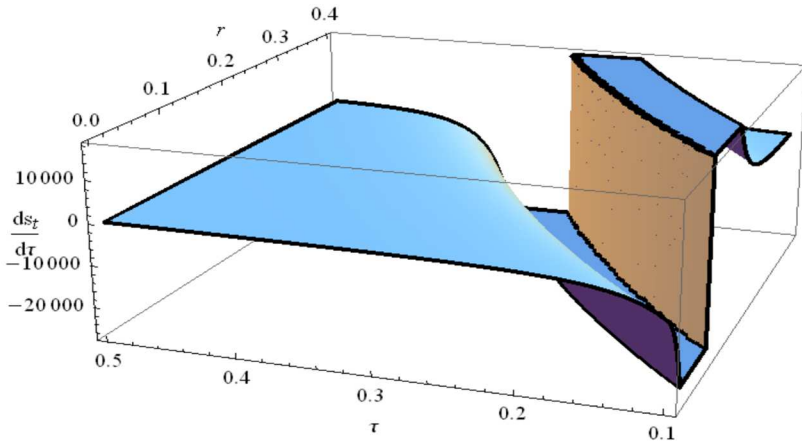

The derivative $\frac{\partial I}{\partial \tau}$ is also negative.
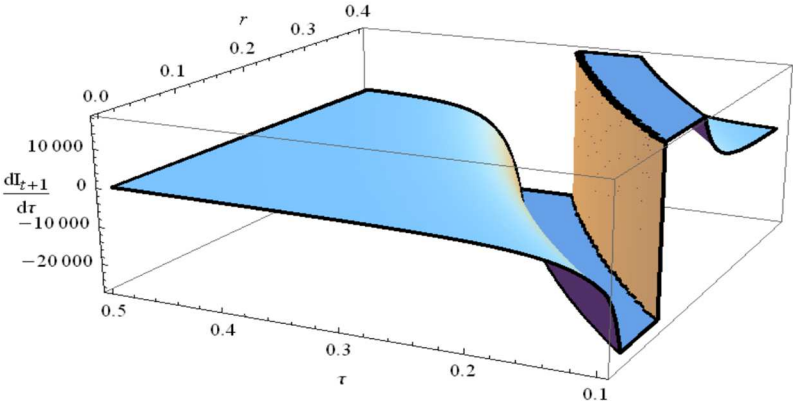
Model of exchange

The objective function: $F = \text{Log}(c_t) - 100(i_{t+1})^{0.5} + \beta q[\text{Log}(c_{t+1}) + 100(i_{t+1})^{0.5}]$

Figure 6. The impact of τ on $s\frac{\partial s}{\partial \tau}$ in the exchange model

The derivative $\frac{\partial s}{\partial \tau}$ is negative for $r < \frac{\tau}{1-\tau}$ and positive otherwise.

Figure 7. The impact of τ on $I\frac{\partial I}{\partial \tau}$ in the exchange model

The derivative $\frac{\partial I}{\partial \tau}$ is also negative for $r < \frac{\tau}{1-\tau}$ and positive otherwise.