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Neutrosophic data envelopment analysis based on the possibilistic mean approach

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Abstract

Data envelopment analysis (DEA) is a non-parametric approach for the estimation of production frontier that is used to calculate the performance of a group of similar decision-making units (DMUs) which employ comparable inputs to produce related outputs. However, observed values might occasionally be confusing, imprecise, ambiguous, inadequate, and inconsistent in real-world applications. Thus, disregarding these factors may result in incorrect decision-making. Thus neutrosophic sets have been created as an extension of intuitionistic fuzzy sets to represent ambiguous, erroneous, missing, and inaccurate information in real-world applications. In this study, we have proposed a technique for solving the neutrosophic form of the Charnes–Cooper–Rhodes (CCR) model based on single-value trapezoidal neutrosophic numbers (SVTrNNs). The possibilistic mean for SVTrNNs is redefined and applied the Mehar approach to transforming the neutrosophic DEA (Neu-DEA) model into its corresponding crisp DEA model. As a result, the efficiency scores of the DMUs are calculated using different risk parameter values lying in [0, 1]. A numerical example is given to analyze the performance of the all India institutes of medical sciences and compared it with Abdelfattah's ranking approach.

Keywords: *efficiency analysis, single value, trapezoidal neutrosophic number, data envelopment analysis, possibilistic mean, Mehar approach*

1. Introduction

One of the most difficult tasks in today's highly competitive world is to monitor the peers' performance and consistently improve since competition is high and increasing by the day. The performance evaluation demands every decision-making unit (DMU) to constantly adapt and improve in order to compete and succeed in today's highly competitive market. Data envelopment analysis (DEA) is a non-parametric, linear programming technique for performance analysis that generates an empirical production frontier that estimates the comparative performance of the DMUs using several inputs and outputs data. It calculates the best practices of the DMUs in such a way that no other DMU has the same quantity of input as

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the provided inputs. Using the assumption of constant returns to scale (CRS), Charnes et al. [10] developed a linear mathematical programming model to estimate the comparative efficiency of DMUs based on Farrell's mathematical model [18]. To investigate relative efficiency under the assumption of variable returns to scale (VRS), Banker et al. [8] expanded a pioneering work [10] by developing a mathematical model called the BCC model.

The DEA is a powerful and efficient MCDM approach that has been widely implemented in various fields such as a variety of industries, including banking institutions [32], the insurance business [26], financial services [33], education [39], supply chain management [22], health care management [37], sustainability [3], energy [19], agriculture [11], and health-care services [31]. The data utilized in the traditional DEA models are numerical/crisp values. However, in real-world applications, the observed data of inputs and/or outputs are frequently unquantifiable, ambiguous, confusing, and non-obtainable in the absence of information [23]. Traditional DEA models cannot be used to evaluate and rank DMU performance when the data are imprecise and ambiguous [17]. Therefore, developing DEA models to deal with this issue is necessary. Otherwise, the efficiency score of the DMU and ranking may become untrustworthy and invalid [42]. As a result, various researchers have proposed different variations of fuzzy data envelopment analysis (FDEA) models using fuzzy programming techniques in recent years. Zadeh [52] in 1965 was the first to develop the notion of the fuzzy set (FS) as a modification and improvisation of traditional set theory. The fuzzy set theory offers a foundation for mathematical modeling of real-world situations with a degree of imprecision, uncertainty, or ambiguity in their description. This theory can be used in various fields, such as engineering, mathematics, and computer science. Sengupta [46] in 1992 used a fuzzy set in DEA for the first time. The fuzzy DEA (FDEA) is a relatively new subject that has dragged the attention of decision-makers, academics, and the scientific community to develop this attractive topic in different fuzzy environments. In FDEA, several approaches have been presented to deal with erroneous, unclear, incomplete, and/or missing data. Stochastic [40] and interval DEA models are usually used to detect inaccurate inputs and outputs data. Figure 1 shows that the fuzzy DEA models are classified into six approaches [17, 23]. Zhou and Xu [53] summarized the development of FDEA and its successful implementations.

It is impossible to simulate all kinds of uncertainty seen in real-world issues, such as incomplete sets. To address this knowledge gap, Atanassov [6] developed an intuitionistic fuzzy set (IFS) in 1986 to further extend the fuzzy set. Instead of a single membership grade, each element in IFS has a non-membership grade. Furthermore, the sum of these two membership grades must be less than or equal to one. When available information is insufficient to describe imprecision via typical fuzzy sets, the idea of IFS might be seen as an appropriate/alternative solution. Later, interval-valued IFS were added to IFS. A number of research publications have been published in DEA that use intuitionistic fuzzy sets. Gandotra et al. [20] proposed the DEA in the context of the intuitionistic fuzzy weighted entropy approach. Sahil et al. [44] proposed the parabolic intuitionistic fuzzy-based DEA based on a parametric approach. Puri and Yadav [43] presented the optimistic and pessimistic efficiencies with intuitionistic fuzzy input/output data in DEA. Arya and Yadav [5] proposed the intuitionistic fuzzy data envelopment analysis (IFDEA) and dual IFDEA (DIFDEA) models based on α and β-cuts, and the index ranking approach is used to rank the DMUs. Javaherian et al. [25] proposed the fuzzy network two-stage DEA model based on the expected value of the intuitionistic fuzzy inputs and outputs. Shakouri et al. [47]

proposed the intuitionistic fuzzy network DEA model based on a parametric approach. Santos Arteaga et al. [45] proposed a novel method for solving intuitionistic fuzzy DEA. Edalatpanah [16] proposed a ranking approach for solving the intuitionistic fuzzy DEA model.

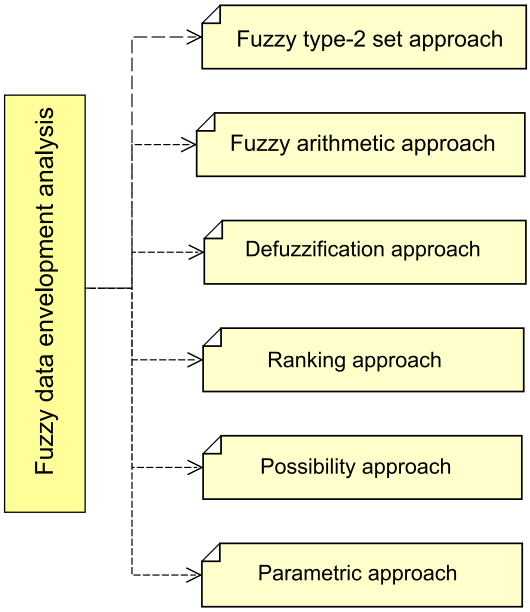

Figure 1. Approaches for solving FDEA

Neutrosophic set (NS), an extended version of FS and IFS presented by Smarandache [48] in 1999, is a strong tool for handling unclear, partial, and unpredictable data in the actual world. It overcomes some of the drawbacks of prior techniques to depicting uncertain decision information by successfully representing ambiguous, incomplete, and inconsistent data with quantified indeterminacy and completely independent truth, indeterminacy, and falsity memberships. It is also closer to human thinking since it simulates human decision-making processes better by considering indeterminacy-related facts. Neutrosophic set and their expansions have subsequently been used in several domains, including computer science [7, 50], mathematics [35], engineering [30], medical [29], etc. NSs have also been used in several MCDM approaches, including AHP, VIKOR, TOPSIS, ELECTRE, PROMETHEE, and others [21, 27]. Recently, Akram et al. [4] proposed Fermatean fuzzy DEA (FFDEA) technique to solve the Fermatean fuzzy multi-objective transportation problem (FFMOTP). Mohanta et al. [36, 38] developed the mathematical technique to handle the DEA model when data are in Spherical fuzzy numbers. Jaberi Hafshjani et al. [24] used a hybrid BSC-DEA technique to evaluate the performance of 20 bank branches using neutrosophic numbers as input-output data. Öztaş et al. [41] used plithogenic set in DEA to measure the performance of the hotels. Edalatpanah [15] studied the DEA model for the first time in 2018 using neutrosophic input and output data. Following then, many authors studied the neutrosophic DEA (Neu-DEA) using various approaches, as shown in Table 1.

In this study, the possibilistic mean for a single-valued trapezoidal neutrosophic number (SVTrNN) is redefined. A novel and efficient solution technique for the Neu-DEA model with SVTrNN inputs-outputs is provided by using the possibilistic mean which is employed to convert the neutrosophic DEA model into the corresponding crisp DEA model. The main advantage of the proposed Neu-DEA solving tech-

Table 1. Neutrosophic data envelopment analysis

Concept	Title of the article	Reference
Score and accuracy function	*Neutrosophic perspective on DEA*	Edalatpanah [15]
Deneutrosophication	*An integrated AHP & DEA methodology with neutrosophic sets*	Kahraman et al. [28]
Parametric approach	*Data envelopment analysis with neutrosophic inputs and outputs*	Abdelfattah [1]
Logarithm approach	*Data envelopment analysis for simplified neutrosophic sets*	Edalatpanah and Smarandache [14]
Ranking approach	*Data envelopment analysis based on triangular neutrosophic numbers*	Edalatpanah [13]
Logarithm approach	*A neutrosophic-based approach in data envelopment analysis with undesirable outputs*	Mao et al. [34]
Ranking approach	*Triangular single valued neutrosophic data envelopment analysis. Application to hospital performance measurement*	Yang et al. [51]
Robust tolerance approach	*Evaluating negative emissions technologies using neutrosophic data envelopment analysis*	Tapia [49]
Ranking and parametric approach	*Neutrosophic data envelopment analysis. An application to regional hospitals in Tunisia*	Abdelfattah [2]

nique is that it allows the decision-maker to be more flexible in determining the efficiency score of the DMUs with risk parameters. The risk parameter shows whether decision-makers believe they should be pessimistic, optimistic, or neutral in an uncertain environment. The DMUs are ranked based on the arithmetic mean of the efficiency score of different risk parameters. A numerical example is given to measure the performance of the seven AIIMS in India. The rest of the paper is arranged as follows: Section 2 discusses some advanced knowledge, concepts, and arithmetic operations on SVTrNNs and possibilistic mean. The development of neutrosophic DEA from the conventional DEA model is discussed in Section 3. Section 4 investigates the step=wise solution procedure of the suggested neutrosophic DEA model, and the Mehar approach [9] is employed to covert into the crisp DEA model. Section 5 gives a numerical example to measure the efficiency score of the AIIMS in India for the suggested model. Section 6 concludes with findings and future directions.

2. Preliminary

Definition 1. [48] Let U be a universe. A neutrosophic set (NS) $\widehat{X}$ over U is defined by

$$\widehat{X} = \{\langle x; \phi_x,\, \varphi_x,\, \psi_x\rangle : x \in U\} \tag{1}$$

where $\phi_x,\ \varphi_x,$ and ψ_x are called membership function, non-membership function, and hesitancy function, respectively.

They are defined by $\phi_x, \varphi_x, \psi_x : U \to [0,\ 1]$ such that

$$0 \le \phi_x + \varphi_x + \psi_x \le 3.$$

Definition 2. [12] The single-value trapezoidal neutrosophic number (SVTrNNs) is defined as

$$\widehat{X} = \langle x^L,\ x^{M_1}, x^{M_2},\ x^U; \phi_x,\ \varphi_x,\ \psi_x\rangle$$

where the truth, indeterminacy, and falsehood degree of x have the following three membership grades:

$$\tau(x) = \begin{cases} \dfrac{x - x^L}{x^{M_1} - x^L}\phi_x, \ x \in [x^L, x^{M_1}] \\ \phi_x, \ x \in [x^{M_1}, x^{M_2}] \\ \dfrac{x^U - x}{x^U - x^{M_2}}\phi_x, \ x \in [x^{M_2}, x^U] \\ 0, \quad \text{otherwise} \end{cases} \tag{2}$$

$$\iota(x) = \begin{cases} \dfrac{x - x^L}{x^{M_1} - x^L}\varphi_x, \ x \in [x^L, x^{M_1}] \\ \varphi_x, \ x \in [x^{M_1}, x^{M_2}] \\ \dfrac{x^U - x}{x^U - x^{M_2}}\varphi_x \ x \in [x^{M_2}, x^U] \\ 1, \quad \text{otherwise} \end{cases} \tag{3}$$

$$\nu(x) = \begin{cases} \dfrac{x - x^L}{x^{M_1} - x^L}\psi_x, \ x \in [x^L, x^{M_1}] \\ \psi_x, \ x \in [x^{M_1}, x^{M_2}] \\ \dfrac{x^U - x}{x^U - x^{M_2}}\psi_x, \ x \in [x^{M_2}, x^U] \\ 1, \quad \text{otherwise} \end{cases} \tag{4}$$

where $0 \leq \tau(x) + \iota(x) + \nu(x) \leq 3, \ \forall \, x \in U$.

Definition 3. [12] Suppose $\widehat{X}_1 = \langle x_1^L, \, x_1^{M_1}, x_1^{M_2}, \, x_1^U; \phi_{x_1}, \, \varphi_{x_1}, \, \psi_{x_1}\rangle$ and $\widehat{X}_2 = \langle x_2^L, \, x_2^{M_1}, x_2^{M_2}, \, x_2^U;$ $\phi_{x_2}, \varphi_{x_2}, \psi_{x_2}\rangle$ two SVTrNNs. Then the arithmetic relations are defined as

1. $\widehat{X}_1 \oplus \widehat{X}_2 = \langle x_1^L + x_2^L, x_1^{M_1} + x_2^{M_1}, x_1^{M_2} + x_2^{M_2}, x_1^U + x_2^U; \phi_{x_1} \wedge \phi_{x_2}, \varphi_{x_1} \vee \varphi_{x_2}, \psi_{x_1} \vee \psi_{x_2}\rangle$.
2. $\widehat{X}_1 - \widehat{X}_1 = \langle x_1^L - x_2^L, x_1^{M_1} - x_2^{M_1}, x_1^{M_2} - x_2^{M_2}, x_1^U - x_2^U; \phi_{x_1} \wedge \phi_{x_2}, \varphi_{x_1} \vee \varphi_{x_2}, \psi_{x_1} \vee \psi_{x_2}\rangle$.
3. $\widehat{X}_1 \otimes \widehat{X}_1 = \langle x_1^L x_2^L, x_1^{M_1} x_2^{M_1}, x_1^{M_2} x_2^{M_2}, x_1^U x_2^U; \phi_{x_1} \wedge \phi_{x_2}, \varphi_{x_1} \vee \varphi_{x_2}, \psi_{x_1} \vee \psi_{x_2}\rangle$.
4. $\alpha\widehat{X}_1 = \begin{cases} \langle \alpha x_1^L, \alpha x_1^{M_1}, \alpha x_1^{M_2}, \alpha x_1^U; \phi_{x_1}, \varphi_{x_1}, \psi_{x_1}\rangle, \ \alpha > 0 \\ \langle \alpha x_1^U, \alpha x_1^{M_2}, \alpha x_1^{M_1}, \alpha x_1^L; \phi_{x_1}, \varphi_{x_1}, \psi_{x_1}\rangle, \ \alpha < 0 \end{cases}$

where $a \wedge b = \min(a, b)$ and $a \vee b = \max(a, b)$.

Definition 4. [9] Let $\widehat{X} = \langle x^L, x^{M_1}, x^{M_2}, x^U; \phi_x, \varphi_x, \psi_x\rangle$ be an SVTrNNs, the possibilistic mean of truth, indeterminacy and falsity degree can be defined as

$$V(\widehat{X}) = \lambda m(\widehat{X}, \phi) + (1 - \lambda)\Big(m(\widehat{X}, \varphi) + m(\widehat{X}, \psi)\Big) \tag{5}$$

where

$$m(\widehat{X}, \phi) = \frac{1}{6}\big(x^L + 2x^{M_1} + 2x^{M_2} + x^U\big)\phi_x^2$$

$$m(\widehat{X},\varphi)=\frac{1}{6}\Big(\big(2x^L+x^{M_1}+x^{M_2}+2x^U\big)-\big(x^L-x^{M_1}-x^{M_2}+x^U\big)\varphi_x - \big(x^L+2x^{M_1}+2x^{M_2}+x^U\big)\varphi_x^2\Big)$$

$$m(\widehat{X},\psi)=\frac{1}{6}\Big(\big(2x^L+x^{M_1}+x^{M_2}+2x^U\big)-\big(x^L-x^{M_1}-x^{M_2}+x^U\big)\psi_x - \big(x^L+2x^{M_1}+2x^{M_2}+x^U\big)\psi_x^2\Big)$$

are the possibilistic means of truth, indeterminacy, and falsity membership degree, respectively, and λ reflects decision-maker's attitude towards taking risks:

1. $\lambda \in [0, 0.5)$ shows the expert is a risk taker who prefers uncertainty.
2. $\lambda = 0.5$ shows the expert's decision on the parameter selection is neutral.
3. $\lambda \in (0.5, 1]$ shows the expert's sensitivity to taking risks while deciding.

Example 1. If $\widehat{X} = a$ be any real number, then in SVTrNN form $\widehat{X} = \langle a, a, a, a; 1, 0, 0\rangle$ then $V(\widehat{X}) = \lambda a + (1-\lambda)2a = 2a - a\lambda \neq a$.

Definition 5. Suppose $\widehat{X_1}$ and $\widehat{X_2}$ be two SVTrNNs, then two SVTrNNs can be compared by

1. $\widehat{X_1} \le \widehat{X_2}$ if and only if $V(\widehat{X_1}) \le V(\widehat{X_2})$,
2. $\widehat{X_1} < \widehat{X_2}$ if and only if $V(\widehat{X_1}) < V(\widehat{X_2})$,

where $V(.)$ is the possibilistic mean.

Definition 6. The possibilistic mean of truth, indeterminacy, and falsity membership degree of $\widehat{X} = \langle x^L, x^{M_1}, x^{M_2}, x^U;\ \phi_x, \varphi_x, \psi_x\rangle$ are redefined as

$$V(\widehat{X}) = \lambda m(\widehat{X},\, \phi) + (1-\lambda)\Big(\frac{m(\widehat{X},\varphi)+m(\widehat{X},\psi)}{2}\Big) \tag{6}$$

where

$$m(\widehat{X},\phi)=\frac{1}{6}\big(x^L+2x^{M_1}+2x^{M_2}+x^U\big)\phi_x^2$$

$$m(\widehat{X},\varphi)=\frac{1}{6}\Big(\big(2x^L+x^{M_1}+x^{M_2}+2x^U\big)-\big(x^L-x^{M_1}-x^{M_2}+x^U\big)\varphi_x - \big(x^L+2x^{M_1}+2x^{M_2}+x^U\big)\varphi_x^2\Big)$$

$$m(\widehat{X},\psi)=\frac{1}{6}\Big([2x^L+x^{M_1}+x^{M_2}+2x^U]-[x^L-x^{M_1}-x^{M_2}+x^U]\psi_x - [x^L+2x^{M_1}+2x^{M_2}+x^U]\psi_x^2\Big)$$

are the possibilistic mean of truth, indeterminacy, and falsity membership degree, respectively.

That implies

$$\begin{aligned}\widetilde{V}(\widehat{X}) = &\frac{\lambda}{6}\Big(\big(x^L + 2x^{M_1} + 2x^{M_2} + x^U\big)\phi_x^2 \\ &+ \frac{1-\lambda}{2}\Big(\big(2x^L + x^{M_1} + x^{M_2} + 2x^U\big) - \big(x^L - x^{M_1} - x^{M_2} + x^U\big)\varphi_x \\ &- \big(x^L + 2x^{M_1} + 2x^{M_2} + x^U\big)\varphi_x^2\Big) + \big(2x^L + x^{M_1} + x^{M_2} + 2x^U\big) \\ &- \big(x^{M_1} - x^{M_2} + x^U\big)\psi_x - \big(x^L + 2x^{M_1} + 2x^{M_2} + x^U\big)\psi_x^2\Big)\end{aligned} \tag{7}$$

Thus the possibilistic mean of any real number $a \in \mathbb{R}$, which can be written in SVTrNN form $\widehat{X} = \langle a, a, a, a; 1, 0, 0\rangle$. Thus, $\widetilde{V}(a) = a$.

Lemma 1. Let us consider $\widehat{X_i} = \langle x_i^L, x_i^{M_1}, x_i^{M_2}, x_i^U;\ \phi_{x_i}, \varphi_{x_i}, \psi_{x_i}\rangle$ be n SVTrNNs and $\alpha_i \in R$. Then the possibilistic mean of the aggregation of the following expression can be defined as

$$\begin{aligned}\widetilde{V}\sum_{i=1}^{n}\alpha_i\widehat{X_i} = &\frac{1}{6}\sum_{i=1}^{n}\Bigg(\lambda\big(x_i^L + 2x_i^{M_1} + 2x_i^{M_2} + x_i^U\big)\Big(\bigwedge_{i=1}^{n}\phi_{x_i}\Big)^2 \\ &\frac{1-\lambda}{2}\Bigg(\big(2x_i^L + x_i^{M_1} + x_i^{M_2} + 2x_i^U\big) - \big(x_i^L - x_i^{M_1} - x_i^{M_2} + x_i^U\big)\Big(\bigvee_{i=1}^{n}\varphi_{x_i}\Big) \\ &+ \Big(2x_i^L + x_i^{M_1} + x_i^{M_2} + 2x_i^U\Big) - \Big(x_i^L - x_i^{M_1} - x_i^{M_2} + x_i^U\Big)\Big(\bigvee_{i=1}^{n}\psi_{x_i}\Big) \\ &- \Big(x_i^L + 2x_i^{M_1} + 2x_i^{M_2} + x_i^U\Big)\Big(\bigvee_{i=1}^{n}\psi_{x_i}\Big)^2\Bigg)\Bigg)\alpha_i\end{aligned} \tag{8}$$

Proof.

$$\sum_{i=1}^{n}\alpha_i\widehat{X_i} = \Big\langle \sum_{i=1}^{n}\alpha_i x_i^L,\ \sum_{i=1}^{n}\alpha_i x_i^{M_1},\ \sum_{i=1}^{n}\alpha_i x_i^{M_2},\ \sum_{i=1}^{n}\alpha_i x_i^U;\ \bigwedge_{i=1}^{n}\phi_{x_i},\ \bigvee_{i=1}^{n}\varphi_{x_i},\ \bigvee_{i=1}^{n}\psi_{x_i}\Big\rangle$$

Then from definition (6), we have

$$\begin{aligned}\widetilde{V}\sum_{i=1}^{n}\alpha_i\widehat{X_i} = &\frac{1}{6}\Bigg(\lambda\Big(\sum_{i=1}^{n}\alpha_i x_i^L + 2\sum_{i=1}^{n}\alpha_i x_i^{M_1} + 2\sum_{i=1}^{n}\alpha_i x_i^{M_2} + \sum_{i=1}^{n}\alpha_i x_i^U\Big)\Big(\bigwedge_{i=1}^{n}\phi_{x_i}\Big)^2 \\ &+ \frac{1-\lambda}{2}\Bigg(\Big(2\sum_{i=1}^{n}\alpha_i x_i^L + \sum_{i=1}^{n}\alpha_i x_i^{M_1} + \sum_{i=1}^{n}\alpha_i x_i^{M_2} + 2\sum_{i=1}^{n}\alpha_i x_i^U\Big) \\ &- \Big(\sum_{i=1}^{n}\alpha_i x_i^L - \sum_{i=1}^{n}\alpha_i x_i^{M_1} - \sum_{i=1}^{n}\alpha_i x_i^{M_2} + \sum_{i=1}^{n}\alpha_i x_i^U\Big)\Big(\bigvee_{i=1}^{n}\varphi_{x_i}\Big) \\ &- \Big(\sum_{i=1}^{n}\alpha_i x_i^L + 2\sum_{i=1}^{n}\alpha_i x_i^{M_1} + 2\sum_{i=1}^{n}\alpha_i x_i^{M_2} + \sum_{i=1}^{n}\alpha_i x_i^U\Big]\Big(\bigvee_{i=1}^{n}\varphi_{x_i}\Big)^2\Bigg)\end{aligned}$$

$$+\Big(2\sum_{i=1}^{n}\alpha_i x_i^L+\sum_{i=1}^{n}\alpha_i x_i^{M_1}+\sum_{i=1}^{n}\alpha_i x_i^{M_2}+2\sum_{i=1}^{n}\alpha_i x_i^U\Big)$$

$$-\Big(\sum_{i=1}^{n}\alpha_i x_i^L-\sum_{i=1}^{n}\alpha_i x_i^{M_1}-\sum_{i=1}^{n}\alpha_i x_i^{M_2}+\sum_{i=1}^{n}\alpha_i x_i^U\Big]\Big(\bigvee_{i=1}^{n}\psi_{x_i}\Big)$$

$$-\Big(\sum_{i=1}^{n}\alpha_i x_i^L+2\sum_{i=1}^{n}\alpha_i x_i^{M_1}+2\sum_{i=1}^{n}\alpha_i x_i^{M_2}+\sum_{i=1}^{n}\alpha_i x_i^U\Big)\Big(\bigvee_{i=1}^{n}\psi_{x_i}\Big)^2\Big)\Big)$$

that implies

$$\widetilde{V}\sum_{i=1}^{n}\alpha_i\widehat{X_i}=\frac{1}{6}\sum_{i=1}^{n}\Bigg(\lambda\Big(x_i^L+2x_i^{M_1}+2x_i^{M_2}+x_i^U\Big)\Big(\bigwedge_{i=1}^{n}\phi_{x_i}\Big)^2$$

$$+\frac{1-\lambda}{2}\Bigg(\Big(2x_i^L+x_i^{M_1}+x_i^{M_2}+2x_i^U\Big)-\Big(x_i^L-x_i^{M_1}-x_i^{M_2}+x_i^U\Big)\Big(\bigvee_{i=1}^{n}\varphi_{x_i}\Big)$$

$$-\Big(x_i^L+2x_i^{M_1}+2x_i^{M_2}+x_i^U\Big)\Big(\bigvee_{i=1}^{n}\varphi_{x_i}\Big)^2+\Big(2x_i^L+x_i^{M_1}+x_i^{M_2}+2x_i^U\Big)$$

$$-\Big(x_i^L-x_i^{M_1}-x_i^{M_2}+x_i^U\Big)\Big(\bigvee_{i=1}^{n}\psi_{x_i}\Big)-\Big(x_i^L+2x_i^{M_1}+2x_i^{M_2}+x_i^U\Big)\Big(\bigvee_{i=1}^{n}\psi_{x_i}\Big)^2\Bigg)\Bigg)\alpha_i$$

$\square$

3. Neutrosophic data envelopment analysis (Neu-DEA)

Suppose that there are n DMUs, each of m inputs and r outputs represented by the vectors $\mathbf{x}\in\mathbb{R}^m$ and $\mathbf{y}\in\mathbb{R}^r$, respectively. We define the input matrix X as $X=[x_1,\ldots,x_m]\in\mathbb{R}^{m\times n}$, and the output matrix Y as $Y=[y_l,\ldots,y_r]\in\mathbb{R}^{r\times n}, x_i\in\mathbb{R}^m\forall i=1,2,\ldots,m,\ y_k\in\mathbb{R}^r,\forall k=1,2,3,\ldots,r$. Assume that $X>0$ and $Y>0$. Charnes et al. [10] developed this model for measuring the efficiency of DMU_o, i.e.,

$$\begin{aligned}
\max_{u,v}\ \theta=&\frac{\sum_{k=1}^{r}u_k y_{ko}}{\sum_{i=1}^{m}v_i x_{io}}\\
\text{subject to}\ &\frac{\sum_{k=1}^{r}u_k y_{kj}}{\sum_{i=1}^{m}v_i x_{ij}}\leq 1,\ j=1,2,\ldots,n\\
&u_k\geq 0,\ k=1,2,\ldots,r\\
&v_i\geq 0,\ i=1,2,\ldots,m
\end{aligned}\tag{9}$$

which is equivalent to the linear program (LP_o), i.e.,

$$\begin{aligned} \max_{u,v} \theta = & \sum_{k=1}^{r} u_k y_{ko} \\ \text{subject to } & \sum_{i=1}^{m} v_i x_{io} = 1 \\ & \sum_{k=1}^{r} u_k y_{kj} \leq \sum_{i=1}^{m} v_i x_{ij},\ j = 1, 2,\ \ldots, n \\ & u_k \geq 0,\ k = 1, 2,\ \ldots, r \\ & v_i \geq 0,\ i = 1, 2,\ \ldots, m \end{aligned} \tag{10}$$

which is called the CCR model.

If any of the observed data for inputs and/or outputs in this model are inaccurate, unclear, or ambiguous, the efficiency score of the DMU_o will be inaccurate. Furthermore, if this DMU is on an efficient production function, it will serve as a shaky reference unit for the other inefficient DMUs. A strong technique for dealing with this type of situation is to use neutrosophic set theory.

Assuming inputs and outputs are SVTrNNs while the variables u_r and v_i are real numbers. Thus, the neutrosophic CCR (Neu-CCR) model will be written as follows:

$$\begin{aligned} \max_{u,v} \theta = & \sum_{k=1}^{r} u_k \widehat{y_{ko}} \\ \text{subject to } & \sum_{i=1}^{m} v_i \widehat{x_{io}} = \widehat{1} \\ & \sum_{k=1}^{r} u_k \widehat{y_{kj}} \leq \sum_{i=1}^{m} v_i, \widehat{x_{ij}},\ j = 1, 2,\ \ldots, n \\ & u_k \geq 0,\ k = 1, 2,\ \ldots, r \\ & v_i \geq 0,\ i = 1, 2,\ \ldots, m \end{aligned} \tag{11}$$

where $\widehat{x_{ij}} = \langle x_{ij}^L, x_{ij}^{M_1}, x_{ij}^{M_2}, x_{ij}^U; \phi_{x_{ij}}, \varphi_{x_{ij}}, \psi_{x_{ij}} \rangle$ and $\widehat{y_{kj}} = \langle y_{kj}^L, y_{kj}^{M_1}, y_{kj}^{M_2}, y_{kj}^U; \phi_{y_{kj}}, \varphi_{y_{kj}}, \psi_{y_{kj}} \rangle$ for $i = 1, 2, 3, \ldots, n,\ j = 1, 2, 3, \ldots, m$ and $k = 1, 2, 3, \ldots, r$, are the SVTrNNs, and $\widehat{1} = \langle 1, 1, 1, 1; 1, 0, 0 \rangle$. The efficiency score of the Neu-CCR model is $\theta^* \in [0,\ 1]$.

Theorem 2. The CCR model given in equation (10) and the Neu-CCR model in equation (11) are equivalent.

Proof. When the aggregation operator is applied, it is easy to see that every Neu-CCR model's optimum feasible solution is also an optimum feasible solution for the CCR model, and vice versa. □

4. Method for solving neutrosophic DEA (Neu-DEA) model

Fuzzifier or fuzzification is the process of changing crisp input-output data into fuzzy input-output data using information from a knowledge base. Fuzzification is necessary at an early stage of the uncertainty theory. As a result, the fuzzifier may be defined as a mapping from an observable crisp data space to a fuzzy data space in a given discourse universe. Trapezoidal membership functions are the most widely employed in the fuzzification process because they are easily implemented by embedded controllers. In this case, a trapezoidal neutrosophic fuzzy set is employed based on the expert decision. The solution technique of the neutrosophic DEA model is represented by the flowchart in Figure 2.

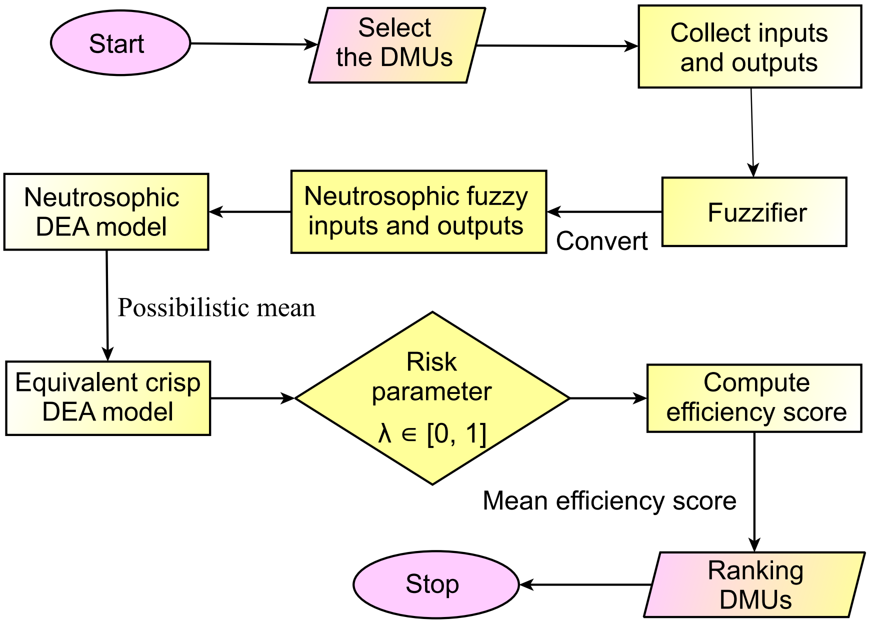

Figure 2. Technique for solving the neutrosophic DEA model

Let us consider the inputs $\widehat{x_{ij}}$ and outputs $\widehat{y_{kj}}$ are the SVTrNNs that is

$$\widehat{x_{ij}} = \langle x_{ij}^L, x_{ij}^{M_1}, x_{ij}^{M_2}, x_{ij}^U; \phi_{x_{ij}}, \varphi_{x_{ij}}, \psi_{x_{ij}} \rangle$$

and

$$\widehat{y_{kj}} = \langle y_{kj}^L, y_{kj}^{M_1}, y_{kj}^{M_2}, y_{kj}^U; \phi_{y_{kj}}, \varphi_{y_{kj}}, \psi_{y_{kj}} \rangle$$

for $i = 1,\ 2,\ 3,\ \dots,\ m,\ j = 1, 2, 3,\ \dots,\ n$ and $k = 1,\ 2,\ 3,\ \dots,\ r$.

The following steps are performed to solve the Neu-DEA model.

Step 1. Transform the DEA model into the Neu-DEA model as shown in equation (11).

Step 2. Apply the possibilistic mean function $(\widetilde{V})$ in the Neu-DEA model. The Neu-CCR models is

$$\begin{aligned}
\max_{u,v}\ \theta =& \widetilde{V} \sum_{k=1}^{r} u_k \widehat{y_{ko}} \\
\text{subject to } & \widetilde{V}\left(\sum_{i=1}^{m} v_i \widehat{x_{io}}\right) = \widetilde{V}(\widehat{1}) \\
& \widetilde{V}\left(\sum_{k=1}^{r} u_k \widehat{y_{kj}}\right) \leq \widetilde{V}\left(\sum_{i=1}^{m} v_i \widehat{x_{ij}}\right),\ j = 1, 2,\ \dots,\ n \\
& u_k \geq 0,\ k = 1, 2,\ \dots,\ r \\
& v_i \geq 0,\ i = 1, 2,\ \dots,\ m
\end{aligned} \tag{12}$$

Step 3. Mehar's approach [9] is used to convert the Neu-CCR model into an equivalent crisp CCR model. From equation (12), the possibilistic mean of the aggregation of the neutrosophic trapezoidal fuzzy environment can be written as

$$\begin{aligned}
\max_{u,v}\ \theta =&\widetilde{V}\Bigg(\Bigg\langle \sum_{k=1}^{r} u_k y_{ko}^{L}, \sum_{k=1}^{r} u_k y_{ko}^{M_1}, \sum_{k=1}^{r} u_k y_{ko}^{M_2}, \sum_{k=1}^{r} u_k y_{ko}^{U}; \bigwedge_{k=1}^{r} \phi_{y_{ko}}, \bigvee_{k=1}^{r} \varphi_{y_{ko}}, \bigvee_{k=1}^{r} \psi_{y_{ko}} \Bigg\rangle\Bigg)\\
\text{s.t. } &\widetilde{V}\Bigg(\Bigg\langle \sum_{i=1}^{m} v_i x_{io}^{L}, \sum_{i=1}^{m} v_i x_{io}^{M_1}, \sum_{i=1}^{m} v_i x_{io}^{M_2}, \sum_{i=1}^{m} v_i x_{io}^{U}; \bigwedge_{i=1}^{m} \phi_{x_{io}}, \bigvee_{i=1}^{m} \varphi_{x_{io}}, \bigvee_{i=1}^{m} \psi_{x_{io}} \Bigg\rangle\Bigg) = \widetilde{V}(\widehat{1})\\
&\widetilde{V}\Bigg(\Bigg\langle \sum_{k=1}^{r} u_k y_{kj}^{L}, \sum_{k=1}^{r} u_k y_{kj}^{M_1}, \sum_{k=1}^{r} u_k y_{kj}^{M_2}, \sum_{k=1}^{r} u_k y_{kj}^{U}; \bigwedge_{k=1}^{r} \phi_{y_{kj}}, \bigvee_{k=1}^{r} \varphi_{y_{kj}}, \bigvee_{k=1}^{r} \psi_{y_{kj}} \Bigg\rangle\Bigg)\\
&\le \widetilde{V}\Bigg(\Bigg\langle \sum_{i=1}^{m} v_i x_{ij}^{L}, \sum_{i=1}^{m} v_i x_{ij}^{M_1}, \sum_{i=1}^{m} v_i x_{ij}^{M_2}, \sum_{i=1}^{m} v_i x_{ij}^{U}; \bigwedge_{i=1}^{m} \phi_{x_{ij}}, \bigvee_{i=1}^{m} \varphi_{x_{ij}}, \bigvee_{i=1}^{m} \psi_{x_{ij}} \Bigg\rangle\Bigg)\\
&j = 1,2,\ \dots,\ n, \text{and}\ u_k \ge 0,\ k = 1,2,\ \dots,\ r,\ v_i \ge 0,\ i = 1,2,\ \dots,\ m
\end{aligned}$$

Now from Lemma (1) we have

$$\begin{aligned}
\max_{u,v} \theta =& \frac{1}{6}\sum_{k=1}^{r}\Bigg(\lambda\Big(y_{ko}^{L} + 2y_{ko}^{M_1} + 2y_{ko}^{M_2} + y_{ko}^{U}\Big)\Big(\bigwedge_{k=1}^{r}\phi_{y_{ko}}\Big)^2\\
&+ \frac{1-\lambda}{2}\Bigg(\Big(2y_{ko}^{L} + y_{ko}^{M_1} + y_{ko}^{M_2} + 2y_{ko}^{U}\Big) - \Big(y_{ko}^{L} - y_{ko}^{M_1} - y_{ko}^{M_2} + y_{ko}^{U}\Big)\Big(\bigvee_{k=1}^{r}\varphi_{y_{ko}}\Big)\\
&- \Big(y_{ko}^{L} + 2y_{ko}^{M_1} + 2y_{ko}^{M_2} + y_{ko}^{U}\Big)\Big(\bigvee_{k=1}^{r}\varphi_{y_{ko}}\Big)^2 + \Big(2y_{ko}^{L} + y_{ko}^{M_1} + y_{ko}^{M_2} + 2y_{ko}^{U}\Big)\\
&- \Big(y_{ko}^{L} - y_{ko}^{M_1} - y_{ko}^{M_2} + y_{ko}^{U}\Big)\Big(\bigvee_{k=1}^{r}\psi_{y_{ko}}\Big) - \Big(y_{ko}^{L} + 2y_{ko}^{M_1} + 2y_{ko}^{M_2} + y_{ko}^{U}\Big)\Big(\bigvee_{k=1}^{r}\psi_{y_{ko}}\Big)^2\Bigg)\Bigg)u_k\\
\text{s.t. } & \frac{1}{6}\sum_{i=1}^{m}\Bigg(\lambda\Big(x_{io}^{L} + 2x_{io}^{M_1} + 2x_{io}^{M_2} + x_{io}^{U}\Big)\Big(\bigwedge_{k=1}^{r}\phi_{x_{io}}\Big)^2\\
&+ \frac{1-\lambda}{2}\Bigg(\Big(2x_{io}^{L} + x_{io}^{M_1} + x_{io}^{M_2} + 2x_{io}^{U}\Big) - \Big(x_{io}^{L} - x_{io}^{M_1} - x_{io}^{M_2} + x_{io}^{U}\Big)\Big(\bigvee_{k=1}^{r}\varphi_{x_{io}}\Big)\\
&- \Big(x_{io}^{L} + 2x_{io}^{M_1} + 2x_{io}^{M_2} + x_{io}^{U}\Big)\Big(\bigvee_{i=1}^{m}\varphi_{x_{io}}\Big)^2 + \Big(2x_{io}^{L} + x_{io}^{M_1} + x_{io}^{M_2} + 2x_{io}^{U}\Big)\\
&- \Big(x_{io}^{L} - x_{io}^{M_1} - x_{io}^{M_2} + x_{io}^{U}\Big)\Big(\bigvee_{k=1}^{r}\psi_{x_{io}}\Big) - \Big(x_{io}^{L} + 2x_{io}^{M_1} + 2x_{io}^{M_2} + x_{io}^{U}\Big)\Big(\bigvee_{i=1}^{m}\psi_{x_{io}}\Big)^2\Bigg)\Bigg)v_i = 1\\
& \frac{1}{6}\sum_{k=1}^{r}\Bigg(\lambda\Big(y_{kj}^{L} + 2y_{kj}^{M_1} + 2y_{kj}^{M_2} + y_{kj}^{U}\Big)\Big(\bigwedge_{k=1}^{r}\phi_{y_{kj}}\Big)^2\\
&+ \frac{1-\lambda}{2}\Bigg(\Big(2y_{kj}^{L} + y_{kj}^{M_1} + y_{kj}^{M_2} + 2y_{kj}^{U}\Big) - \Big(y_{kj}^{L} - y_{kj}^{M_1} - y_{kj}^{M_2} + y_{kj}^{U}\Big)\Big(\bigvee_{k=1}^{r}\varphi_{y_{kj}}\Big)
\end{aligned}$$

$$
\begin{aligned}
&-\left(y_{kj}^{L}+2y_{kj}^{M_1}+2y_{kj}^{M_2}+y_{kj}^{U}\right)\Big(\bigvee_{k=1}^{r}\varphi_{y_{kj}}\Big)^2+\left(2y_{kj}^{L}+y_{kj}^{M_1}+y_{kj}^{M_2}+2y_{kj}^{U}\right)\\
&-\left(y_{kj}^{L}-y_{kj}^{M_1}-y_{kj}^{M_2}+y_{kj}^{U}\right)\Big(\bigvee_{k=1}^{r}\psi_{y_{kj}}\Big)-\left(y_{kj}^{L}+2y_{kj}^{M_1}+2y_{kj}^{M_2}+y_{kj}^{U}\right)\Big(\bigvee_{k=1}^{r}\psi_{y_{kj}}\Big)^2\Big)\Big)u_k\\
&\le \frac{1}{6}\sum_{i=1}^{m}\Big(\lambda\left(x_{ij}^{L}+2x_{ij}^{M_1}+2x_{ij}^{M_2}+x_{ij}^{U}\right)\Big(\bigwedge_{k=1}^{r}\phi_{x_{ij}}\Big)^2\\
&+\frac{1-\lambda}{2}\Big(\left(2x_{ij}^{L}+x_{ij}^{M_1}+x_{ij}^{M_2}+2x_{ij}^{U}\right)-\left(x_{ij}^{L}-x_{ij}^{M_1}-x_{ij}^{M_2}+x_{ij}^{U}\right)\Big(\bigvee_{k=1}^{r}\varphi_{x_{ij}}\Big)\\
&-\left(x_{ij}^{L}+2x_{ij}^{M_1}+2x_{ij}^{M_2}+x_{ij}^{U}\right)\Big(\bigvee_{i=1}^{m}\varphi_{x_{ij}}\Big)^2+\left(2x_{ij}^{L}+x_{ij}^{M_1}+x_{ij}^{M_2}+2x_{ij}^{U}\right)\\
&-\left(x_{ij}^{L}-x_{ij}^{M_1}-x_{ij}^{M_2}+x_{ij}^{U}\right)\Big(\bigvee_{k=1}^{r}\psi_{x_{ij}}\Big)-\left(x_{ij}^{L}+2x_{ij}^{M_1}+2x_{ij}^{M_2}+x_{ij}^{U}\right)\Big(\bigvee_{i=1}^{m}\psi_{x_{ij}}\Big)^2\Big)\Big)v_i\\
&j=1,2,\,\dots,\,n,\quad u_k\ge 0,\ k=1,2,\,\dots,\,r,\ v_i\ge 0,\ i=1,2,\,\dots,\,m
\end{aligned}
$$

which is the corresponding crisp CCR model.

Step 4. Solve this crips CCR model and find the optimal solution θ^* for each $\lambda\in[0,\,1]$ which represents the attitude of the DM regarding the risk:

1. $\lambda\in[0,0.5)$ shows that the expert is a risk taker who prefers uncertainty.
2. $\lambda=0.5$ shows that the expert's decision on the parameter selection is neutral.
3. $\lambda\in(0.5,1]$ shows that the expert's sensitivity to taking risks while making a decision.

Step 5. The DMUs are ranked based on the arithmetic mean of efficiency scores.

5. Numerical example

In this example, we calculate the efficiency score of the All India Institute of Medical Sciences (AIIMS) in India. The input parameters are the number of faculty in hundred, the number of departments, and the number of beds in hundred, while the output parameters are outpatients in lacs and inpatients in thousand. The single-value trapezoidal neutrosophic numbers (SVTrNNs) are used to represent the fuzzy input and output data, as shown in Tables 2 and 3. The proposed crisp DEA model is feasible because the number of inputs and outputs ($3+2=5$) is less than the number of DMUs (7).

Table 2. Inputs data of the AIIMS in India

AIIMS	Faculty (in 100)	Department	Beds (in 100)
Bhopal	$\langle 0.54, 0.58, 0.63, 0.65; 0.7, 0.6, 0.4\rangle$	$\langle 26, 26, 26, 26; 1, 0, 0\rangle$	$\langle 2.6, 2.9, 3.1, 3.2; 0.6, 0.4, 0.6\rangle$
Bhubaneswar	$\langle 1.25, 1.28, 1.32, 1.35; 0.9, 0.2, 0.3\rangle$	$\langle 41, 41, 41, 41; 1, 0, 0\rangle$	$\langle 4.8, 5, 5.4, 5.6; 0.8, 0.5, 0.3\rangle$
Jodhpur	$\langle 0.98, 1.02, 1.05, 1.08; 0.8, 0.5, 0.4\rangle$	$\langle 34, 34, 34, 34; 1, 0, 0\rangle$	$\langle 3.2, 3.5, 3.9, 4.2; 0.9, 0.3, 0.2\rangle$
New Delhi	$\langle 6.5, 6.8, 7.2, 7.4; 0.8, 0.5, 0.2\rangle$	$\langle 57, 57, 57, 57; 1, 0, 0\rangle$	$\langle 23.4, 23.7, 23.8, 24.1; 0.7, 0.4, 0.7\rangle$
Patna	$\langle 1.34, 1.37, 1.39, 1.42; 0.9, 0.3, 0.3\rangle$	$\langle 45, 45, 45, 45; 1, 0, 0\rangle$	$\langle 8.9, 9.1, 9.3, 9.5; 0.2, 0.8, 0.6\rangle$
Raipur	$\langle 0.68, 0.71, 0.73, 0.76; 0.9, 0.5, 0.1\rangle$	$\langle 26, 26, 26, 26; 1, 0, 0\rangle$	$\langle 3.56, 3.6, 3.62, 3.64; 0.5, 0.8, 0.4\rangle$
Rishikesh	$\langle 0.89, 0.91, 0.93, 0.95; 0.8, 0.4, 0.6\rangle$	$\langle 31, 31, 31, 31; 1, 0, 0\rangle$	$\langle 6.5, 6.9, 7.1, 7.4; 0.8, 0.1, 0.7\rangle$

Table 3. Outputs data of the AIIMS in India

AIIMS	Outpatients (in lacs)	Inpatients (in 1000)
Bhopal	$\langle 2.56, 2.59, 2.63, 2.69; 0.8, 0.5, 0.7\rangle$	$\langle 3.14, 3.18, 3.21, 3.24; 0.8, 0.5, 0.3\rangle$
Bhubaneswar	$\langle 4.16, 4.21, 4.26, 4.28; 0.7, 0.6, 0.5\rangle$	$\langle 15.25, 15.28, 15.32, 15.34; 0.9, 0.4, 0.1\rangle$
Jodhpur	$\langle 2.02, 2.08, 2.12, 2.15; 0.5, 0.8, 0.2\rangle$	$\langle 9.14, 9.19, 9.2, 9.24; 0.8, 0.4, 0.3\rangle$
New Delhi	$\langle 41.34, 41.38, 41.4, 41.41; 0.9, 0.3, 0.3\rangle$	$\langle 234, 234.1, 234.16, 234.2; 0.7, 0.2, 0.4\rangle$
Patna	$\langle 5.21, 5.26, 5.29, 5.31; 0.4, 0.5, 0.8\rangle$	$\langle 18.4, 18.8, 19.1, 19.3; 0.7, 0.6, 0.7\rangle$
Raipur	$\langle 2.2, 2.24, 2.27, 2.31; 0.8, 0.4, 0.4\rangle$	$\langle 6, 6.04, 6.1, 6.15; 0.7, 0.5, 0.6\rangle$
Rishikesh	$\langle 3.18, 3.25, 3.28, 3.3; 0.7, 0.3, 0.4\rangle$	$\langle 6.95, 6.98, 7.04, 7.1; 0.6, 0.8, 0.3\rangle$

We have used the solution technique outlined in Section 4 to measure the efficiency of each AIIMS in India. The mathematical model shown in Step 3 is employed in MATLAB R2013a to determine the relative efficiency scores of the DMUs which were calculated and ranked the DMUs based on the mean efficiency scores of the DMUs as shown in Table 4. Also, Table 4 shows how the risk parameter $\lambda \in [0, 1]$ influences the efficiency score of the DMUs. When $\lambda = 0$, AIIMS Delhi is more efficient than other AIIMS. When $\lambda = 0.5$, AIIMS Delhi is more efficient than other AIIMS. When $\lambda = 0.75$, AIIMS Delhi and AIIMS Bhopal are more efficient than other AIIMS, and when $\lambda = 1$, AIIMS Delhi and AIIMS Patna are more efficient than other AIIMS. After analyzing the efficiency scores of AIIMS in India (Table 4 and Figure 3), we find that AIIMS Delhi is fully efficient with an efficiency score of 1, whereas the efficiency scores of the other DMUs decrease in the following order: AIIMS Bhopal, AIIMS Patna, AIIMS Raipur, AIIMS Bhubaneswar, AIIMS Rishikesh, and AIIMS Jodhpur. The mean efficiency score of various risk levels is determined, and AIIMS Delhi is efficient, whereas other AIIMS are inefficient. The trends of efficiency score changes with respect to risk parameter $\lambda \in [0, 1]$ are shown in Figure 4, which show how the efficiency score affects when the risk level changes from optimistic to pessimistic level. The efficiency score of the three DMUs (AIIMS Bhubaneswar, AIIMS Rishikesh, and AIIMS Jodhpur) decreases when the risk level changes from optimistic to pessimistic, while it increases for AIIMS Patna, AIIMS Delhi performs constantly, and AIIMS Bhopal and AIIMS Raipur increase from 0 to 0.75 after that efficiency score decreases.

Table 4. Efficiency score of the AIIMS in India

AIIMS	λ						Ranking	Abdelfattah [2]
	0	0.25	0.5	0.75	1.0	Mean		
Bhopal	0.6165	0.7166	0.8498	1	0.6839	0.77336	2	0.8387
Bhubaneswar	0.3905	0.396	0.4025	0.405	0.2729	0.37338	5	0.4846
Jodhpur	0.232	0.2136	0.1918	0.165	0.1083	0.18214	7	0.3269
AIIMS New Delhi	1	1	1	1	1	1	1	1
Patna	0.5476	0.6091	0.7164	0.9523	1	0.76508	3	0.5694
Raipur	0.5204	0.5823	0.6703	0.7871	0.5348	0.61898	4	0.4663
Rishikesh	0.4502	0.4238	0.3961	0.358	0.1304	0.3517	6	0.6471

The ranking function defined by Abdelfattah [2] for a SVTrNN $\widehat{X} = \langle x^L, x^{M_1}, x^{M_2}, x^U; \phi_x, \varphi_x, \psi_x\rangle$, is

$$R(\widehat{X}) = \frac{x^L + x^U + 2(x^{M_1} + x^{M_2})}{2} + (\phi_x - \varphi_x - \psi_x)$$

It is used to convert each SVTrNN input and output into an equivalent crisp input and output respectively. The crisp input and output values are then employed in a traditional DEA model to calculate the relative

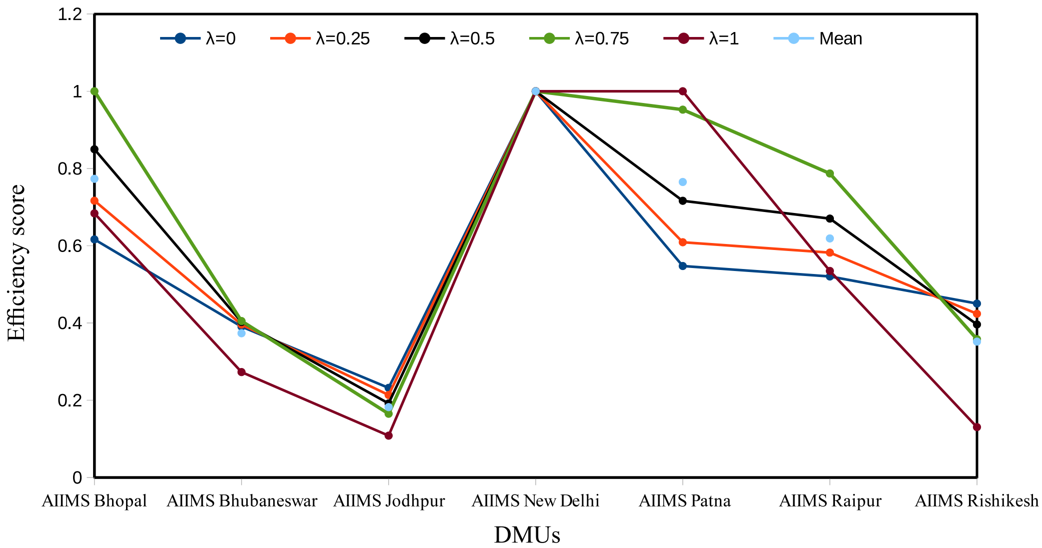

Figure 3. Efficiency score of the AIIMS in India with different risk parameters

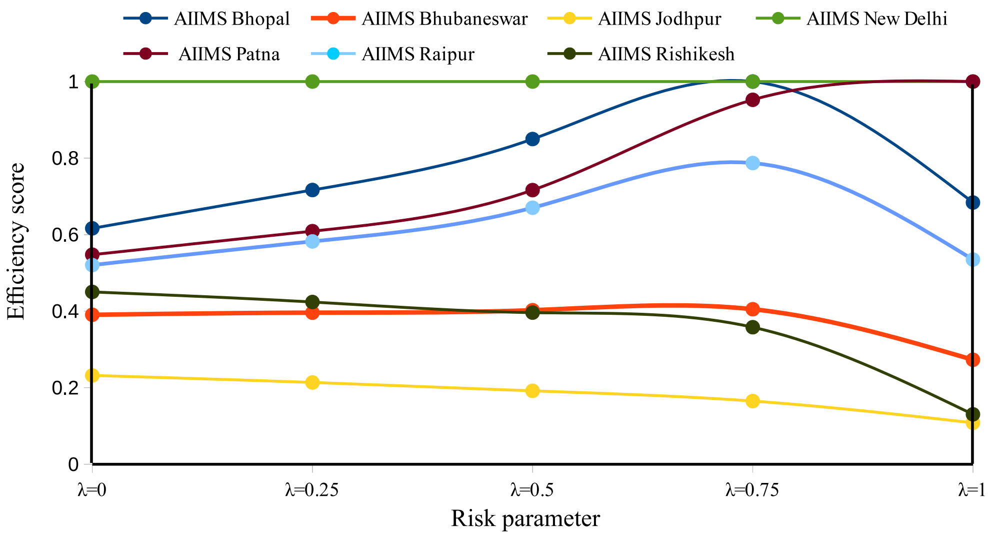

Figure 4. Efficiency score changes with optimistic to pessimistic decision

efficiency of the DMUs. This ranking method cannot be used directly to the neutrosophic constraints to convert equivalent crisp constraints. However, the set of neutrosophic constraints is converted into an equivalent set of crisp constraints through the possibilistic mean approach. Furthermore, the corresponding crisp constraints associated with the risk parameter $\lambda \in [0, 1]$, which expresses the attitude of the decision-maker toward taking risks. In Table 4, the proposed approach and Abdelfattah's ranking approach [2] were used to determine the efficiency score of the DMUs. Also illustrates the relative efficiencies of all AIIMS in India at various risk levels as calculated by the suggested neutrosophic DEA model. We compared in Figure 5, the mean efficiency score of the suggested approach with Abdelfattah's ranking approach [2].

6. Conclusion

The conventional DEA model evaluates the performance of the DMUs when input and output data are accurately measured. It is a challenge to quantify inputs and outputs correctly since they are imprecise, unclear, partial, complicated, confusing, and occasionally linguistic. It is difficult to accomplish this in a real-world investigation because data is often inaccurate or ambiguous. Therefore, it is necessary to develop new approaches and ideas to handle this situation. A new theory, neutrosophic set theory, which is an extension of FS and IFS theories, has recently emerged as a highly effective tool for achieving this goal due to its ability to manage ambiguity as well as indeterminate and inconsistent data.

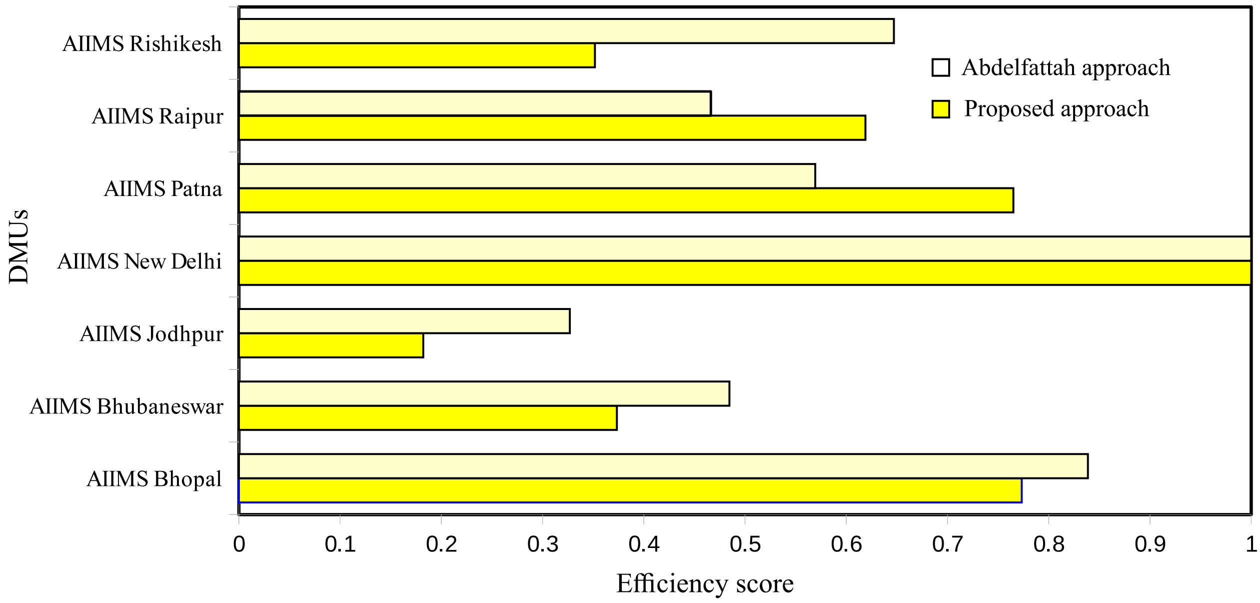

Figure 5. Comparison with Abdelfattah ranking approach citeabdelfattah2021neutrosophic

This article presented a novel technique for solving a neutrosophic data envelopment analysis model with single-value trapezoidal neutrosophic inputs and outputs. This proposed approach converted the Neu-DEA model into the corresponding crisp DEA model using the possibilistic mean. The suggested DEA approach is unique in its capability to handle inputs and outputs easily and effectively in comparison to the existing Neu-DEA approaches. The proposed technique has shown promising results in computing and analyzing the performance of the DMUs. It is important to note that in this research discussion of uncertainty, ambiguity, and indeterminacy only applies to single-valued trapezoidal neutrosophic numbers. The proposed method also takes into consideration the decision maker's preference parameters or risk attitude. This risk parameter reflects the decision-maker's attitude toward taking risks. However, a practical application is used in this work to show that the neutrosophic DEA model can handle real-world applications by evaluating the efficiency of seven All-India Institutes of Medical Sciences (AIIMS) in India in a neutrosophic environment. The presented technique shows that AIIMS Delhi is efficient with optimistic to pessimistic decisions, and the efficiency score of other AIIMS are shown in Table 4. The proposed approach may be considered efficient and effective in the context of the finding.

The performance of each AIIMS in India is evaluated using a limited number of variables (inputs and outputs). The primary limitation of the study is the increase in inputs and/or outputs, which may lead to a different efficiency score. Future research is suggested in order to determine whether there are other effective parameters that might be used to improve the performance of AIIMS in India. Also, one of the fascinating areas for future research is when the inputs and outputs are not always homogeneous. This proposed possibilistic mean approach will be solve several other DEA models, including BCC, SBM, Additive, Super efficiency, and Undesirable DEA models, with impressive outcomes.

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